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Exponential Stability of Uncertain Switched System with Time-Varying Delay

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1. Introduction

During the past decades, many researchers have investigated stability of switched systems; due to its potential for real world application such as transportation systems, computer systems, communication systems, control of mechanical systems, etc. A switched systems is composed of a family of continuous time (Alan & Lib, 2008; Alan & Lib, 2009, Alan et al., 2008; Hien et al., 2009; Hien & Phat, 2009; Kim et al., 2006; Li et al., 2009; Niamsup, 2008; Li et al., 2009; Lien et al., 2009; Lib et al., 2008) or discrete time systems (Wu et al., 2004) and a switching condition determining at any time instant which subsystem is activated.

In recent years, the stability of systems with time delay has received considerable attention. Switched system in which all subsystems are stable was studied in (Lien et al., 2009) and switched system in which subsystems are both stable and unstable was studied in (Alan & Lib, 2008; Alan & Lib, 2009, Alan et al., 2008). The commonly used approach to stability analysis of switched systems is Lyapunov theory and some important preliminaries results have been applied to obtain sufficient conditions for stability of switched systems. A single Lyapunov function approach is used in (Alan & Lib, 2008) and a multiple Lyapunov functions approach is used in (Hien et al., 2009; Kim et al., 2006; Li et al., 2009; Lien et al., 2009; Lib et al., 2008) and the references therein. The asymptotical stability of the linear with time delay and uncertainties has been considered in (Lien et al., 2009). In (L.V.Hien et al., 2009), the authors investigated the exponential stability and stabilization of switched linear systems with time varying delay and uncertainties by using the strictly complete systems of matrices approach. The strictly complete of the matrices has been also used for the switching condition, see (Hien et al., 2009; Huang et al., 2005; Niamsup, 2008; Lib et al., 2008; Wu et al., 2004). In this paper, stability analysis for switched linear and nonlinear systems with uncertainties and time-varying delay are studied. We obtain the new conditions for exponential stability of switched system in which subsystems consist of stable and unstable subsystems. The stability conditions are derived in terms of linear matrix inequality (LMI) by using a new Lyapunov

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function. The free weighting matrices and Newton-Leibniz formula are applied. As a results, the obtained stability conditions are less conservative comparing to some previous existing results in the literatures. In particular, comparing to (Alan & Lib, 2008), our results give a much less conservative results, namely, for stable subsystems, the condition that state matrices are Hurwitz stable is not required. Moreover, advantages of the paper are that the delay is time-varying and switched system may have uncertainties. The paper is organized as follows. In section 1, problem formulation and introduction is addressed. In section 2, we give some notations, definitions and the preliminary results that will be used in this paper. Switching design for the exponential stability of the switched system is presented in Section 3. In section 4, numerical examples are given to illustrate the theoretical results. The paper ends with conclusions and cited references.

2. Preliminaries

The following notations will be used throughout this paper. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space. $\mathbb{R}^{n\times n}$ denotes the space of all matrices of $n \times n$-dimensions. A^T denotes the transpose of A. I denotes the identity matrix. $\lambda(A), \lambda_M(A), \lambda_m(A)$ denote the set of all eigenvalues of A, the maximum eigenvalue of A, and the minimum eigenvalue of A, respectively. For all real symmetric matrix X, the notation $X > 0 (X \geq 0, X < 0, X \leq 0)$ means that X is positive definite (positive semidefinite, negative definite, negative semidefinite, respectively.) For a vector x, $\|x_t\| = \sup_{s\in[-h_M,0]} \|x(t+s)\|$ with $\|x\|$ being the Euclidean norm of vector x.

