Cellular Automata based Artificial Financial Market

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1. Introduction

Rational investor hypothesis, efficient markets hypothesis(EMH), and random walk of yield rate are three basic concepts of modern capital market theory. However, it could not be proved that real capital markets are full with rational investors. The theory, which regards the price movement of capital market as random walks, and regards the yield time series as a normal distribution, is not supported by the real statistics data usually. A capital market, in essence, could be regarded as a complex system, which consists of masses of investors. Investors make investment decision basing on the public or private information inside or outside the market. The movement of price and volume is the emergency of investors' group behavior. With the sustained growth of computational capabilities and the appearance of complexity science, artificial life, multi-agent system (MAS), and cellular automata (CA) are provided for the modeling of complex system. Researchers got powerful tools to build discrete dynamics model for the capital market for the first time. The Santa Fe artificial stock market(SF-ASM), which is presented by Santa Fe institution in 1970s, is the original version of the artificial financial market(AFM). Modeling for the microstructure of the capital market, made the verification and falsification of economics theories possible. On the part of macroscopic statistical data of the market, a series non-linear dynamic analysis method, such as fractal statistics, had been applied to analysis of financial time series. New research methods, which are used both in microscopic and macroscopic aspects of capital market, help us build brand new dynamic models for capital markets.

The appearance of SF-ASM has influenced this area deeply. Most successors are the variety or improvement of SF-ASM. SF-ASM is a kind of MAS, which focuses on simulating heterogeneous investors' investment behaviours. In my opinion, the investment process of an investor can be divided into 2 steps: forecasting and decision. The forecasting step is how an investor considers public or private information inside or outside of the market. And the decision step is how an investor reacts to the prediction. Rational investor hypothesis and various investment decision processes in SF-ASM are just different ways to deal with information. Basing on neoclassicism economics, EMH announce that the price in the market reflects all information, or at least all public information, and that rational investors react to these information in the same way. Multi-Agent based SF-ASM supports heterogeneous investors in reacting to information in various ways, but provides public price as the only information. The fact that information relating to the market is homogeneous and public to each investor can be compared to the gas filling the whole

"container of market". However, as we know, in real capital markets, except public information including the announcement, annual report, interest rate etc., there are also inside information, individual attitudes or predictions, and even emotions, which can influence investors' investment. What's more, information is time sensitive. Because non-public information may reach investors in different time, the situation of real capital market could be more complex. So SF-ASM is more "efficient" than real capital markets for it's simplifying the description of information.

If we describe the non-public information in an AFM model, the interoperation among individual investors can be expressed certainly. As a result, the cellular automaton (CA) is adopted. Classical CA is a kind of large scale discrete dynamical systems. Each cell in CA can interoperate with neighbors in a local scope, which is defined by CA's neighborhood. Yi-ming Wei, Shang-jun Ying, Ying Fan, and Bing-Hong Wang presented a CA based AFM in 2003. In this model, the local interoperation of CA is used to describe the spread of the herd behavior in capital markets. However, the neighborhood of this CA based AFM is still classical Moore neighborhood. All the investors in this AFM have the same simple investment behavior rule. The pricing mechanism of the market is far from the realistic markets. In real capital markets, as we know, the non-public information spreads through the investors' social network, rather than 2-D lattice. The connectivity, diameter, and degree distribution of the social network can decide the speed and scope of the information spreading. Furthermore, social network is not a fix, but dynamic structure.

According to the above reasons, combining the feature of multi-agent system and complex network, we extend the definition of CA in following aspects in this chapter: Neighborhood with network topology is adopted in CA; Structure of neighborhood is no more fixed, and will change following the neighborhood evolution rule; Cells in CA are no more homogeneous, and each cell has its own state transfer function with the same interoperation interface. Combining the above extensions of CA, as well as the other researchers' research on cellular automata on networks (or graph automata), we present a formal definition of CA on networks. On the basis of CA on networks, a new artificial financial market modeling framework, Emergency-AFM (E-AFM), is introduced in this chapter. E-AFM provides all standard interfaces and full functional components of AFM modeling. It includes classification and expression of information, uniform interfaces for investors' prediction and decision process, uniform interface for pricing mechanism, and analysis tools for time series.

E-AFM is a modeling framework for any kind of AFM. By instantiating the investors' asset structure, neighborhood network, behavior rules of investors, and pricing mechanism, we can get a specific AFM model. After an AFM model is simulated, we can get a price and volume time series in standard format just like real capital markets. Analysis tools provided by E-AFM, such as Hurst exponent and Lyapunov exponent, can be used to measure the fluctuation feature of price/yield time series. We can compare the simulation data with the real capital market data. Also we can find the relationship between the fluctuation feature and the topology of social networks.

In the rest of this chapter, an E-AFM based AFM model is introduced. This model is a simple model which is designed to find the relationship between the fluctuation feature of price time series and the degree distribution of the social network (neighborhood of CA). The statistics feature of neighbourhood structure is observed and compared with the fluctuation feature of price/yield time series. It is not a perfect model to get a new capital theory, but we can still realize how cellular automata can help us to do research in financial area.

2. The capital market in viewpoint of complex system

The capital market has existed for hundreds of years. And it is one of the most essential part of modern societies. However, people know little about capital market till today, even through which influences everyone's benifit. There is still no capital market theory which can explain the inner dynamic mechanism of capital market strictly. The capital market is one of the complex systems, which created by human being self, but are difficult to understand by us. Traditionally, when we discribe a uncertain system, we consider it a stochastic system. For example, modern capital market theory is based on probability statistics theory. Actually, however, there are many strict conditions for stochastic systems, such as, independence assumption. So, it is not rigorous to classify any uncertain behaviour to stochastic system. In order to apply stochastic process tools into modern capital market theory, its founder made many strong assumptions, such as, Rational investor hypothesis, efficient markets hypothesis(EMH), and random walk of yield rate. Unfortunately, neither these assumptions could be supported by investor psychology and behaviour analysis, nor their conclusions could be proved by market statistics data. If we investigate the capital market in viewpoint of complex system, we can find that the complexity of this system's behaviour is never as simple as random walk, but comes from extremely complex internal structure of capital market. We need some approaches to find out what assumptions are reasonable, what caused the fluctuation in price, and what theory is reliable and verifiable.

