

A Pursuit-Rendezvous Approach for Robotic Tracking

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1. Introduction

Robot navigation in dynamic environments and tracking of moving objects are among the most important topics addressed by the robotics community. These problems are more difficult than classical navigation problems, where the robot navigates to reach a stationary object. More interesting and complex applications involve moving targets. For example, applications such as dynamic surveillance can benefit from the tracking and navigation towards a moving goal. In surveillance problems the aim is for a robot to keep an evader in the field of view of the robot's sensory system (which consists of vision sensors in most cases). While adequate solutions to the problem of navigation towards a stationary goal have been elaborated, the problem of navigation and tracking of moving objects is still an open problem. This problem is fairly new and much more difficult. Algorithms developed in the motion planning community are highly effective at computing open loop controls, but cannot provide closed loop systems. This makes these algorithms less appropriate for tracking and navigation towards a moving object. This problem is a real-time problem that requires a closed loop strategy. The problem of navigation towards a moving goal in the presence of obstacles is a more difficult problem. The problem combines both local and global aspects. The local navigation aspect deals with the navigation on a small scale, where the primary problem is obstacle avoidance. The global navigation aspect deals with a larger scale, where the problem resides in reaching the goal.

Various methods based on different strategies such as computer vision, fuzzy logic, and Lyapunov theory have been suggested to solve this problem. Methods used for tracking moving targets can be classified into two different families: model-based methods and feature-based methods. Model-based methods aim to build a model of the tracking problem. Feature-based methods track the features of the object. Vision-based methods

are among the most important feature-based methods. These methods are widely used for tracking and reaching moving objects (Tsai et al., 2003; Oh & Allen, 2001). Other authors consider the problem of tracking humans using a wheeled mobile robot. The suggested algorithms can be used in different surveillance applications. Feyrer and Zell (Feyrer & Zell, 2001) suggested an algorithm that allows the detection, tracking and pursuing of humans in real time. The navigation is based on a potential field method and the detection process is based on an approach that combines colour, motion, and contour information. An algorithm for tracking humans from a moving platform is suggested in (Davis et al., 2000). Visual servoing methods are also used to keep the target in the field of vision of the robot (Thuilot et al., 2002). This problem is related to the problem of positioning and localization of the robot with respect to the moving object (Kim et al., 2001; Chaumette et al, 1999).

Even though vision-based methods are widely used, they may suffer from the following drawbacks:

- Most vision-based methods use complex algorithms that are computationally expensive, especially to track fast moving objects.
- It is necessary to keep the moving object in the field of view of the camera. This requires camera calibration. However, this task is difficult for manoeuvring and fast moving targets.

Several solutions have been suggested to solve these problems. Data reduction and the use of fast algorithms are among the most used solutions.

The problem of cooperative hunting behaviour by mobile robots troops is considered in (Yamaguchi, 2003). Clearly, this problem evolves navigation towards the prey. This task is accomplished using a model-based method.

Even though vision sensors are the most used, other sensors such as LADAR sensors, and acoustic sensors (Parker et al., 2003) are also used for target tracking and interception. Sensor planning and control for optimally tracking of targets is discussed in (Spletzer & Taylor, 2003). The existence of a strategy to maintain a moving target within the sensing range of an observer is discussed in (Murrieta et al., 2003).

Methods from control theory are also used for target-tracking. In (Lee et al., 2000), Lyapunov theory is used to derive an asymptotically stable solution to the tracking problem. A combination between control theory and artificial potential field methods is discussed in (Adams, 1999), where the asymptotic stability is discussed in details.

Model-based methods can be divided into three main families:

- Methods based on artificial intelligence.
- Methods based on optimal control and differential games theory.
- Methods based on geometric and kinematics equations.

In many situations, the robot tracks an intelligent evader. In this case more elaborate control strategies are needed. Methods based on artificial intelligence are used mainly by researchers in the robotics community to pursue and keep a moving target in the field of view of the robot. Optimal control methods are widely used in the aerospace community. These methods require an estimation of the time-to-go, which is a difficult task in practice. Kinematics-based methods are based on the derivation of a kinematics model for the motion. Kinematics-based methods can be used in various applications in surveillance and domain coverage. One important advantage of model-based methods is that it is possible to implement these methods using different types of sensors.

Here, we suggest a novel approach for tracking of and navigation towards a moving object. Our approach is based on a combination of the kinematics equations with geometric rules. The goal is to derive a closed loop control law for the robot's orientation angle. Since the target's motion is not a-priori known to the robot, it is necessary to design an online control law. Our approach combines the pursuit law with a rendezvous strategy and consists of a closed loop strategy. Thus, the control law for the robot's orientation angle has two terms. The first term corresponds to the pursuit, and the second term corresponds to the rendezvous. In the pursuit behavior the robot tracks or follows the path of the target. In the rendezvous behavior the robot does not follow or move towards the goal, but it moves towards a point that both the robot and the goal will reach simultaneously.

Our navigation law depends on a real variable, which we call the navigation parameter. This parameter allows controlling the navigation law. Thus, we can obtain a pure pursuit behavior or a pure rendezvous behavior, or a combination between the pursuit and the rendezvous. The navigation parameter can be time-varying too. In the presence of obstacles, the navigation law is combined with an obstacle avoidance algorithm; therefore, the robot moves in two modes: the tracking mode and the obstacle avoidance mode. We also suggest studying the control law in terms of the optimality of the path traveled by the robot. The method presents various advantages over other classical methods such as:

- Robustness: Kinematics-based methods are well-known by their robustness.
- Model-based: Our method is model-based, which means that the method can be implemented using different types of sensors.
- Proof of correctness: Our method allows us to rigorously prove that the robot navigating under our control law will reach the target successfully.

The algorithm discussed here relies on a localization technique to determine the visibility angle. However, only the control loop is discussed. The localization problem and the influence of the sensory system are beyond the scope of this study.