The switched system under the consideration is described by

$$\begin{aligned}\dot{x}(t) &= [A_\sigma + \Delta A_\sigma(t)]x(t) + [B_\sigma + \Delta B_\sigma(t)]x(t-h(t)) \\ &\quad + f_\sigma(t, x(t), x(t-h(t))),\ t > 0, \\ x(t) &= \phi(t),\ t \in [-h_M, 0], \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector. $\sigma(\cdot) : \mathbb{R}^n \to S = \{1, 2, ..., N\}$ is the switching function. Let $i \in S = S_u \cup S_s$ such that $S_u = \{1, 2, ..., r\}$ and $S_s = \{r+1, r+2, ..., N\}$ be the set of the unstable and stable modes, respectively. N denotes the number of subsystems. $A_i, B_i \in \mathbb{R}^{n\times n}$ are given constant matrices. $\Delta A_i(t), \Delta B_i(t)$ are uncertain matrices satisfying the following conditions:

$$\Delta A_i(t) = E_{1i}F_{1i}(t)H_{1i},\ \Delta B_i(t) = E_{2i}F_{2i}(t)H_{2i}, \tag{2}$$

where $E_{ji}, H_{ji}, j = 1, 2, i = 1, 2, ..., N$ are given constant matrices with appropriate dimensions. $F_{ji}(t)$ are unknown, real matrices satisfying:

$$F_{ji}^T(t)F_{ji}(t) \leq I,\ j = 1, 2,\ i = 1, 2, ..., N,\ \forall t \geq 0, \tag{3}$$

where I is the identity matrix of appropriate dimension.
The nonlinear perturbation $f_i(t, x(t), x(t-h(t)))$, $i = 1, 2, ..., N$ satisfies the following condition:

$$\| f_i(t, x(t), x(t-h(t))) \| \leq \gamma_i \| x(t) \| + \delta_i \| x(t-h(t)) \| \tag{4}$$

for some γ_i, $\delta_i > 0$. The time-varying delay function $h(t)$ is assumed to satisfy one of the following conditions:

(i) when $\Delta A_i(t) = 0$ and $\Delta B_i(t) = 0$ and $f_i(t, x(t), x(t-h(t))) = 0$

$$0 \leq h_m \leq h(t) \leq h_M,\ \dot{h}(t) \leq \mu,\ t \geq 0,$$

(ii) when $\Delta A_i(t) \neq 0$ or $\Delta B_i(t) \neq 0$ or $f_i(t, x(t), x(t-h(t))) \neq 0$

$$0 \leq h_m \leq h(t) \leq h_M,\ \dot{h}(t) \leq \mu < 1,\ t \geq 0,$$

where h_m, h_M and μ are given constants.

Definition 2.1 (Hien et al., 2009) Given $\beta > 0$. The system (1) is $\beta-$exponentially stable if there exists a switching function $\sigma(\cdot)$ and positive number γ such that any solution $x(t, \phi)$ of the system satisfies

$$\| x(t,\phi) \| \leq \gamma e^{-\beta t} \| \phi \|,\ \forall t \in \mathbb{R}^+,$$

for all the uncertainties.

Lemma 2.1 (Hien et al., 2009) For any $x, y \in \mathbb{R}^n$, matrices W, E, F, H with $W > 0, F^T F \leq I$, and scalar $\varepsilon > 0$, one has

(1.) $EFH + H^T F^T E^T \leq \varepsilon^{-1} E E^T + \varepsilon H^T H,$

(2.) $2x^T y \leq x^T W^{-1} x + y^T W y.$

Lemma 2.2 (Alan & Lib, 2008) Let $u : [t_0, \infty] \to \mathbb{R}$ satisfy the following delay differential inequality:

$$\dot{u}(t) \leq \alpha u(t) + \beta \sup_{\theta \in [t-\tau, t]} u(\theta),\ t \geq t_0.$$

Assume that $\alpha + \beta > 0$. Then, there exist positive constant ξ and k such that

$$u(t) \leq k e^{\xi(t-t_0)},\ t \geq t_0,$$

where $\xi = \alpha + \beta$ and $k = \sup_{\theta \in [t_0-\tau, t_0]} u(\theta)$.

Lemma 2.3 (Alan & Lib, 2008) Let the following differential inequality:

$$\dot{u} \leq -\alpha u(t) + \beta \sup_{\theta \in [t-\tau, t]} u(\theta),\ t \geq t_0,$$

hold. If $\alpha > \beta > 0$, then there exist positive k and ζ such that

$$u(t) \leq k e^{-\zeta(t-t_0)},\ t \geq t_0,$$

where $\zeta = \alpha - \beta$ and $k = \sup_{\theta \in [t_0-\tau, t_0]} u(\theta)$.

