Robust Linear Control of Nonlinear Flat Systems

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1. Introduction

Asymptotic estimation of external, unstructured, perturbation inputs, with the aim of exactly, or approximately, canceling their influences at the controller stage, has been treated in the existing literature under several headings. The outstanding work of professor C.D. Johnson in this respect, under the name of *Disturbance Accommodation Control* (DAC), dates from the nineteen seventies (see Johnson (1971)). Ever since, the theory and practical aspects of DAC theory have been actively evolving, as evidenced by the survey paper by Johnson Johnson (2008). The theory enjoys an interesting and useful extension to discrete-time systems, as demonstrated in the book chapter Johnson (1982). In a recent article, by Parker and Johnson Parker & Johnson (2009), an application of DAC is made to the problem of decoupling two nonlinearly coupled linear systems. An early application of disturbance accommodation control in the area of Power Systems is exemplified by the work of Mohadjer and Johnson in Mohadjer & Johnson (1983), where the operation of an interconnected power system is approached from the perspective of load frequency control.

A closely related vein to DAC is represented by the sustained efforts of the late Professor Jingqing Han, summarized in the posthumous paper, Han Han (2009), and known as: *Active Disturbance Estimation and Rejection* (ADER). The numerous and original developments of Prof. Han, with many laboratory and industrial applications, have not been translated into English and his seminal contributions remain written in Chinese (see the references in Han (2009)). Although the main idea of observer-based disturbance estimation, and subsequent cancelation via the control law, is similar to that advocated in DAC, the emphasis in ADER lies, mainly, on *nonlinear* observer based disturbance estimation, with necessary developments related to: efficient time derivative computation, practical relative degree computation and nonlinear PID control extensions. The work, and inspiration, of Professor Han has found interesting developments and applications in the work of Professor Z. Gao and his colleagues (see Gao et al. (2001), Gao (2006), also, in the work by Sun and Gao Sun & Gao (2005) and in the article by Sun Sun (2007)). In a recent article, a closely related idea, proposed by Prof. M. Fliess and C. Join in Fliess & Join (2008), is at the core of *Intelligent PID Control*(IPIDC). The mainstream of the IPIDC developments makes use of the Algebraic Method and it implies to resort to first order, or at most second order, *non-phenomenological* plant models. The interesting aspect of this method resides in using suitable algebraic manipulations to

locally deprive the system description of the effects of nonlinear uncertain additive terms and, via further special algebraic manipulations, to efficiently identify time-varying control gains as piece-wise constant control input gains (see Fliess et al. (2008)). An entirely algebraic approach for the control of synchronous generator was presented in Fliess and Sira-Ramírez, Sira-Ramírez & Fliess (2004).

In this chapter, we advocate, within the context of trajectory tracking control for nonlinear flat systems, the use of approximate, yet accurate, state dependent disturbance estimation via linear Generalized Proportional Integral (GPI) observers. GPI observers are the dual counterpart of GPI controllers, developed by M. Fliess *et al.* in Fliess et al. (2002). A high gain GPI observer naturally includes a, self-updating, lumped, time-polynomial model of the nonlinear state-dependent perturbation; it estimates it and delivers the time signal to the controller for on-line cancelation while simultaneously estimating the phase variables related to the measured output. The scheme is, however, approximate since only a small as desired reconstruction error is guaranteed at the expense of high, noise-sensitive, gains. The on-line approximate estimation is suitably combined with linear, estimation-based, output feedback control with the appropriate, on-line, disturbance cancelation. The many similarities and the few differences with the DAC and ADER techniques probably lie in 1) the fact that we do not discriminate between *exogenous* (i.e., external) unstructured perturbation inputs and *endogenous* (i.e., state-dependent) perturbation inputs in the nonlinear input-output model. These perturbations are all lumped into a simplifying time-varying signal that needs to be linearly estimated. Notice that plant nonlinearities generate time functions that are *exogenous* to any observer and, hence, algebraic loops are naturally avoided 2) We emphasize the natural possibilities of *differentially flat systems* in the use of linear disturbance estimation and linear output feedback control with disturbance cancelation (For the concept of flatness see Fliess *et al.* Fliess et al. (1995)) and the book Sira-Ramírez & Agrawal (2004).

This chapter is organized as follows: Section 2 presents an introduction to linear control of nonlinear differentially flat systems via (high-gain) GPI observers and suitable linear controllers feeding back the phase variables related to the output function. The single input-single output synchronous generator model in the form a *swing* equation, is described in Section 3. Here, we formulate the reference trajectory tracking problem under a number of information restrictions about the system. The linear observer-linear controller output feedback control scheme is designed for lowering the deviation angle of the generator. We carry out a robustness test regarding the response to a three phase short circuit. We also carry an evaluation of the performance of the control scheme under significant variations of the two control gain parameters required for an exact cancelation of the gain. Section 4 is devoted to present an experimental illustrative example concerning the non-holonomic car which is also a multivariable nonlinear system with input gain matrix depending on the estimated phase variables associated with the flat outputs.

2. Linear GPI observer-based control of nonlinear systems

Consider the following perturbed nonlinear single-input single input-output, smooth, nonlinear system,

$$y^{(n)} = \psi(t, y, \dot{y}, ..., y^{(n-1)}) + \phi(t, y)u + \zeta(t) \tag{1}$$

The unperturbed system, $(\zeta(t) \equiv 0)$ is evidently flat, as all variables in the system are expressible as differential functions of the flat output y.

We assume that the exogenous perturbation $\zeta(t)$ is uniformly absolutely bounded, i.e., it an L_∞ scalar function. Similarly, we assume that for all bounded solutions, $y(t)$, of (1),

obtained by means of suitable control input u, the additive, endogenous, perturbation input, $\psi(t, y(t), \dot{y}(t), ..., y^{(n-1)}(t))$, viewed as a time signal is uniformly absolutely bounded.

We also assume that the nonlinear gain function $\phi(t, y(t))$ is L_∞ and uniformly bounded away from zero, i.e., there exists a strictly positive constant μ such that

$$\inf_t |\phi(t, y(t))| \geq \mu \tag{2}$$

for all smooth, bounded solutions, $y(t)$, of (1) obtained with a suitable control input u. Although the results below can be extended when the input gain function ϕ depends on the time derivatives of y, we let, motivated by the synchronous generator case study to be presented, ϕ to be an explicit function of time and of the measured flat output y. This is equivalent to saying the $\phi(t, y(t))$ is perfectly known.

We have the following formulation of the problem:

Given a desired flat output reference trajectory, $y^*(t)$, devise a linear output feedback controller for system (1) so that regardless of the endogenous perturbation signal $\psi(t, y(t), \dot{y}(t), ..., y^{(n-1)}(t))$ and of the exogenous perturbation input $\zeta(t)$, the flat output y tracks the desired reference signal $y^*(t)$ even if in an approximate fashion. This approximate character specifically means that the tracking error, $e(t) = y - y^*(t)$, and its first, n, time derivatives, globally asymptotically exponentially converge towards a small as desired neighborhood of the origin in the reference trajectory tracking error phase space.

The solution to the problem is achieved in an entirely linear fashion if one conceptually considers the nonlinear model (1) as the following linear perturbed system

$$y^{(n)} = v + \xi(t) \tag{3}$$

where $v = \phi(t, y)u$, and $\xi(t) = \psi(t, y(t), \dot{y}(t), ..., y^{(n-1)}(t)) + \zeta(t)$.

