

Spatial Optimization and Resource Allocation in a Cellular Automata Framework

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1. Introduction

Land management is a complex activity associated with the determination of land uses, the placement of activities and facilities and the distribution of resources over extensive territories with a view to satisfying one or more criteria of economical and/or ecological character. It follows from this descriptive definition that in land management it is not sufficient to distribute goods or commodities to a number of beneficiaries, but, mainly, to carry out planning with respect to space and location, thus intervening and shaping the local geography of economical and environmental characteristics.

An important part of land management is land use planning, in which a given area is divided into land blocks with each one of them being assigned a specific land use, taken from a set of possible land uses. The search for suitable combinations of land uses, so as to attain given objectives, constitutes a computationally intensive optimization problem. A related problem concerns spatial resource allocation, in which one or several resources have to be allocated to each one of the described land blocks, again in order to attain preset objectives and possibly satisfy constraints. The sought for distribution and nature of these resources bears a strong relation to the land uses of the respective blocks. This fact gives rise to even more difficult, but also more realistic optimization problems.

A basic resource to be managed is water. Allocating water may not simply involve its unit price, but also the estimation of transportation and extraction costs. In the latter case physical modeling of groundwater movement and pumping is needed and this contributes to the complexity and nonlinearity of the problem. This fact makes the present problem different from the typical allocation problems. Problems of land use planning and water allocation combined with water extraction have been presented by Sidiropoulos & Fotakis (2009) and Fotakis (2009) and are reviewed in this chapter, along with new results concerning a cellular - genetic approach.

Genetic algorithms and cellular automata will be the basic tools to be implemented in the present approach. Genetic algorithms are well-known biologically-inspired meta-heuristics. Their properties and characteristics are described in textbooks such as Michalewicz (1992) and Goldberg (1989). Applications abound in the literature.

Cellular automata date back to von Neumann. Their fundamental importance was demonstrated by Wolfram (2002). They have been used as a background for modeling a great diversity of natural, as well as social and economic systems. Cellular automata have been used for simulating natural phenomena. Also, numerous applications have been

presented for the spatial analysis of ecosystems. Hogeweg (1988) used them to simulate changes in landscape. Green et al (1985), Karafyllidis and Thanailakis (1997), Karafyllidis (2004) and Supratid & Sunanda (2004) employed cellular automata to simulate the spread of fire in a forest ecosystem, while Sole and Manrubia (1995) simulated the dynamics of forest openings by means of cellular automata. Also, cellular automata have been used for the simulation of succession and spatial analysis of vegetation growth (Colasanti and Grime, 1993). A useful application of cellular automata concerns the study of spatial characteristics of the socio-economic development. Balmann (1997) analyzed the structural change in a rural landscape with the help of a two-dimensional cellular automaton, and Deadman et al (1993) used cellular automata to model the development of rural settlements, while Jennerette and Wu (2001) used them to model urban development, along with producing possible land-use scenaria. Finally, Prasad (1988) described the economy as a cellular automaton where the self-organization responds to the evolution of the system seeking a more efficient provision of social resources.

Recently, cellular automata are used for various optimization problems such as finding the optimal path (Adamatzky 1996), designing of sewerage systems (Guo etc., 2007), management of groundwater aquifers (Sidiropoulos and Tolikas, 2008), reservoir management (Afshar and Shahidi, 2009) and optimization of forest planning (Strange et al. 2001, Heinonen and Pukkala 2007, Mathey et al. 2008) and afforestation (Strange et al. 2002). The use of evolutionary methods in spatial optimization problems such as the ones outlined here is called for by their complexity and nonlinearity. Additionally, a particular characteristic of these problems is the relation between local interactions and global system behavior. These considerations lead to the introduction of cellular automata.

The area under study may be modeled as a cellular automaton with the land blocks being represented by the cells of the automaton. The actual spatial arrangement of the land blocks provides the neighborhood structure of the cells. The states of each cell represent the land uses or the water sources corresponding to the land block represented by the particular cell. In the typical cellular automaton, a transition rule is required operating on each cell as a function of the states of the cell and of its neighbors. In the literature such rules have been determined in order to construct cellular automata that perform certain computational tasks (Mitchel et al., 1994 and many subsequent reports along the same basic idea). In the present approach no constant rule is sought. Instead, genetic algorithms will be embedded into the cellular automaton in order to guide its evolution. More specifically, two types of genetic algorithm will be implemented:

1. The operative genetic algorithm, which will indicate each time a replacement rule for each block. No constant rule will be sought and no decomposition of the objective function will be involved.
2. The natural genetic algorithm endowed with a neighborhood rule. This rule will operate on a neighborhood level and on the basis of local values of the objective function for the purpose of enhancing the performance of the natural genetic algorithm.

Both these approaches have been presented in different publications (Sidiropoulos and Fotakis, 2009; Fotakis, 2009) but have not been compared or combined. This is done by their juxtaposition in the present chapter.

The natural genetic algorithm works on the whole configuration and its genetic operators are not based on local interactions among neighboring cells. Therefore, despite the cellular background, it would not by itself qualify as a cellular – genetic scheme. The addition of the

neighborhood rule introduces the local element into the computational scheme and, therefore, it brings the whole scheme closer to the cellular prototype.

The operative genetic algorithm, on the other hand, is fully consistent with the cellular automaton model. This algorithm defines each time a renewed rule for synchronous changes to each cell on the basis of the neighboring states.