Lemma 2.4 (Schur Complement Lemma) (Boyd et al., 1985) Given constant symmetric Q, S and $R \in \mathbb{R}^{n \times n}$ where $R > 0, Q = Q^T$ and $R = R^T$ we have

$$\begin{bmatrix} Q & S \\ S^T & -R \end{bmatrix} < 0 \Leftrightarrow Q + S R^{-1} S^T < 0.$$

3. Main results

In this section, we establish exponential stability of uncertain switched system with time-varying delay. For simplicity of later presentation, we use the following notations:

$\lambda^+ = \max_i \{\xi_i, \forall i \in S_u\}$, ξ_i denotes the growth rates of the unstable modes.

$\lambda^- = \min_i \{\zeta_i, \forall i \in S_s\}$, ζ_i denotes the decay rates of the stable modes.

$T^{+}(t_0, t)$ denotes the total activation times of the unstable modes over $[t_0, t)$.
$T^{-}(t_0, t)$ denotes the total activation times of the stable modes over $[t_0, t)$.
$N(t)$ denotes the number of times the system is switched on $[t_0, t)$.
$l(t)$ denotes the number of times the unstable subsystems are activated on $[t_0, t)$.
$N(t) - l(t)$ denotes the number of times the stable subsystems are activated on $[t_0, t)$.

$\psi = \dfrac{\max\limits_i\{\lambda_M(P_i)\}}{\min\limits_j\{\lambda_m(P_j)\}}$.

$\alpha_1 = \min\limits_i\{\lambda_m(P_i)\}$.

$$\alpha_2 = \max_i\{\lambda_M(P_i)\} + h_M \max_i\{\lambda_M(Q_i)\} + \frac{h_M^2}{2}\max_i\{\lambda_M(R_i)\} + h_M^2 \max_i\{\lambda_M(\begin{bmatrix} S_{11,i} & S_{12,i} \\ S_{12,i}^T & S_{22,i} \end{bmatrix})\} + 2h_M^2 \max_i\{\lambda_M(A_i^T T_i A_i), \lambda_M(A_i^T T_i B_i), \lambda_M(B_i^T T_i A_i), \lambda_M(B_i^T T_i B_i)\},$$

$$\alpha_3 = \max_i\{\lambda_M(P_i)\} + h_M \max_i\{\lambda_M(Q_i)\} + \frac{h_M^2}{2}\max_i\{\lambda_M(R_i)\} + h_M^2 \max_i\{\lambda_M(\begin{bmatrix} S_{11,i} & S_{12,i} \\ S_{12,i}^T & S_{22,i} \end{bmatrix})\}.$$

$$\Omega_{1,i} = \begin{bmatrix} \Phi_{11,i} & \Phi_{12,i} \\ * & \Phi_{13,i} \end{bmatrix},$$

$$\Phi_{11,i} = A_i^T P_i + P_i A_i + Q_i + h_M R_i + h_M S_{11,i} + h_M A_i^T T_i A_i,$$
$$\Phi_{12,i} = B_i^T P_i + h_M S_{12,i} + h_M A_i^T T_i B_i,$$
$$\Phi_{13,i} = -(1-\mu)e^{-2\beta h_M} Q_i + h_M S_{22,i} + h_M B_i^T T_i B_i.$$

$$\Omega_{2,i} = \begin{bmatrix} \Phi_{21,i} & \Phi_{22,i} \\ * & \Phi_{23,i} \end{bmatrix},$$

$$\Phi_{21,i} = A_i^T P_i + P_i A_i + Q_i + h_M R_i + h_M S_{11,i} + h_M A_i^T T_i A_i + h_M X_{11,i} + Y_i + Y_i^T,$$
$$\Phi_{22,i} = B_i^T P_i + h_M S_{12,i} + h_M A_i^T T_i B_i + h_M X_{12,i} - Y_i + Z_i^T,$$
$$\Phi_{23,i} = -(1-\mu)e^{-2\beta h_M} Q_i + h_M S_{22,i} + h_M B_i^T T_i B_i + h_M X_{22,i} - Z_i - Z_i^T.$$

