

The Nonlinear Absorption of a Strong Electromagnetic Wave in Low-dimensional Systems

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1. Introduction

It is well known that in low-dimensional systems, the motion of electrons is restricted. The confinement of electron in these systems has changed the electron mobility remarkably. This has resulted in a number of new phenomena, which concern a reduction of sample dimensions. These effects differ from those in bulk semiconductors, for example, electron-phonon interaction effects in two-dimensional electron gases (Mori & Ando, 1989; Rucker et al., 1992; Butscher & Knorr, 2006), electron-phonon interaction and scattering rates in one-dimensional systems (Antonyuk et al., 2004; Kim et al., 1991) and dc electrical conductivity (Vasilopoulos et al., 1987; Suzuki, 1992), the electronic structure (Gaggero-Sager et al., 2007), the wave function distribution (Samuel & Patil, 2008) and electron subband structure and mobility trends in quantum wells (Ariza-Flores & Rodriguez-Vargas, 2008). The absorption of electromagnetic wave in bulk semiconductors, as well as low dimensional systems has also been investigated (Shmelev et al., 1978; Bau & Phong, 1998; Bau et al., 2002; 2007). However, in these articles, the author was only interested in linear absorption, namely the linear absorption of a weak electromagnetic wave has been considered in normal bulk semiconductors (Shmelev et al., 1978), the absorption coefficient of a weak electromagnetic wave by free carriers for the case of electron-optical phonon scattering in quantum wells are calculated by the Kubo-Mori method in quantum wells (Bau & Phong, 1998) and in doped superlattices (Bau et al., 2002), and the quantum theory of the absorption of weak electromagnetic waves caused by confined electrons in quantum wires has been studied based on Kubo's linear response theory and Mori's projection operator method (Bau et al., 2007); the nonlinear absorption of a strong electromagnetic wave by free electrons in the normal bulk semiconductors has been studied by using the quantum kinetic equation method (Pavlovich & Epshtein, 1977). However, the nonlinear absorption problem of an electromagnetic wave, which has strong intensity and high frequency, in low dimensional systems is still open for study.

In this book chapter, we study the nonlinear absorption of a strong electromagnetic wave in low dimensional systems (quantum wells, doped superlattices, cylindrical quantum wires and rectangular quantum wires) by using the quantum kinetic equation method. Starting from the kinetic equation for electrons, we calculate to obtain the electron distribution functions in low dimensional systems. Then we find the expression for current density vector and the nonlinear absorption coefficient of a strong electromagnetic wave in low dimensional

systems. The problem is considered in two cases: electron-optical phonon scattering and electron-acoustic phonon scattering. Numerical calculations are carried out with a AlAs/GaAs/AlAs quantum well, a compensated n-p n-GaAs/p-GaAs doped superlattices, a specific GaAs/GaAsAl quantum wire.

This book chapter is organized as follows: In section 2, we study the nonlinear absorption of a strong electromagnetic wave by confined electrons in a quantum well. Section 3 presents the nonlinear absorption of a strong electromagnetic wave by confined electrons in a doped superlattice. The nonlinear absorption of a strong electromagnetic wave by confined electrons in a cylindrical quantum wire and in a rectangular quantum wire is presented in section 4 and section 5. Conclusions are given in the section 6.

2 The nonlinear absorption of a strong electromagnetic wave by confined electrons in a quantum well

2.1 The electron distribution function in a quantum well

It is well known that in quantum wells, the motion of electrons is restricted in one dimension, so that they can flow freely in two dimension. The Hamiltonian of the electron - phonon system in a quantum wells in the second quantization representation can be written as (in this chapter, we we select $\hbar=1$)

$$H = H_0 = \sum_{n, \vec{p}_\perp} \varepsilon_n(\vec{p}_\perp - \frac{e}{c} \vec{A}(t)) a_{n, \vec{p}_\perp}^+ a_{n, \vec{p}_\perp} + \sum_{\vec{q}} \omega_{\vec{q}} b_{\vec{q}}^+ b_{\vec{q}} + \sum_{n, n', \vec{p}_\perp, \vec{q}} C_{\vec{q}} I_{n, n'}(q_z) a_{n', \vec{p}_\perp + \vec{q}_\perp}^+ a_{n, \vec{p}_\perp} (b_{\vec{q}} + b_{-\vec{q}}^+), \quad (1)$$

where e is the electron charge, c is the velocity of light, n denotes the quantization of the energy spectrum in the z direction ($n = 1, 2, \dots$), (n, \vec{p}_\perp) and $(n', \vec{p}_\perp + \vec{q}_\perp)$ are electron states before and after scattering, respectively. \vec{p}_\perp (\vec{q}_\perp) is the in plane (x, y) wave vector of the electron (phonon), a_{n, \vec{p}_\perp}^+ and a_{n, \vec{p}_\perp} ($b_{\vec{q}}^+$ and $b_{\vec{q}}$) are the creation and the annihilation operators of electron (phonon), respectively. $\vec{q} = (\vec{q}_\perp, q_z)$, $\vec{A}(t) = \frac{c}{\Omega} \vec{E}_0 \cos(\Omega t)$ is the vector potential, \vec{E}_0 and Ω are the intensity and the frequency of the EMW, $\omega_{\vec{q}}$ is the frequency of a phonon, $C_{\vec{q}}$ is the electron-phonon interaction constants, $I_{n', n}(q_z)$ is the electron form factor in quantum wells.

In order to establish the quantum kinetic equations for electrons in a quantum well, we use the general quantum equation for the particle number operator (or electron distribution function)

$$n_{n, \vec{p}_\perp}(t) = \langle a_{n, \vec{p}_\perp}^+ a_{n, \vec{p}_\perp} \rangle_t$$

$$i \frac{\partial n_{n, \vec{p}_\perp}(t)}{\partial t} = \langle [a_{n, \vec{p}_\perp}^+ a_{n, \vec{p}_\perp}, H] \rangle_t \quad (2)$$

where $\langle \psi \rangle_t$ denotes a statistical average value at the moment t , and $\langle \psi \rangle_t = \text{Tr}(\widehat{W} \widehat{\psi})$ (\widehat{W} being the density matrix operator). Starting from the Hamiltonian Eq. (1) and using the commutative relations of the creation and the annihilation operators, we obtain the quantum kinetic equation for electrons in quantum wells:

$$\frac{\partial n_{n, \vec{p}_\perp}(t)}{\partial t} = - \sum_{\vec{q}, n'} |C_{\vec{q}}|^2 |I_{n, n'}|^2 \sum_{k, s = -\infty}^{\infty} J_k(\frac{e \vec{E}_0 \vec{q}_\perp}{m \Omega^2}) J_s(\frac{e \vec{E}_0 \vec{q}_\perp}{m \Omega^2}) \exp[-i(s - k)\Omega] \int_{-\infty}^t dt'$$

$$\begin{aligned} & \times \left\{ [n_{n,\vec{p}_\perp}(t')N_{\vec{q}} - n_{n',\vec{p}_\perp+\vec{q}_\perp}(t')(N_{\vec{q}}+1)]\exp[i(\varepsilon_{n',\vec{p}_\perp+\vec{q}_\perp} - \varepsilon_{n,\vec{p}_\perp} - \omega_{\vec{q}} - k\Omega + i\delta)(t-t')] \right. \\ & + [n_{n,\vec{p}_\perp}(t')(N_{\vec{q}}+1) - n_{n',\vec{p}_\perp+\vec{q}_\perp}(t')N_{\vec{q}}]\exp[i(\varepsilon_{n',\vec{p}_\perp+\vec{q}_\perp} - \varepsilon_{n,\vec{p}_\perp} + \omega_{\vec{q}} - k\Omega + i\delta)(t-t')] \\ & - [n_{n',\vec{p}_\perp-\vec{q}_\perp}(t')N_{\vec{q}} - n_{n,\vec{p}_\perp}(t')(N_{\vec{q}}+1)]\exp[i(\varepsilon_{n,\vec{p}_\perp} - \varepsilon_{n',\vec{p}_\perp-\vec{q}_\perp} - \omega_{\vec{q}} - k\Omega + i\delta)(t-t')] \\ & \left. - [n_{n',\vec{p}_\perp-\vec{q}_\perp}(t')(N_{\vec{q}}+1) - n_{n,\vec{p}_\perp}(t')N_{\vec{q}}]\exp[i(\varepsilon_{n,\vec{p}_\perp} - \varepsilon_{n',\vec{p}_\perp-\vec{q}_\perp} + \omega_{\vec{q}} - k\Omega + i\delta)(t-t')] \right\}. \end{aligned} \tag{3}$$

where $J_k(x)$ is the Bessel function, m is the effective mass of the electron, $N_{\vec{q}}$ is the time - independent component of the phonon distribution function, and the quantity δ is infinitesimal and appears due to the assumption of an adiabatic interaction of the electromagnetic wave.