The complexity of system can be classified into time and space complexity. That is to say, we can investigate it in behaviour and structure aspects. From the standpoint of time, some extremely simple system, such as nonlinear dynamic systems like Logistic equation, can present extremely complex dynamic behaviour. This kind of dynamic systems, however, which have explicit equations, could be investigated in mathematical methods. The degree of freedom of this kind of system is finite and knowable. And their behaviours are still reproduceable in controlled conditions. The behaviour complexity of a real complex system comes from its structure complexity. What we call structure complexity means that the degree of freedom is too complex to reproduce its dynamic feature in classic analytical way. The structure complex could be reflected in the uncertainty of degrees of freedom, as well as in the interdependence of the components. The Name of two books: "Hidden Order: How Adaptation Builds Complexity" (Holland, 1996) and "Emergence: From Chaos To Order "(Holland, 1999), are good summary of the formation of the complex system. When individuals in a system interact with each other, their adaptive behaviours are the inner rules, or "hidden order", of system dynamics. Due to the quite huge amount of the individuals, and intricate interdependence within them, the whole system would represent some complex dynamic feature which could be observed by us. This process is called by John Holland "Emergence". John Holland's viewpoint explained how complex systems appear. The adaptive individuals are not organized in some regular or linear way. They don't act randomly and independently with each other either. The individuals with their autonomous targets in a system, may form some stable structures which are hard to know, during their adaptive behaviours. These stable structures make these complex system much harder to investigate than both absolutely ordered systems and absolutely disordered systems. John Holland calls these stable structures, which formed during adaptive interactive behaviours, "patterns". For a complex system, pattern is key to explore the relationship between microstructure and macrodynamics of the system.

From above discuss, we can conclude some essential conditions of an complex system:

- The system is composed of a large amount of individuals with their autonomous targets. An individual's target is the reason of its adaptive behaviour.
- Individuals in the system would interact with each other in a local scope. Interactions within individuals made the system an organic whole. The locality of these interaction is the condition of patterns in the system.
- The feature of the patterns decides the complexity of the system dynamics.

From our experiential knowledge about capital market, it satisfies above conditions exactly. The macrodynamics of capital market is price and volume movement. And the movement of transaction data comes from the trading orders quoted by masses of investors. Most investors participate in the capital market to earn profit. There still may be some investors with other targets. But at least all participants of capital market have their target. So capital market satisfies the first condition.

The investment decisions of investors are based on the predictions on the future price. Investors' predictions come from their judgement on different kinds of informations, such as macro-economy policy, profitability of the company, history transaction data, important news, influences from other investors etc. Some kinds of informations are public informations, the others spread through the investors' interactions. The interactions within investors are direct and local, just like other kinds of social networks. Capital market satisfies the second condition too.

An capital market is comprised of masses of investors with heterogeneous features, which involving the investor's condition of assets, information source, and risk preference etc. We call all about these "market structure". Different market structure can decide the complexity of macrodynamics of capital market. This matches the third condition of complex system. Actually, different hypotheses about market structures decide different capital market theories. For example, the rational investor hypothesis assumes that investors are seeking effectiveness of mean/variance in Markowitz meaning; efficient markets hypothesis assumes that investors in capital market can get infinite risk-free credit, which means investors can buy or sell as long as they wish; and only public informations, which had been reflected in market price already, can influence investors' decisions. In this kind of market structure, the dynamic feature of market price or yield is a random walk. In later sections of the chapter, we can see more models, in which market structure decides the feature of price fluctuation.

According to above discuss, we consider that treating capital market as an complex system is reasonable to explore its dynamic mechanism. Building models for an complex system is the best way to research it. Neoclassic financial theory can be treated as a kind of system model of capital market without direct interaction within investors. The subsequent theories, such as Coherent Market Hypothesis (CMH) or Fractal Market Hypothesis (FMH), could be treated as other kinds of capital market models which emphasize heterogeneity and direct interactions of investors. When we build models for complex system, we just can design interaction rules according to our experience, logical reasoning, or conclusions from psychology and behavioristics. If we want to verify the rationality and correctness of a model, we must evolute it and compare the macrodynamics of the model with the real system. Fortunately, masses of transaction data had been accumulated in real capital markets, and there are some effective methods to analyse time series. The conditions to build a verification system for capital market theories are equipped now. The approaches to verify capital market theories are usually collectively called Experimental Finance.

3. Introduction to previous works on artificial financial market

In the development of complexity science, some modeling tools such as cellular automata and muti-agent system appeared. In the 1980s, because of the influence of artificial life (Christopher Langton, 1986), ideas like complexity, evolution, self-organization, and emergence are applied into the modeling of social system. Researchers in Santa Fe Institute first introduced Agent-based Computational Economics into financial area. Their Santa Fe Institute Artificial Stock Market(SFI-ASM) was the pioneer of artificial financial market. In recent years, group behaviours in capital market attract many researchers, and cellular automata was introduced to build artificial financial market models.

3.1 Senta Fe Institute Artificial Stock Market

A classic Santa Fe Institute Artificial Stock Market(SFI-ASM) includes *N* interactive agents, and a stock market, or an exchange, which is avialable to perform stock exchange. Agents in SFI-ASM could belong to different categories, or in a sense, they are heterogeneous. There is not direct interaction within agents in SFI-ASM. They just interact with each other through trading in the exchange. Time in SFI-ASM is discrete. Period *t* lasts from time t to *t+1*. At end of each period, bonus would be allocated to each share, following time series *d(t+1)*. The bonus time series is a stochastic process, which is independent of the stock market or the agents. Ornstein-Uhlenbeck process is often adopted as the bonus time series. There are even a fixed-income asset with fixed interest rate *r*, such as bank, in the market. Agents can decide invest how much money into the stock market or left it in bank. At any time *t*, *agent i* holds some shares of stock *h_i(t)*, and lefts a part of cash in bank *M_i(t)*, then the total assets of *agent i* is:

$$w_i(t) = M_i(t) + h_i(t)p(t) \tag{1}$$

Where *p(t)* is the price at time *t*. After a time step, the value of the asset portfolio is:

$$\overline{w}_i(t+1) = (1+r)M_i(t) + h_i(t)p(t+1) + h_i(t)d(t+1) \tag{2}$$

Note: $\overline{w}_i$ *(t+1)*is not *w_i(t+1)*, $\overline{w}_i$ *(t+1)* does not includes transaction in next time step, which could cause changes of the cash in bank or the shares held by the agent.