$$\Omega_{3,i} = \begin{bmatrix} X_{11,i} & X_{12,i} & Y_i \\ * & X_{22,i} & Z_i \\ * & * & \frac{T_i}{2} \end{bmatrix}.$$

$$\Xi_i = \begin{bmatrix} \Phi_{31,i} & \Phi_{32,i} \\ * & \Phi_{33,i} \end{bmatrix},$$

$$\Phi_{31,i} = A_i^T P_i + P_i A_i + Q_i + h_M R_i + h_M S_{11,i} + \varepsilon_{1i}^{-1} H_{1i}^T H_{1i} + \varepsilon_{1i} P_i E_{1i}^T E_{1i} P_i + \varepsilon_{2i} P_i E_{2i}^T E_{2i} P_i,$$
$$\Phi_{32,i} = B_i^T P_i + h_M S_{12,i},$$
$$\Phi_{33,i} = -(1-\mu)e^{-2\beta h_M} Q_i + h_M S_{22,i} + \varepsilon_{2i}^{-1} H_{2i}^T H_{2i}.$$

$$\Theta_i = \begin{bmatrix} \Phi_{41,i} & \Phi_{42,i} \\ * & \Upsilon_{43,i} \end{bmatrix},$$

$$\Phi_{41,i} = A_i^T P_i + P_i A_i + Q_i + h_M R_i + h_M S_{11,i} + \varepsilon_{3i}^{-1}\gamma_i I + \varepsilon_{3i} P_i P_i + \varepsilon_{4i}^{-1} H_{4i}^T H_{4i} + \varepsilon_{4i} P_i E_{4i}^T E_{4i} P_i + \varepsilon_{6i} P_i E_{5i}^T E_{5i} P_i,$$
$$\Phi_{42,i} = B_i^T P_i + h_M S_{12,i},$$

$\Phi_{43,i} = -(1-\mu)e^{-2\beta h_M}Q_i + h_M S_{22,i} + \varepsilon_{3i}^{-1}\delta_i I + \varepsilon_{5i}^{-1}H_{5i}^T H_{5i}.$

3.1 Exponential stability of linear switched system with time-varying delay

In this section, we deal with the problem for exponential stability of the zero solution of system (1) without the uncertainties and nonlinear perturbation ($\Delta A_i(t) = \Delta B_i(t) = 0,\ f_i(t, x(t), x(t-h(t))) = 0$).

Theorem 3.1 *The zero solution of system (1) with* $\Delta A_i(t) = \Delta B_i(t) = 0$ *and* $f_i(t, x(t), x(t-h(t))) = 0$ *is exponentially stable if there exist symmetric positive definite matrices* $P_i, Q_i, R_i,$ $\begin{bmatrix} S_{11,i} & S_{12,i} \\ S_{12,i}^T & S_{22,i} \end{bmatrix}$, T_i *and appropriate dimension matrices* Y_i, Z_i *such that the following conditions hold:*

$A1$. (i) *For* $i \in S_u$,

$$\Omega_{1,i} > 0. \tag{5}$$

(ii) *For* $i \in S_s$,

$$\Omega_{2,i} < 0 \ \text{ and } \ \Omega_{3,i} \geq 0. \tag{6}$$

$A2$. *Assume that, for any* t_0 *the switching law guarantees that*

$$\inf_{t \geq t_0} \frac{T^-(t_0, t)}{T^+(t_0, t)} \geq \frac{\lambda^+ + \lambda^*}{\lambda^- - \lambda^*} \tag{7}$$

where $\lambda^* \in (0, \lambda^-)$. *Furthermore, there exists* $0 < \nu < \lambda^*$ *such that*

(i) *If the subsystem* $i \in S_u$ *is activated in time intervals* $[t_{i_k - 1}, t_{i_k}), k = 1, 2, ...,$ *then*

$$ln\ \psi - \nu(t_{i_k} - t_{i_k - 1}) \leq 0,\ k = 1, 2, ..., l(t). \tag{8}$$

(ii) *If the subsystem* $j \in S_s$ *is activated in time intervals* $[t_{j_k - 1}, t_{j_k}), k = 1, 2, ...,$ *then*

$$ln\ \psi + \zeta_j h_M - \nu(t_{j_k} - t_{j_k - 1}) \leq 0,\ k = 1, 2, ..., N(t) - 1. \tag{9}$$