It is well known that to obtain the explicit solutions from Eq. (3) is very difficult. In this paper, we use the first - order tautology approximation method (Pavlovich & Epshtein, 1977; Malevich & Epstein, 1974; Epstein, 1975) to solve this equation. In detail, in Eq. (3), we use the approximation:

$$n_{n,\vec{p}_\perp}(t') \approx \bar{n}_{n,\vec{p}_\perp}, \quad n_{n,\vec{p}_\perp+\vec{q}_\perp}(t') \approx \bar{n}_{n,\vec{p}_\perp+\vec{q}_\perp}, \quad n_{n,\vec{p}_\perp-\vec{q}_\perp}(t') \approx \bar{n}_{n,\vec{p}_\perp-\vec{q}_\perp}.$$

where $\bar{n}_{n,\vec{p}_\perp}$ is the time - independent component of the electron distribution function. The approximation is also applied for a similar exercise in bulk semiconductors (Pavlovich & Epshtein, 1977; Malevich & Epstein, 1974). We perform the integral with respect to t . Next, we perform the integral with respect to t of Eq. (3). The expression of electron distribution function can be written as

$$\begin{aligned} n_{n,\vec{p}_\perp}(t) = & - \sum_{\vec{q},n'} |C_{\vec{q}}|^2 |I_{n,n'}|^2 \sum_{k,l=-\infty}^{\infty} J_k\left(\frac{e\vec{E}_0\vec{q}_\perp}{m\Omega^2}\right) J_{k+l}\left(\frac{e\vec{E}_0\vec{q}_\perp}{m\Omega^2}\right) \frac{1}{l\Omega} e^{-il\Omega t} \\ & \times \left\{ - \frac{\bar{n}_{n,\vec{p}_\perp}N_{\vec{q}} - \bar{n}_{n',\vec{p}_\perp+\vec{q}_\perp}(N_{\vec{q}}+1)}{\varepsilon_{n',\vec{p}_\perp+\vec{q}_\perp} - \varepsilon_{n,\vec{p}_\perp} - \omega_{\vec{q}} - k\Omega + i\delta} - \frac{\bar{n}_{n,\vec{p}_\perp}(N_{\vec{q}}+1) - \bar{n}_{n',\vec{p}_\perp+\vec{q}_\perp}N_{\vec{q}}}{\varepsilon_{n',\vec{p}_\perp+\vec{q}_\perp} - \varepsilon_{n,\vec{p}_\perp} + \omega_{\vec{q}} - k\Omega + i\delta} \right. \\ & \left. + \frac{\bar{n}_{n',\vec{p}_\perp-\vec{q}_\perp}N_{\vec{q}} - \bar{n}_{n,\vec{p}_\perp}(N_{\vec{q}}+1)}{\varepsilon_{n,\vec{p}_\perp} - \varepsilon_{n',\vec{p}_\perp-\vec{q}_\perp} - \omega_{\vec{q}} - k\Omega + i\delta} + \frac{\bar{n}_{n',\vec{p}_\perp-\vec{q}_\perp}(N_{\vec{q}}+1) - \bar{n}_{n,\vec{p}_\perp}N_{\vec{q}}}{\varepsilon_{n,\vec{p}_\perp} - \varepsilon_{n',\vec{p}_\perp-\vec{q}_\perp} + \omega_{\vec{q}} - k\Omega + i\delta} \right\}. \end{aligned} \tag{4}$$

From Eq.(4) we see that the electron distribution function depends on the constant in the case of electron - phonon interaction, the electron form factor and the electron energy spectrum in quantum wells. Eq.(4) also can be considered a general expression of the electron distribution function in two dimensional systems with the electron form factor and the electron energy spectrum of each systems.

2.2 Calculations of the nonlinear absorption coefficient of a strong electromagnetic wave by confined electrons in a quantum well

In a quantum well, the motion of electrons is confined and that energy spectrum of electron is quantized into discrete levels. We assume that the quantization direction is the z direction. The total wave function of electrons can be written as

$$\psi_n(\vec{r}) = \psi_0 e^{i\vec{p}_\perp\vec{r}_\perp} \sin(p_z^n z), \tag{5}$$

where ψ_0 is normalization constant, the electron energy spectrum takes the simple form:

$$\epsilon_{n,\bar{p}_\perp} = \frac{1}{2m}(p_\perp^2 + p_z^2). \tag{6}$$

where p_z^n takes discrete values: $p_z^n = n\pi/L$, L is width of a quantum well. The electron form factor can be written as

$$I_{n',n}(qz) = \frac{2}{L} \int_0^L \sin(p_z^{n'}z)\sin(p_z^n z)e^{iqz} dz \tag{7}$$

The carrier current density formula in quantum wells takes the form (Pavlovich & Epshtein, 1977)

$$\vec{j}_\perp(t) = \frac{e}{m} \sum_{n,\bar{p}_\perp} (\bar{p}_\perp - \frac{e}{c}\vec{A}(t))n_{n,\bar{p}_\perp}(t). \tag{8}$$

Because the motion of electrons is confined along the z direction in a quantum well, we only consider the in - plane (x,y) current density vector of electrons, $\vec{j}_\perp(t)$. Using Eq. (4), we find the expression for current density vector:

$$\vec{j}_\perp(t) = -\frac{e^2}{mc} \sum_{n,\bar{p}_\perp} \vec{A}(t)n_{n,\bar{p}_\perp}(t) + \sum_{l=1}^\infty \vec{j}_l \sin(l\Omega t). \tag{9}$$

Here,

$$\begin{aligned} \vec{j}_l = & 2\pi \frac{e}{m} \frac{1}{l\Omega} \sum_{n,n'} \sum_{\bar{p},\bar{p}_\perp} |C_{\bar{q}}|^2 |I_{n,n'}|^2 \sum_{k=-\infty}^\infty \bar{q}_\perp J_k \left(\frac{e\vec{E}_0\bar{q}_\perp}{m\Omega^2} \right) \left[J_{k+l} \left(\frac{e\vec{E}_0\bar{q}_\perp}{m\Omega^2} \right) + J_{k-l} \left(\frac{e\vec{E}_0\bar{q}_\perp}{m\Omega^2} \right) \right] \\ & \times N_{\bar{q}}(\bar{n}_{n,\bar{p}_\perp} - \bar{n}_{n',\bar{p}_\perp + \bar{q}_\perp}) \{ \delta(\epsilon_{n',\bar{p}_\perp + \bar{q}_\perp} - \epsilon_{n,\bar{p}_\perp} + \omega_{\bar{q}} - k\Omega) + [\omega_{\bar{q}} \rightarrow -\omega_{\bar{q}}] \}. \end{aligned} \tag{10}$$

Using the expression of the nonlinear absorption coefficient of a strong electromagnetic wave (Pavlovich & Epshtein, 1977)

$$\alpha = \frac{8\pi}{c\sqrt{\chi_\infty}E_0^2} \left\langle \vec{j}_\perp(t) \vec{E}_0 \sin \Omega t \right\rangle_t, \tag{11}$$

and properties of Bessel function, we obtain the nonlinear absorption coefficient of a strong electromagnetic wave by confined electrons in quantum well

$$\begin{aligned} \alpha = & \frac{8\pi^2\Omega}{c\sqrt{\chi_\infty}E_0^2} \sum_{n,n'} |I_{n,n'}|^2 \sum_{\bar{q},\bar{p}} |C_{\bar{q}}|^2 N_{\bar{q}} \sum_{k=-\infty}^\infty [\bar{n}_{n,\bar{p}} - \bar{n}_{n',\bar{p}+\bar{q}}] \times \\ & \times k J_k^2 \left(\frac{eE_0\bar{q}}{m\Omega^2} \right) \{ \delta(\epsilon_{n',\bar{p}+\bar{q}} - \epsilon_{n,\bar{p}} + \omega_{\bar{q}} - k\Omega) + [\omega_{\bar{q}} \rightarrow -\omega_{\bar{q}}] \} \end{aligned} \tag{12}$$

In the following, we study the problem with different electron-phonon scattering mechanisms. We only consider the absorption close to its threshold because in the rest case (the absorption far away from its threshold) α is very smaller. In the case, the condition $|k\Omega - \omega_{\bar{q}}| \ll \bar{\epsilon}$ must be satisfied (Pavlovich & Epshtein, 1977). We restrict the problem to the case of one photon absorption and consider the electron gas to be non-degenerate:

$$\bar{n}_{n,\vec{p}} = n_0^* \exp\left(-\frac{\varepsilon_{n,\vec{p}}}{k_b T}\right), \text{ with } n_0^* = \frac{n_0(e\pi)^{\frac{3}{2}}}{V(m_0 k_b T)^{\frac{3}{2}}} \quad (13)$$

where, V is the normalization volume, n_0 is the electron density in quantum well, m_0 is the mass of free electron, k_b is Boltzmann constant.

2.2.1 Electron - optical phonon scattering

In this case, The electron-optical phonon interaction constants can be taken as (Shmelev et al., 1978; Pavlovich & Epshtein, 1977) $|C_{\vec{q}}^{op}|^2 \equiv |C_{\vec{q}}^{op}|^2 = 2\pi e^2 \omega_0 (1/\chi_\infty - 1/\chi_0) / \epsilon_0 (q_\perp^2 + q_z^2) V$, here V is the volume, ϵ_0 is the permittivity of free space, χ_∞ and χ_0 are the high and low-frequency dielectric constants, respectively. $\omega_{\vec{q}} \equiv \omega_0$ is the frequency of the optical phonon in the equilibrium state. By using the electron - optical phonon interaction factor $C_{\vec{q}}^{op}$, the Bessel function and the electron distribution function n_{n,\vec{p}_\perp} , from the general expression for the nonlinear absorption coefficient of a strong electromagnetic wave in a quantum well Eq.(12), we obtain the explicit expression of the nonlinear absorption coefficient α in quantum well for the case electron-optical phonon scattering:

$$\alpha = \frac{\alpha_0}{\pi L} \sum_{nn'} \exp\left(-\frac{\pi^2 n'^2}{2mk_B T L^2}\right) \left\{ \left[\exp\left(\frac{\Omega - \omega_0}{k_B T}\right) - 1 \right] \left\{ 1 + \frac{e^2 E_0^2}{m\Omega^3} \frac{3k_B T}{8\Omega} \left[1 + \frac{1}{2k_B T} \left((\omega_0 - \Omega) + \frac{\pi^2 (n'^2 - n^2)}{2mL^2} \right) \right] \right\} + [\omega_0 \rightarrow -\omega_0] \right\}, \quad (14)$$

where

$$\alpha_0 = \frac{\pi e^4 n_0^* (k_B T)^2}{2\epsilon_0 c \sqrt{\chi_\infty} \Omega^3} \left(\frac{1}{\chi_\infty} - \frac{1}{\chi_0} \right). \quad (15)$$

In bulk materials, there is a strong dispersion when the phonon energy is close to the optical phonon energy. However, in a quantum well, we will see an increase in the absorption coefficient of Electromagnetic Wave (see the numerical calculation and the discussion sections). This is due to the surprising changes in the electron spectrum and the wave function in quantum system. This also results in an significant property for low - dimensional materials.