A central issue in spatial optimization is the balance of the local versus the global aspects of the problem. The introduction of the above local rules serves the purpose of guiding iteratively the whole cellular automaton to optimal conditions. But the definition of such rules is not self-evident. The objective function of the problem generally depends on the entire configuration and decomposition or reduction to cell contributions is not evident or even feasible. Both schemes deal with this issue, which is also discussed in relation to the relevant literature.

In both genetic-cellular schemes the resulting arrangements of the cellular automaton present greater compactness, in comparison to the natural genetic algorithm, with respect to subareas with the same land use or water source. This is a significant qualitative result for land management (Vanegas et al, 2010). Moreover, in the present approach this characteristic is obtained as an emergent result without the addition of any special constraints or the modification of the objective function, as in similar problems of the literature.

2. Description and formulation of the problem

A hypothetical problem is considered in order to illustrate land management combined with groundwater allocation (Sidiropoulos and Fotakis, 2009). The terrain is represented in the form of a two-dimensional grid, the nodes of which correspond to land blocks.

The resource to be allocated is water and the cost involved will consist of two parts related to extraction and transport, respectively. Each one of the blocks is connected to one of the wells. These connections are the decision variables of the problem. They can be depicted as in Figure 1. The color of each cell indicates the well to which it is connected.

The transport cost for each block is taken as proportional to the distance of the block from the respective well. The total transport cost results from summing over all land blocks. The pumping cost is estimated via a steady-state groundwater phreatic aquifer model. The sum of the two costs forms the objective function to be minimized.

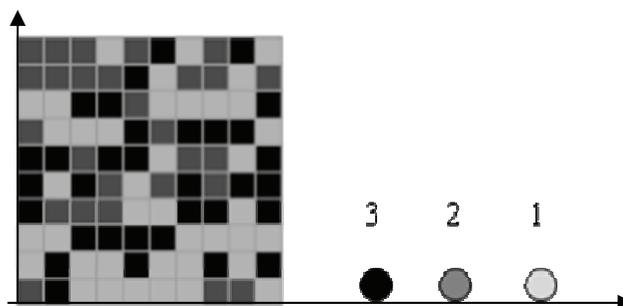


Fig. 1. Two-dimensional mosaic with three wells

Let (i, j) be the coordinates of the center of the typical block with $i=1,2,\dots,a$ and $j=1,2,\dots,b$, where a and b are the lengths of the two sides of the orthogonal grid. The blocks can be numbered consecutively, row by row. If $k=1,2,\dots,a\cdot b$, then

$$i = k - a \left[\frac{k}{a} \right], \quad j = \left[\frac{k}{a} \right] + 1 \quad (1)$$

where the brackets denote the integer part of the enclosed number.

Let m be the number of the wells and let the wells be numbered from 1 to m . Also, let $w_k \in \{1,2,\dots,m\}$ be the number of the well assigned to block k (Fig.2) with $k=1,2,\dots,a\cdot b$, according to the above numbering.

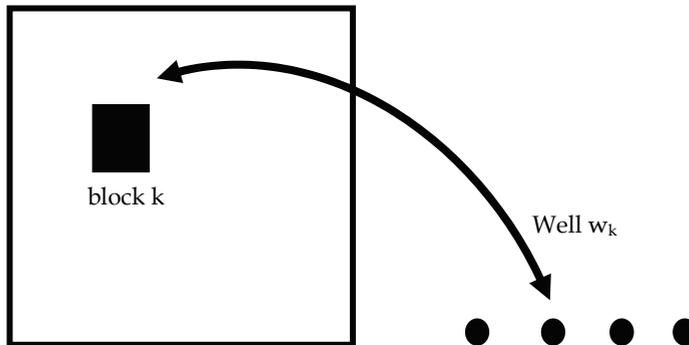


Fig. 2. Correspondence of cell to well

Then the transport cost is

$$F_T = \sum_{k=1}^{a\cdot b} \left[(x_k - x_{w_k})^2 + (y_k - y_{w_k})^2 \right]^{1/2} \quad (2)$$

where (x_k, y_k) are the coordinates of block k and (x_{w_k}, y_{w_k}) the coordinates of the respective well.

The pumping cost is expressed as follows:

Let $s_w, w \in \{1, \dots, m\}$ be the number of blocks irrigated from well w . Then the discharge from well w is equal to

$$Q_w = \sum_{k=1}^{s_w} q_k \quad (3)$$

where q_k is a quantity representing the water needs of block k .

The drawdown at each well is given by

$$\Delta h_w = \frac{1}{2\pi b} \sum_{\substack{w'=1 \\ w' \neq w}}^m \frac{Q_{w'}}{k_{w'}} \ln \frac{\sqrt{(x_w - x_{w'})^2 + (y_w - y_{w'})^2}}{R} + \frac{Q_w}{2\pi b k_w} \ln \frac{r_w}{R} \quad (4)$$

where b is the thickness of the aquifer, assumed constant, R is the influence radius, r_w is the radius of well w and k_w with $w=1,2,..,m$ are the hydraulic conductivities of the areas around each one of the wells.

Finally, the total pumping cost is proportional to the quantity

$$F_p = \sum_{w=1}^m Q_w \Delta h_w \quad (5)$$

where Q_w and Δh_w are given by Equations (3) and (4).

Thus, from Equations (2) and (5), the total cost can be taken to be equal to the sum

$$F = F_T + F_p \quad (6)$$

From the above formulation it can be seen that the present problem differs from classical resource allocation problems, because the cost associated to each cell does not depend only on the quantity of the water to be supplied to the particular block. It also depends on the position of the block itself. In fact this is true both for the transport cost, which depends on distances, and for the pumping cost, which is determined through the aquifer model with its predominantly spatial character. Moreover, the most distinct difference comes from the fact that the pumping cost is influenced not only by the well connected to the particular block, but also by the action of the other wells.