Proof. Consider the following Lyapunov functional:

$$V_i(x_t) = V_{1,i}(x(t)) + V_{2,i}(x_t) + V_{3,i}(x_t) + V_{4,i}(x_t) + V_{5,i}(x_t)$$

where $x_t \in C([-h_M, 0], \mathbb{R}^n),\ x_t(s) = x(t+s),\ s \in [-h_M, 0]$ and

$V_{1,i}(x(t)) = x^T(t)P_i x(t),$

$V_{2,i}(x_t) = \int_{t-h(t)}^{t} e^{2\beta(s-t)} x^T(s) Q_i x(s) ds,$

$V_{3,i}(x_t) = \int_{-h(t)}^{0} \int_{t+s}^{t} e^{2\beta(\xi - t)} x^T(\xi) R_i x(\xi) d\xi ds,$

$V_{4,i}(x_t) = \int_{-h(t)}^{0} \int_{t+s}^{t} e^{2\beta(\xi - t)} \begin{bmatrix} x(\xi) \\ x(\xi - h(\xi)) \end{bmatrix}^T \begin{bmatrix} S_{11,i} & S_{12,i} \\ S_{12,i}^T & S_{22,i} \end{bmatrix} \begin{bmatrix} x(\xi) \\ x(\xi - h(\xi)) \end{bmatrix} d\xi ds,$

$V_{5,i}(x_t) = \int_{-h(t)}^{0} \int_{t+s}^{t} \dot{x}^T(\xi) T_i \dot{x}(\xi) d\xi ds.$

It is easy to verify that

$$\alpha_1 \parallel x(t) \parallel^2 \leq V_i(x_t) \leq \alpha_2 \parallel x_t \parallel^2,\ t \geq 0. \tag{10}$$

We have

$$\begin{aligned}V_{1,i}(x(t)) &\leq \max_i\{\lambda_M(P_i)\} \parallel x(t) \parallel^2 \\ &= \frac{\max_i\{\lambda_M(P_i)\}}{\min_j\{\lambda_m(P_j)\}} \min_j\{\lambda_m(P_j)\}x^T(t)x(t) \\ &\leq \frac{\max_i\{\lambda_M(P_i)\}}{\min_j\{\lambda_m(P_j)\}} x^T(t)P_jx(t) \\ &= \frac{\max_i\{\lambda_M(P_i)\}}{\min_j\{\lambda_m(P_j)\}} V_{1,j}(x(t)).\end{aligned}$$

Let $\psi = \frac{\max_i\{\lambda_M(P_i)\}}{\min_j\{\lambda_m(P_j)\}}$. Obviously $\psi \geq 1$ and we get

$$V_i(x_t) \leq \psi V_j(x_t),\ \forall i,j \in S. \tag{11}$$

Taking derivative of $V_{1,i}(x(t))$ along trajectories of any subsystem ith we have

$$\begin{aligned}\dot{V}_{1,i}(x(t)) &= \dot{x}^T(t)P_ix(t) + x^T(t)P_i\dot{x}(t) \\ &= \sum_{i=1}^{N} \lambda_i(t)[x^T(t)A_i^TP_ix(t) + x^T(t-h(t))B_i^TP_ix(t) \\ &\quad +x^T(t)P_iA_ix(t) + x^T(t)P_iB_ix(t-h(t))].\end{aligned}$$

Next, by taking derivative of $V_{2,i}(x_t), V_{3,i}(x_t), V_{4,i}(x_t)$ and $V_{5,i}(x_t)$, respectively, along the system trajectories yields

$$\begin{aligned}\dot{V}_{2,i}(x_t) &= x^T(t)Q_ix(t) - (1-\dot{h}(t))e^{-2\beta h(t)}x^T(t-h(t))Q_ix(t-h(t)) - 2\beta V_{2,i}(x_t) \\ &\leq x^T(t)Q_ix(t) - (1-\mu)e^{-2\beta h(t)}x^T(t-h(t))Q_ix(t-h(t)) - 2\beta V_{2,i}(x_t),\end{aligned}$$