2.2.2 Electron - acoustic phonon scattering

In the case, $\omega_{\vec{q}} \ll \Omega$ ($\omega_{\vec{q}}$ is the frequency of acoustic phonons), so we let it pass. The electron-acoustic phonon interaction constants can be taken as (Mori & Ando, 1989; Shmelev et al., 1978) $|C_{\vec{q}}^{ac}|^2 \equiv |C_{\vec{q}}^{ac}|^2 = \xi^2 q / 2\rho v_s V$, here V , ρ , v_s , and ξ are the volume, the density, the acoustic velocity and the deformation potential constant, respectively. In this case, we obtain the explicit expression of α in quantum well for the case electron-acoustic phonon scattering:

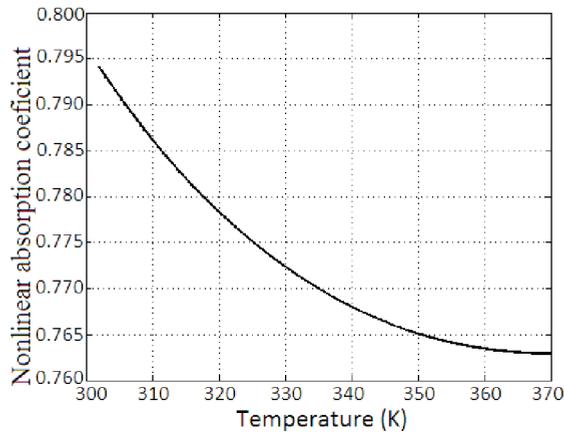


Fig. 1. The dependence of α on T in a quantum well

$$\alpha = \frac{(k_B T)^3 e^2 m n_0 \zeta^2}{c \sqrt{x_\infty} v_s^2 \rho \Omega^3 L} \sum_{n, n'} \exp \left[-\frac{1}{k_B T} \left(\frac{\pi^2 (n'^2 - n^2)}{2mL^2} \right) \right] \times \left[\exp \left(\frac{\Omega}{k_B T} \right) - 1 \right] \left\{ 1 + \frac{\beta}{2k_B T} + \frac{3}{32} \frac{e^2 E_0^2}{m \Omega^4 k_B T} (\beta^2 + 3\beta k_B T + 12(k_B T)^2) \right\} \quad (16)$$

$$\text{with } \beta = \frac{\pi^2 (n'^2 - n^2)}{2mL^2} - \Omega. \quad (17)$$

From Eqs. 14-17 we see that the nonlinear absorption coefficient are complex and has difference from those obtained in normal bulk semiconductors. the nonlinear absorption coefficient has the sum over the quantum number n . In addition, when the term in proportion to a quadratic in the intensity of the electromagnetic wave (E_0^2) tend toward zero, the nonlinear result will turn back to the linear case which was calculated by another method-the Kubo - Mori (Bau & Phong, 1998).

2.3 Numerical results and discussion

In order to clarify the mechanism for the nonlinear absorption of a strong electromagnetic wave in a quantum well, we will evaluate, plot, and discuss the expression of the nonlinear absorption coefficient for the case of a specific quantum well: AlAs/GaAs/AlAs. The parameters used in the calculations are as follows (Bau et al., 2002; Pavlovich & Epshtein, 1977): $\chi_\infty = 10.9$, $\chi_0 = 12.9$, $n_0 = 10^{23} \text{ m}^{-3}$, $L = 100 \text{ \AA}$, $m = 0.067m_0$, m_0 being the mass of a free electron, $\hbar\omega_0 = 36.25 \text{ meV}$, and $\Omega = 2.10^{14} \text{ s}^{-1}$.

2.3.1 Electron - optical phonon scattering

Figure 1 show the nonlinear absorption in quantum wells. When the temperature T of the system rises, its absorption coefficient decreased. However, for the case of bulk semiconductors, the absorption coefficient increases following its temperature. In addition, the absorption coefficient in bulk semiconductors is smaller than in quantum wells. The fact

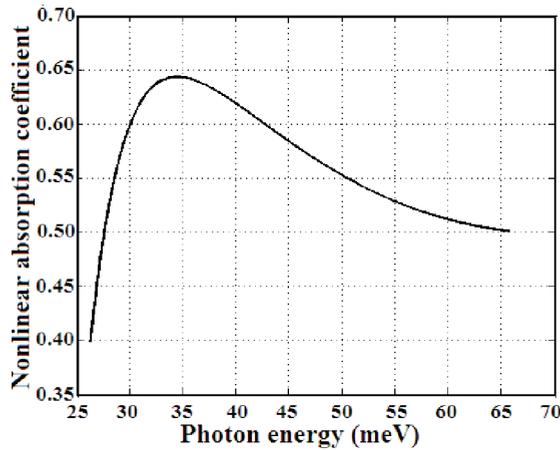


Fig. 2. The dependence of α on photon energy in a quantum well

proves that confined electrons in quantum wells have enhanced electromagnetic absorption ability.

Figure 2 shows the nonlinear absorption coefficient as a function of the electromagnetic wave energy (photon energy) for the case electron - optical phonon scattering. This figure shows that the curve has a maximum where $\Omega = \omega_0$.

Figure 3 shows the dependence of the nonlinear absorption coefficient depends on well's width L at different values of the electromagnetic wave energy, each curve has one maximum peak. The resonance peak only appears when $20 \text{ nm} < L < 40 \text{ nm}$, and it will be sharper if the frequency of the electromagnetic wave is close to the frequency of the optical phonon $\Omega = \omega_0$. This suggests that when external parameters are not changed, we can change the width of quantum well to get the absorption of a strong electromagnetic wave the best.

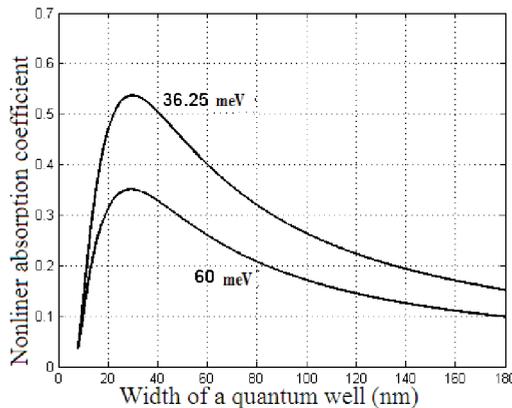


Fig. 3. The dependence of α on L (well's width) in a quantum well

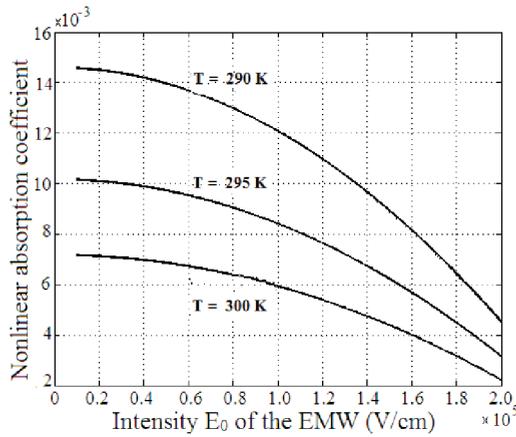


Fig. 4. The dependence of α on E_0 in a quantum well

2.3.2 Electron - acoustic phonon scattering

The parameters used in the calculations are as follow (Bau et al., 2002; Pavlovich & Epshtein, 1977): $\chi_\infty = 10,9, \zeta = 13.5$ eV, $v_s = 5370$ m/s, $\rho = 5320$ kg/m³, $n_0 = 10^{23}$ m⁻³, $L = 100$ Å, $m = 0,067m_0$, m_0 being the mass of free electron.

Figure 4 and figure 5 shows the nonlinear absorption coefficient in quantum wells for the case electron - acoustic phonon scattering. The most important point is that in this case, the absorption coefficient is very small. Figure. 5 shows the nonlinear absorption coefficient dependence on the electromagnetic wave energy for the case electron - acoustic phonon scattering. Different from the case electron - optical phonon scattering, the nonlinear absorption coefficient α in this case has not maximum values (peaks).