All agents in SFI-ASM have the same utility function and risk preference. But each agent has a condition-forecast rule itself. The form of condition-forecast rule is as follows:

if (condition fulfilled), then (derive forecast).

It can be seen that it is a general form for condition-forecast rules. Different agents can use different rules, such as basic analysis or technical analysis, to predict trend of future price. And then, agents can make invest decision based on the predictions. So the agents in SFI-ASM are heterogeneous fundamentally. Artificial intelligence methods like artificial neural network and genetic algorithm can be applied to condition-forecast rules to provide agents self-learning and self-adaption abilities.

In SFI-ASM, there is a "specialist" who controls the Trading Process. He decides the price of next time step (*p(t+1)*) according to supply and demand in the market. When in oversupply, the price would drop, when in short supply, the price would rise. The specialist influences the investment decision of agents by the fluctuation in prices. Price, bonus, the size of all bid

and ask orders, compose the global information variable of SFI-ASM. The global information variable is the foundation of agents' prediction of next time step.

SFI-ASM is the pioneer of artificial financial market and has revolutionary influence on experimental finance. Many models derived from SFI-ASM appeared in its long-term development. Creators of SFI-ASM introduced the methodology of complex system modeling into financial area. In SFI-ASM, investors are regarded as initiative individuals (agents) in the system. The investors' investment behaviours on the market are regarded as the interaction within them. The investment behaviour of an investor was divided into three stages: price prediction, making judgement by utility function, makeing investment decision according to risk preference. The heterogeneity of agents is reflected in the price prediction stage. The other two stages keep homogeneous.

However, some shortcomings of SFI-ASM come into our notice. Information is vital important to real capital market. Because all investment behaviours are based on predictions, and information is the fundamental of predictions. As we know, the basic viewpoint of efficient markets hypothesis is that the price had reflected all information related to the market. The sceptics of efficient markets hypothesis queried this assumption greatly. In real market, the spread of information is complex. There is public global information which can reach all investors at the same time. Such as trading data, financial policy, news of the company are this kind of information. There is also non public information in the market. The personal viewpoints, emotion, insider information, are just trasfered from individual to individual in a local area. Some individuals respond the information immediately after they receive it; others may just wait till the information is verified. The delayed responses may cause more complex phenomenon in capital market. As the information is time sensitive, different investors with different "investment start point"(Peters, 1996) would be interested in different kinds of information.

The SFI-ASM, which based on muti-agent system, focuses on heterogeneous investment behaviours of investors. But only simple, public, global information without delay, was adopted in SFI-ASM. In SFI-ASM, information spreads in the market at the same time, and would be handled by agents immediately. Excessive simplification of information and ignoring local direct interaction may make SFI-ASM close to efficient markets hypothesis. Or in other words, the complexity of SFI-ASM comes from complex individual behaviours, other than inner structure formed by self-organization of individuals.

3.2 The classic cellular automata based capital market model

Recent years, the non public information's influence on capital market came into researchers' sights. Especially, under some specific culture environments or the market is not developed or mature enough, public information is not transparent or reliable, non public personal information could be decisive. Group psychology and herd behaviour appeared frequently in the emerging market like Chinese capital market.

It is necessary to describe the interaction within individuals if we want to build a model for the spread of non public information. Cellular automata is superior in this aspect. The classic cellular automaton is kind of discrete dynamic system which is composed with masses of individuals. The behaviour rule of the individuals is simple and unique in a cellular automaton. The interactions within individuals rely on neighbourhood structure in the cellular automata. These features can be used to express direct non public information exchange within investors.

One of the typical cellular automata based artificial financial markets is "the cellular automaton model of investment behavior in the stock market"(Wei et al., 2003). In this model, stock market is regarded as a cellular automaton. And the investors are regarded as cells in 2D lattice space. The neighbourhood of a cell follows the Moore's definition (Fig. 1.).
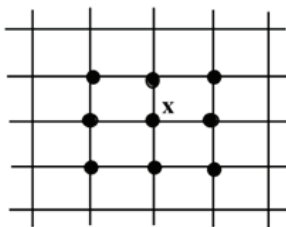

Fig. 1. Moore's neighbourhood

"The cellular automaton model of investment behavior in the stock market" focused on the influence of herd behaviour on the capital market. In the model, a cell have just three states (attitude): buying, holding and selling. In this model, the unit of time is step. At step t, a cell's state would be decided by states of neighbours at step $t-1$, according to its state transition rule. The state transition rule would calculate the distribution of buying, selling, holding neighbours, and decide the state at step t itself. In each step, the model would figure out a price according to the distribution of cells' states.

Compare with SFI-ASM, "The cellular automaton model of investment behavior in the stock market" has totally different standpoint about market information. In this model, only local information inside the neighbourhood can influence a cell's investment decision. No public information is taken into account. The primary importance of "The cellular automaton model of investment behavior in the stock market" lies in introducing the local interaction of investors into capital market models, and comparing the relativity between group psychology and VAR(Value-at-Risk). However, this model just focused on group psychology in the market, ignored all other factors involved with price fluctuation. Its pricing mechanism is too subjective, and it is much less mature than SFI-ASM. Even regarding the interaction within cells, the 2D lattice space and Moore's neighbourhood definition are not suitable for social relationship. Actually, social relationship is usually a network. Its structure influence the spread dynamic feature deeply.

4. The formal definition of cellular automata based artificial financial market

As discussed above, the capital market is a dynamic system with masses of individuals interacting with each other. Individuals have their own target. The behaviour of an individual relys on information based prediction. In essential, the difference in different capital market theories and models lies in different standpoint about information's category, spread, and handling. Further more, we consider that the complexity of the capital market dynamic, comes from the inner structure which is formed in the process of the individuals' self-organization. Both muti-agent and cellular automata are suitable for modeling of capital market. As there is neighbourhood definition in cellular automata to limit the interaction scope of cells, it is superior in describing non public information in capital market. If we extend the definition of classic cellular automata, make it can contain heterogeneous cells

and social relationship neighbourhood, it would be a better choice to build artificial financial market based on cellular automata. Before we can do so, it is necessary to extend classic cellular automata in some aspects.