Consider the following preliminary result:

Proposition 1. *The unknown perturbation vector of time signals, $\xi(t)$, in the simplified tracking error dynamics (3), is* observable *in the sense of Diop and Fliess (see Diop & Fliess (1991))).*

Proof The proof of this fact is immediate after writing (3) as

$$\xi(t) = y^{(n)} - v = y^{(n)} - \phi(t, y)u \tag{4}$$

i.e., $\xi(t)$ can be written in terms of the output vector y, a finite number of its time derivatives and the control input u. Hence, $\xi(t)$ is observable.

Remark 2. *This means, in particular, that if $\xi(t)$ is bestowed with an exact linear model; an exact asymptotic estimation of $\xi(t)$ is possible via a linear observer. If, on the other hand, the linear model is only approximately locally valid, then the estimation obtained via a linear observer is asymptotically convergent towards an equally approximately locally valid estimate.*

We assume that the perturbation input $\xi(t)$ may be locally modeled as a $p-1$-th degree time polynomial z_1 plus a residual term, $r(t)$, i.e.,

$$\xi(t) = z_1 + r(t) = a_0 + a_1 t + \cdots + a_{p-1} t^{p-1} + r(t), \text{ for all } t \tag{5}$$

The time polynomial model, z_1, (also called: a Taylor polynomial) is invariant with respect to time shifts and it defines a family of $p-1$ degree Taylor polynomials with arbitrary real

coefficients. We incorporate z_1 as an internal model of the additive perturbation input (see Johnson (1971)).

The perturbation model z_1 will acquire a *self updating* character when incorporated as part of a linear asymptotic observer whose estimation error is forced to converge to a small vicinity of zero. As a consequence of this, we may safely assume that the self-updating residual function, $r(t)$, and its time derivatives, say $r^{(p)}(t)$, are uniformly absolutely bounded. To precisely state this, let us denote by y_j an estimate of $y^{(j-1)}$ for $j = 1, ..., n$.

We have the following general result:

Theorem 3. *The GPI observer-based dynamical feedback controller:*

$$u = \frac{1}{\phi(t,y)} \left[[y^*(t)]^{(n)} - \sum_{j=0}^{n-1} \left(\kappa_j [y_j - (y^*(t))^{(j)}] \right) - \hat{\xi}(t) \right] \tag{6}$$

$$\hat{\xi}(t) = z_1$$

$$\dot{y}_1 = y_2 + \lambda_{p+n-1}(y - y_1)$$
$$\dot{y}_2 = y_3 + \lambda_{p+n-2}(y - y_1)$$
$$\vdots$$
$$\dot{y}_n = v + z_1 + \lambda_p(y - y_1)$$
$$\dot{z}_1 = z_2 + \lambda_{p-1}(y - y_1)$$
$$\vdots$$
$$\dot{z}_{p-1} = z_p + \lambda_1(y - y_1)$$
$$\dot{z}_p = \lambda_0(y - y_1) \tag{7}$$

asymptotically exponentially drives the tracking error phase variables, $e_y^{(k)} = y^{(k)} - [y^*(t)]^{(k)}$, $k = 0, 1, .., n-1$ to an arbitrary small neighborhood of the origin, of the tracking error phase space, which can be made as small as desired from the appropriate choice of the controller gain parameters $\{\kappa_0, ..., \kappa_{n-1}\}$. Moreover, the estimation errors: $\tilde{e}^{(i)} = y^{(i)} - y_i$, $i = 0, ..., n-1$ and the perturbation estimation error: $z_m - \xi^{m-1}(t)$, $m = 1, ..., p$ asymptotically exponentially converge towards a small as desired neighborhood of the origin of the reconstruction error space which can be made as small as desired from the appropriate choice of the controller gain parameters $\{\lambda_0, ..., \lambda_{p+n-1}\}$.

Proof The proof is based on the fact that the estimation error $\tilde{e}$ satisfies the perturbed linear differential equation

$$\tilde{e}^{(p+n)} + \lambda_{p+n-1} e^{(p+n-1)} + \cdots + \lambda_0 \tilde{e} = r^{(p)}(t) \tag{8}$$

Since $r^{(p)}(t)$ is assumed to be uniformly absolutely bounded then there exists coefficients λ_k such that $\tilde{e}$ converges to a small vicinity of zero, provided the roots of the associated characteristic polynomial in the complex variable s:

$$s^{p+n} + \lambda_{p+n-1} s^{p+n-1} + \cdots + \lambda_1 s + \lambda_0 \tag{9}$$

are all located deep into the left half of the complex plane. The further away from the imaginary axis, of the complex plane, are these roots located, the smaller the neighborhood of the origin, in the estimation error phase space, where the estimation error $\tilde{e}$ will remain ultimately bounded (see Kailath Kailath (1979)). Clearly, if $\tilde{e}$ and its time derivatives converge to a neighborhood of the origin, then $z_j - \xi^{(j)}$, $j = 1, 2, ...$, also converge towards a small vicinity of zero.

The tracking error $e_y = y - y^*(t)$ evolves according to the following linear perturbed dynamics

$$e_y^{(n)} + \kappa_{n-1} e_y^{(n-1)} + \cdots + \kappa_0 e_y = \xi(t) - \hat{\xi}(t) \tag{10}$$

Choosing the controller coefficients $\{\kappa_0, \cdots, \kappa_{n-1}\}$, so that the associated characteristic polynomial

$$s^n + \kappa_{n-1} s^{n-1} + \cdots + \kappa_0 \tag{11}$$

exhibits its roots sufficiently far from the imaginary axis in the left half portion of the complex plane, the tracking error, and its various time derivatives, are guaranteed to converge asymptotically exponentially towards a vicinity of the tracking error phase space. Note that, according to the observer expected performance, the right hand side of (10) is represented by a uniformly absolutely bounded signal already evolving on a small vicinity of the origin. For this reason the roots of (11) may be located closer to the imaginary axis than those of (9). A rather detailed proof of this theorem may be found in the article by Luviano *et al.* Luviano-Juárez et al. (2010)

Remark 4. *The proposed GPI observer (7) is a high gain observer which is prone to exhibiting the "peaking" phenomena at the initial time. We use a suitable "clutch" to smooth out these transient peaking responses in all observer variables that need to be used by the controller. This is accomplished by means of a factor function smoothly interpolating between an initial value of zero and a final value of unity. We denote this clutching function as $s_f(t) \in [0, 1]$ and define it in the following (non-unique) way*

$$s_f(t) = \begin{cases} 1 & \text{for } t > \epsilon \\ \sin^q\left(\frac{\pi t}{2\epsilon}\right) & \text{for } t \leq \epsilon \end{cases} \tag{12}$$

where q is a su itably large positive even integer.

2.1 Generalized proportional integral observer with integral injection

Let $\xi(t)$ be a measured signal with an uniformly absolutely bounded iterated integral of order m. The function $\xi(t)$ is a measured signal, whose first few time derivatives are required for some purpose.

Definition 5. *We say that a signal $\rho_1(t)$ converges to a neighborhood of $\xi(t)$ whenever the error signal, $\xi(t) - \rho_1(t)$, is ultimately uniformly absolutely bounded inside a small vicinity of the origin.*

The following proposition aims at the design of a GPI observer based estimation of time derivatives of a signal, $\xi(t)$, where $\xi(t)$ is possibly corrupted by a zero mean stochastic process whose statistics are unknown. In order to smooth out the noise effects on the on-line computation of the time derivative, we carry out a double iterated integration of the measured signal, $\xi(t)$, thus assuming the second integral of $\xi(t)$ is uniformly absolutely bounded (i.e., $m = 2$).