$$\begin{aligned}\dot{V}_{3,i}(x_t) &= \int_{-h(t)}^{0}[x^T(t)R_ix(t) - e^{2\beta s}x^T(t+s)R_ix(t+s)]ds - 2\beta V_{3,i}(x_t) \\ &\leq h_Mx^T(t)R_ix(t) - \int_{t-h(t)}^{t} e^{2\beta(s-t)}x^T(s)R_ix(s)ds - 2\beta V_{3,i}(x_t),\end{aligned}$$

$$\begin{aligned}
\dot{V}_{4,i}(x_t) = & \int_{-h(t)}^{0} [\begin{bmatrix} x(\xi) \\ x(\xi - h(\xi)) \end{bmatrix}^T \begin{bmatrix} S_{11,i} & S_{12,i} \\ S_{12,i}^T & S_{22,i} \end{bmatrix} \begin{bmatrix} x(\xi) \\ x(\xi - h(\xi)) \end{bmatrix} \\
& - e^{2\beta s} \begin{bmatrix} x(t+s) \\ x(t+s-h(t+s)) \end{bmatrix}^T \begin{bmatrix} S_{11,i} & S_{12,i} \\ S_{12,i}^T & S_{22,i} \end{bmatrix} \begin{bmatrix} x(t+s) \\ x(t+s-h(t+s)) \end{bmatrix}] ds \\
& - e^{2\beta s} \begin{bmatrix} x(t+s) \\ x(t+s-h(t+s)) \end{bmatrix}^T \begin{bmatrix} S_{11,i} & S_{12,i} \\ S_{12,i}^T & S_{22,i} \end{bmatrix} \begin{bmatrix} x(t+s) \\ x(t+s-h(t+s)) \end{bmatrix}] ds \\
& - 2\beta V_{4,i}(x_t) \\
\leq & \; h_M \begin{bmatrix} x(t) \\ x(t-h(t)) \end{bmatrix}^T \begin{bmatrix} S_{11,i} & S_{12,i} \\ S_{12,i}^T & S_{22,i} \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-h(t)) \end{bmatrix} \\
& - \int_{t-h(t)}^{t} e^{2\beta(s-t)} \begin{bmatrix} x(s) \\ x(s-h(s)) \end{bmatrix}^T \begin{bmatrix} S_{11,i} & S_{12,i} \\ S_{12,i}^T & S_{22,i} \end{bmatrix} \begin{bmatrix} x(s) \\ x(s-h(s)) \end{bmatrix} ds \\
& - e^{2\beta s} \begin{bmatrix} x(t+s) \\ x(t+s-h(t+s)) \end{bmatrix}^T \begin{bmatrix} S_{11,i} & S_{12,i} \\ S_{12,i}^T & S_{22,i} \end{bmatrix} \begin{bmatrix} x(t+s) \\ x(t+s-h(t+s)) \end{bmatrix}] ds \\
& - 2\beta V_{4,i}(x_t) \\
\leq & \; h_M \begin{bmatrix} x(t) \\ x(t-h(t)) \end{bmatrix}^T \begin{bmatrix} S_{11,i} & S_{12,i} \\ S_{12,i}^T & S_{22,i} \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-h(t)) \end{bmatrix} \\
& - \int_{t-h(t)}^{t} e^{2\beta(s-t)} \begin{bmatrix} x(s) \\ x(s-h(s)) \end{bmatrix}^T \begin{bmatrix} S_{11,i} & S_{12,i} \\ S_{12,i}^T & S_{22,i} \end{bmatrix} \begin{bmatrix} x(s) \\ x(s-h(s)) \end{bmatrix} ds \\
& - 2\beta V_{4,i}(x_t),
\end{aligned}$$

$$\begin{aligned}
\dot{V}_{5,i}(x_t) = & \int_{-h(t)}^{0} [\dot{x}^T(t) T_i \dot{x}(t) - \dot{x}^T(t+s) T_i \dot{x}(t+s)] ds \\
\leq & \; h_M \dot{x}^T(t) T_i \dot{x}(t) - \int_{t-h(t)}^{t} \dot{x}^T(s) T_i \dot{x}(s) ds \\
= & \; h_M \dot{x}^T(t) T_i \dot{x}(t) - \frac{1}{2} \int_{t-h(t)}^{t} \dot{x}^T(s) T_i \dot{x}(s) ds - \frac{1}{2} \int_{t-h(t)}^{t} \dot{x}^T(s) T_i \dot{x}(s) ds.
\end{aligned}$$