Questions of spatial decomposition will be addressed in a subsequent section, along with a comparison of the present problem to typical problems of spatial resource allocation, as they appear in the relevant literature. It needs to be noted that typical problems are based on separability of the individual cell contributions, which does not hold true for the present problem.

The solution methods to be described in the following sections are specially adjusted to the spatial - cellular character of the problem domain. The present approach is further compared to the treatment of various types of resource allocation problems of the recent literature, in particular to problems of forest planning.

3. Method of solution

3.1 A natural genetic algorithm

The objective function, Equation (6), is to be minimized as a function of the connections of the various blocks to the water wells. The objective function is nonlinear and the problem is one of combinatorial optimization.

Evolutionary algorithms are particularly suited for this kind of resource allocation problem. Indeed, such methods have been applied to resource allocation problems (Khan et al. 2008, Magalhaes-Mendes 2008), as well as problems involving both water allocation and crop planning (Ortega et al. 2004, Zhou et al. 2007, Khan et al. 2008).

A natural way of encoding the problem in a genetic algorithm framework has been presented by Sidiropoulos and Fotakis (2009). Here, it is cast into a more general framework. Let L be the set of cells as numbered according to the above scheme:

$$L = \{1, 2, \dots, a, b\}$$

and W the set of the m wells numbered from 1 to m :

$$W = \{1, 2, \dots, m\}.$$

Let $p : L \rightarrow W$ be a function assigning a well to each cell. This function gives rise to a set

$$C = p(L) \equiv \{p(1), p(2), \dots, p(a \cdot b)\} = \{w_1, w_2, \dots, w_{a \cdot b}\} \quad (7)$$

where $w_k = p(k) \in W$, $k = 1, 2, \dots, a \cdot b$

The set C is called a configuration of the cellular automaton.

Consider N such functions and let

$$C_i = p_i(L), \quad i = 1, 2, \dots, N$$

The C_i 's represent possible configurations and they can be taken as the chromosomes of the natural genetic algorithm, with each chromosome expressing a distribution of the water sources among the cells.

The genetic operators of selection, crossover and mutation (Michalewicz, 1992) are applied to the above population of the N chromosomes. In particular, the crossover operator follows a two-dimensional pattern (e.g. Moon et al, 1997; Sidiropoulos and Fotakis, 2009), as shown in Fig. 3

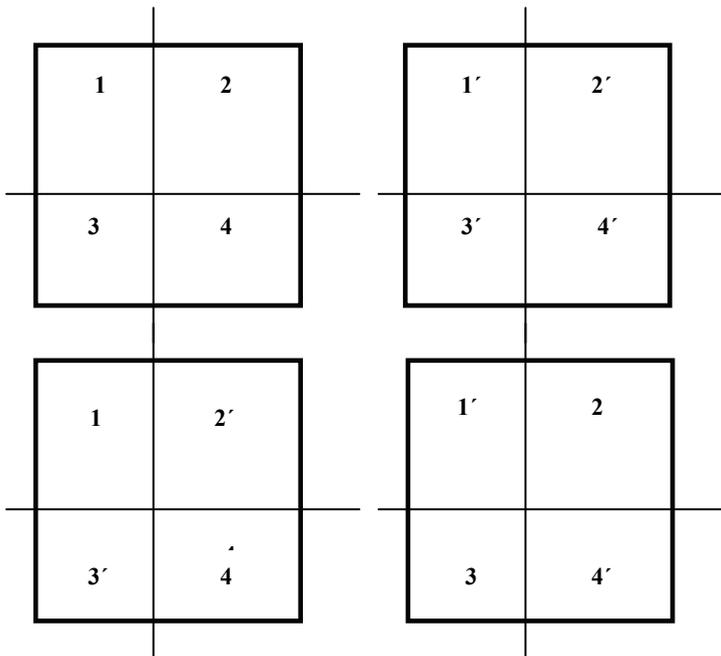


Fig. 3. Parents (above) and offspring (below)

3.2 An operative genetic approach

The alternative scheme is based on the notion of neighborhood and is thus consistent with the cellular automaton approach. A neighborhood is assigned to each block. The

neighborhood is defined in the sense of von Neumann (east – west and north – south cells, e.g. Gaylor and Nishidate,1996).

Let N_k be the neighborhood of cell k and let $n_k \in N_k$ be functionally related to k : $k \mapsto n_k$. Then L is transformed to

$$L_n = \{n_1, n_2, \dots, n_{a \cdot b}\} \tag{8}$$

and a new configuration can be defined as

$$\bar{C} = \{\bar{w}_1, \bar{w}_2, \dots, \bar{w}_{a \cdot b}\} \tag{9}$$

where $\bar{w}_k = p(n_k)$, as prescribed by Equation (7).