Proposition 6. *Consider the following perturbed second order integration system, where the input signal, $\xi(t)$, is a measured (zero-mean) noise corrupted signal satisfying the above assumptions:*

$$\dot{y}_0 = y_1, \quad \dot{y}_1 = \xi(t) \tag{13}$$

Consider the following integral injection GPI observer for (13) *including an internal time polynomial model of degree r for the signal $\xi(t)$ and expressed as ρ_1,*

$$
\begin{aligned}
\dot{\hat{y}}_0 &= \hat{y}_1 + \lambda_{r+1}(y_0 - \hat{y}_0) \\
\dot{\hat{y}}_1 &= \rho_1 + \lambda_r(y_0 - \hat{y}_0) \\
\dot{\rho}_1 &= \rho_2 + \lambda_{r-1}(y_0 - \hat{y}_0)
\end{aligned}
\tag{14}
$$

$$\vdots$$

$$\dot{\rho}_r = \lambda_0(y_0 - \hat{y}_0) \tag{15}$$

Then, the observer variables, ρ_1, ρ_2, ρ_3, ..., respectively, asymptotically converge towards a small as desired neighborhood of the disturbance input, $\xi(t)$, and of its time derivatives: $\dot{\xi}(t)$, $\ddot{\xi}(t)$,... provided the observer gains, $\{\lambda_0, ..., \lambda_{r+2}\}$, are chosen so that the roots of the polynomial in the complex variable s.

$$P(s) = s^{r+2} + \lambda_{r+1}s^{r+1} + \cdots + \lambda_1 s + \lambda_0 \tag{16}$$

are located deep into the left half of the complex plane. The further the distance of such roots from the imaginary axis of the complex plane, the smaller the neighborhood of the origin bounding the reconstruction errors.

Proof. Define the twice iterated integral injection error as, $\varepsilon = y_0 - \hat{y}_0$. The injection error dynamics is found to be described by the perturbed linear differential equation

$$\varepsilon^{(r+2)} + \lambda_{r+1}\varepsilon^{(r+1)} + \cdots + \lambda_1\dot{\varepsilon} + \lambda_0\varepsilon = \xi^{(r)}(t) \tag{17}$$

By choosing the observer parameters, $\lambda_0, \lambda_1, \cdots, \lambda_{r+1}$, so that the polynomial (16) is Hurwitz, with roots located deep into the left half of the complex plane, then, according to well known results of solutions of perturbed high gain linear differential equations, the injection error ε and its time derivatives are ultimately uniformly bounded by a small vicinity of the origin of the reconstruction error phase space whose radius of containment fundamentally depends on the smallest real part of all the eigenvalues of the dominantly linear closed loop dynamics (see Luviano *et al.* Luviano-Juárez et al. (2010) and also Fliess and Rudolph Fliess & Rudolph (1997)).   □

3. Controlling the single synchronous generator model

In this section, we advocate, within the context of the angular deviation trajectory control for a single synchronous generator model, the use of approximate, yet accurate, state dependent disturbance estimation via linear Generalized Proportional Integral (GPI) observers. GPI observers are the dual counterpart of GPI controllers, developed by M. Fliess *et al.* in Fliess et al. (2002). A high gain GPI observer naturally includes a, self-updating, lumped, time-polynomial model of the nonlinear state-dependent perturbation; it estimates it and delivers the time signal to the controller for on-line cancelation while simultaneously estimating the phase variables related to the measured output. The scheme is, however, approximate since only a small as desired reconstruction error is guaranteed at the expense

of high, noise-sensitive, gains. The on-line approximate estimation is suitably combined with linear, estimation-based, output feedback control with the appropriate, on-line, disturbance cancelation. The many similarities and the few differences with the DAC and ADER techniques probably lie in 1) the fact that we do not discriminate between *exogenous* (i.e., external) unstructured perturbation inputs and *endogenous* (i.e., state-dependent) perturbation inputs in the nonlinear input-output model. These perturbations are all lumped into a simplifying time-varying signal that needs to be linearly estimated. Notice that plant nonlinearities generate time functions that are *exogenous* to any observer and, hence, algebraic loops are naturally avoided 2) We emphasize the natural possibilities of *differentially flat systems* in the use of linear disturbance estimation and linear output feedback control with disturbance cancelation (For the concept of flatness see Fliess *et al.* Fliess et al. (1995)) and the book Sira-Ramírez & Agrawal (2004).

3.1 The single synchronous generator model

Consider the swing equation of a synchronous generator, connected to an *infinite bus*, with a series capacitor connected with the help of a thyristor bridge (See Hingorani Hingorani & Gyugyi (2000)),

$$
\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= P_m - b_1 x_2 - b_2 x_3 \sin(x_1) \\
\dot{x}_3 &= b_3(-x_3 + x_3^*(t) + u + \zeta(t))
\end{aligned}
\tag{18}
$$

x_1 is the load angle, considered to be the measured output. The variable, x_2, is the deviation from nominal, synchronous, speed at the shaft, while x_3 stands for the admittance of the system. The control input, u, is usually interpreted as a quantity related to the fire angle of the switch. $\zeta(t)$ is an unknown, external, perturbation input. The static equilibrium point of the system, which may be parameterized in terms of the equilibrium position for the angular deviation, $\overline{x}_1$, is given by,

$$
x_1 = \overline{x}_1, \ \overline{x}_2 = 0, \ \overline{x}_3 = \overline{x}_3^*(t) = \frac{P_m}{b_2 \sin(\overline{x}_1)}
\tag{19}
$$

We assume that the system parameters, b_2, and, b_3, are known. The constant quantities P_m, b_1 and the time varying quantity, $x_3^*(t)$, are assumed to be completely unknown.

3.2 Problem formulation

It is desired to have the load angular deviation, $y = x_1$, track a given reference trajectory, $y^*(t) = x_1^*(t)$, which remains bounded away from zero, independently of the unknown system parameters and in spite of possible external system disturbances (such as short circuits in the three phase line, setting, momentarily, the mechanical power, P_m, to zero), and other unknown, or un-modeled, perturbation inputs comprised in $\zeta(t)$.

3.3 Main results

The unperturbed system in (18) is *flat*, with flat output given by the load angle deviation $y = x_1$. Indeed, all system variables are differentially parameterizable in terms of the load

angle and its time derivatives. We have:

$$x_1 = y$$
$$x_2 = \dot{y}$$
$$x_3 = \frac{P_m - b_1\dot{y} - \ddot{y}}{b_2\sin(y)}$$
$$u = -\frac{b_1\ddot{y} + y^{(3)}}{b_3 b_2\sin(y)} - \frac{P_m - b_1\dot{y} - \ddot{y}}{b_3 b_2\sin^2(y)}\dot{y}\cos(y)$$
$$+\frac{P_m - b_1\dot{y} - \ddot{y}}{b_2\sin(y)} - x_3^*(t) \tag{20}$$

The perturbed input-output dynamics, devoid of any zero dynamics, is readily obtained with the help the control input differential parametrization (20). One obtains the following simplified, perturbed, system dynamics, including $\zeta(t)$, as:

$$y^{(3)} = -\left[b_3 b_2\sin(y)\right]u + \xi(t) \tag{21}$$

where $\xi(t)$ is given by

$$\xi(t) = -b_1\ddot{y} + b_3\left(P_m - b_1\dot{y} - \ddot{y}\right)\left(1 - \frac{\dot{y}\cos(y)}{b_3\sin(y)}\right)$$
$$-b_3 b_2\sin(y)\left(x_3^*(t) + \zeta(t)\right) \tag{22}$$

We consider $\xi(t)$ as an unknown but uniformly absolutely bounded disturbance input that needs to be on-line estimated by means of an observer and, subsequently, canceled from the simplified system dynamics via feedback in order to regulate the load angle variable y towards the desired reference trajectory $y^*(t)$. It is assumed that the gain parameters b_2 and b_3 are known.