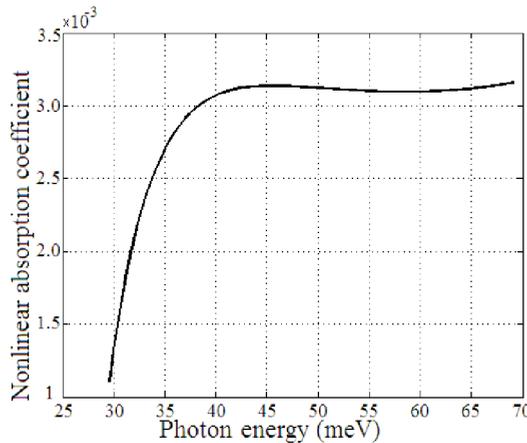


Fig. 5. The dependence of α on photon energy in a quantum well

3. The nonlinear absorption of a strong electromagnetic wave by confined electrons in a doped superlattice

3.1 Calculations of the nonlinear absorption coefficient of a strong electromagnetic wave by confined electrons in a doped superlattice

The total wave function of electrons and the electron energy spectrum in a doped superlattice can be written as (Bau et al., 2002)

$$\psi_{n,p_z}(z) = \sum_{j=0}^{s_0} e^{ip_z jz} \psi_n(z - jd), \tag{18}$$

$$\epsilon_n(\vec{p}_\perp) = \frac{\vec{p}_\perp^2}{2m} + \omega_p \left(n + \frac{1}{2} \right), \tag{19}$$

Here, $\psi_n(z)$ is the wave function of the n -th state in one of the one-dimensional potential wells which compose the doped superlattice potential, d is the doped superlattice period, s_0 is the number of doped superlattice period, $\omega_p = \left(\frac{4\pi e^2 n_D}{\chi_0 m} \right)^{1/2}$ is the frequency plasma caused by donor doping concentration, n_D is the doping concentration. The electron form factor, $I_{n,n'}(q_z)$, is written as

$$I_{n,n'}(q_z) = \sum_{j=1}^{s_0} \int_0^d e^{iq_z d} \psi_n(z - jd) \psi_{n'}(z - jd) dz. \tag{20}$$

In order to establish analytical expressions for the nonlinear absorption coefficient of a strong electromagnetic wave by confined electrons in a doped superlattice, We insert the expression for $n_{n,\ell,\vec{p}}(t)$ into the expression for $\vec{j}(t)$ and then insert the expression for $\vec{j}(t)$ into the expression for α in Eq.(11). Using the properties of Bessel function and realizing the calculations, we obtain the nonlinear absorption coefficient of a strong electromagnetic wave by confined electrons in a doped superlattice as

$$\begin{aligned} \alpha = & \frac{8\pi^2 \Omega}{c\sqrt{\chi_\infty} E_0^2} \sum_{n,n'} |I_{n,n'}|^2 \sum_{\vec{q},\vec{p}} |C_{\vec{q}}|^2 N_{\vec{q}} \sum_{k=-\infty}^{\infty} [\bar{n}_{n,\vec{p}} - \bar{n}_{n',\vec{p}+\vec{q}}] k J_k^2 \left(\frac{eE_0 \vec{q}}{m\Omega^2} \right) \times \\ & \times \delta \left(\frac{(\vec{p} + \vec{q})^2}{2m} + \omega_p(n' + 1/2) - \frac{\vec{p}_\perp^2}{2m} - \omega_p(n + 1/2) + \omega_{\vec{q}} - k\Omega \right) \end{aligned} \tag{21}$$

Using the time - independent component of the electron distribution function, the Bessel function and the electron-optical phonon interaction constants, we can calculate to obtain expression of the carrier current density and the nonlinear absorption coefficient of a strong electromagnetic wave by confined electrons in a doped superlattice. we obtain the explicit expression of α in a doped superlattice for the case electron-optical phonon scattering:

$$\begin{aligned} \alpha = & \frac{\sqrt{2}\pi n_0^* (k_B T)^2 e^4}{8c\sqrt{m}\chi_\infty \hbar^3 \Omega^3} \left(\frac{1}{\chi_\infty} - \frac{1}{\chi_0} \right) \sum_{n,n'} |I_{n,n'}(q_z)|^2 \exp \left(-\frac{\hbar\omega_p \left(n + \frac{1}{2} \right) + \frac{\xi}{2}}{k_B T} \right) \\ & \times e^{-2\sqrt{\rho\sigma}} \left(\frac{\rho}{|\xi|\sigma} \right)^{\frac{1}{2}} \left\{ 1 + \frac{3}{16\sqrt{\rho\sigma}} + \frac{3e^2 E_0^2}{32m^2 \Omega^4} \left(\frac{\rho}{\sigma} \right)^{\frac{1}{2}} \left[1 + \frac{1}{\sqrt{\rho\sigma}} + \frac{1}{16\rho\sigma} \right] \right\} \end{aligned} \tag{22}$$

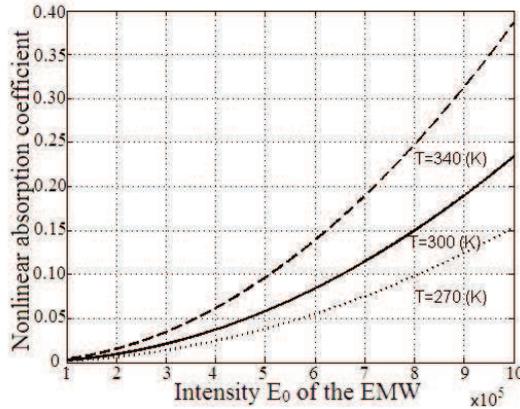


Fig. 6. The dependence of α on the E_0 in a doped superlattice

Here: $\zeta = \hbar\omega_p (n' - n) + \hbar\omega_0 - \hbar\Omega$; $a = \frac{3}{8} \left(\frac{eE_0}{2m\Omega^2} \right)^2$; $\rho = \frac{m\zeta^2}{2\hbar^2 k_B T}$; $\sigma = \frac{\hbar^2}{8mk_B T}$.

In a doped superlattice, the nonlinear absorption coefficient is more complex than those obtained in quantum well. The term in proportion to quadratic intensity of a strong electromagnetic wave tends toward zero, the nonlinear result will turn back to the linear case which was calculated by another method—the Kubo - Mori (Bau et al., 2002)

3.2 Numerical results and discussion

In order to clarify the mechanism for the nonlinear absorption of a strong electromagnetic wave in a doped superlattice, we will evaluate, plot, and discuss the expression of the nonlinear absorption coefficient for the case of a specific doped superlattice: n-GaAs/p-GaAs. Figure 6 shows the dependence of the nonlinear absorption coefficient on intensity E_0 of electromagnetic wave in a doped superlattice. When intensity E_0 of the electromagnetic wave rises up, its absorption coefficient speeds up too. The absorption coefficient in bulk semiconductors is smaller than it is in a doped superlattice. Otherwise, the absorption

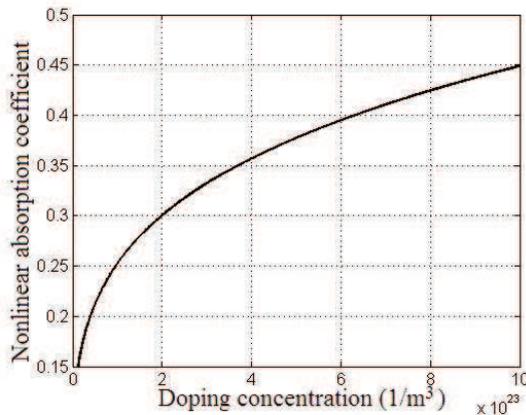


Fig. 7. The dependence of α on the n_D in a doped superlattice

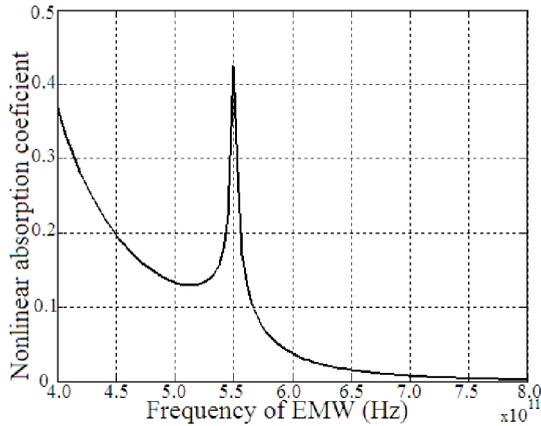


Fig. 8. The dependence of α on the Ω in a doped superlattice

coefficient changes insignificantly in bulk semiconductor. Figure 7 shows that the nonlinear absorption coefficient in a doped superlattice depends strongly on the doping concentration n_D . When the doping concentration of the system rises up, its absorption coefficient speeds up too.

Figure 8 presents the dependence of the nonlinear absorption coefficient on the frequency of the electromagnetic wave. This figure shows that the curve has a maximum coincide with the case $\Omega = \omega_0$. That is, appear a resonance peak at $\Omega = \omega_0$. However, compared with quantum well, these absorption peaks are sharper.