The above can be written more succinctly in operative form:

Let $q: L \rightarrow L$ be the above function relating k to n_k such that

$$q(k) \in N_k \quad \forall k \in L.$$

Then

$$n_k = q(k), \quad L_n = q(L) \quad \text{and} \quad \bar{w}_k = p(q(k)).$$

Thus an operator O can be defined as

$$O[p(k)] = p(q(k)) \quad \text{and} \quad \bar{C} = O[C]$$

where \bar{C} is the transformed configuration

Therefore, starting from a configuration

$$C = \{w_k \mid w_k \in W \quad \text{and} \quad k = 1, 2, \dots, a \cdot b\}$$

the transformed configuration

$$\bar{C} = C_n = \{w_{n_1}, w_{n_2}, \dots, w_{n_{a \cdot b}}\} \tag{9}$$

is obtained via the operator O . The list L_n of Equation (8) induces the function $q(k)$ and thus it generates the operator O . L_n will become the typical chromosome:

$$L_n = \{n_k \mid n_k \in N_k \quad \text{and} \quad k = 1, 2, \dots, a \cdot b\} \tag{10}$$

Considering N functions q_i , $i=1,2,\dots,N$, like the function q , N corresponding operators O_i result with N new transformed \bar{C}_i configurations.

The generating lists L_{ni} , $i=1,2,\dots,N$ constitute the chromosome population in the operative genetic algorithm.

Also, according to this notation, let $w_{n_k} \in \{1, 2, \dots, m\}$ be the number of the well assigned to the neighbor cell n_k .

The chromosome L_{ni} of Equation (10) can be considered as a replacement rule that dictates to the block k to replace the well w_k assigned to it by the the well w_{n_k} of the selected neighboring block n_k (Fig.2).

The appropriate choice of the suitable neighbor n_k will be put forward by the genetic algorithm.

The algorithm can be summarized as follows:

- An initial reference configuration is formed by taking $w_k = \text{Random}(1,m)$ for $k = 1, 2, \dots, a.b$.
- An initial population is created consisting of chromosome operators of the type (10).
- The operators act on the reference configuration resulting in N new configurations emanating from these N transformations of the reference configuration.
- The N configurations of (c) are evaluated and the best one becomes the new reference configuration.
- Stopping criteria are applied on best configurations obtained up to the current generation.
- The evaluations of (d) are assigned to the chromosomes of (b) and the operations of selection, crossover and mutation are applied on them and a new population of operators results. Control is transferred to step (c).

The algorithm is shown schematically in Fig. 4.

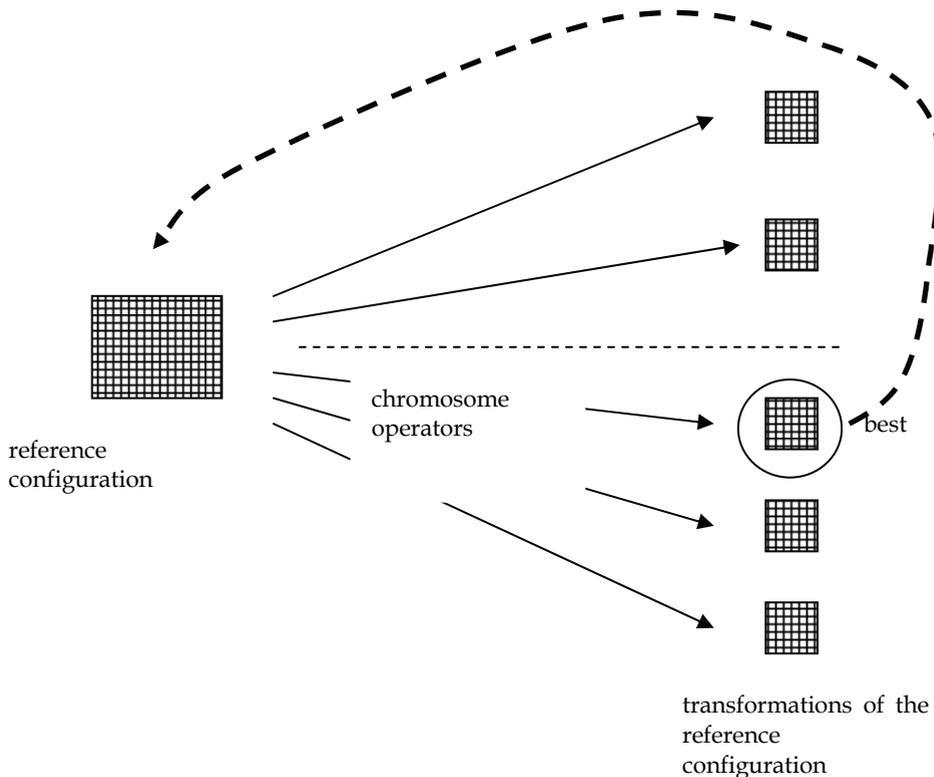


Fig. 4. Schematic representation of the cell-based operative genetic algorithm

4. Local versus global objectives

A basic issue in spatial optimization concerns the formulation of local objectives in relation to the overall global objectives, as the latter are expressed through the objective function. Local objectives permit the design of local transition rules in cellular automata. The optimization problem presented above was treated by means of operators applied to the individual cells in the local sense without decomposing the objective function into local contributions. Another approach would be to define local components of the objectives and then attempt to reduce the overall problem to the solution of the partial corresponding problems at the neighborhood level of each cell. In the literature various approaches can be noted in relation to this issue.

Strange et al (2001) were among the first researchers to employ cellular automata concepts in spatial optimization. They considered individual cell contributions of the objective function. However, some of these contributions, although referred to a particular cell, relate to the wider configuration as they express neighborhood properties. Optimization is performed on the basis of optimizing each one of these cell components.

In Heinonen and Pukkala (2007) the objectives are distinguished into local and global ones and a composite objective function is used consisting of the weighted sum of a local and a global term. The weighting coefficient of the global term is gradually increased in the course of a successive local and global solution of the optimization problem. The local part is treated by means of updating, via mutations, the cells of an underlying automaton.