2. Preliminaries: Geometry and kinematics

In this section, we introduce several important concepts and definitions. The workspace consists of a subset χ of IR^2 . The robot and the goal are shown in figure 1. The reference point of the robot is denoted by R. The goal point is defined to be $(x_G, y_G) \in \chi$. It is denoted by G. Let point O be the origin of an inertial reference frame of coordinates. With reference to figure 1, we define the following quantities:

1. The visibility line robot-goal: This is the imaginary straight line that starts at the robot's reference point and is directed towards the goal. This line is defined even in the presence of obscurities.
2. Based on the visibility line, we define the visibility angle denoted by η as shown in figure 1. η is a function of the robot's and the goal's coordinates.
3. The visibility angles for the robot and the goal are given by η_R and η_G , respectively.
4. r_R and r_G denote the Euclidian distances from the origin to the reference points of the robot and the goal, respectively. The relative distance robot-goal is denoted by r .
5. The robot's coordinates in the Cartesian frame are given by (x_R, y_R) , and the goal's coordinates are given by (x_G, y_G) . The position error vector is given by $[x_e, y_e]^T$, with $x_e = x_G - x_R$, $y_e = y_G - y_R$.

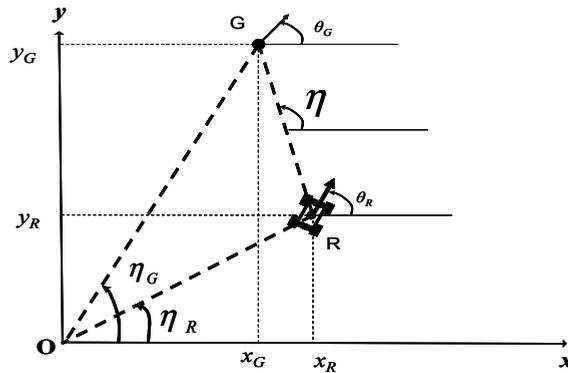


Fig. 1. A representation of the geometry.

The movement of the robot is controlled by its parameterized linear velocity $v_R(t)$ and angular velocity $\omega_R(t)$. The kinematics of the robot are described by the following equation:

$$\begin{aligned} \dot{x}_R &= v_R \cos(\theta_R) \\ \dot{y}_R &= v_R \sin(\theta_R) \\ x_R(t_0) &= x_{R0}, y_R(t_0) = y_{R0} \end{aligned} \tag{1}$$

with $\dot{\theta}_R = \omega_R$, where θ_R is the robot's orientation angle with respect to the positive x-axis. The kinematics equations describe the relationship between the control functions and the resulting trajectories. The goal is a moving point that moves according to the following kinematics equations:

$$\begin{aligned} \dot{x}_G &= v_G \cos(\theta_G) \\ \dot{y}_G &= v_G \sin(\theta_G) \\ x_G(t_0) &= x_{G0}, y_G(t_0) = y_{G0} \end{aligned} \tag{2}$$

where v_G is the goal's linear velocity and θ_G is the goal's orientation angle. v_G and θ_G are not a-priori known to the robot. However, it is assumed that the robot has a sensory system that allows to obtain these quantities in addition to the goal's position in real time. The kinematics model given by system (1) is in fact the kinematics model of a wheeled mobile robot of the unicycle type. The position of the robot in the reference frame of coordinates is given by the vector:

$$\begin{aligned} \vec{r}_R &= \vec{OR} = x_R \vec{u}_x + y_R \vec{u}_y \\ \vec{r}_R &= (r_R \cos \eta_R) \vec{u}_x + (r_R \sin \eta_R) \vec{u}_y \end{aligned} \tag{3}$$

In a similar way, the position of the goal is given by the vector:

$$\begin{aligned} \vec{r}_G &= \vec{OG} = x_G \vec{u}_x + y_G \vec{u}_y \\ \vec{r}_G &= (r_G \cos \eta_G) \vec{u}_x + (r_G \sin \eta_G) \vec{u}_y \end{aligned} \tag{4}$$

where \vec{u}_x, \vec{u}_y are the unit vectors along the x- and y-axes, respectively. The time derivative of \vec{r}_R and \vec{r}_G gives the velocity vectors as follows:

$$\begin{aligned} \dot{\vec{r}}_R &= \dot{x}_R \vec{u}_x + \dot{y}_R \vec{u}_y \\ \dot{\vec{r}}_G &= \dot{x}_G \vec{u}_x + \dot{y}_G \vec{u}_y \end{aligned} \tag{5}$$

By taking the time derivative of $x_R, y_R, x_G,$ and $y_G,$ we obtain:

$$\begin{aligned}\dot{x}_R &= \dot{r}_R \cos \eta_R - r_R \dot{\eta}_R \sin \eta_R \\ \dot{y}_R &= \dot{r}_R \sin \eta_R + r_R \dot{\eta}_R \cos \eta_R \\ \dot{x}_G &= \dot{r}_G \cos \eta_G - r_G \dot{\eta}_G \sin \eta_G \\ \dot{y}_G &= \dot{r}_G \sin \eta_G + r_G \dot{\eta}_G \cos \eta_G\end{aligned}\quad (6)$$

By replacing \dot{x}_R, \dot{y}_R by their values in the kinematics model of the robot, we obtain:

$$\begin{aligned}\dot{r}_R &= v_R \cos(\theta_R - \eta_R) \\ \dot{\eta}_R &= v_R \sin(\theta_R - \eta_R) r_R^{-1}\end{aligned}\quad (7)$$

By replacing \dot{x}_G, \dot{y}_G by their values in the kinematics model of the goal, we obtain:

$$\begin{aligned}\dot{r}_G &= v_G \cos(\theta_G - \eta_G) \\ \dot{\eta}_G &= v_G \sin(\theta_G - \eta_G) r_G^{-1}\end{aligned}\quad (8)$$