4. The nonlinear absorption of a strong electromagnetic wave by confined electrons in a cylindrical quantum wire

4.1 The electron distribution function in a cylindrical quantum wire

The Hamiltonian of the electron-phonon system in quantum wires. in the presence of a laser field $\vec{E}(t) = \vec{E}_0 \sin(\Omega t)$, can be written as

$$H = \sum_{n,l,\vec{p}} \varepsilon_{n,l} (\vec{p} - \frac{e}{c} \vec{A}(t)) a_{n,l,\vec{p}}^+ a_{n,l,\vec{p}} + \sum_{\vec{q}} \omega_{\vec{q}} b_{\vec{q}}^+ b_{\vec{q}} + \sum_{n,l,n',l',\vec{p},\vec{q}} C_{\vec{q}} I_{n,l,n',l'}(\vec{q}) a_{n,l,\vec{p}+\vec{q}}^+ a_{n',l',\vec{p}} (b_{\vec{q}} + b_{-\vec{q}}^+), \quad (23)$$

where e is the electron charge, c is the velocity of light, $\vec{A}(t) = \frac{c}{\Omega} \vec{E}_0 \cos(\Omega t)$ is the vector potential, \vec{E}_0 and Ω are the intensity and the frequency of the electromagnetic wave, $a_{n,\ell,\vec{p}}^+$ ($a_{n,\ell,\vec{p}}$) is the creation (annihilation) operator of an electron, $b_{\vec{q}}^+$ ($b_{\vec{q}}$) is the creation (annihilation) operator of a phonon for a state having wave vector \vec{q} , $\omega_{\vec{q}}$ is the frequency of a phonon, $C_{\vec{q}}$ is the electron-phonon interaction constants. $I_{n,l,n',l'}(\vec{q})$ is the electron form factor. In order to establish expressions for the electron distribution function in quantum wires, we use the quantum kinetic equation for particle number operator of an electron, $n_{n,\ell,\vec{p}}(t) = \langle a_{n,\ell,\vec{p}}^+ a_{n,\ell,\vec{p}} \rangle_t$:

$$i \frac{\partial n_{n,\ell,\vec{p}}(t)}{\partial t} = \langle [a_{n,\ell,\vec{p}}^+ a_{n,\ell,\vec{p}}, H] \rangle_t. \tag{24}$$

From Eq. (24), using the Hamiltonian in Eq. (23) and realizing the calculations, we obtain the quantum kinetic equation for the confined electrons in a cylindrical quantum wire. Using the first-order tautology approximation method to solve this equation, we obtain the expression of electron distribution function in cylindrical quantum wires, $n_{n,\ell,\vec{p}}(t)$:

$$n_{n,\ell,\vec{p}}(t) = - \sum_{\vec{q},n',\ell'} |C_{\vec{q}}|^2 |I_{n,\ell,n',\ell'}|^2 \sum_{k,l=-\infty}^{\infty} J_k \left(\frac{e\vec{E}_0 \cdot \vec{q}}{m\Omega^2} \right) J_{k+l} \left(\frac{e\vec{E}_0 \cdot \vec{q}}{m\Omega^2} \right) \frac{1}{l\Omega} e^{-il\Omega t} \times$$

$$\times \left\{ - \frac{\bar{n}_{n,\ell,\vec{p}}(N_{\vec{q}} + 1) - \bar{n}_{n',\ell',\vec{p}+\vec{q}} N_{\vec{q}}}{\varepsilon_{n',\ell',\vec{p}+\vec{q}} - \varepsilon_{n,\ell,\vec{p}} + \omega_{\vec{q}} - k\Omega + i\delta} - \frac{\bar{n}_{n,\ell,\vec{p}} N_{\vec{q}} - \bar{n}_{n',\ell',\vec{p}+\vec{q}}(N_{\vec{q}} + 1)}{\varepsilon_{n',\ell',\vec{p}+\vec{q}} - \varepsilon_{n,\ell,\vec{p}} - \omega_{\vec{q}} - k\Omega + i\delta} + \right.$$

$$\left. + \frac{\bar{n}_{n',\ell',\vec{p}-\vec{q}}(N_{\vec{q}} + 1) - \bar{n}_{n,\ell,\vec{p}} N_{\vec{q}}}{\varepsilon_{n,\ell,\vec{p}} - \varepsilon_{n',\ell',\vec{p}-\vec{q}} + \omega_{\vec{q}} - k\Omega + i\delta} + \frac{\bar{n}_{n',\ell',\vec{p}-\vec{q}} N_{\vec{q}} - \bar{n}_{n,\ell,\vec{p}}(N_{\vec{q}} + 1)}{\varepsilon_{n,\ell,\vec{p}} - \varepsilon_{n',\ell',\vec{p}-\vec{q}} - \omega_{\vec{q}} - k\Omega + i\delta} \right\}, \tag{25}$$

where $N_{\vec{q}}$ ($\bar{n}_{n,\vec{p}}$) is the time-independent component of the phonon (electron) distribution function, $J_k(x)$ is the Bessel function, and the quantity δ is infinitesimal and appears due to the assumption of an adiabatic interaction of the electromagnetic wave. Eq.(25) also can be considered a general expression of the electron distribution function in quantum wires.

4.2 Calculations of the nonlinear absorption coefficient of a strong electromagnetic wave by confined electrons in a cylindrical quantum wire

We consider a wire of GaAs with a circular cross section with a radius R and a length L_z embedded in AlAs. The carriers (electrons) are assumed to be confined by infinite potential barriers and to be free along the wire’s axis (Oz). It is noted that a cylindrical quantum wire with radius $R \sim 160 \text{ \AA}$ has already been fabricated experimentally. In this case, the total wave function of electrons in cylindrical coordinates (r, ϕ, z) takes the form (Zakhleniuk et al., 1996)

$$\psi_{n,\ell,\vec{p}}(r, \phi, z) = \frac{1}{\sqrt{V_0}} e^{in\phi} e^{ip_z z} \psi_{n,\ell}(r), \quad r < R, \tag{26}$$

where $V_0 = \pi R^2 L_z$ is the wire volume, $n = 0, \pm 1, \pm 2, \dots$ is the azimuthal quantum number, $\ell = 1, 2, 3, \dots$ is the radial quantum number, $\vec{p} = (0, 0, p_z)$ is the electron wave vector (along the wire’s z axis), and $\psi_{n,\ell}(r)$ is the wave function of electron moving in the (x, y) plane and takes the form

$$\psi_{n,\ell}(r) = \frac{1}{J_{n+1}(B_{n,\ell})} J_n(B_{n,\ell} \frac{r}{R}), \tag{27}$$

with $B_{n,\ell}$ being the ℓ -th root of the n -th order Bessel function, corresponding to the equation $J_n(B_{n,\ell}) = 0$, for example, $B_{01} = 2.405$ and $B_{11} = 3.832$. The electron energy spectrum takes the form [18]

$$\varepsilon_{n,\ell}(\vec{p}) = \varepsilon(p_z) + \varepsilon_{n,\ell}, \tag{28}$$

where $\varepsilon(p_z) = p_z^2/2m$ is the electron kinetic energy in the z -direction and $\varepsilon_{n,\ell} = B_{n,\ell}^2/2mR^2$ is the quantized energy in the other directions, m is the effective mass of the electron.

The electron form factor can be written as (Wang & Lei, 1994)

$$I_{n,\ell,n',\ell'}(q_{\perp}) = \frac{2}{R^2} \int_0^R J_{|n-n'|}(q_{\perp}R) \psi_{n',\ell'}^*(r) \psi_{n,\ell}(q_{\perp}R) r dr. \tag{29}$$

Due to the complexity of the expression for the radial function in Eq. (26), the integral in Eq. (29) cannot be calculated analytically. However, according to (Gold & Ghazali, 1990), it can be calculated for ground states of electrons by applying the approximate expression for the wave function and for the energies of states: namely,

$$\psi_{0,1} \approx \sqrt{3}(1 - x^2), \quad I_{01,0,1}(q_{\perp}) = 24 \frac{J_3(q_{\perp}R)}{(q_{\perp}R)^3} \tag{30}$$

$$\psi_{\pm 1,1} \approx \sqrt{12}(x - x^2), \quad I_{\pm 1,1,0,1}(q_{\perp}) = 48 \frac{J_4(q_{\perp}R)}{(q_{\perp}R)^3}. \tag{31}$$

The carrier current density $\vec{j}(t)$ and the nonlinear absorption coefficient of a strong electromagnetic wave α take the form

$$\vec{j}(t) = \frac{e}{m} \sum_{n,\ell,\vec{p}} (\vec{p} - \frac{e}{c} \vec{A}(t)) n_{n,\ell,\vec{p}}(t); \quad \alpha = \frac{8\pi}{c\sqrt{\chi_{\infty}}E_0^2} \langle \vec{j}(t) \vec{E}_0 \sin \Omega t \rangle_t, \tag{32}$$

In order to establish analytical expressions for the nonlinear absorption coefficient of a strong electromagnetic wave by confined electrons in a cylindrical quantum wire, We insert the expression for $n_{n,\ell,\vec{p}}(t)$ into the expression for $\vec{j}(t)$ and then insert the expression for $\vec{j}(t)$ into the expression for α in Eq.(32). Using the properties of Bessel function and realizing the calculations, we obtain the nonlinear absorption coefficient of a strong electromagnetic wave by confined electrons in a cylindrical quantum wire as

$$\alpha = \frac{8\pi^2\Omega}{c\sqrt{\chi_{\infty}}E_0^2} \sum_{n,\ell,n',\ell'} |I_{n,\ell,n',\ell'}|^2 \sum_{\vec{q},\vec{p}} |C_{\vec{q}}|^2 N_{\vec{q}} \sum_{k=-\infty}^{\infty} [\bar{n}_{n,\ell,\vec{p}} - \bar{n}_{n',\ell',\vec{p}+\vec{q}}] \times \\ \times k J_k^2 \left(\frac{eE_0\vec{q}}{m\Omega^2} \right) \left\{ \delta \left(\frac{(\vec{p} + \vec{q})^2}{2m} + \frac{B_{n',\ell'}^2}{2mR^2} - \frac{\vec{p}^2}{2m} + \frac{B_{n,\ell}^2}{2mR^2} + \omega_{\vec{q}} - k\Omega \right) + [\omega_{\vec{q}} \rightarrow -\omega_{\vec{q}}] \right\}, \tag{33}$$

where $\delta(x)$ is the Dirac delta function.