The problem is then reduced to the trajectory tracking problem defined on the perturbed third order, predominantly, linear system (21) with measurable state dependent input gain and unknown, but uniformly bounded, disturbance input.

We propose the following estimated state feedback controller with a smoothed (i.e., "clutched") disturbance cancelation term, $z_{1s}(t) = s_f(t)z_1(t)$, and smoothed estimated phase variables $y_{js} = s_f(t)y_j(t)$, $j = 1, 2, 3$ with $s_f(t)$ as in equation (12) with a suitable ϵ value.

$$u = -\frac{1}{b_3 b_2\sin(y)}\left[(y^*(t))^{(3)} - k_2(y_{3s} - \ddot{y}^*(t))\right.$$
$$\left. -k_1(y_{2s} - \dot{y}^*(t)) - k_0(y - y^*(t)) - z_{1s}\right]$$

The corresponding variables, y_3, y_2 and z_1, are generated by the following linear GPI observer:

$$\dot{y}_1 = y_2 + \lambda_5(y - y_1)$$
$$\dot{y}_2 = y_3 + \lambda_4(y - y_1)$$
$$\dot{y}_3 = -(b_3 b_2\sin(y))u + z_1 + \lambda_3(y - y_1)$$
$$\dot{z}_1 = z_2 + \lambda_2(y - y_1)$$
$$\dot{z}_2 = z_3 + \lambda_1(y - y_1)$$
$$\dot{z}_3 = \lambda_0(y - y_1) \tag{23}$$

where y_1 is the redundant estimate of the output y, y_2 is the shaft velocity estimate and y_3 is the shaft acceleration estimate. The variable z_1 estimates the perturbation input $\xi(t)$ by means of a local, self updating, polynomial model of third order, taken as an internal model of the state dependent additive perturbation affecting the input-output dynamics (21).

The clutched observer variables z_{1s}, y_{2s} and y_{3s} are defined by

$$\theta_s = s_f(t)\theta, \quad s_f(t) = \begin{cases} \sin^8(\frac{\pi t}{2\epsilon}) & \text{for } t \leq \epsilon \\ 1 & \text{for } t > \epsilon \end{cases} \tag{24}$$

with θ_s being either z_{1s}, y_{2s} or y_{3s}

The reconstruction error system is obtained by subtracting the observer model from the perturbed simplified linear system model. We have, letting $\tilde{e} = e_1 = y - y_1$, $e_2 = \dot{y} - y_2$, etc.

$$\begin{aligned} \dot{e}_1 &= e_2 - \lambda_5 e_1 \\ \dot{e}_2 &= e_3 - \lambda_4 e_1 \\ \dot{e}_3 &= \xi(t) - z_1 - \lambda_3 e_1 \\ \dot{z}_1 &= z_2 + \lambda_2(y - y_1) \\ \dot{z}_2 &= z_3 + \lambda_1(y - y_1) \\ \dot{z}_3 &= \lambda_0(y - y_1) \end{aligned} \tag{25}$$

The reconstruction error, $\tilde{e} = e_1 = y - y_1$, is seen to satisfy the following linear, perturbed, dynamics

$$\tilde{e}^{(6)} + \lambda_5 \tilde{e}^{(5)} + \lambda_4 \tilde{e}^{(4)} + \cdots + \lambda_1 \dot{\tilde{e}} + \lambda_0 \tilde{e} = \xi^{(3)}(t) \tag{26}$$

Choosing the gains $\{\lambda_5, \cdots, \lambda_0\}$ so that the roots of the characteristic polynomial,

$$p_o(s) = s^6 + \lambda_5 s^5 + \lambda_4 s^4 + \cdots + \lambda_1 s + \lambda_0, \tag{27}$$

are located deep into the left half of the complex plane, it follows from the bounded input, bounded output stability theory that the trajectories of the reconstruction error $\tilde{e}$ and those of its time derivatives $\tilde{e}^{(j)}$, $j = 1, 2, ...$ are uniformly ultimately bounded by a disk, centered at the origin in the reconstruction error phase space, whose radius can be made arbitrarily small as the roots of $p_o(s)$ are pushed further to the left of the complex plane.

The closed loop tracking error dynamics satisfies

$$e_y^{(3)} + \kappa_2 e_y^{(2)} + \kappa_1 \dot{e}_y + \kappa_0 e_y = \xi(t) - z_{1s} \tag{28}$$

The difference, $\xi(t) - z_{1s}$, being arbitrarily small after some time, produces a reference trajectory tracking error, $e_y = y - y^*(t)$, that also asymptotically exponentially converges towards a small vicinity of the origin of the tracking error phase space.

The characteristic polynomial of the predominant linear component of the closed loop system may be set to have poles placed in the left half of the complex plane at moderate locations

$$p_c(s) = s^3 + \kappa_2 s^2 + \kappa_1 s + \kappa_0 \tag{29}$$

3.4 Simulation results

3.4.1 A desired rest-to-rest maneuver

It is desired to smoothly lower the load angle, $y_1 = x_1$, from an equilibrium value of $y = 1$ [rad] towards a smaller value, say, $y = 0.6$ [rad] in a reasonable amount of time, say, $T = 5$ [s], starting at $t = 5$ [s] of an equilibrium operation characterized by (see Bazanella *et al.* Bazanella et al. (1999) and Pai Pai (1989))

$$x_1 = 1, \ x_2 = 0, \ x_3 = 0.8912$$

We used the following parameter values for the system

$$b_1 = 1, \ b_2 = 21.3360, \ b_3 = 20$$

We set the external perturbation input, $\zeta(t)$, as the time signal,

$$\zeta(t) = 0.005e^{(\sin^2(3t)\cos(3t))}\cos(0.3t)$$

The observer parameters were set in accordance with the following desired characteristic polynomial $p_o(s)$ for the, predominantly, linear reconstruction error dynamics. We set $p_o(s) = (s^2 + 2\zeta_o\omega_{no}s + \omega_{no}^2)^3$, with

$$\zeta_o = 1, \ \omega_{no} = 20$$

The controller gains $\kappa_2, \kappa_1, \kappa_0$ were set so that the following closed loop characteristic polynomial, $p_c(s)$, was enforced on the tracking error dynamics,

$$p_c(s) = (s^2 + 2\zeta_c\omega_{nc}s + \omega_{nc}^2)(s + p_c)$$

with

$$p_c = 3, \ \omega_{nc} = 3, \ \zeta_c = 1$$

The trajectory for the load angle, $y^*(t)$, was set to be

$$y^*(t) = \overline{x}_{1,\text{initial}} + (\rho(t, t_1, t_2))(\overline{x}_{1,\text{final}} - \overline{x}_{1,\text{initial}})$$

with $\rho(t, t_1, t_2)$ being a smooth Bèzier polynomial achieving a smooth rest-to-rest trajectory for the nominal load angle $y^*(t)$ from the initial equilibrium value $y^*(t_1) = \overline{x}_{1,\text{initial}} = 1$ [rad] towards the final desired equilibrium value $y^*(t_2) = \overline{x}_{1,\text{final}} = 0.6$ [rad]. We set $t_1 = 5.0$ [s], $t_2 = 10.0$ [s]; $\epsilon = 3.0$