Now we consider the relative range between the robot and the goal which is given by

$$\bar{r} = \bar{r}_G - \bar{r}_R \quad (9)$$

Its time derivative gives the relative velocity

$$\dot{\bar{r}} = \dot{\bar{r}}_G - \dot{\bar{r}}_R \quad (10)$$

The relative velocity can be decomposed into two components along and across the visibility line robot-goal. This allows us to obtain:

$$\begin{aligned}\dot{r} &= v_G \cos(\theta_G - \eta) - v_R \cos(\theta_R - \eta) \\ \dot{\eta} &= (v_G \sin(\theta_G - \eta) - v_R \sin(\theta_R - \eta)) r^{-1}\end{aligned}\quad (11)$$

The relative kinematics model given by system (11) is very important in the formulation of our navigation-tracking problem. This model gives a good description of the motion of the goal as seen by the robot. The first equation gives the relative distance robot-goal. A decreasing range corresponds to $\dot{r} < 0$. The second equation gives the rate of turn of the goal with respect to the robot. The sign of $\theta_G - \eta$ indicates whether the goal is approaching or moving away from the robot. For $\theta_G - \eta \in]-\pi/2, \pi/2[$ the goal is moving away from the robot. For $\theta_G - \eta \in]\pi/2, 3\pi/2[$ the goal is approaching from the robot. System (11) is highly nonlinear. Its solution gives the robot's path in the plane (r, η) . However, the analytical solution is difficult except in a few particular cases.

3. Problem statement

The workspace is cluttered with N stationary obstacles denoted by B_i ($i=1, \dots, N$). The robot moves in the workspace according to the kinematics equations given by (1). The path of the robot is given by $P_R(t)$. The path of the moving goal is given by $P_G(t)$. This path is not a-priori known to the robot. Our goal is to design a closed loop control law for the robot in order to reach the goal and avoid possible collisions with obstacles. This can be stated as follows:

$$\begin{aligned} \|P_G(t_f) - P_R(t_f)\| < \varepsilon \\ P_R(t) \cap B_i = \Phi \end{aligned} \quad (12)$$

where ε is a small real number and t_f is the interception time. We will also design a control law for the robot linear velocity to keep the goal within a given coverage range from the robot.

It is assumed that

1. The control input for the robot is $[v_R, \theta_R]$ instead of $[v_R, \omega_R]$.
2. The robot is faster than the goal. This means that $v_R > v_G$.
3. The robot does not have a pre-decided knowledge of the environment. However, it has a sensory system that allows detecting obstacles and obtaining the necessary information on the goal. As we mentioned previously, the influence of the sensory system is beyond the scope of this study.

This problem is difficult, because it combines two different aspects, navigation towards the goal and obstacle avoidance. Navigation towards the goal has a global aspect while obstacle avoidance has a local aspect. The goal can perform two different types of motion, namely accelerating motion, and non-accelerating motion. In the case of an accelerating motion, either the linear velocity or the orientation angle of the goal is time-varying. In the case of a non-accelerating motion, both θ_G and v_G are constant. It is more difficult for the robot to reach an accelerating goal.

4. The control law

As we mentioned previously there exist two approaches for navigating a robot towards a moving object. The first approach is the pursuit (Belkhouche & Belkhouche, 2005), the second approach is the rendezvous. In the pursuit, the robot follows the moving object directly. That is the robot is always heading towards the goal at any time. This is the most obvious way to reach a moving goal. Various sensor-based algorithms use the pursuit even though they do not model the problem using the pursuit equations. The rendezvous approach uses a completely different principle which is the opposite extreme of the pursuit. In the rendezvous, the robot does not follow the path of the goal, but it moves towards a point where the robot and the goal will arrive at the same time. To accomplish this task the robot moves in lines that are parallel to the initial line of sight. The strategy discussed in this chapter is a new strategy that combines the pursuit and the rendezvous. In this approach, the robot orientation angle is given by the following equation:

$$\theta_R = \eta + c \sin^{-1}(K_v \sin(\theta_G - \eta)) \quad (13)$$

where c is a real number that satisfies $0 \leq c \leq 1$. c is the control variable of the robot's orientation angle. $K_v = v_G/v_R$ is the speed ratio. Equation (13) is a closed loop control law, where the control input depends on the state of the system, mainly the state of the goal. For $c = 0$, the control law acts like the pure pursuit. For $c = 1$, the control law acts like the pure rendezvous. For $0 < c < 1$, the control law has a behaviour that combines the rendezvous and the pursuit. For example, $c=0.9$ corresponds to 90% rendezvous and 10% pursuit. The control law given by (13) allows to reach the goal from any arbitrary initial position when the assumptions stated previously are satisfied. The main result concerning the navigation using the control law given by (13) is stated as follows:

Proposition:

Under the control law given by (13), the robot reaches the goal from any initial state when $v_R > v_G$.

Proof

The proof is based on the differential equation of the relative range rate; recall that $\dot{r}_R < 0$ corresponds to a decreasing range. We put

$$\lambda = \sin^{-1}(K_v \sin(\theta_G - \eta)) \quad (14)$$

Under the control law the relative range varies as follows:

$$\dot{r} = v_G \cos(\theta_G - \eta) - v_R \cos(c\lambda) \quad (15)$$

Recall that under the assumption that the robot is faster than the goal we have $K_v < 1$. The inverse of the sine function maps the domain $[-1, 1]$ to $[-\pi/2, \pi/2]$, and since $K_v < 1$, we have

$$\lambda = \sin^{-1}(K_v \sin(\theta_G - \eta)) \in]-\pi/2, \pi/2[\quad (16)$$

The cosine function of λ is strictly positive, i.e.,

$$\cos(\lambda) > 0 \quad (17)$$

Since the cosine function of λ is strictly positive and $0 \leq c \leq 1$, it turns out that

$$\cos(\lambda) \leq \cos(c\lambda) \quad (18)$$

Since $K_v < 1$, we have

$$\cos(\theta_G - \eta) < \cos(c\lambda) \quad (19)$$

Therefore

$$\begin{aligned} \dot{r} &= v_G \cos(\theta_G - \eta) - v_R \cos(c\lambda) < 0 \\ &\forall \theta_G \end{aligned} \quad (20)$$

Thus, since $\dot{r} < 0$ under the control law, the relative distance between the robot and the goal is a decreasing function of time. ■

There exist major differences in the behaviour of the control law for different values of c . For $c = 0$, the path of the robot is more curved near the interception. Thus more corrections are required near the interception. The opposite is true for $c = 1$, where the path of the robot is more curved at the beginning. This requires more corrections at the beginning of the navigation process. These aspects are discussed in the simulation. The control law becomes a simple pure pursuit when the goal is stationary.