Seppelt and Voinov (2002) in a grid search approach present a clear distinction between a local and a global method. Their objective function consists of a sum of cell-dependent terms and the solution consists of two stages, a local and a global one. The latter is performed through a genetic algorithm. Although a grid forms the basis of the problem formulation, no complete cellular automaton characteristics, such as local transition, appear in the whole treatment.

Aerts and Heuvelink (2002) and Aerts et al. (2005) again represent their studied area in the form of a grid of cells. They employ simulated annealing and genetic algorithms for the spatial optimization problem. No cellular automaton procedure can be discerned, although their special crossover operator is designed in a way that prevents fragmentation of the grid domain. Various criteria for compactness and contiguity are considered by suitably augmenting the objective function.

Specially designed operators are also presented by Datta et al (2007) in order to favor more compact configurations.

Ligmann-Zielinska et al.(2008) describe the SMOLA (Sustainable Multiobjective Land Use Allocation) software. The spatial optimization problems treated under this software belong to the class of linear and integer programming. Contiguity and compactness are handled by means of separate constraints.

The present problem differs from the typical problems of spatial analysis by the fact that a large part of the computational effort is placed on the allocation of the resource and on modeling its extraction and transport. Thus optimization has to be achieved both in terms of the spatial arrangement and with respect to water extraction efficiency. It is worth noting that there are two points of view, one emphasizing spatial optimization and treating necessary resources as optimization parameters and the other one emphasizing resource management and regulating spatial arrangements toward economy in resource spending. The latter approach may be encountered in the hydraulic engineering literature.

Characteristically, Yeo and Guldmann (2010) investigate land use allocation with a view to minimize watershed peak runoff. Similarly, the same authors (Yeo et al., 2004) optimize land use patterns to reduce both peak runoff and nonpoint source pollution. Notably, Sadeghi et al (2009) optimize land uses with a view both to reducing soil erosion and maximizing benefit. However, their problem is formulated in terms of linear programming.

In general, the objective functions of these composite problems are both nonlinear and non-separable with respect to partial cell contributions. However, local objective functions are defined in the present approach without a strictly local character. These local functions will not supplant the objective function for the optimization procedure. They will only be used for the formulation of an auxiliary operator, called the neighborhood rule (Fotakis, 2009, Fotakis and Sidiropoulos, 2010). This operator falls into the category of learning operators (Hinton & Nowlan, 1987, Krzanowski & Raper, 2001). These operators are characterized by their action on the phenotype and not on the genetic composition of the objects under study. Learning operators have been demonstrated to facilitate the evolutionary process. The definitions of local objectives for the problem under study are given in the next section.

4.1 Local objectives

The transport cost F_T (Equation 2) can be decomposed into cell contributions as follows:

$$f_{Tk} = [(x_k - x_{wk})^2 + (y_k - y_{wk})^2]^{1/2} \quad (11)$$

for $k=1,2,\dots,a,b$.

The pumping cost F_P (Equation 5) does not lend itself to such a direct decomposition. Let q_k denote the discharge corresponding to the given water needs of block k . The subscript j indicates that this block is connected to well j .

Let S_j be a subset of the set L of the cells, characterized by the fact that its members are connected to well j . Because each one of the cells is connected to one well only,

$$\bigcup_{j=1}^m S_j = L, \quad S_i \cap S_j = \emptyset \quad i \neq j, \quad i, j = 1, 2, \dots, m$$

Then the discharge of well j is equal to

$$Q_j = \sum_{k \in S_j} q_k \quad (12)$$

It follows from Equation (4) that the drawdown at the position of well j , $j=1,2,\dots,m$, is a nonlinear function of Q_1, Q_2, \dots, Q_m .

The pumping cost F_P , from Equation 5 and Fig. 2 relating k to w_k , can be rewritten as

$$F_P = \sum_{j=1}^m Q_j \Delta h_j = \sum_{j=1}^m \left(\sum_{k \in S_j} q_k \right) \Delta h_j = \sum_{k=1}^{a \cdot b} q_k \Delta h_{w_k} \quad (13)$$

The local value of the pumping cost can now be written as follows based on equation (13):

$$f_{Pk} = q_k \Delta h_{w_k}, \quad k = 1, 2, \dots, a \cdot b \quad (14)$$

It must be noted again that Equation 14 does not represent a strict separation into cell contributions. However this expression will be used beneficially in the local sense, as it will be shown below.

The local objective function may now be defined as

$$f_k = f_{T_k} + f_{P_k}, \quad k = 1, 2, \dots, a \cdot b \tag{15}$$

The nature of the above decomposition will now be examined more closely.

4.2 Decomposition and deviation from the classical resource allocation problem

For the typical resource allocation problem the following basic formulation, with certain variations, is given in the pertinent literature (Aerts & Heuveling, 2002, Aerts et al, 2008, Datta et al, 2007, Lingmann - Zielinska et al, 2010):

Minimize
$$F = \sum_i \sum_j \sum_\ell C_{i,j,\ell} x_{ij\ell} \tag{16}$$

where $C_{i,j,\ell}$ is the cost associated with the land use ℓ and

$$x_{ij\ell} = \begin{cases} 1 & \text{if block } i, j \text{ is assigned land use } \ell \\ 0 & \text{otherwise} \end{cases}$$

The subscripts i and j run over the coordinates of all cells and the subscript ℓ runs over all possible land uses.