The interpolating polynomial $\rho(t, t_1, t_2)$, is of the form:

$$\rho(t) = \tau^8 \left[r_1 - r_2\tau + r_3\tau^2 - r_4\tau^3 + r_5\tau^4 \right.$$
$$\left. - r_6\tau^5 + r_7\tau^6 - r_8\tau^7 + r_9\tau^8 \right]$$

with,

$$\tau = \frac{t - t_1}{t_2 - t_1}$$

The choice,

$$r_1 = 12870, \ r_2 = 91520, \ r_3 = 288288$$
$$r_4 = 524160, \ r_5 = 600600, \ r_6 = 443520$$
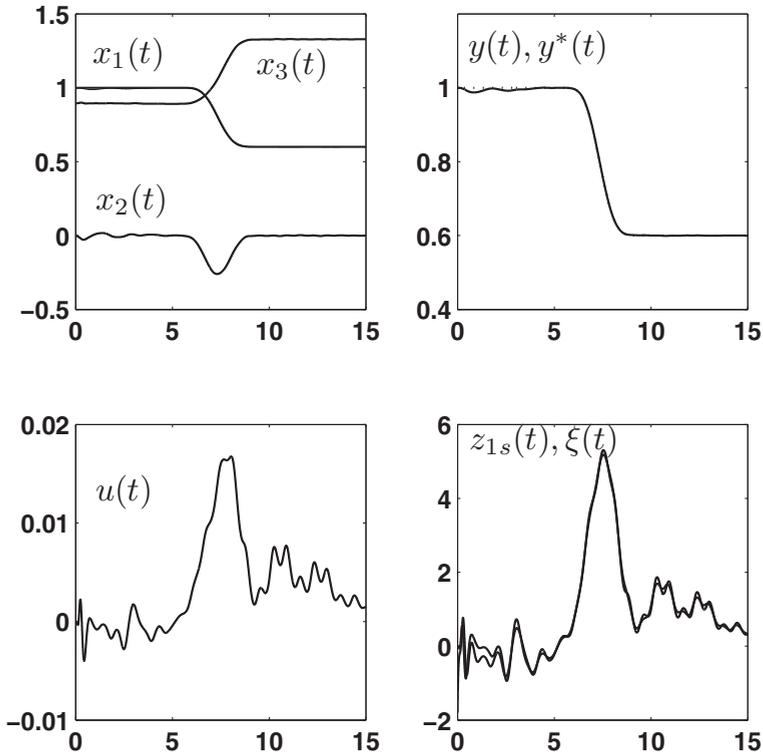$$r_7 = 205920, \ r_8 = 54912, \ r_9 = 6435$$

Fig. 1. Performance of GPI observer based linear controller for load angle rest-to-rest trajectory tracking in a perturbed synchronous generator.

renders a time polynomial which is guaranteed to have enough derivatives being zero, both, at the beginning and at the end of the desired rest to rest maneuver.

Figure 1 depicts the closed loop performance of the proposed GPI observer based linear output feedback controller for the forced evolution of the synchronous generator load angle trajectory following a desired rest to rest maneuver.

3.4.2 Robustness with respect to controller gain mismatches

We simulated the behavior of the closed loop system when the gain parameters product, $b_3 b_2$, is not precisely known and the controller is implemented with an estimated (guessed) value of this product, denoted by $\widehat{b_2 b_3}$, and set to be $\widehat{b_2 b_3} = \kappa b_2 b_3$. We determined that κ is a positive factor ranging in the interval $[0.95, \infty]$. However, if we allow independent estimates of the parameters in the form $\hat{b}_2 = \kappa_{b2} b_2$ and $\hat{b}_3 = \kappa_{b3} b_3$, we found that a larger robustness interval of mismatches is allowed by satisfying the empirical relation $\kappa_{b2} \kappa_{b3} \geq 0.95$. The assessment
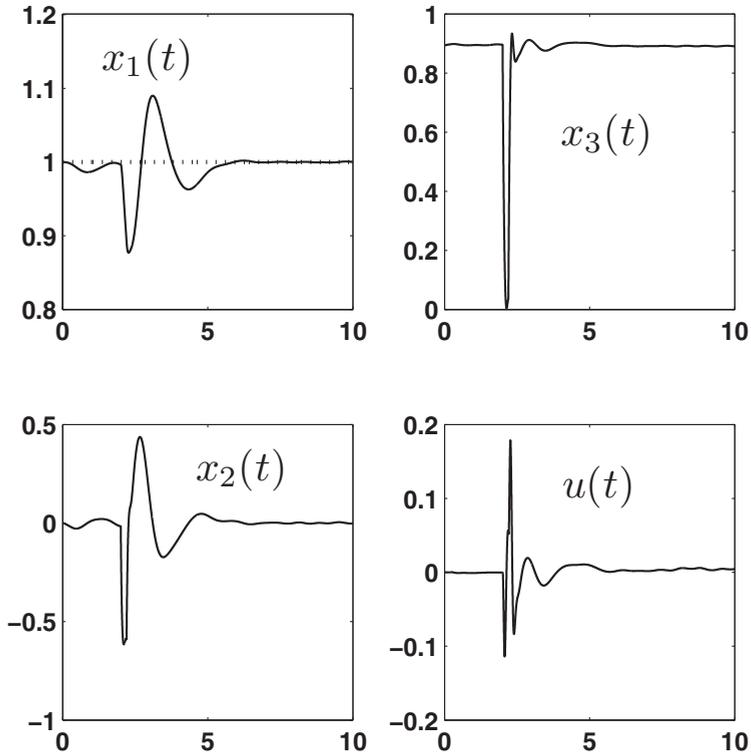
Fig. 2. Performance of GPI observer based controller under a sudden loss of power at t=2 [sec] during 0.2 [sec].

was made in terms of the proposed rest to rest maneuver and possible simulations look about the same.

3.4.3 Robustness with respect to sudden power failures

We simulated an un-modeled sudden three phase short circuit occurring at time $t = 2$ [s]. The power failure lasts for $t = 0.2$ [s]. Figure 3 depicts the performance of the GPI observer based controller in the rapid transient occurring during the recovery of the prevailing equilibrium conditions.

4. Controlling the non-holonomic car

Controlling non-holonomic mobile robots has been an active topic of research during the past three decades due to the wide variety of applications. Several methods have been proposed, and applied, to solve the regulation and trajectory tracking tasks in mobile robots. These methods range from sliding mode techniques Aguilar et al. (1997), Wang et al. (2009),

Yang & Kim (1999), backstepping Hou et al. (2009), neural networks approaches (see Peng et al. (2007) and references therein), linearization techniques Kim & Oh (1999), and classical control approaches (see Sugisaka & Hazry (2007)) among many other possibilities. A classical contribution to this area is given in the work of Canudas de Wit Wit & Sordalen (1992). An excellent book, dealing with some appropriate control techniques for this class of systems, is that of Dixon *et al.* Dixon et al. (2001). A useful approach to control non-holonomic mechanical systems is based on linear time-varying control schemes (see Pomet (1992); Tian & Cao (2007)). In the pioneering work of Samson Samson (1991), smooth feedback controls (depending on an exogenous time variable) are proposed to stabilize a wheeled cart.