4.1 Heading regulation

The initial state of the robot's orientation angle is given by $\theta_{R0} = \theta_R(t_0)$. In most cases, the value of θ_R given by the control law is different from θ_{R0} . Our goal is to design a smooth feedback plan that solves the planar navigation problem. For this reason a heading regulation is necessary. The heading regulation is accomplished by using the following formula:

$$\begin{aligned} \dot{\theta}_R &= -K_\theta (\theta_R - \theta_R^{des}) \\ \theta_R(t_0) &= \theta_{R0} \end{aligned} \tag{21}$$

where θ_R^{des} is given by the control law in equation (13). K_θ is a real positive number. The heading regulation is a transition phase that allows putting the robot in a configuration where the application of the control law is possible. The heading regulation phase is used whenever modes are switched during the collision avoidance process. Heading regulation is illustrated in figure 2 for both the pure pursuit (PP) and pure rendezvous (PR). The robot initial orientation angle is given by 90° .

4.2 The pure rendezvous

As we mentioned previously, the pure rendezvous corresponds to $c = 1$. By replacing θ_R by its value in the equation of the visibility angle rate, we obtain

$$\dot{\eta} = 0 \tag{22}$$

which implies that the visibility angle is constant, i.e., $\eta = constant$. This is the most important characterization of the pure rendezvous law. As a result, the motion of the goal as seen by the robot is linear, meaning that the robot moves in a straight line if the goal is moving in a straight line. The visibility angle is given by

$$\tan \eta = \frac{y_e}{x_e} \tag{23}$$

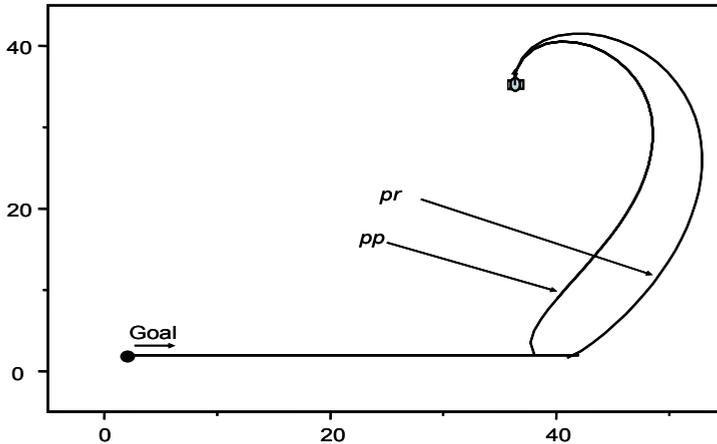


Fig. 2. An illustration of the heading regulation for the PP and PR. The robot's initial orientation angle is 90° .

which is constant under the pure rendezvous. By taking the time derivative we obtain

$$\tan \eta = \frac{\dot{y}_e}{\dot{x}_e} \tag{24}$$

This allows us to write

$$\frac{y_e}{x_e} = \frac{\dot{y}_e}{\dot{x}_e} \tag{25}$$

This equation is another important equation that characterizes the pure rendezvous. The orientation angle under the pure rendezvous is constant when the goal is not accelerating, and the robot is moving with a constant linear velocity. This is stated in the following result.

Proposition:

Under the pure rendezvous with $v_R = \text{constant}$, the robot's orientation angle is constant for non-maneuvring goals.

Proof:

The first step resides in proving that the visibility angle is constant under the pure rendezvous. By replacing θ_R by its value, we obtain

$$\dot{\eta} = \left(v_G \sin(\theta_G - \eta) - v_R \sin \left(c \sin^{-1} (K_v \sin(\theta_G - \eta)) \right) \right) r^{-1} \quad (26)$$

It turns out that under the pure rendezvous ($c=1$), we have $\dot{\eta} = 0$, therefore $\eta = \text{const}$. As a result, for a non-maneuvring target and a robot moving with constant speed we have $\theta_R = \text{constant}$. Thus the robot moves in a straight line. ■

4.3 The pure pursuit

The pure pursuit is another important particular case. It corresponds to $c = 0$, and thus the robot's orientation angle is equal to the visibility angle. The kinematics equations under the pure pursuit are given by

$$\begin{aligned} \dot{r} &= v_G \cos(\theta_G - \eta) - v_R \\ \dot{\eta} &= (v_G \sin(\theta_G - \eta)) r^{-1} \end{aligned} \quad (27)$$

It is clear from the first equation in the system that the range rate is negative when $v_G < v_R$. Unlike the pure rendezvous, the visibility angle is not constant in the case of the pure pursuit. In fact, in the pure pursuit the visibility angle tracks the goal's orientation angle with time.

The pure pursuit and the pure rendezvous are illustrated in figures 3 and 4. The difference in the path is obvious. Note that in the case of the pure rendezvous, the visibility line angles are parallel to each other. A more detailed comparison is shown in our simulation.

4.4 Navigation with time varying navigation parameter

We have seen that the navigation parameter enables us to control the navigation law between two extreme strategies. Previously we have considered only constant values of c . However the navigation parameter can be time varying too. This property is used to combine the advantages of the pure pursuit with those of the rendezvous in one navigation law. It is possible to use different formulae for $c(t)$. Two possibilities are the following:

$$c(t) = 1 - e^{-bt} \quad (28)$$

and

$$c(t) = e^{-bt} \quad (29)$$

where b is a real positive number. By using equation (28), the navigation law acts like the pure pursuit near the initial state and like the pure rendezvous near the interception. The opposite is true with equation (29). It is also important to note that equations (28) and (29) can be used for transition between the pure pursuit and the pure rendezvous. Smaller values of b are required in this case.