4.2.1 Electron-acoustic phonon scattering

Using the electron - acoustic phonon interaction factor $C_{\vec{q}}^{ac}$, the time-independent component of the electron distribution function $n_{n,\vec{p}_{\perp}}$ the Bessel function and the energy spectrum of an electron in a cylindrical quantum wire, we obtain an explicit expression for α in a cylindrical quantum wire for the case of electron-acoustic phonon scattering:

$$\alpha = \frac{\sqrt{2m\pi}e^2n_0^*\xi^2(k_bT)^{5/2}}{4c\sqrt{\chi_{\infty}}\rho v_s^2\Omega^3V} \sum_{n,\ell,n',\ell'} |I_{n,\ell,n',\ell'}|^2 \exp \left\{ \frac{1}{2k_bT} D_1 \right\} \left[1 - \exp \left\{ \frac{\Omega}{k_bT} \right\} \right] \times \\ \times \frac{D_1}{2k_bT} \left[1 + \frac{3e^2E_0^2(k_bT)^2}{4m\Omega^4D_1} \left(\frac{D_1^2}{4(k_bT)^2} + \frac{3D_1}{4k_bT} + 3 \right) \right], \tag{34}$$

where $D_1 = (B_{n',\ell'}^2 - B_{n,\ell}^2)/2mR^2 - \Omega$.

4.2.2 Electron-optical Phonon Scattering

By using the electron - optical phonon interaction factor $C_{\vec{q}}^{op}$, the Bessel function and the time-independent component of the electron distribution function n_{n,\vec{p}_\perp} , from the general expression for the nonlinear absorption coefficient of a strong electromagnetic wave in a quantum well (Eq.33), we obtain the explicit expression for α in a cylindrical quantum wire for the case of electron-optical phonon scattering:

$$\alpha = \frac{\sqrt{2\pi}e^4 n_0^*(k_b T)^{3/2}}{4c\epsilon_0 \sqrt{m\chi_\infty} \Omega^3 V} \left(\frac{1}{\chi_\infty} - \frac{1}{\chi_0} \right) \sum_{n\ell, \hat{n}, \hat{\ell}} |I_{n\ell, \hat{n}, \hat{\ell}}|^2 \left\{ \left[\exp\left\{ \frac{1}{k_b T} (\omega_0 - \Omega) \right\} - 1 \right] \times \right. \\ \left. \times \exp\left\{ \frac{1}{k_b T} B_1 \right\} \left[1 + \frac{3e^2 E_0^2 k_b T}{8m\Omega^4} \left(1 + \frac{B_1}{2k_b T} \right) \right] \right] + [\omega_0 \rightarrow -\omega_0] \right\}, \quad (35)$$

where $B_1 = (B_{n,\ell'}^2 - B_{n,\ell}^2)/2mR^2 + \omega_0 - \Omega$.

From the analytic expressions for the nonlinear absorption coefficient of a strong electromagnetic wave caused by confined electrons in a cylindrical quantum wire with an infinite potential (Eq. 34 and Eq. 35), we can see that when the term proportional to the quadratic intensity of the electromagnetic wave (E_0^2) tends to zero, the nonlinear result will become a linear result (Bau et al., 2007).

4.3 Numerical results and discussions

In order to clarify the results that have been obtained, in this section, we numerically calculate the nonlinear absorption coefficient of a strong electromagnetic wave for a *GaAs/GaAsAl* cylindrical quantum wire. The nonlinear absorption coefficient is considered as a function of the intensity E_0 and energy of strong electromagnetic wave, the temperature T of the system, the radius R of cylindrical quantum wire. The parameters used in the numerical calculations (Ariza-Flores & Rodriguez-Vargas, 2008; Bau et al., 2002) are $\zeta = 13.5eV$, $\rho = 5.32gcm^{-3}$, $v_s = 5378ms^{-1}$, $\epsilon_0 = 12.5$, $\chi_\infty = 10.9$, $\chi_0 = 13.1$, $m = 0.066m_0$, m_0 being the mass of free electron, $\hbar\omega_0 = 36.25meV$, $k_b = 1.3807 \times 10^{-23}j/K$, $n_0 = 10^{23}m^{-3}$, $e = 1.60219 \times 10^{-19}C$, $\hbar = 1.05459 \times 10^{-34}j.s$.

4.3.1 Electron-acoustic phonon scattering

Figure 9 shows the dependence of the nonlinear absorption coefficient of a strong electromagnetic wave on the wire's radius at different values of the intensity, E_0 , of the electromagnetic wave. It can be seen from this figure that the absorption coefficient depends strongly and nonlinearly on the radius R of the wire. The absorption has the same maximum values (peaks), but with different values of the radius of the wire. For example, at $E_0 = 1,6 \times 10^6(V/m)$ and $E_0 = 3,6 \times 10^6(V/m)$, the peaks correspond to $R \sim 23nm$ and $R \sim 28nm$ respectively. The absorption coefficient has negative values, which was seen in the case linear absorption (Bau et al., 2007) and is the difference between quantum wires and bulk semiconductors (Pavlovich & Epshtein, 1977) as well as quantum wells and doped duperlattices. α was changed strongly by the confinement of electron in a cylindrical quantum wire.

The Figure 10 presents the dependence of the nonlinear absorption coefficient α on the electromagnetic wave energy at different values of the wire's radius R . It is seen that different from the normal bulk semiconductors(Pavlovich & Epshtein, 1977) and two-dimensional systems, the nonlinear absorption coefficient α in quantum wire has the maximum values

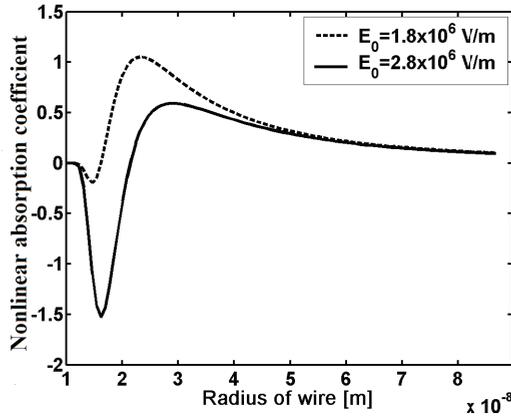


Fig. 9. The dependence of α on R in a cylindrical quantum wire (electron-acoustic phonon scattering)

(peaks). The electromagnetic wave energy at which α has a maximum are changed as the radius R of wire is varied.

Figure 11 shows the dependence of the nonlinear absorption coefficient α on the temperature T of the system at different values of the wire's radius R . It can be seen from this figure that the nonlinear absorption coefficient α has depends strongly and nonlinear on T . The nonlinear absorption coefficient α has the same maximum value, but with different values of T . For example, at $R = 15\text{nm}$ and $R = 25\text{nm}$, the peaks correspond to $T \sim 135\text{K}$ and 120K , respectively, it is also a difference compared to the normal bulk semiconductors (Pavlovich & Epshtein, 1977), quantum wells and doped superlattices . To start from the maximum value, the nonlinear absorption coefficient α decreases when the temperature T rises.

Figure 12 presents the dependence of the nonlinear absorption coefficient α on the intensity E_0 of electromagnetic wave. This dependence shows that the nonlinear absorption coefficient α is descending when the intensity E_0 of electromagnetic wave increases. Different from

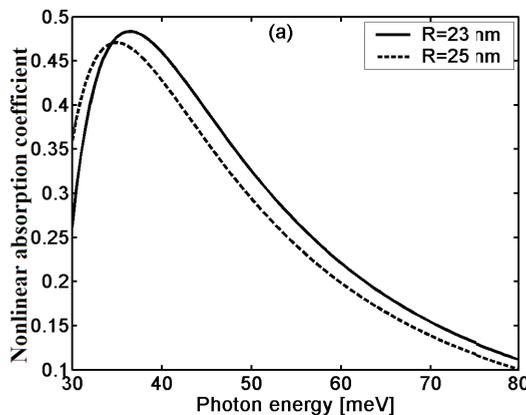


Fig. 10. The dependence of α on $\hbar\Omega$ in a cylindrical quantum wire (electron-acoustic phonon scattering)

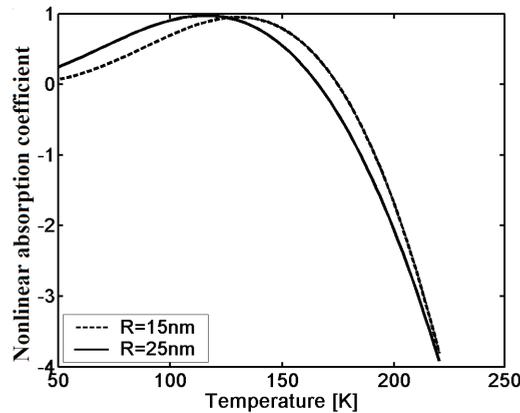


Fig. 11. The dependence of α on T in a cylindrical quantum wire (electron-acoustic phonon scattering)

normal bulk semiconduction (Pavlovich & Epshtein, 1977) and two-dimensional systems, the nonlinear absorption coefficient α in quantum wire is bigger. This is explained that when electrons are confined in quantum wire, the electron energy spectrum continue to be quantized. So the absorption of a strong electromagnetic wave is better. This fact is also reflected in the expressions of the nonlinear absorption coefficient (Eqs 34-35). Besides the sum over quantum n (as in quantum well), the expressions of the nonlinear absorption coefficient in quantum wire have the sum over the quantum number ℓ .