Then, the derivative of $V_i(x_t)$ along the any trajectory of solution of (1) is estimated by

$$\begin{aligned}
\dot{V}_i(x_t) \leq & \sum_{i=1}^{N} \lambda_i(t) \begin{bmatrix} x(t) \\ x(t-h(t)) \end{bmatrix}^T \Omega_{1,i}^{\star} \begin{bmatrix} x(t) \\ x(t-h(t)) \end{bmatrix} - 2\beta V_{2,i}(x_t) \\
& - \int_{t-h(t)}^{t} e^{2\beta(s-t)} x^T(s) R_i x(s) ds - 2\beta V_{3,i}(x_t) \\
& - \int_{t-h(t)}^{t} e^{2\beta(s-t)} \begin{bmatrix} x(s) \\ x(s-h(s)) \end{bmatrix}^T \begin{bmatrix} S_{11,i} & S_{12,i} \\ S_{12,i}^T & S_{22,i} \end{bmatrix} \begin{bmatrix} x(s) \\ x(s-h(s)) \end{bmatrix} ds \\
& - 2\beta V_{4,i}(x_t) + h_M \dot{x}(t)^T T_i \dot{x}(t) - \frac{1}{2} \int_{t-h(t)}^{t} \dot{x}^T(s) T_i \dot{x}(s) ds \\
& - \frac{1}{2} \int_{t-h(t)}^{t} \dot{x}^T(s) T_i \dot{x}(s) ds, \qquad (12)
\end{aligned}$$

where

$$\Omega^{\star}_{1,i} = \begin{bmatrix} A_i^T P_i + P_i A_i + Q_i + h_M R_i + h_M S_{11,i} & B_i^T P_i + h_M S_{12,i} \\ * & -(1-\mu)e^{-2\beta h_M} Q_i + h_M S_{22,i} \end{bmatrix}$$

Since

$$\begin{aligned}\int_{-h(t)}^{0}\int_{t+s}^{t} e^{2\beta(\xi-t)}x^T(\xi)R_i x(\xi)d\xi ds &\le \int_{-h(t)}^{0}\int_{t-h(t)}^{t} e^{2\beta(\xi-t)}x^T(\xi)R_i x(\xi)d\xi ds \\ &\le h_M \int_{t-h(t)}^{t} e^{2\beta(s-t)}x^T(s)R_i x(s)ds,\end{aligned}$$

we have

$$\begin{aligned}-\int_{t-h(t)}^{t} e^{2\beta(s-t)}x^T(s)R_i x(s)ds &\le -\frac{1}{h_M}\int_{-h(t)}^{0}\int_{t+s}^{t} e^{2\beta(\xi-t)}x^T(\xi)R_i x(\xi)d\xi ds \\ &= -\frac{1}{h_M}V_{3,i}(x_t). \end{aligned} \tag{13}$$

Similarly, we have

$$-\int_{t-h(t)}^{t} e^{2\beta(s-t)} \begin{bmatrix} x(s) \\ x(s-h(s)) \end{bmatrix}^T \begin{bmatrix} S_{11,i} & S_{12,i} \\ S_{12,i}^T & S_{22,i} \end{bmatrix} \begin{bmatrix} x(s) \\ x(s-h(s)) \end{bmatrix} ds \le -\frac{1}{h_M}V_{4,i}(x_t), \tag{14}$$

and

$$-\frac{1}{2}\int_{t-h(t)}^{t} \dot{x}(s)T_i\dot{x}(s)ds \le -\frac{1}{2h_M}V_{5,i}(x_t). \tag{15}$$