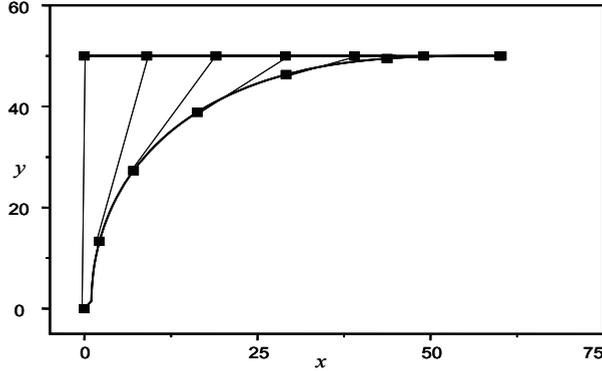


Fig.3. An illustration of the pure pursuit.

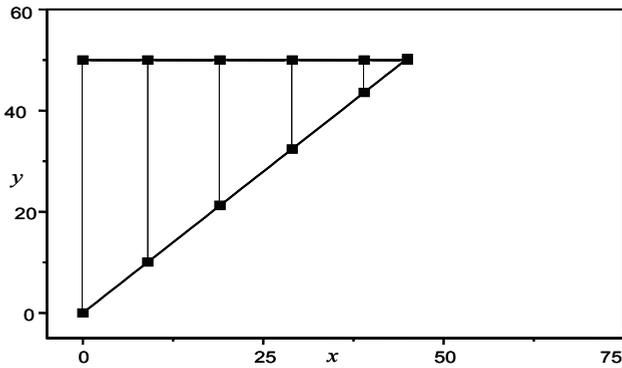


Fig. 4. An illustration of the pure rendezvous.

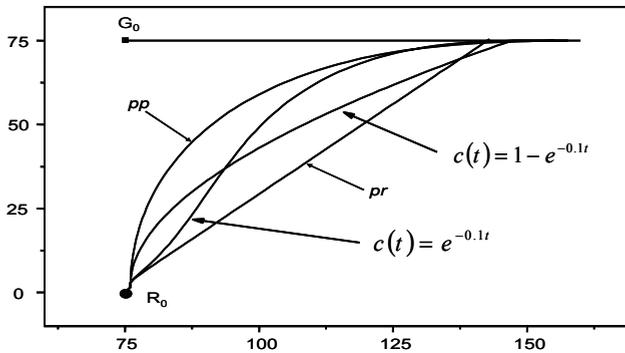


Fig. 5. An illustration of the of $c(t)$ to switch between the pure pursuit (PP) and the pure rendezvous (PR).

4.5 The time-to-go

The time-to-go is the time it takes the robot to reach the moving goal. The time-to-go is very important for any comparison between control strategies. The time-to-go can be estimated by the following equation:

$$t_{to-go} = -\frac{r}{\dot{r}} \tag{30}$$

In general, it is difficult to estimate the time-to-go since it depends on many factors that are time-varying. The most important factors are the velocity ratio, and the target manoeuvres. The time-to-go may be used to determine the appropriate value of b to adjust $c(t)$. The only case where it is possible to find the time-to-go analytically is when the goal moves in a straight line, ($\theta_G = const$), v_R and v_G are constant, and the robot is applying a pure rendezvous approach. In this case, the time-to-go is given by

$$t_{to-go} = -\frac{r_0}{v_G \cos(\theta_G - \eta) - v_R \cos(c\lambda)} \tag{31}$$

It is obvious that the time-to-go is proportional to the initial range.

5. In the presence of obstacles

It is clear that the problems of navigation and reaching a moving object in the presence of obstacles are among the most difficult problems in robotic navigation. They combine local path planning for collision avoidance with global path planning for reaching the goal. In our formulation, the robot moves in two modes, the navigation mode and the obstacle avoidance mode. Clearly, the obstacle avoidance mode has the priority. The collision avoidance is accomplished by building a polar histogram of the environment. The polar histogram is based on the angular information obtained from the sensory system. Only obstacles that appear within a given region called the active region are considered. The polar histogram allows determining free directions and directions corresponding to the obstacles. A snapshot of the local environment from a given position of the robot is a characterization of the visible obstacles and the angles they make with the robot.

The first stage in the polar histogram is to represent the robot’s surrounding environment using angular information provided by the robot’s onboard sensors. The angles λ_{i1} and λ_{i2} are the limit angles characterizing obstacle B_i as shown in figure 7. The polar diagram denoted by D is obtained as follows:

$$D = \sum_{i=1}^k d_i \tag{32}$$

where k denotes the number of obstacles in the active region, and d_i is given as follows:

$$d_i = 1 \text{ if } \theta_R - \eta \in [\lambda_{i1} - \eta, \lambda_{i2} - \eta] \tag{33}$$

$d_i = 0$ otherwise

Note that the polar histogram is constructed based on the angle $\beta = \theta_R - \eta$, therefore the pure pursuit corresponds to $\beta = 0$, and the pure rendezvous corresponds to $\beta = \lambda$. The obstacle avoidance mode is activated when at least one obstacle appears in the active region, and the robot navigates by using the polar histogram. It is also easy to represent the goal’s

orientation angle in the polar histogram. The robot deviates from its nominal path only if an obstacle appears in its path. The algorithm for collision avoidance is the following:

Procedure Deviation

1. Choose an intermediary point M such that $\eta_M - \eta$ has the same sign as $\theta_G - \eta$. η_M is the visibility angle between the robot and point M .
2. Navigate towards this point using the pure pursuit. A heading regulation procedure is used to keep the smoothness of the path. The equation for the heading regulation is similar to (21).

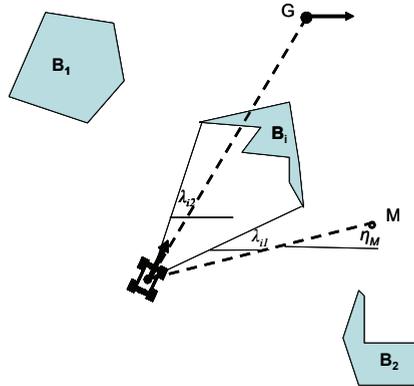


Fig. 6. Collision avoidance.