In the literature a number of constraints is added to the above formulation expressing e.g. allowable percentages in the distribution of land uses, demands on compactness, restrictions on fragmentation etc.

The above expression (16) may assume the following equivalent and somewhat simpler form under the present notation:

Minimize
$$F = \sum_k \sum_\ell C_{k,\ell} x_{k\ell} \tag{17}$$

and
$$x_{ij\ell} = \begin{cases} 1 & \text{if block } k \text{ is assigned land use } \ell \\ 0 & \text{otherwise} \end{cases}$$

where now k runs over all cells under the numbering introduced in section 2.

Typically, $C_{k\ell}$ depends solely on the cell's position and on the land use attached to it. In that case the problem can be solved by linear programming, provided the constraints are also linear.

Expression (17) for the objective function can be rewritten as follows, in order to allow comparisons with the present problem formulation:

$$F = \sum_{\ell=1}^m \sum_{k \in S_\ell} C_{k,\ell} \tag{18}$$

where S_ℓ is the set of cells assigned to the land use ℓ and m is the number of possible land uses.

In the present problem the land uses are identified with the water sources and expression (18) can be compared to equation (13), which gives the objective function for the pumping cost, i.e.

$$C_{k,\ell} = q_k \Delta h_{\ell_k} \quad (19)$$

It is important to note here that the coefficient $C_{k\ell}$ does not depend solely on the land use of cell k . Indeed, by virtue of Equation (4), Δh_{ℓ_k} depends on the discharges of all the wells and the discharges again are determined by the distribution of the wells among the cells. Therefore, $C_{k\ell}$ depends on the land uses of the other cells as well as on the land use of cell k . This fact differentiates the present problem from the typical case of the spatial optimization literature. The cell decomposition discussed above is utilized in order to define a local neighborhood operator.

4.3 Neighborhood rule

An operator acting on the neighborhood of each cell is called the neighborhood rule and is defined as follows:

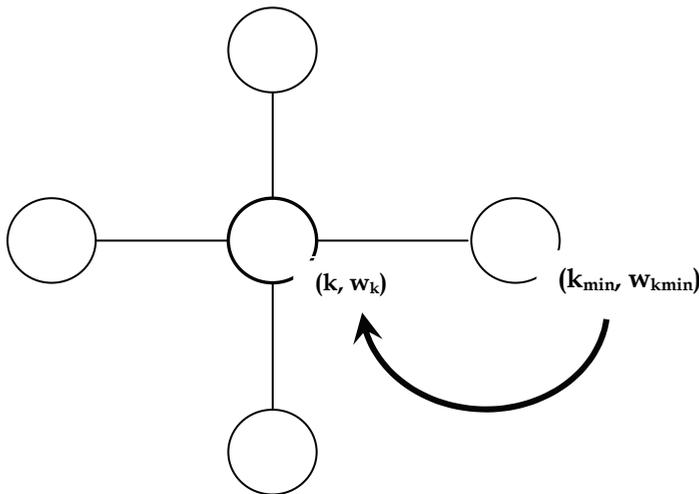


Fig. 5. Neighborhood rule

Let N_k be the neighborhood of cell k , consisting of 4 elements:

$$N_k = \{k_1, k_2, \dots, k_4\}$$

For each one of the neighborhood elements the local objective function (15) is evaluated.

Let k_{\min} be the element of N_k with the minimum value of the local objective function:

$$f_{k_{\min}} = \min\{f_{k_1}, f_{k_2}, \dots, f_{k_4}\}.$$

Then the state of the current cell k is replaced by the state of the state of the cell k_{\min} (Fig.5):

$$w_k \leftarrow w_{k_{\min}}.$$

The operation just described may be considered as a kind of mutation applied on the phenotype. It is applied with a certain probability to all genes (cells) of all chromosomes.

4.4 Neighborhood mutation

Another operator that does not depend on the local objective functions and acts on the phenotype is now introduced under the name neighborhood mutation. It differs from the classical version by the fact that it is tied to the neighborhood of the current gene-cell. Thus it is also consistent with the notion of the cellular automaton.

More specifically, for every cell a random number r is taken between 0 and 1 and if $r < p_{nm}$, where p_{nm} is the preset neighborhood mutation probability, then the state of the cell is replaced by the state of one of its neighbors. The latter is again chosen at random (Fig.6).

In the notation of the previous section,

For each k ($k=1,2,\dots,a.b$)

if $r < p_{nm}$,

let s be a random integer between 1 and l and replace

$$w_k \leftarrow w_{k_s} .$$

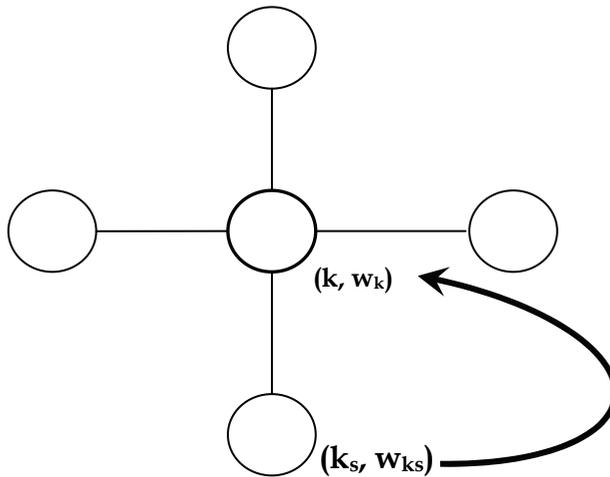


Fig. 6. Neighborhood mutation

Both of the above kinds of mutation will be tried and the results will be presented in the next section.