It has been shown that some mobile robotic systems are differentially flat when slippage is not allowed in the model (see Leroquais & d'Andrea Novel (1999)). The differential flatness property allows a complete parametrization of all system variables in terms of the flat outputs an a and a finite number of their time derivatives. Flat outputs constitute a limited set of special, differentially independent, output variables. The reader is referred to the work of Fliess *et al.* Fliess et al. (1995) for the original introduction of the idea in the control systems literature.

From the flatness of the non-holonomic car system, it is possible to reduce the control task to that of a linearizable, extended, multivariable input-output system. The linearization of the flat output dynamics requires the cancelation of the nonlinear input gain matrix, which depends only on the cartesian velocities of the car. To obtain this set of noisy unmeasured state variables, we propose linear Generalized Proportional Integral (GPI) observers consisting of linear, high gain Luenberger-like observers Luenberger (1971) exhibiting an internal polynomial model for the measured signal. These GPI observers, introduced in Sira-Ramírez & Feliu-Battle (2010), can provide accurate, filtered, time derivatives of the injected output signals via an appropriate iterated integral estimation error injection (see also Cortés-Romero et al. (2009)). Since high-gain observers are known to be sensitive to noisy measurements, the iterated integral injection error achieves a desirable low pass filtering effect.

The idealized model of a single axis two wheeled vehicle is depicted in figure 3. The axis is of length L and each wheel of radius R is powered by a direct current motor yielding variable angular speeds ω_1, ω_2 respectively. The position variables are (x_1, x_2) and θ is the orientation angle of the robot. The linear velocities of the points of contact of the wheels respect to the ground are given by $v_1 = \omega_1 R$ and $v_2 = \omega_2 R$. In this case, the only measurable variables are x_1, x_2. This system is subject to non-holonomic restrictions.

The kinematic model of the system is stated as follows

$$\begin{cases} \dot{x}_1 = u_1 \cos\theta, \\ \dot{x}_2 = u_1 \sin\theta, \\ \dot{\theta} = u_2 \end{cases} \tag{30}$$

where:

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} R/2 & R/2 \\ -R/L & R/L \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \end{bmatrix}$$

The control objective is stated as follows: given a desired trajectory $(x_1^*(t), x_2^*(t))$, devise feedback control laws, u_1, u_2, such that the flat output coordinates, (x_1, x_2), perform an asymptotic tracking while rejecting the un-modeled additive disturbances.
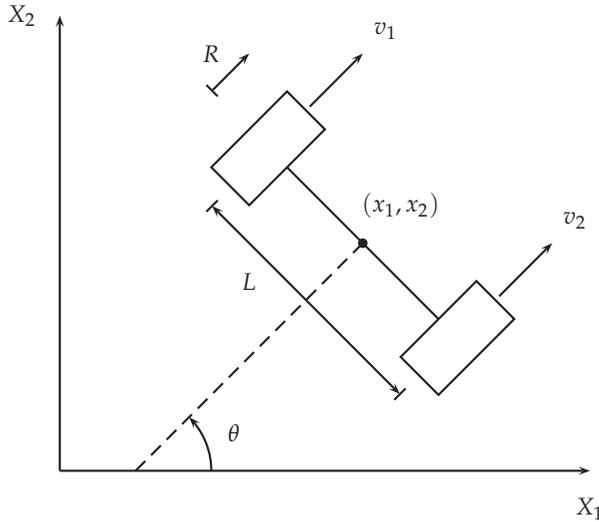
Fig. 3. The one axis car

4.1 Controller design

System (30) is differentially flat, with flat outputs given by the pair of coordinates: (x_1, x_2), which describes the position of the rear axis middle point. Indeed the rest of the system variables, including the inputs are differentially parameterized as follows:

$$\theta = \arctan\left(\frac{\dot{x}_2}{\dot{x}_1}\right), \ u_1 = \sqrt{\dot{x}_1^2 + \dot{x}_2^2}, \ u_2 = \frac{\ddot{x}_2\dot{x}_1 - \dot{x}_2\ddot{x}_1}{\dot{x}_1^2 + \dot{x}_2^2}$$

Note that the relation between the inputs and the flat outputs highest derivatives is not invertible due to an ill defined relative degree. To overcome this obstacle to feedback linearization, we introduce, as an extended auxiliary control input, the time derivative of u_1. We have:

$$\dot{u}_1 = \frac{\dot{x}_1\ddot{x}_1 + \dot{x}_2\ddot{x}_2}{\sqrt{\dot{x}_1^2 + \dot{x}_2^2}}$$

This control input extension yields now an invertible control input-to-flat outputs highest derivatives relation, of the form:

$$\begin{bmatrix} \dot{u}_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} \frac{\dot{x}_1}{\sqrt{\dot{x}_1^2 + \dot{x}_2^2}} & \frac{\dot{x}_2}{\sqrt{\dot{x}_1^2 + \dot{x}_2^2}} \\ \frac{-\dot{x}_2}{\dot{x}_1^2 + \dot{x}_2^2} & \frac{\dot{x}_1}{\dot{x}_1^2 + \dot{x}_2^2} \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} \tag{31}$$

4.2 Observer-based GPI controller design

Consider the following multivariable feedback controller based on linear GPI controllers and estimated cancelation of the nonlinear input matrix gain:

$$\begin{bmatrix} \dot{u}_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} \dfrac{\hat{x}_1}{\sqrt{(\hat{x}_1)^2 + (\hat{x}_2)^2}} & \dfrac{\hat{x}_2}{\sqrt{(\hat{x}_1)^2 + (\hat{x}_2)^2}} \\ \dfrac{-\hat{x}_2}{(\hat{x}_1)^2 + (\hat{x}_2)^2} & \dfrac{\hat{x}_1}{(\hat{x}_1)^2 + (\hat{x}_2)^2} \end{bmatrix} \begin{bmatrix} \nu_1 \\ \nu_2 \end{bmatrix} \tag{32}$$

with the auxiliary control variables, ν_1, ν_2, given by[1]:

$$\nu_1 = \ddot{x}_1^*(t) - \left[\frac{k_{12}s^2 + k_{11}s + k_{10}}{s(s + k_{13})} \right] (x_1 - x_1^*(t))$$

$$\nu_2 = \ddot{x}_2^*(t) - \left[\frac{k_{22}s^2 + k_{21}s + k_{20}}{s(s + k_{23})} \right] (x_2 - x_2^*(t)) \tag{33}$$

and where the estimated velocity variables: $\hat{x}_1, \hat{x}_2$, are generated, respectively, by the variables ρ_{11} and ρ_{12} in the following single iterated integral injection GPI observers (i.e., with $m = 1$),

$$\begin{aligned}
\dot{\hat{y}}_{10} &= \hat{y}_1 + \lambda_{13}(y_{10} - \hat{y}_{10}) \\
\dot{\hat{y}}_1 &= \rho_{11} + \lambda_{12}(y_{10} - \hat{y}_{10}) \\
\dot{\rho}_{11} &= \rho_{21} + \lambda_{11}(y_{10} - \hat{y}_{10}) \\
\dot{\rho}_{21} &= \lambda_{10}(y_{10} - \hat{y}_{10}) \\
y_{10} &= \int_0^t x_1(\tau)d\tau
\end{aligned} \tag{34}$$

$$\begin{aligned}
\dot{\hat{y}}_{20} &= \hat{y}_2 + \lambda_{23}(y_{20} - \hat{y}_{20}) \\
\dot{\hat{y}}_2 &= \rho_{12} + \lambda_{22}(y_{20} - \hat{y}_{20}) \\
\dot{\rho}_{12} &= \rho_{22} + \lambda_{21}(y_{20} - \hat{y}_{20}) \\
\dot{\rho}_{22} &= \lambda_{20}(y_{20} - \hat{y}_{20}) \\
y_{20} &= \int_0^t x_2(\tau)d\tau
\end{aligned} \tag{35}$$