Collision avoidance algorithm:

1. If obstacle detected within the active region, then the collision avoidance mode is activated.
2. If the robot is in a collision course with obstacle B_i , then call procedure deviation
3. After obstacle passed go back to the pursuit-rendezvous mode. Since $\eta_M - \eta$ and $\theta_G - \eta$ have the same sign, the robot orientation angle and the goal orientation angle are on the same side of the visibility line.

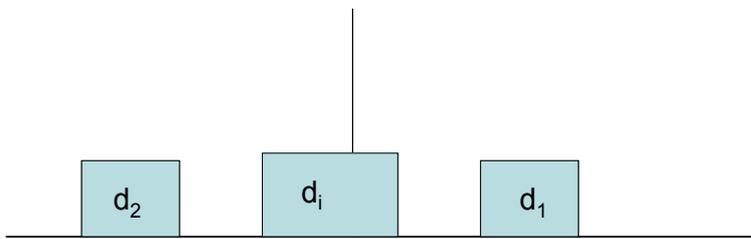


Fig. 7. Polar histogram for the environment in figure 6.

6. Pursuit-rendezvous for target dynamic coverage

Dynamic target coverage by a wheeled mobile robot or a group of mobile robots has been considered in the literature recently. This problem is important in various applications, such

as cleaning, security and patrolling, and sensor network deployment. Dynamic target coverage aims to generate a trajectory and the corresponding linear velocities. In the previous section, we designed a control law that allows the robot to reach the moving goal from an arbitrary initial state. In this section our goal is to design a second control law to keep the moving object within a given distance from the robot so that the goal stays in the robot’s field of view. That is,

$$r_1^{des} \leq r(t) \leq r_2^{des} \tag{34}$$

with $r_1^{des} \leq r^{des} \leq r_2^{des}$. r^{des} is the desired value of the coverage range, r_1^{des} and r_2^{des} are the range limits for r^{des} . The coverage range is represented by a circle as shown in figure 8. Dynamic coverage is necessary in various surveillance and tracking applications. For example, in many situations it is important to keep the goal in the field of view of the robot’s sensory system. To accomplish this task, it is necessary to design a control law for the robot’s linear velocity. Note that a constant range between the robot and the moving object corresponds to $\dot{r} = 0$; that is,

$$v_G \cos(\theta_G - \eta) = v_R \cos(c\lambda) \tag{35}$$

In order to combine the navigation mode with the tracking at a constant distance mode, we use the method which is known as feedback linearization (Drakunov et. 1991) in combination with backstepping or block control (Drakunov et. 1991) which gives

$$\dot{r} = -K_r (r - r^{des}) \tag{36}$$

where K_r is a real positive number. Equation (36) allows to drive the relative range smoothly to its desired coverage range. By replacing \dot{r} by its value, we obtain

$$v_G \cos(\theta_G - \eta) - v_R \cos(c\lambda) = -K_r (r - r^{des}) \tag{37}$$

From which the relative velocity of the robot can be obtained as follows:

$$v_R = \frac{K_r (r - r^{des}) + v_G \cos(\theta_G - \eta)}{\cos(c\lambda)} \tag{38}$$

The system converges to a steady state that satisfies equation (35). We have the following remarks concerning equation (38):

1. The term $K_r (r - r^{des})$ goes to zero with time.
2. If the goal applies a pure escape strategy, then $\theta_R = \eta$ and $v_R = v_G$. This is true for both the pure pursuit and the pure rendezvous.
3. In general, the required value of v_R is smaller in the case of the pure pursuit.

In the case of the pure pursuit, the dynamic coverage of a target is characterized by an important property, which can be states as follows:

Proposition

Under the pure pursuit, the dynamic coverage is characterized by $\theta_R \rightarrow \eta$ and $v_R \rightarrow v_G$. This means that the robot’s orientation angle will track the target’s orientation angle, and the robot’s linear velocity will track the target’s linear velocity.

Proof

The kinematics model under the pure pursuit is written as

$$\begin{aligned} \dot{r} &= v_G \cos(\theta_G - \eta) - v_R \\ \dot{\eta} &= (v_G \sin(\theta_G - \eta))r^{-1} \end{aligned} \quad (39)$$

The equilibrium position for the second equation is given by $\eta^* = \theta_T$. By using the classical linearization, it turns out that this equilibrium position is asymptotically stable. Therefore, $\eta \rightarrow \theta_G$, since $\eta = \theta_R$ under the pure pursuit, which gives $\theta_R \rightarrow \theta_G$. From equation (38) under the pure pursuit ($c = 0$), we have $v_R \rightarrow v_G$ as $\theta_R \rightarrow \theta_G$.

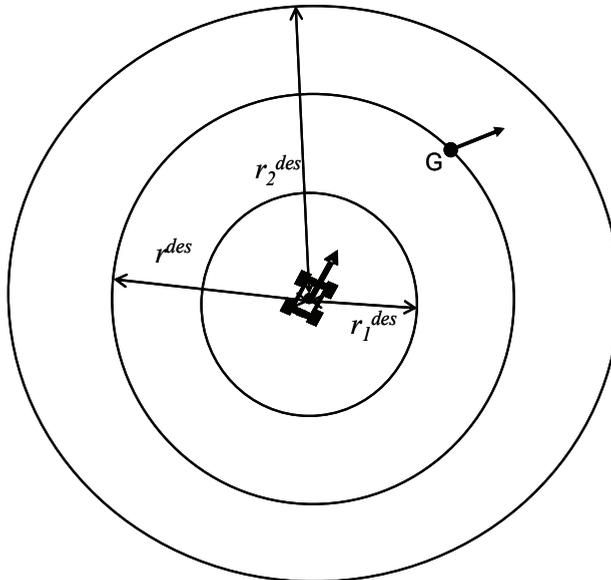


Fig. 8. An illustration of the dynamic coverage ranges.