4.3.2 Electron-optical phonon scattering

Figures 13 shows the dependence of α on the radius R of wires in the case electron- optical phonon scattering. It can be seen from this figure that like in the case electron- acoustic phonon scattering, the nonlinear absorption coefficient α has the peak. But the absorption coefficient

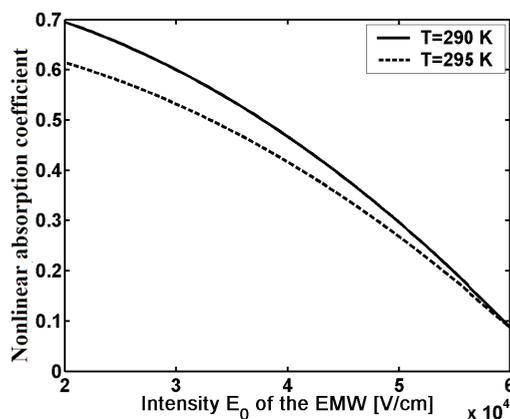


Fig. 12. The dependence of α on E_0 in a cylindrical quantum wire (electron-acoustic phonon scattering)

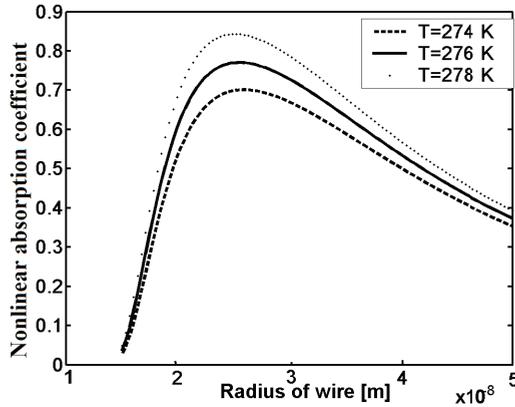


Fig. 13. The dependence of α on radius R in a cylindrical quantum wire (electron-optical phonon scattering)

does not have not negative values. Figure 14 presents the dependence of α on the intensity E_0 of electromagnetic wave. Different from the case electron - acoustic phonon scattering, in this case, α increases when the intensity E_0 of electromagnetic wave increases. Figure 15 presents the dependence of α on the electromagnetic wave energy at different values of the radius of wire. It is seen that α has the same maximum values (peaks) at $\Omega \equiv \omega$. The electromagnetic wave energy at which α has a maximum are not changed as the radius of wire is varied. This means that α depends strongly on the frequency Ω of the electromagnetic wave and resonance conditions are determined by the electromagnetic wave energy.

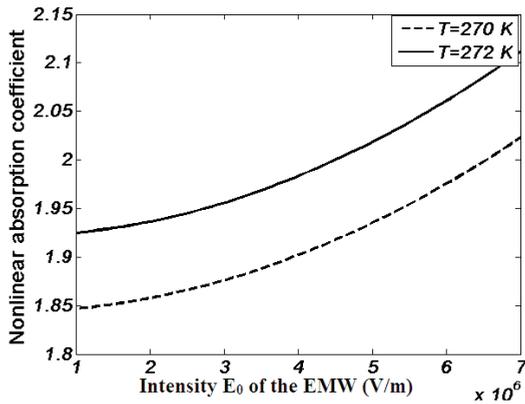


Fig. 14. The dependence of α on the intensity E_0 in a cylindrical quantum wire (electron-optical phonon scattering)

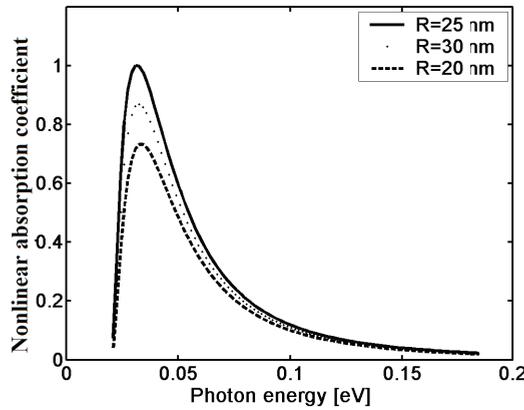


Fig. 15. The dependence of α on $\hbar\Omega$ in a cylindrical quantum wire (electron-optical phonon scattering)

5. The nonlinear absorption of a strong electromagnetic wave by confined electrons in a rectangular quantum wire

5.1 Calculations of the nonlinear absorption coefficient of a strong electromagnetic wave by confined electrons in a rectangular quantum wire

In our model, we consider a wire of GaAs with rectangular cross section ($L_x \times L_y$) and length L_z , embedded in GaAlAs. The carriers (electron gas) are assumed to be confined by an infinite potential in the (x, y) plane and are free in the z direction in Cartesian coordinates (x, y, z) . The laser field propagates along the x direction. In this case, the state and the electron energy spectra have the form (Mickevicius & Mitin, 1993)

$$|n, \ell, \vec{p}\rangle = \frac{2e^{ip_z z}}{\sqrt{L_z L_x L_y}} \sin\left(\frac{\pi n x}{L_x}\right) \sin\left(\frac{\pi \ell y}{L_y}\right); \epsilon_{n, \ell}(\vec{p}) = \frac{p^2}{2m} + \frac{\pi^2}{2m} \left(\frac{n^2}{L_x^2} + \frac{\ell^2}{L_y^2}\right) \quad (36)$$

where n and ℓ ($n, \ell=1, 2, 3, \dots$) denote the quantization of the energy spectrum in the x and y direction, $\vec{p} = (0, 0, p_z)$ is the electron wave vector (along the wire's z axis), m is the effective mass of electron. The electron form factor, it is written as (Mickevicius & Mitin, 1993)

$$I_{n, \ell, \hat{n}, \hat{\ell}}(\vec{q}) = \frac{32\pi^4 (q_x L_x n \hat{n})^2 (1 - (-1)^{n+\hat{n}} \cos(q_x L_x))}{[(q_x L_x)^4 - 2\pi^2 (q_x L_x)^2 (n^2 + \hat{n}^2) + \pi^4 (n^2 - \hat{n}^2)^2]^2} \times \frac{32\pi^4 (q_y L_y \ell \hat{\ell})^2 (1 - (-1)^{\ell+\hat{\ell}} \cos(q_y L_y))}{[(q_y L_y)^4 - 2\pi^2 (q_y L_y)^2 (\ell^2 + \hat{\ell}^2) + \pi^4 (\ell^2 - \hat{\ell}^2)^2]^2} \quad (37)$$

In order to establish analytical expressions for the nonlinear absorption coefficient of a strong electromagnetic wave by confined electrons in a rectangular quantum wire, we insert the expression of $n_{n, \ell, \vec{p}}(t)$ into the expression of $\vec{j}(t)$ and then insert the expression of $\vec{j}(t)$ into the expression of α . Using properties of Bessel function and realizing calculations, we obtain the nonlinear absorption coefficient of a strong electromagnetic wave by confined electrons in a rectangular quantum wire.

$$\alpha = \frac{8\pi^2\Omega}{c\sqrt{\chi_\infty}E_0^2} \sum_{n,\ell,n',\ell'} |I_{n,\ell,n',\ell'}|^2 \sum_{\vec{q},\vec{p}} |C_{\vec{q}}|^2 N_{\vec{q}} \sum_{k=-\infty}^{\infty} [\bar{n}_{n,\ell,\vec{p}} - \bar{n}_{n',\ell',\vec{p}+\vec{q}}] \times$$

$$\times k J_k^2\left(\frac{eE_0\vec{q}}{m\Omega^2}\right) \left\{ \delta\left(\frac{(\vec{p}+\vec{q})^2 - \vec{p}^2}{2m} + \frac{\pi^2}{2m} \left(\frac{n'^2 - n^2}{L_x^2} + \frac{\ell'^2 - \ell^2}{L_y^2}\right) + \omega_{\vec{q}} - k\Omega\right) + [\omega_{\vec{q}} \rightarrow -\omega_{\vec{q}}] \right\}$$
(38)

In the following, we study the problem with different electron-phonon scattering mechanisms.

5.1.1 Electron-optical phonon scattering

In this case, $\omega_{\vec{q}} \equiv \omega_0$ is the frequency of the optical phonon in the equilibrium state. Using the electron-optical phonon interaction constants $C_{\vec{q}}^{op}$, Bessel function and Fermi-Dirac distribution function for electron, we obtain the explicit expression of α in a rectangular quantum wire for the case electron-optical phonon scattering

$$\alpha = \frac{\sqrt{2\pi}e^4 n_0^* (k_b T)^{3/2}}{4c\epsilon_0 \sqrt{m\chi_\infty} \Omega^3 V} \left(\frac{1}{\chi_\infty} - \frac{1}{\chi_0}\right) \sum_{n,\ell,\hat{n},\hat{\ell}} |I_{n,\ell,\hat{n},\hat{\ell}}|^2 \left\{ \left[\exp\left\{\frac{1}{k_b T}(\omega_0 - \Omega)\right\} - 1 \right] \times \right.$$

$$\left. \times \exp\left\{\frac{1}{k_b T} \frac{\pi^2}{2m} \left(\frac{\hat{n}^2}{L_x^2} + \frac{\hat{\ell}^2}{L_y^2}\right)\right\} \left[1 + \frac{3e^2 E_0^2 k_b T}{8m\Omega^4} \left(1 + \frac{B}{2k_b T}\right) \right] \right\} + [\omega_0 \rightarrow -\omega_0]$$
(39)

where $B = \pi^2[(\hat{n}^2 - n^2)/L_x^2 + (\hat{\ell}^2 - \ell^2)/L_y^2]/2m + \omega_0 - \Omega$.