Then, the following theorem describes the effect of the proposed integral injection observers, and of the GPI controllers, on the closed loop system:

Theorem 7. Given a set of desired reference trajectories, $(x^*(t), y^*(t))$, for the desired position in the plane of the kinematic model of the car, described by (30); given a set initial conditions, $(x(0), y(0))$, sufficiently close to the initial value of the desired nominal trajectories, $(x^*(0), y^*(0))$, then, the above described GPI observers and the linear multi-variable dynamical feedback controllers, (32)-(35), forces the closed loop controlled system trajectories to asymptotically converge towards a small as desired neighborhood of the desired reference trajectories, $(x_1^*(t), x_2^*(t))$, provided the observer and controller gains

---

[1] Here we have combined, with an abuse of notation, frequency domain and time domain signals.

are chosen so that the roots of the corresponding characteristic polynomials describing, respectively, the integral injection estimation error dynamics and the closed loop system, are located deep into the left half of the complex plane. Moreover, the greater the distance of these assigned poles to the imaginary axis of the complex plane, the smaller the neighborhood that ultimately bounds the reconstruction errors, the trajectory tracking errors, and their time derivatives.

Proof. Since the system is differentially flat, in accordance with the results in Maggiore & Passino (2005), it is valid to make use of the separation principle, which allows us to propose the above described GPI observers. The characteristic polynomials associated with the perturbed integral injection error dynamics of the above GPI observers, are given by,

$$P_{\varepsilon 1}(s) = s^4 + \lambda_{13}s^3 + \lambda_{12}s^2 + \lambda_{11}s + \lambda_{10}$$
$$P_{\varepsilon 2}(s) = s^4 + \lambda_{23}s^3 + \lambda_{22}s^2 + \lambda_{21}s + \lambda_{20}$$
$$s \in \mathbb{C}$$

thus, the $\lambda_{i,j}$, $i = 1, 2, j = 0, \cdots, 3$, are chosen to identify, term by term, the above estimation error characteristic polynomials with the following desired stable injection error characteristic polynomials,

$$P_{\varepsilon 1}(s) = P_{\varepsilon 2}(s) = (s + 2\mu_1\sigma_1 s + \sigma_1^2)(s + 2\mu_2\sigma_2 s + \sigma_2^2)$$
$$s \in \mathbb{C}, \mu_1, \mu_2, \sigma_1, \sigma_2 \in \mathbb{R}^+$$

Since the estimated states, $\hat{x}_1 = \rho_{11}$, $\hat{x}_2 = \rho_{12}$, asymptotically exponentially converge towards a small as desired vicinity of the actual states: $\dot{x}_1$, $\dot{x}_2$, substituting (32) into (31), transforms the control problem into one of controlling two decoupled double chains of integrators. One obtains the following dominant linear dynamics for the closed loop tracking errors:

$$e_1^{(4)} + k_{13}e_1^{(3)} + k_{12}\ddot{e}_1 + k_{11}\dot{e}_1 + k_{10}e_1 = 0 \tag{36}$$
$$e_2^{(4)} + k_{23}e_2^{(2)} + k_{22}\ddot{e}_2 + k_{21}\dot{e}_2 + k_{20}e_2 = 0 \tag{37}$$

The pole placement for such dynamics has to be such that both corresponding associated characteristic equations guarantee a dominant exponentially asymptotic convergence. Setting the roots of these characteristic polynomials to lie deep into the left half of the complex plane one guarantees an asymptotic convergence of the perturbed dynamics to a small as desired vicinity of the origin of the tracking error phase space.   □

4.3 Experimental results

An experimental implementation of the proposed controller design method was carried out to illustrate the performance of the proposed linear control approach. The used experimental prototype was a parallax "Boe-Bot" mobile robot (see figure 5). The robot parameters are the following: The wheels radius is $R = 0.7$ $[m]$; its axis length is $L = 0.125$ $[m]$. Each wheel radius includes a rubber band to reduce slippage. The motion system is constituted by two servo motors supplied with 6 V dc current. The position acquisition system is achieved by means of a color web cam whose resolution is 352×288 pixels. The image processing was carried out by the MATLAB image acquisition toolbox and the control signal was sent to the robot micro-controller by means of a wireless communication scheme. The main function of

the robot micro-controller was to modulate the control signals into a PWM input for the motor. The used micro-controller was a BASIC Stamp 2 with a blue-tooth communication card. Figure 4 shows a block diagram of the experimental framework. The proposed tracking tasks was a six-leaved "rose" defined as follows:

$$x_1^*(t) = \sin(3\omega t + \eta)\sin(2\omega t + \eta)$$
$$x_2^*(t) = \sin(3\omega t + \eta)\cos(2\omega t + \eta)$$

The design parameters for the observers were set to be, $\mu_1 = 1.8$, $\mu_2 = 2.3$, $\sigma_1 = 3$, $\sigma_2 = 4$ and for the corresponding parameters for the controllers, $\zeta_1 = \zeta_3 = 1.2$, $\zeta_2 = \zeta_4 = 1.5$, $\omega_{n1} = \omega_{n3} = 1.8$, $\omega_{n2} = \omega_{n4} = 1.9$. Also, we compared the observer response with that of a GPI observer without the integral injection $(x_1\_, x_2\_)$ Luviano-Juárez et al. (2010). The experimental implementation results of the control law are depicted in figures, 6 and 7, where the control inputs and the tracking task are depicted. Notice that in the case of figure 8, there is a clear difference between the integral injection observer and the usual observer; the filtering effect of the integral observer helped to reduce the high noisy fluctuations of the control input due to measurement noises. On the average, the absolute error for the tracking task, for booth schemes, is less than 1 [cm]. This is quite a reasonable performance considering the height of the camera location and its relatively low resolution.
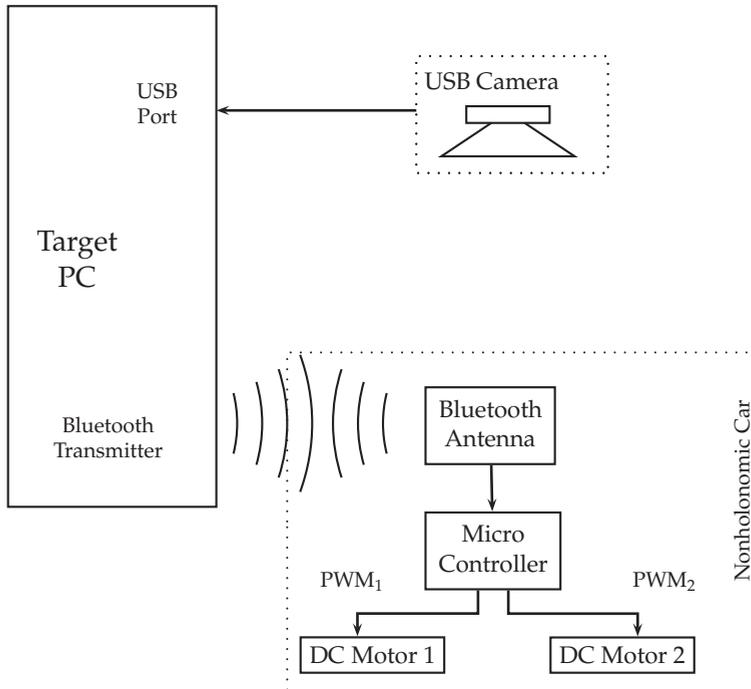
Fig. 4. Experimental control schematics