7. Simulation

Here we consider several simulation examples to illustrate the suggested approach.

Example 1: A comparison between the pure pursuit (PP) and the pure rendezvous (PR)

Three scenarios are shown here. The first scenario shown in figure 9 corresponds to a goal moving in a straight line. The second scenario shown in figure 12 corresponds to a goal moving in a circle. The third scenario is shown in figure 13, the goal moves in a sinusoidal motion, which is among the most difficult paths to reach. Note that the path of the goal is not a-priori known to the robot. For the scenario of figure 9, the visibility angle is shown in figure 10, and the robot orientation angle in figure 11. From figure 10, the visibility angle is constant under the PR. From figure 11, it is clear that more corrections and manoeuvres are required under the PP. Figure 14 shows the robot path for different values of c . In all cases the robot reaches the goal successfully.

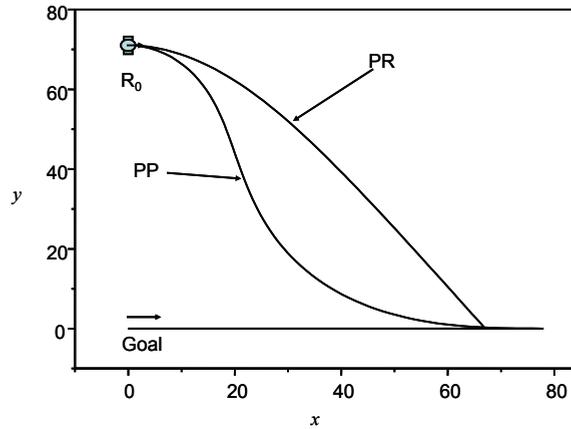


Fig. 9. Reaching a goal moving in a line.

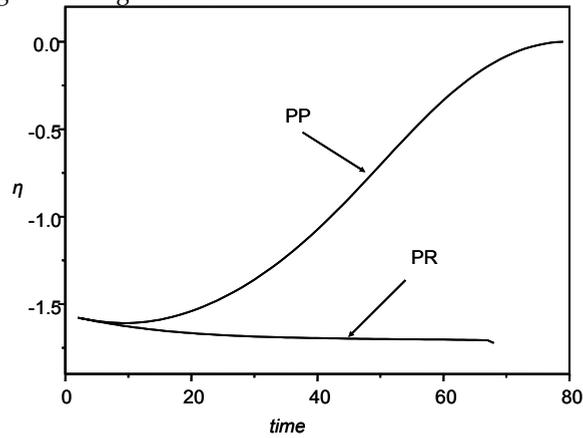


Fig. 10. Evolution of the visibility angle for the scenario of figure 9.

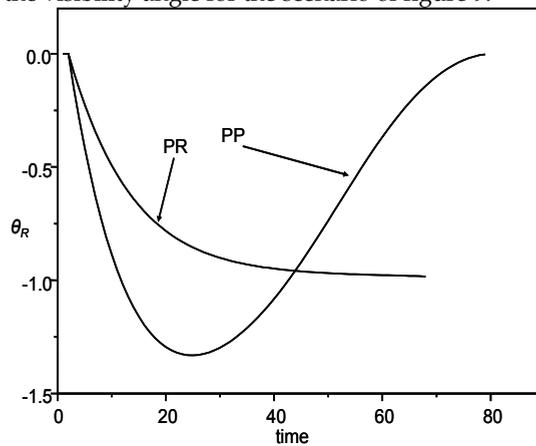


Fig. 11. Evolution of the robot's orientation angle for the scenario of figure 9.

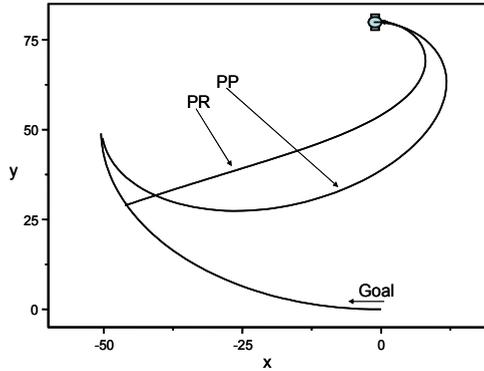


Fig. 12. Reaching a goal moving in circle.

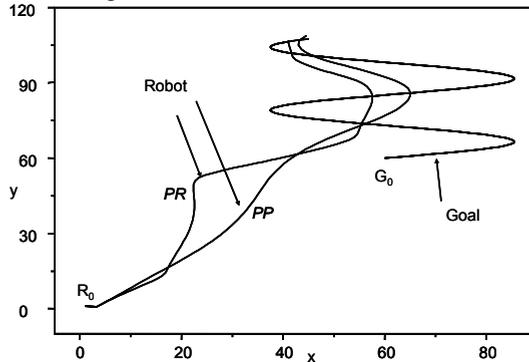


Fig. 13. Reaching a goal moving in a sinusoidal motion.

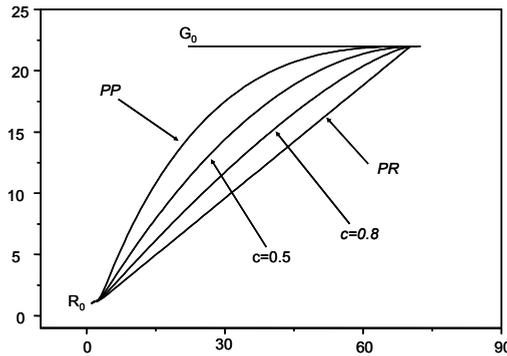


Fig. 14. Robot' path for different Values of c.

Example 2: in the presence of obstacles:

Two scenarios are shown in figures 14 and 15 to illustrate the navigation towards a moving goal in the presence of obstacles. The paths of the robot under the PP and the PR are different as shown in the figures. The robot accomplish the navigation and obstacle avoidance tasks successfully.