A d-dimensional classic cellular automaton could be defined as a quadri-tuple:

$$\wedge = \left(Z^d, S, N, \delta \right) \tag{3}$$

Z^d stands for a d-dimensional discrete lattice space. It's the space structure of d-dimensional classic cellular automata.

S is the finite states set of cells.

$N = \{ n_j = (x_{1j}, \ldots, x_{dj}), j \in \{1, \ldots, n\} \}$ is the finite ordered subset of Z^d. N is called the neighbourhood of cellular automata.

$\delta: S^{n+1} \rightarrow S$ is the local state transition function of Λ.

We can find from this definition that the essential feature of cellular automata is its discrete space-time and local interaction. If we want to apply it to social system modeling, we must extend its definition in four aspects.

The cellular automata focus on how individuals' adaptive behaviors result in complexity of the system. But when we build some models for real world, there is public information which can influence individuals behaviours as well as interaction within them. If we adopted public information in cellular automata, it would become an open system.

Traditionally, cells in the cellular automata are homogeneous. That means all cells in a cellular automata have the same state transition function. But sometimes, we need to include individuals who would respond to the information in various way. The problem of heterogeneous cells is that cells must interact with neighbours who may have different state transition function. So we must guarantee the S in the quadri-tuple can be accepted by all cells' state transition function, even though they may have different logic.

The neighbourhood of cellular automata represents interaction scope of a cell. The space of social system is not like physical system. The relationship within social members is some kind of networks. So d-dimensional discrete lattice space must be replaced by network space. In fact, network is a universal description for discrete space. The d-dimensional discrete lattice space is just an example of it.

In classic cellular automata, the neighbourhood is fixed. In social system, however, the relationship between two members is not so stable. The adaptive behaviors of individuals are even the cause of formation of the system's inner structure. Margolus designed odd-even neighbourhood for odd-even steps, then realized the change of neighbourhood. In the cellular automata based 2-dimensional fluid model: HPP Lattice Gas Automata(Hardy et al., 1973), Margolus neighbourhood is adopted. The successor of HPP model: FHP Lattice Gas Automata (Frisch et al., 1986), change the lattice into hexagon. The neighbourhood of FHP model is alterable too. Network dynamics plays an increasingly important role in social networks modeling. We could add network dynamics as the neighbourhood transformation rule into the definition of cellular automata.

Considering the four extends, we can get a new definition for cellular automata:

$$\wedge = \left(Z, S, N, P, \delta, \sigma \right) \tag{4}$$

Because the public information and neighbourhood transformation function are supported in the cellular automata, the new definition becomes a six-tuple. In the new definition, Z

insteads the original Z^d, which means the space of cellular automata dosen't have to be Euclid space. It could be a network or graph structure. Accordingly the N in the cellular automata may follow the graph's neighbourhood definition. Further more, the N dosen't have to be stable. $\sigma: N{\rightarrow}N$ is the neighbourhood transformation function, which can change the neighbourhood in every evolution step of the cellular automata. P stands for the public information. Accordingly the state transition function becomes $\delta: P, S^{n+1}{\rightarrow}S$.

Although we extended the definition from classic cellular automata, we still kept its essential features. The new kinds of cellular automata are still time-space discrete system. Each cell decides its state in next step according to the states of neighbours and itself in current step. The macrodynamics of cellular automata is the emergence of masses of cells' adaptive behaviours. The classic cellular automata could be regarded as an instance of the new definition. Because the cells could be heterogeneous, a cellular automaton under new definition could also be a multi-agent system.

New definition of cellular automata gives us a foundation to define a cellular automata based artificial financial market. Because there could be many artificial financial markets under different assumptions, we just define the general part of them. In a cellular automata based artificial financial market, each cell represents an investor. Z in the formula (4) is the set of cells. In cellular automata based artificial financial market, the finite states set S is a 6-tuple:

$$S = (C_u, C_f, S_u, S_f, Q, E) \tag{5}$$

C_u and C_f stand for usable and frozen cash respectively. S_u and S_f are usable and frozen stock respectively. Q is the set of orders which have been quoted to exchange house but have not been completed or canceled yet. Each order includes direction (ask or bid), price and amount. E, which valued rise, fall, or keeping, is the price prediction of an investor. C_p is the total property of an investor. Given P as current stock price, C_p can be expressed as follows:

$$C_p = C_u + C_f + P\left(S_u + S_f\right) \tag{6}$$

Maximization of C_p is the only goal for all investors.

N is the neighborhood in cellular automata. In this ASM, N is a directed graph. When $Cell_i$ is making a prediction, a directed edge $<i, j>$ between $Cell_i$ and $Cell_j$ exists only if the $S.E$ value of $Cell_j$ can affect the $S.E$ value of $Cell_i$. In this condition, $Cell_j$ is defined as a neighbor of $Cell_i$. The neighbour relationship between these two cells is not self reciprocal.

In the cellular automata based artificial financial market, public information includes trading data, public financial policy, such as risk free rate, and company news, such as financial reports and bonus. As the definition of cellular automata, public information is represented by P. The state transition function decide a cell's state according to the public information and the states of the cell self and the neighbours. So is defined as:

$$\delta : P, S^{n+1} \rightarrow S \tag{7}$$

The neighborhood transition function is:

$$\sigma : N \rightarrow N \tag{8}$$

σ is the variance of the dependency relationship between cells. Based on different assumptions, methods to rebuild the dependency networks can be different. σ would be performed after each trading day.

Once we gave the extended definition of cellular automata specific meaning, we defined a cellular automata based artificial financial market. We try our best to abstract the essential of the capital market. We emphasize the heterogeneous individual behaviours, as well as the complex information spread in the market. We believe the information is the decisive factor for a predictive system. The Emergence-Artificial Financial Market Framework, which would be introduced later, is a realization of the cellular automata based artificial financial market.