5.1.2 Electron-acoustic phonon scattering

In the case, $\omega_{\vec{q}} \ll \Omega$ ($\omega_{\vec{q}}$ is the frequency of acoustic phonon), so we let it pass. Using electron-acoustic phonon interaction constants $C_{\vec{q}}^{ac}$, we obtain the explicit expression of α in a rectangular quantum wire for the case electron-acoustic phonon scattering

$$\alpha = \frac{\sqrt{2m\pi}e^2 n_0^* \zeta_5^2 (k_b T)^{5/2}}{4c\sqrt{\chi_\infty} \rho v_s^2 \Omega^3 V} \sum_{n,\ell,n',\ell'} |I_{n,\ell,n',\ell'}|^2 \exp\left\{\frac{1}{k_b T} \frac{\pi^2}{2m} \left(\frac{\hat{n}^2}{L_x^2} + \frac{\hat{\ell}^2}{L_y^2}\right)\right\} \times$$

$$\times \left[\exp\left\{\frac{\Omega}{k_b T}\right\} - 1 \right] \left[1 + \frac{D}{2k_b T} \left[1 + \frac{3e^2 E_0^2 (k_b T)^2}{4m\Omega^4 D} \left(\frac{D^2}{4(k_b T)^2} + \frac{3D}{4k_b T} + 3\right) \right] \right]$$
(40)

where $D = \pi^2[(\hat{n}^2 - n^2)/L_x^2 + (\hat{\ell}^2 - \ell^2)/L_y^2] - \Omega$

5.2 Numerical results and discussions

In order to clarify the results that have been obtained, in this section, we numerically calculate the nonlinear absorption coefficient of a strong electromagnetic wave for a *GaAs/GaAsAl* rectangular quantum wire. The nonlinear absorption coefficient is considered as a function of the intensity E_0 and energy of strong electromagnetic wave, the temperature T of the system, and the parameters of a rectangular quantum wire.

Figure 16 shows the dependence of α of a strong electromagnetic wave on the size L (L_y and L_x) of wire. It can be seen from this figure that α depends strongly and nonlinearly on size L of

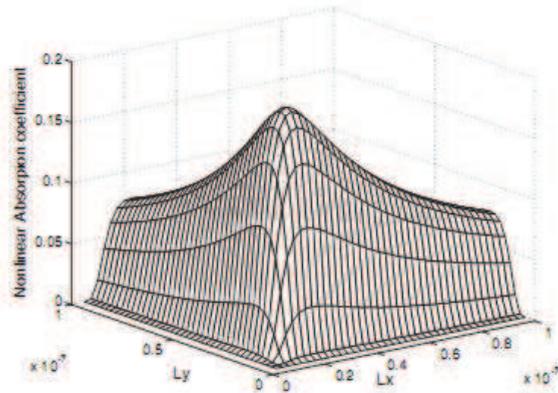


Fig. 16. The dependence of α on L_y and L_x in a rectangular quantum wire (electron-acoustic phonon scattering)

wire. When L decreases, the nonlinear absorption coefficient will increase until its maximum at L_x and $L_y \sim 24nm$ then started to decrease.

Figure 17 presents the dependence of the nonlinear absorption coefficient α on the temperature T of the system at different values of the intensity E_0 of the external strong electromagnetic wave. It can be seen from this figure that the nonlinear absorption coefficient α depends strongly and nonlinearly on the temperature T and it has the same maximum value but with different values of T . For example, at $E_0 = 2.6 \times 10^6 V/m$ and $E_0 = 2 \times 10^6 V/m$, the peaks correspond to $T \sim 170K$ and $190K$, respectively, this fact was not seen in bulk semiconductors (Pavlovich & Epshtein, 1977) as well as quantum wells and doped superlattices, but it fits the case of linear absorption (Bau et al., 2007)

Figure 18 presents the dependence of α on the electromagnetic wave energy at different values of the radius of wire. It is seen that α has the same maximum values (peaks) at $\Omega \equiv \omega_0$. The electromagnetic wave energy at which α has a maximum are not changed as the radius of wire

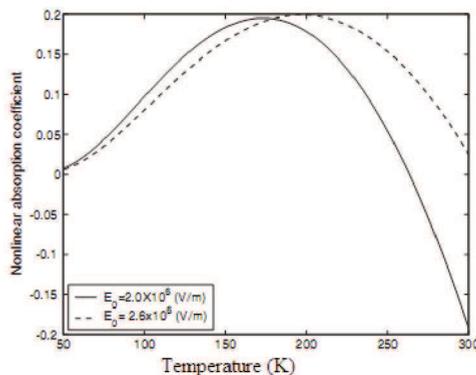


Fig. 17. The dependence of α on T in a rectangular quantum wire (electron-acoustic phonon scattering)

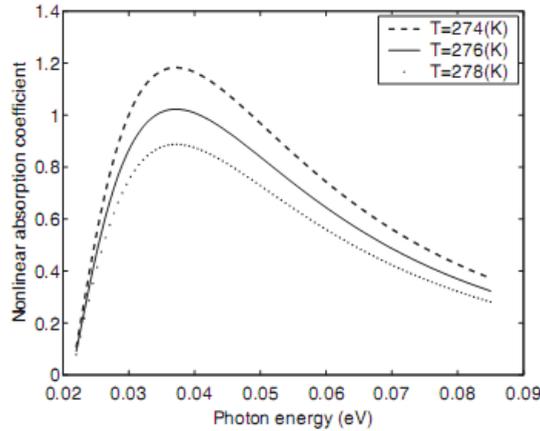


Fig. 18. The dependence of α on $\hbar\Omega$ in a rectangular quantum wire (electron-optical phonon scattering)

is varied. This means that α depends strongly on the frequency Ω of the electromagnetic wave and resonance conditions are determined by the electromagnetic wave energy.

6. Conclusion

In this chapter, the nonlinear absorption of a strong electromagnetic wave by confined electrons in low-dimensional systems is investigated. By using the method of the quantum kinetic equation for electrons, the expressions for the electron distribution function and the nonlinear absorption coefficient in quantum wells, doped superlattices, cylindrical quantum wires and rectangular quantum wires are obtained. The analytic results show that the nonlinear absorption coefficient depends on the intensity E_0 and the frequency Ω of the external strong electromagnetic wave, the temperature T of the system and the parameters of the low-dimensional systems as the width L of quantum well, the doping concentration n_D in doped superlattices, the radius R of cylindrical quantum wires, size L_x and L_y of rectangular quantum wires. This dependence are complex and has difference from those obtained in normal bulk semiconductors (Pavlovich & Epshtein, 1977), the expressions for the nonlinear absorption coefficient has the sum over the quantum number n (in quantum wells and doped superlattices) or the sum over two quantum numbers n and ℓ (in quantum wires). It shows that the electron confinement in low dimensional systems has changed significantly the nonlinear absorption coefficient. In addition, from the analytic results, we see that when the term in proportion to a quadratic in the intensity of the electromagnetic wave (E_0^2)(in the expressions for the nonlinear absorption coefficient of a strong electromagnetic wave) tend toward zero, the nonlinear result will turn back to a linear result (Bau & Phong, 1998; Bau et al., 2002; 2007). The numerical results obtained for a AlAs/GaAs/AlAs quantum well, a n-GaAs/p-GaAs doped superlattice, a GaAs/GaAsAl cylindrical quantum wire and a a GaAs/GaAsAl rectangular quantum wire show that α depends strongly and nonlinearly on the intensity E_0 and the frequency Ω of the external strong electromagnetic wave, the temperature T of the system, the parameters of the low-dimensional systems. In particular, there are differences between the nonlinear absorption of a strong electromagnetic

wave in low-dimensional systems and the nonlinear absorption of a strong electromagnetic wave in normal bulk semiconductors (Pavlovich & Epshtein, 1977), the nonlinear absorption coefficient in a low-dimensional systems has the same maximum values (peaks) at $\Omega \equiv \omega_0$, the electromagnetic wave energies at which α has maxima are not changed as other parameters is varied, the nonlinear absorption coefficient in a low-dimensional systems is bigger. The results show a geometrical dependence of α due to the confinement of electrons in low-dimensional systems. The nonlinear absorption in each low-dimensional systems is also different, for example, these absorption peaks in doped superlattices are sharper than those in quantum wells, the nonlinear absorption coefficient in quantum wires is bigger than those in quantum wells and doped superlattices,... It shows that the nonlinear absorption of a strong electromagnetic wave by confined electrons depends significantly on the structure of each low-dimensional systems.

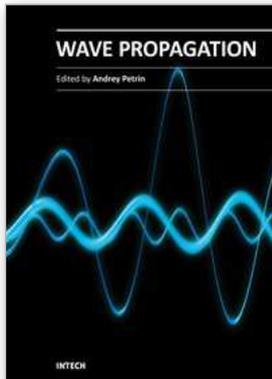
However in this study we have not considered the effect of confined phonon in low-dimensional systems, the influence of external magnetic field (or a weak electromagnetic wave) on the nonlinear absorption of a strong electromagnetic wave. This is still open for further studying.

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