5. Results

A fictive spatial optimization problem (Sidiropoulos & Fotakis, 2009) is formulated in order to illustrate the methods and operators presented here. The area under study is represented as a 10×10 grid with the wells placed in the positions shown in Fig.1. The wells' coordinates are $x_{w1}=20, y_{w1}=0, x_{w2}=18, y_{w2}=0, x_{w3}=15, y_{w3}=0$. The hydraulic conductivities around wells

1, 2, 3 are $k_1=0.05 \times 10^{-3} \text{m/s}$, $k_2=0.5 \times 10^{-3} \text{m/s}$, $k_3=1.2 \times 10^{-3} \text{m/s}$, respectively. The constant thickness of the underlying aquifer is $b=50 \text{m}$, the radius of influence $R=15 \text{m}$ and the radii of the wells all equal to $r_w=0.10 \text{m}$.

The natural GA was first compared to the operative GA, as in Sidiropoulos & Fotakis (2009). The result obtained by the operative GA is clearly superior to the one of the natural GA both in terms of the objective function value and in terms of compactness (Figs. 7 and 8). The arrangement depicted in Figure 8 was reached within 160 generations and stabilized thereafter, while the arrangement of Fig. 7 was the best result obtained by the natural GA within 1000 generations.

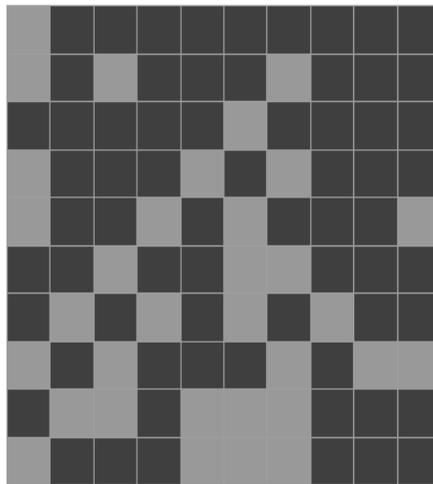


Fig. 7. Natural genetic algorithm

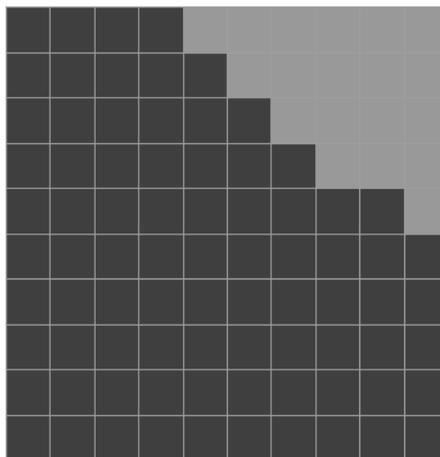


Fig. 8. Operative genetic algorithm

Subsequently, the natural GA was enhanced by the addition of the neighborhood rule (NR) described in section 4.3. The arrangement shown in Figure 9 was obtained within 600 generations. It presents a better picture in terms of compactness compared with the natural GA (Fig. 7), but not as good as the arrangement of the operative GA (Fig. 8). However, the objective function value attained in this case is even better than the one given by the operative GA.

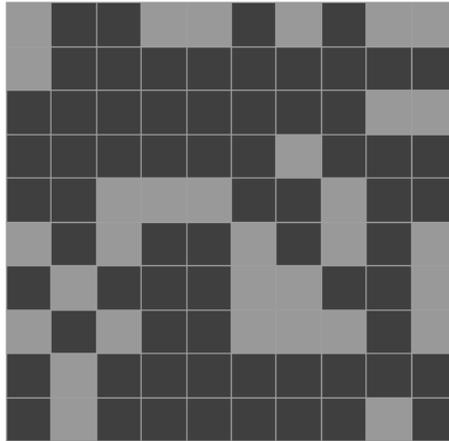


Fig. 9. Natural GA with NR

In a similar fashion the natural GA was combined with the neighborhood mutation (NM) of section 4.4. The result is shown in Figure 10. Concerning compactness it is clearly better than the one of Figure 9 and almost as good as that of Figure 8. Also, the objective function value is not as good as the one of Figure 9 and somewhat inferior to that of Figure 8, meaning that the neighborhood mutation tends to produce compact results. The result of Figure 10 was obtained within 650 generations.

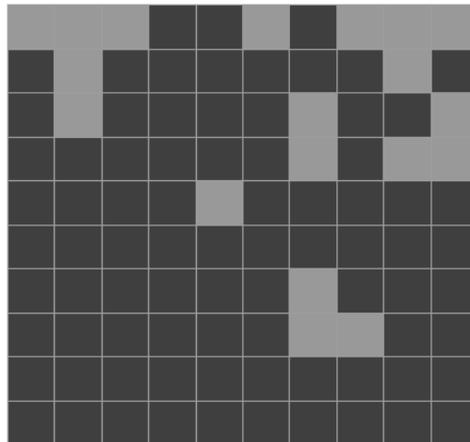


Fig. 10. Natural GA with NM

The above considerations motivate a combined implementation of NR and NM within the setting of the natural GA. The result is shown in Figure 11, in which the resulting arrangement is clearly compact but not connected as the one in Figure 8. Its objective function value is better than the one of Figure 9 and close, although inferior to that of Figure 10.