5. The emergence-artificial financial market framework

Now we can build artificial financial market models under above definition. As we know, the target of artificial financial market is to find out the relationship between macrodynamics and microstructure of the capital market, and verify the financial theories which are based on different assumptions. These assumptions are focus on the investors' behaviours and the spread of information. Other components of the capital market are stable and clear. So, we built a framework, realized the common parts of the capital markets in it, and defined the interfaces of the heterogeneous investors and informations. Because the complex macrodynamics could be regarded as the emergence of the adaptive behaviours of individuals, we named the framework "The Emergence-Artificial Financial Market Framework (E-AFM)".

5.1 The structure of E-AFM

As we defined in section 4, an E-AFM is a cellular automata based artificial market, so, first, we realized a cellular automata library under the extended definition. Then we realized E-AFM as a template instantiation of the cellular automata library. All these frameworks are realized in C++ language, in order to utilize its generic programming mode and parallel technology.

The basic starting point of the cellular automata library is abstraction of the data type of the cell state (personal infermation) , and public information. That's why we use parameterized type feature of C++ template. The base classes of cell, cells' container, neighbourhood, are provided in the library. The base classes of cell and cells' container are both template classes. The template parameters are the abstract data types of cell state and public information. The template parameter StateType is the data type of the cells states. Users can define it according to their needs. In the CellBase<StateType> class, state transition function is declared as a pure virtual function, any class, which derived from CellBase<StateType>, should overwrite the state transition function in its own rule. Two derived classes of CellBase<StateType> were provided in this library, one is for synchronous cellular automata, and another is for asynchronous cellular automata. When we realize a cellular automaton, we just need to define the data type of personal and public information, design a class derived from class SynchCellBase<StateType> or class AsynchCellBase<StateType>, and provide relevant state transition function.

In the cellular automata library, all cells are managed by cell container classes. The design targets of the cell container include following aspects. Firstly, the cell container should provide one or more kinds of traversal methods to access all cells in the cellular automaton. Secondly the cells' random access should be supported, because we can't assume the structure of users' cellular automata, and we need to access a cell through its neighbors. Thirdly, the neighborhood of the cellular automata should have an inner expression in the container. That means, when we access a cell, it is required to get the cell's neighbors

directly. Lastly, both serial and parallel accesses must be supported by the container. In detail, when different threads access different cells without mutex at the same time, the container should be thread safe. When different threads access the same cell at the same time, a mutex would be provided.

In our cellular automata library, the solution to satisfy the requirement of concurrency is class concurrent_vector which is provided in Microsoft Concurrency Runtime technology. If we use this library to build a cellular automaton model, all container classes should derive from class CellContainerBase<StateType>, which uses a thread safe container class concurrent_vector to store cells. So we can access any cell randomly by its index. The class CellContainerBase<StateType> has a member pointer, which points to a derived class of class Neighborhood. Neighborhood class declared two basic abstract functions. The function *AppendItem* is used to add a new cell's index into the relationship structure. The function *Neighbors* is required to return a cell's neighbors' index. The derived classes of neighborhood are required to realize the two functions. The purpose of adopting index to manage cells' neighborhood is to separate the design of container class and neighborhood class. Two derived classes of CellContainerBase<StateType> are provided to perform the evolution of the cellular automata in serial or parallel way.

As discussed above, an artificial financial market would be regarded as a cellular automaton model. So, E-AFM, as a framework, realized the main parts of this kind of cellular automata. The realized parts include the investment process of each investor (cell); the interaction way with cells; the basic mechanism of the artificial financial market; the definitions of evolution step and trading day; the account management etc. But the basic assumptions to the capital market are left to model designers. For example, model designers can define the data type of the public or private information, decide the structure and the evolution of neighborhood, design the behavior of investors, and choose the price formation mechanism of the capital market, such as order-driven or quote-driven rule.

The E-AFM is a template instantiation of the cellular automata library. As discussed above, the state type of an investor has been defined clearly in formula(5). The classes which are related to cash account, position account, trading orders, and investor's attitude are provided in E-AFM as cell state. So we can instant the template classes of the cellular automata library, and provide their derived class in E-AFM. SimInvestorBase class is derived from CellBase<StateType> class. It provided functionality to manage cell state itself, but it is still an abstract class, because the investment process are left to users to realize. There are also some classes derived from neighbourhood class provided in E-AFM. These neighbourhood classes can change their structures after a trading day. Especially some neighborhood transition functions are related with individuals' state. There are other components in E-AFM, which provide stable functionalities such as account management, exchange, quoted order management, and pricing mechanism etc. It is not necessary to derive them usually.