Fig. 5. Mobile Robot Prototype

5. Conclusions

In this chapter, we have proposed a linear observer-linear controller approach for the robust trajectory tracking task in nonlinear differentially flat systems. The nonlinear inputs-to-flat outputs representation is viewed as a linear perturbed system in which only the orders of integration of the Kronecker subsystems and the control input gain matrix of the system are considered to be crucially relevant for the controller design. The additive nonlinear terms in the input output dynamics can be effectively estimated, in an approximate manner, by means of a linear, high gain, Luenberger observer including finite degree, self updating, polynomial models of the additive state dependent perturbation vector components. This perturbation may also include additional unknown external perturbation inputs of uniformly absolutely bounded nature. A close approximate estimate of the additive nonlinearities is guaranteed to be produced by the linear observers thanks to customary, high gain, pole placement procedure. With this information, the controller simply cancels the disturbance vector and regulates the resulting set of decoupled chain of perturbed integrators after a direct nonlinear input gain matrix cancelation. A convincing simulation example has been presented dealing with a rather complex nonlinear physical system. We have also shown that the method efficiently results in a rather accurate trajectory tracking output feedback controller in a real laboratory implementation. A successful experimental illustration was presented which considered a non-holonomic mobile robotic system prototype, controlled by an overhead camera.
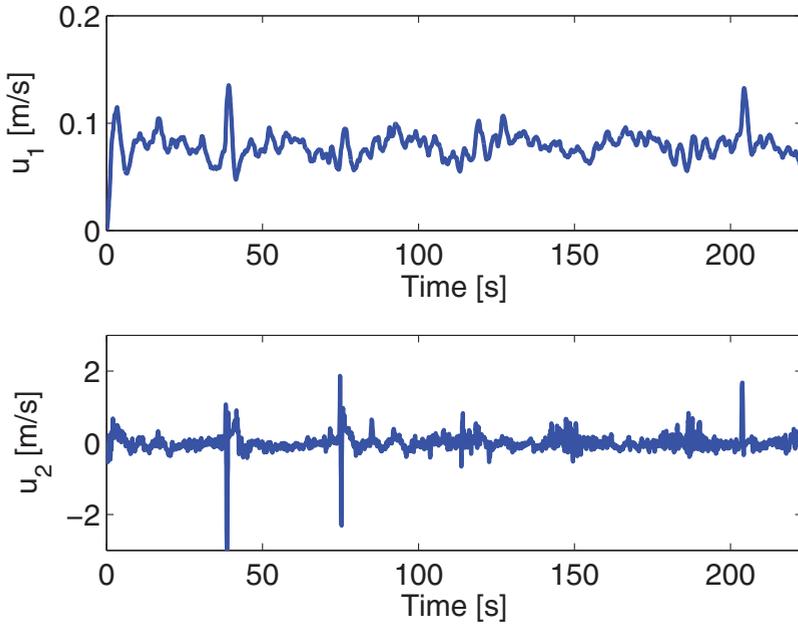
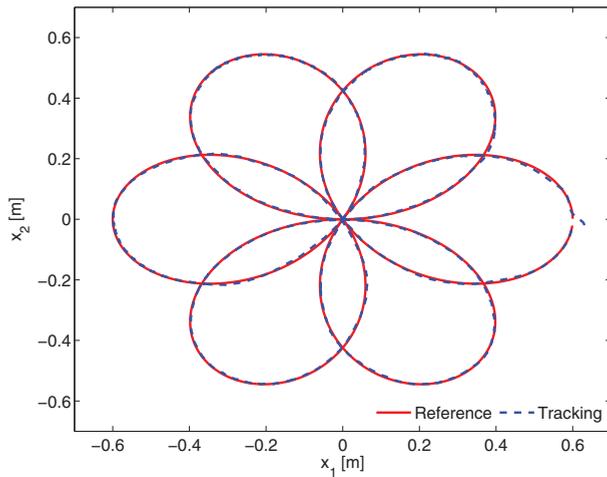Fig. 6. Experimental applied control inputs

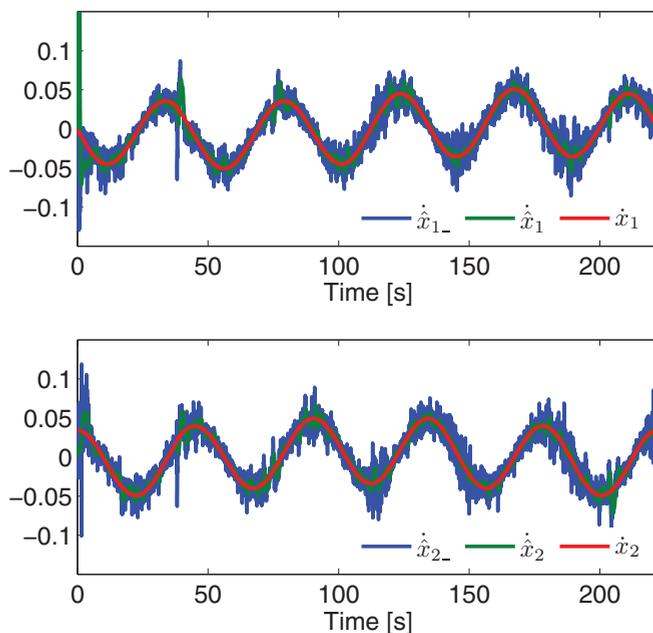Fig. 7. Experimental performance of GPI observer-based control on trajectory tracking task

Fig. 8. Noise reduction effect on state estimations via integral error injection GPI observers

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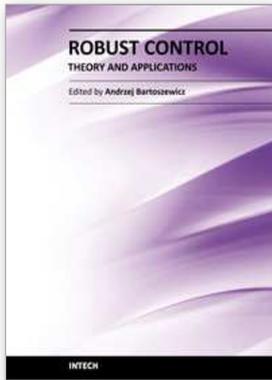
Robust Control, Theory and Applications

Edited by Prof. Andrzej Bartoszewicz

The main objective of this monograph is to present a broad range of well worked out, recent theoretical and application studies in the field of robust control system analysis and design. The contributions presented here include but are not limited to robust PID, H-infinity, sliding mode, fault tolerant, fuzzy and QFT based control systems. They advance the current progress in the field, and motivate and encourage new ideas and solutions in the robust control area.

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