Finally, the operative GA was combined with the NR. As explained in section 3.2, the operative algorithm produces a new configuration after every generation. This configuration was each time subjected to the NR operator and an improved arrangement resulted. The final result is shown in Figure 12 and it is the same as that of Figure 8. The difference lies in the fact that the result is now obtained within 60 instead of 160 generations.

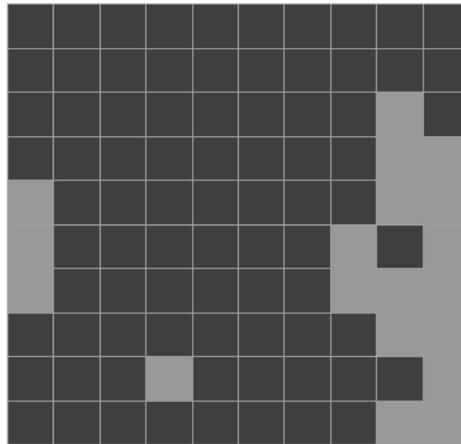


Fig. 11. Natural GA with NR and NM

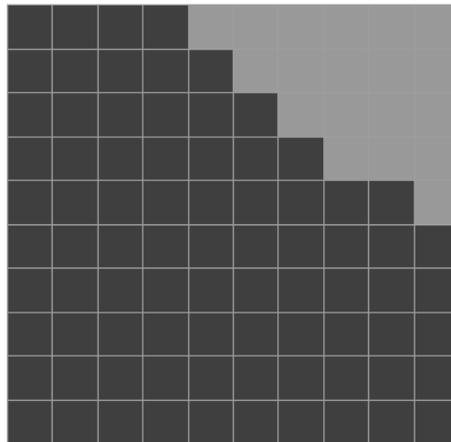


Fig. 12. Operative genetic algorithm with NR

6. Discussion – Conclusions

Spatial optimization problems are computationally intensive and demand the application of heuristic search methods for their solution. However, as it is demonstrated here, these methods have to be designed in accordance with the spatial character of the field under study. This character is fittingly modeled by means of cellular automata.

On a cellular background it is easier to pursue a balance between local and global characteristics. In this chapter two basic approaches are presented along this line. One of them is a natural genetic algorithm and, as described above, its typical chromosome consists of genes corresponding to land blocks. This natural algorithm is further equipped with operators acting on the local level and improving drastically the efficiency of the algorithm. The other approach has been characterized as operative, because the chromosomes of the genetic algorithm act as operators on the current configuration.

The latter method is in full accordance with the cellular automaton model, because the chromosome - operators act on the various cells in a way related to the neighbors of the cell. It is demonstrated that the operative genetic algorithm is more prone to producing compact configurations, although not necessarily yielding globally optimal results. On the other hand, these compact arrangements are highly desirable results in spatial optimization applications.

In summary, the schemes presented here provide the spatial planner with tools for obtaining a variety of alternative solutions with enhanced compactness characteristics without imposing special constraints. Indeed, these desirable results come out in an emergent sense. This means that no explicit provision is made within the computational process for their attainment. Such a provision would be the augmentation of the objective function by suitable penalty terms or the addition of specific constraints in the formulation of the problem. Moreover, the local operators (NR and NM) do not, at least explicitly, contain by their definition any bias or direction toward producing compact or contiguous arrangements. Locality of operation is their only characteristic.

Thus, it appears that methods connected to the cellular nature of the problem produce results more agreeable to a real-world point of view of spatial planning. This fact motivates further research toward a more systematic comparison of the present unconstrained methods to those involving explicit compactness, contiguity or percentage constraints. Clearly, the issue of emergence in the present algorithms calls for more investigation.

On the other hand, there can be constraints in relation to the hydraulic aquifer problem involved in the resource distribution question. Such constraints may answer the demand for ecological considerations. An example of this kind of constraint is the imposition of upper bounds on the pumping drawdowns at selected places of the aquifer. This restriction is aimed at protecting the aquifer from depletion.

Another direction of research concerns the multi-objective versions of the methods presented here. The operative genetic algorithm, as well as the local operators, admit non-trivial extensions into the multi-objective optimization field.

The progress of the cellular automaton toward optimal configurations raises the issue of self-organizing evolutionary methods and the role of an accompanying genetic algorithm in a mixed cellular – genetic scheme. This can be the subject of further follow-on research.

The integration of simulated annealing into the methods presented here is also worth investigating. A first step has already been taken in that direction (Sidiropoulos & Fotakis, 2009).

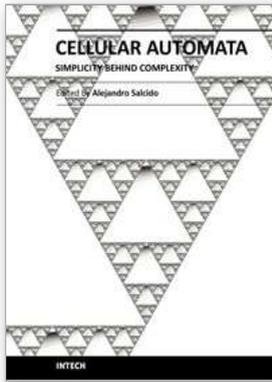
Finally, the present methodological framework may be applied to a variety of composite spatial planning problems arising in forest management, plant location and many other environmental or industrial fields.

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Cellular automata make up a class of completely discrete dynamical systems, which have become a core subject in the sciences of complexity due to their conceptual simplicity, easiness of implementation for computer simulation, and their ability to exhibit a wide variety of amazingly complex behavior. The feature of simplicity behind complexity of cellular automata has attracted the researchers' attention from a wide range of divergent fields of study of science, which extend from the exact disciplines of mathematical physics up to the social ones, and beyond. Numerous complex systems containing many discrete elements with local interactions have been and are being conveniently modelled as cellular automata. In this book, the versatility of cellular automata as models for a wide diversity of complex systems is underlined through the study of a number of outstanding problems using these innovative techniques for modelling and simulation.

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