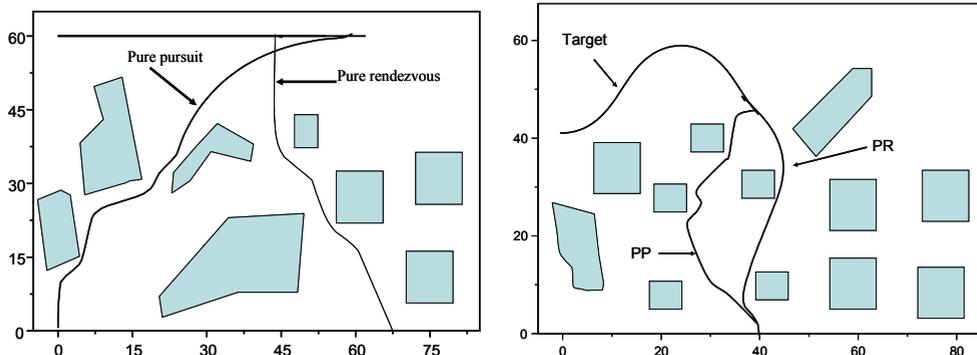


Fig. 15. Tracking and navigation in the presence of obstacles, goal moving in a line presence of obstacles.

8. Conclusion

We presented a method for robotic navigation and tracking of an unpredictably moving object. Our method is kinematics-based, and combines the pursuit law with the rendezvous law. First a kinematics model is derived. This kinematics model gives the motion of the goal with respect to the robot. The first equation gives the range rate between the robot and its goal. The second equation gives the turning rate of the goal with respect to the robot. The control law is then derived based on this kinematics model. This law is controlled by a real variable, which may be constant or time-varying. The most important properties of the control law are discussed. The dynamic coverage of the target is also discussed, where a second law for the robot's linear velocities is derived.

9. References

- Adams, M.D. (1999) High speed target pursuit and asymptotic stability in mobile robotics. *IEEE Transactions on Robotics and Automation*, Vol. 15, No. 2, pp. 230-236.
- Chaumette, F.; Rives P. and Espiau, B. (1991) Positioning of a robot with respect to an object, tracking it and estimating its velocity by visual servoing, *Proceedings of the IEEE International Conference on Robotics and Automation*, pp. 2248-2253 California.
- Davis, L; Philomin, V. and Duraiswami, R. (2000). Tracking humans from a moving platform, *Proceedings of IEEE International Conference on Pattern recognition*, pp. 171-178.
- Drakunov, S.; Izosimov, D.; Lukjanov, A.; Utkin, V.I. and Utkin, V. Block control principle. *Automation and Remote Control*, Vol. 51, No. 6, pp. 737-746, 1991.
- Drakunov, S.; Izosimov, D.; Lukjanov, A.; Utkin, V.I. and Utkin, V. Block control principle. *Automation and Remote Control*, Vol. 51, No. 5, pp. 601-608, 1991.
- Feyrer, S. and Zell, A. (1999). Detection, tracking, and pursuit of humans with an autonomous mobile robot. *Proceedings of the IEEE International Conference on Intelligent Robots and Systems*, pp. 864-869.
- Feyrer, S. and Zell, (1999). Tracking and pursuing persons with a mobile robot *Proceedings of the International Workshop on Recognition, Analysis and Tracking Faces and Gestures in Real time Systems*, pp. 83-88, California.

- Kim, B.H.; Roh, D.K.; Lee, J.M.; Lee, M.H.; Son, K.; Lee, M.C.; Choi, J.W.; and Han, S.H. (2001) Localization of a mobile robot using images of a moving target, *Proceedings of the IEEE International Conference on Robotics and Automation*, pp. 253-258 Seoul.
- Lee, S.O.; Cho, Y.J.; Hwang-Bo, M.; You, B.J. and Oh, S.R.(2000) A stable target-tracking control for unicycle mobile robots, *Proceedings of the IEEE/RSJ International Conference on Intelligent Robots and Systems*, pp. 1822-1827.
- Murrieta, R.; Sarmiento, A. and Hutchinson, S. (2003) On the existence of a strategy to maintain a moving target within the sensing range of an observer reacting with delay, *Proceedings of the IEEE/RSJ International Conference on Intelligent Robots and Systems*, 1184-1191.
- Oh, P.Y. and Allen, P.K. (2001). Visual servoing by partitioning degrees of freedom. *IEEE Transactions on Robotics and Automation*, Vol. 17; No. 1, 1-17.
- Thuilot, B; Martinet, P; Cordesses, L. and Gallice, J. (2002) Position based visual servoing: Keeping the object in the field of vision, *Proceedings of the IEEE International Conference on Robotics and Automation*, pp 1624-1629 Washington DC.
- Tsai, M ; Chen, K; Cheng, M; and Lin, K. (2003). Implementation of a real-time moving object tracking system using visual servoing. *Robotica*, Vol. 21, No. 6, 615-625.
- Parker, L.; Birch, B.; Reardon, C. (2003) Indoor target intercept using an acoustic sensor network and dual wavefront path planning, *Proceedings of IEEE/RSJ International Conference on Intelligent Robots and Systems*, 278-283.
- Spletzer, J. and Taylor, C. (2003) Dynamic Sensor Planning and Control for Optimally Tracking Targets, *The International Journal of Robotics Research*, Vol. 22, No. 1, 7-20.
- Yamaguchi, H. (1999) A cooperative hunting behavior by mobile-robot troops, *The International Journal of Robotics Research*, Vol. 18, No. 9, 931-940.
- Belkhouche F. and Belkhouche B. (2005) A method for robot navigation toward a moving goal with unknown maneuvers, *Robotica*, Vol. 23, No 6, 709-720.



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Today robots navigate autonomously in office environments as well as outdoors. They show their ability to beside mechanical and electronic barriers in building mobile platforms, perceiving the environment and deciding on how to act in a given situation are crucial problems. In this book we focused on these two areas of mobile robotics, Perception and Navigation. This book gives a wide overview over different navigation techniques describing both navigation techniques dealing with local and control aspects of navigation as well as those handling global navigation aspects of a single robot and even for a group of robots.

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