When we simulate the artificial financial market, we are performing the evolution of the cellular automata. Utilizing the Concurrent Runtime technology, the simulation could be parallel. The benefits of parallel simulation are not only higher performance. It is more important that the concurrent evolution is more close to reality. A trading day was defined as an evolution with several steps in the E-AFM. After a trading day, the artificial financial market would be closed, and the accounts' settlement would be performed. The neighbourhood of the cellular automata could be rebuilt too. One simulation of an E-AFM instance could include hundreds of trading days.
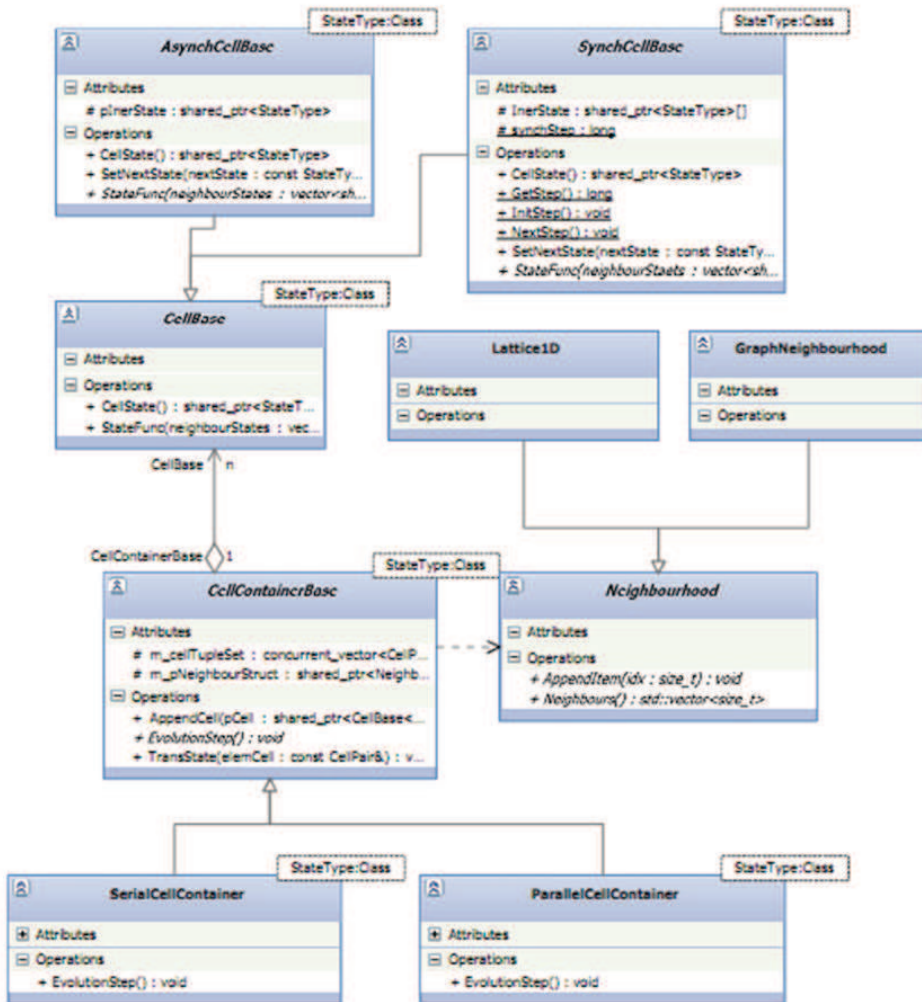
Fig. 2. Static Structure of Cellular Automata Library

5.2 Analysis tools in E-AFM

After a simulation finished, trading data would be produced as real capital market. Some tools are provided in E-AFM to analysis the macrodynamics and microstructure of cellular automata based artificial markets. Some of these tools can also be used to analysis trading data of real capital market.

One of the analysis tools is the Hurst exponent which was introduced into financial time series analysis first by Mandelbrot. Mandelbrot consider the Hurst exponent is better than variance analysis, spectral analysis, and autocorrelation. Hurst exponent is mainly used to estimate the long term memory of time series. R/S analysis (Rescaled Range Analysis)

(Hurst, 1951) is the most classic estimation method of Hurst exponent. Edgar E. Peters used R/S analysis to find the fractal feature of financial time series, and built the Fractal Market Hypothesis (FMH). R/S analysis is also provided in E-AFM.

We have a time series, which length is T. First, we should divided the time series into N adjacent v-length sub-periods, and $N*v=T$. Each sub-period is recorded as I_n, $n = 1,...,N$. Each element in I_n is recorded as $r_{t,n}$, $t= 1,2, ...,v$. M_n is the arithmetic mean value of I_n. We can calculate the the accululated deviation $X_{t,n}$ from the mean using following equation:

$$X_{t,n} = \sum_{u=1}^{t} X_u - M_n \tag{9}$$

Let:

$$R_n = \left(\max(X_{t,n}) - \min(X_{t,n})\right) \tag{10}$$

R_n is called range of I_n. Let S is the standard deviation of the I_n. Then the Rescaled Range is defined as:

$$E\left(R_n/S_n\right) = \left(aN\right)^H \ \text{ as } \ N \rightarrow \infty. \tag{11}$$

or:

$$\log\left(E\left(R_n/S_n\right)\right) = H\log\left(N\right) + \log\left(a\right) \tag{12}$$

The slope H is the Hurst exponent. It can be estimated by least square method or other methods.

One of E-AFM's tasks is to find how does the microstructure of the capital market cause complexity of macrodynamics. For example, the structure of neigbourhood graph can influence the spread of non-public information in the capital market. We use the degree distribution to measure the complexity of the networks, and use the clustering coefficient to measure the dependency level within the investors. The clustering coefficient γ_v of a vertex v in a graph can be defined as:

$$\gamma_v = \frac{\left|E(\Gamma_v)\right|}{\binom{k_v}{2}} \tag{13}$$

Where Γ_v is the neighbourhood of vertex v, and $|E(\Gamma_v)|$ is the number of edges in the neighbourhood. $\binom{k_v}{2}$ is the maximum number of possible edges in the neighbourhood.

There are still many other methods could be used to measure the time-space complexity of the artificial financial market. Due to space limitations, we don't discuss them individually.

6. Dynamic analysis of an Artificial Financial Market

In the last part of this chapter, we'd like to show the readers an example artificial financial market which is based on E-AFM, and analysis the results of simulation. The assumptions of this artificial financial market are not complete enough to proclaim a new capital market

theory. But it can show us that the cellular automata based artificial financial market could be an effective tool to simulate the process of self-organization in the capital market. And it can also show how does the structure of the social network influence the spread of information and then influence the price fluctuation.

In this artificial financial market, we assume that an investor's behaviour can be divided into prediction stage and investment stage. In the prediction stage, the investor predicts the direction future market price (tuple E in equation 5) according to the public and non-public information. The public information is the technical analysis on history trading data, such as moving average convergence/divergence index (MACD). The non-public information is the collection of neighbours' attitudes. Each investor has a weight number δ_i decides the different influence of the public and non-public information on the investor's judgement. The prediction is the base of the investment stage and the non-public information which can be visited by neighbours. Once the prediction is made, the investor would send orders to exchange, according to its condition of assets (S in equation 5). Both the prediction and investment stage are parts of the investor's state transition function. Prediction stage is more important, because it's the stage of information processing. The heterogeneity of the individuals is also reflected in the prediction stage.

The neighbourhood of the cellular automata is defined as a social network. The initial network is a random graph. However, after each trading day, the network would be rebuilt. An individual's history in-degree and its assets condition ranking are two factors influencing its in-degree in next network. There is a weight number ω to accommodate the importance of the two factors. The pricing mechanism in this model adopts the order-driven Electronic Communications Networks (ECNs) Trading mode.

There are 2 critical control parameters in this artificial financial market, ω and δ_i. ω decides the weight coefficient between historical stickiness and profit orientation when rebuilding the dependency network. δ_i decides the weight coefficient between technical analysis and herd psychology in a cell's prediction stage. The simulation results show that the the co-effect of the two key factors caused the different structures of the neighbourhood, and various features of price fluctuation.

When the individuals' history in-degree plays the main role in rebuilding the neighbourhood network, the degree distribution is shown in Figure 3. After enough trading days, the clustering coefficient is close to 0.5. That means the interaction is active under this condition.
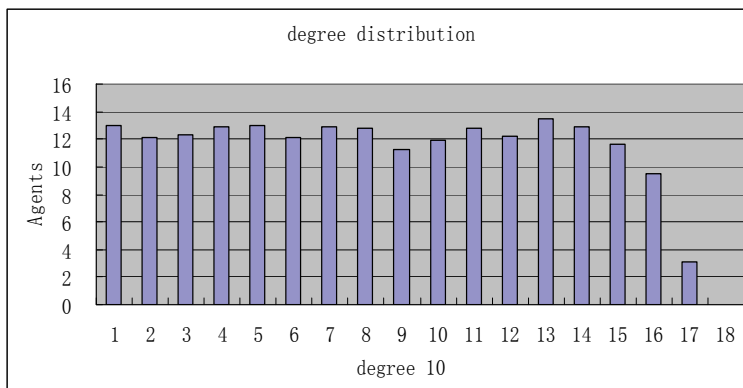

Fig. 3. Degree Distribution of History Degree Ranking Hurt Behaviour

On the other hand, when the individuals' assets condition ranking plays the main role in rebuilding the neighbourhood network, the degree distribution is shown in Figure 4. And the clustering coefficient is close to 0.3. The investors rely more on the judgement of themselves.

Another problem is whether the public or non-public information plays the main role when individuals predict the market price. We simulated the two assumptions both. Combining with the factor of neighbourhood structure, we got four simulation results. We estamited their Hurst exponent using R/S analysis, which are showed in Fig 5, Fig 6, Fig 7, Fig 8.
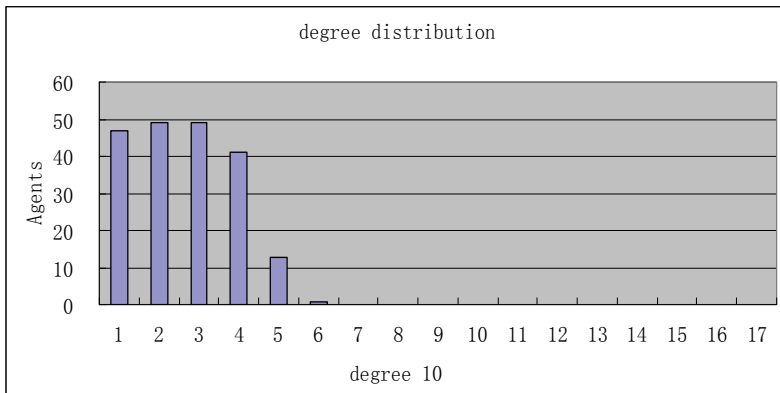
Fig. 4. Degree Distribution of Assets Ranking Hurt Behaviour
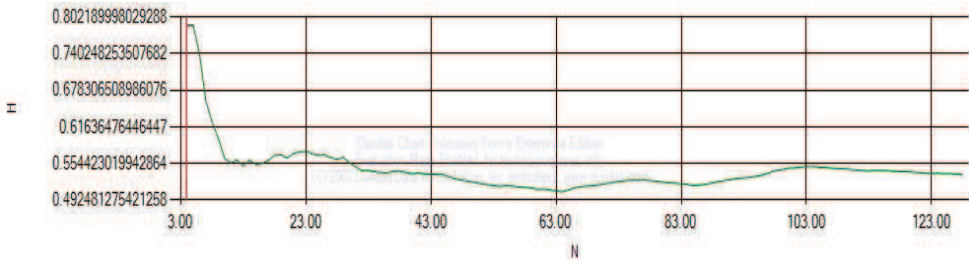
Fig. 5. Assets Ranking Hurt Behaviour & Public information
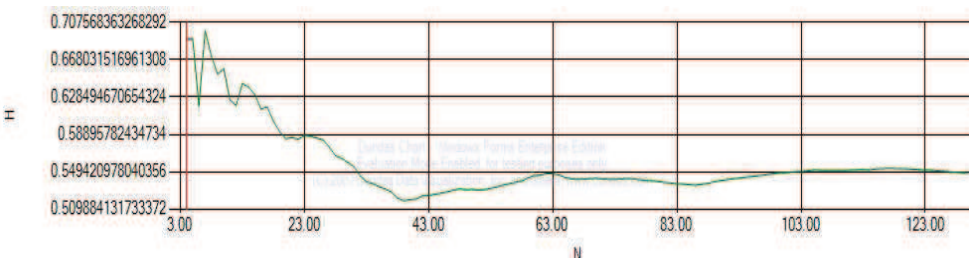
Fig. 6. History Degree Ranking Hurt Behaviour & Public information
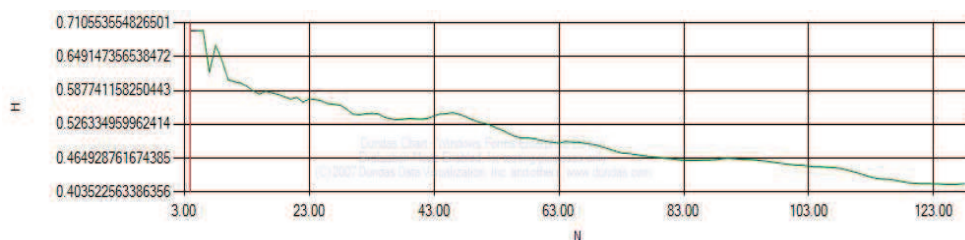
Fig. 7. Assets Ranking Ranking Hurt Behaviour & Non-Public information
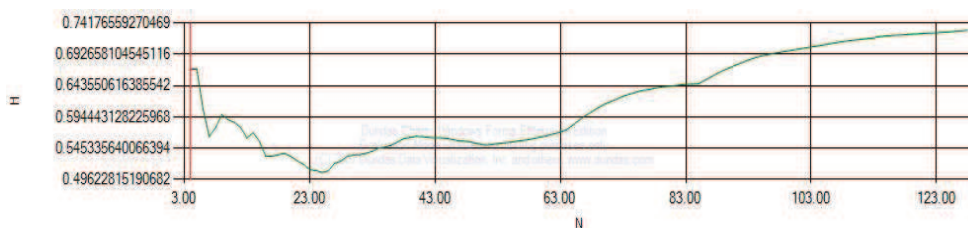
Fig. 8. History Degree Ranking Hurt Behaviour & Non-Public information

It is noticed that the homogeneity of investors is weak when public information plays the primary role in prediction. At this time, especially when assets ranking determines the neighbourhood structure, the Hurst exponent is close to 0.5. This means the volatility of market prices follows Random Walk, just like the hypothesis of classic theory.

When non-public information plays the primary role in prediction., the market mainly consists of herd behavior investors, and the transmission dynamic structure plays the decisive role in determining the durative or anti-durative of the price movement. When assets ranking determine the neighbourhood strucure, the Hurst exponent is close to 0.4 (Figure 7). The remarkable anti-durative of the price movement indicates the collapse of market. When history degree ranking determine the neighbourhood strucure, the historical stickiness causes durative of the price movement, and the Hurst exponent is close to 0.74.

We can also compare the yield rate and the Hurst exponent. As showed in Fig 9, the X-axis is the yield rate and the Y-axis is the Hurst exponent, when Hurst exponent stands low level, yield rate is just a fluctuation around zero. When Hurst exponent is less than 0.5, the system has the feature of anti-persistence, reversals of the price movement would appear frequently.

The simulation result presented above shows that the transmission dynamic structure is of critical importance to the prices movement in a market full of herd behavior investors. Because of the susceptibility of the herd behavior investor, the transmission of the market information could enhance the homogeneity of the investors. If there are some historical sticking authorities trusted by most herd behavior investors in this kind of market, the durative prices movement would appear. If the sticking trust disappears, the herd behavior investor will fall into panic, and the market collapse will come out. It is interesting that both positive and opposite deviation of Hurst exponent from 0.5 is caused by the homogeneity of
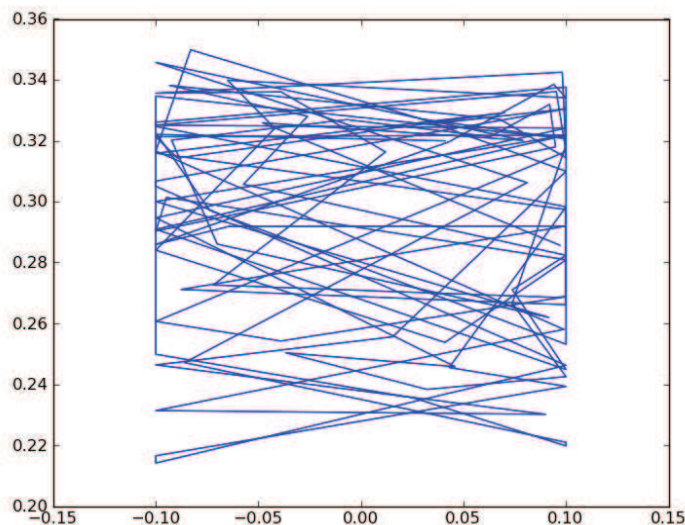
Fig. 9. Phase Diagram of yield rate and the Hurst exponent.

investor structure. The durative or anti-durative of the price movement just depends on whether the information source is keeping changing. In a developed and mature market, however, because there are large amounts of heterogeneous investors, the effect of the transmission dynamic structure is relatively weak.

7. Conclusion and future works

When we build a model for the capital market, tt is difficult to include all essential factors into considering. But we can believe the heterogeneous investors' response to the information spreaded in the market is fundamental motive power of the macrodynamics of the capital market. Investors response to the information and produce information at the same time. The capital market is a kind of self-feedback system. The self-feedback procedure is so complex, and the microstructure formed by the individuals' adaptive behaviour play the primary role. The cellular automata based artificial financial market provided a possibility to describe these factors, and their interaction rules. The self-organization process could be simulated in it. The macrodynamics of the artificial financial market and real capital market can be compared.

However, we must recognize that the cellular automata based artificial market is not mature enough to build a new capital market theory. What is the key feature of the microstructure inside the investors? How can we measure it? There is still no perfect answer. We just have some ideas to do further research. For example, cluster coefficients of network may be related with volatility feature of price; Investors following various behavior rules, may have different average yield rates. But there is still no remarkable result supporting these assumptions.

In the future, the cellular automata based artificial financial market should be extended to describe market factors more particularly. The evolution rule should be more valid. More researches should be focus on the category, form, and spread way of the information. And we should consider more effective way to measure the complexity of the microstructure within the individuals.

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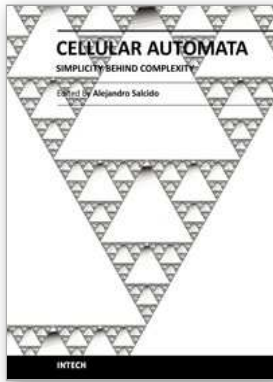
Cellular Automata - Simplicity Behind Complexity

Edited by Dr. Alejandro Salcido

Cellular automata make up a class of completely discrete dynamical systems, which have became a core subject in the sciences of complexity due to their conceptual simplicity, easiness of implementation for computer simulation, and their ability to exhibit a wide variety of amazingly complex behavior. The feature of simplicity behind complexity of cellular automata has attracted the researchers' attention from a wide range of divergent fields of study of science, which extend from the exact disciplines of mathematical physics up to the social ones, and beyond. Numerous complex systems containing many discrete elements with local interactions have been and are being conveniently modelled as cellular automata. In this book, the versatility of cellular automata as models for a wide diversity of complex systems is underlined through the study of a number of outstanding problems using these innovative techniques for modelling and simulation.

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