From (12), (13), (14) and (15), we obtain

$$\begin{aligned}\dot{V}_i(x_t) \le & \sum_{i=1}^{N}\lambda_i(t)\begin{bmatrix} x(t) \\ x(t-h(t)) \end{bmatrix}^T \Omega_{1,i} \begin{bmatrix} x(t) \\ x(t-h(t)) \end{bmatrix} - 2\beta V_{2,i}(x_t) \\ & -(2\beta + \frac{1}{h_M})(V_{3,i}(x_t) + V_{4,i}(x_t)) - \frac{1}{2h_M}V_{5,i}(x_t) \\ & -\frac{1}{2}\int_{t-h(t)}^{t} \dot{x}(s)T_i\dot{x}(s)ds. \end{aligned} \tag{16}$$

For $i \in S_u$, we have

$$\dot{V}_i(x_t) \le \sum_{i=1}^{N}\lambda_i(t)\begin{bmatrix} x(t) \\ x(t-h(t)) \end{bmatrix}^T \Omega_{1,i} \begin{bmatrix} x(t) \\ x(t-h(t)) \end{bmatrix}.$$

By (5), (16) and Lemma 2.2, there exists $\xi_i > 0$ such that

$$V_i(x_t) \le \sum_{i=1}^{N}\lambda_i(t) \parallel V_i(x_{t_0}) \parallel e^{\xi_i(t-t_0)},\ t \ge t_0. \tag{17}$$

where $\xi_i = \dfrac{2\max\limits_i\{\lambda_M(\Omega_{1,i})\}}{\min\limits_i\{\lambda_m(P_i)\}}$.

For $i \in S_s$, we have that when $X_i = \begin{bmatrix} X_{11,i} & X_{12,i} \\ * & X_{22,i} \end{bmatrix} \geq 0$, the following holds:

$$h_M \begin{bmatrix} x(t) \\ x(t-h(t)) \end{bmatrix}^T X_i \begin{bmatrix} x(t) \\ x(t-h(t)) \end{bmatrix}$$
$$-\int_{t-h(t)}^{t} e^{2\beta(s-t)} \begin{bmatrix} x(t) \\ x(t-h(t)) \end{bmatrix}^T X_i \begin{bmatrix} x(t) \\ x(t-h(t)) \end{bmatrix} ds \geq 0. \tag{18}$$

Using the Newton-Leibniz formula, (Wu et al., 2004), we can write

$$x(t-h(t)) = x(t) - \int_{t-h(t)}^{t} \dot{x}(s)ds.$$

Then, for any appropriate dimension matrices Y_i and Z_i, we have

$$2[x^T(t)Y_i + x^T(t-h(t))Z_i][x(t) - \int_{t-h(t)}^{t} \dot{x}(s)ds - x(t-h(t))] = 0.$$

It follows that

$$2x^T(t)Y_i x(t) - 2x^T(t)Y_i \int_{t-h(t)}^{t} \dot{x}(s)ds - 2x^T(t)Y_i x(t-h(t)) + 2x^T(t-h(t))Z_i x(t)$$
$$-2x^T(t-h(t))Z_i \int_{t-h(t)}^{t} \dot{x}(s)ds - 2x^T(t-h(t))Z_i x(t-h(t)) = 0. \tag{19}$$

From (16) with (18) and (19), we have

$$\dot{V}_i(x_t) \leq \sum_{i=1}^{N} \lambda_i(t) \begin{bmatrix} x(t) \\ x(t-h(t)) \end{bmatrix}^T \Omega_{2,i} \begin{bmatrix} x(t) \\ x(t-h(t)) \end{bmatrix} - 2\beta V_{2,i}(x_t)$$
$$-(2\beta + \frac{1}{h_M})(V_{3,i}(x_t) + V_{4,i}(x_t)) - \frac{1}{2h_M} V_{5,i}(x_t)$$
$$-\int_{t-h(t)}^{t} \begin{bmatrix} x(t) \\ x(t-h(t)) \\ \dot{x}(s) \end{bmatrix}^T \Omega_{3,i} \begin{bmatrix} x(t) \\ x(t-h(t)) \\ \dot{x}(s) \end{bmatrix} ds. \tag{20}$$

By (6), (20) and Lemma 2.3, there exist $\zeta_i > 0$ such that

$$V_i(x_t) \leq \sum_{i=1}^{N} \lambda_i(t) \parallel V_i(x_{t_0}) \parallel e^{-\zeta_i(t-t_0)},\ t \geq t_0. \tag{21}$$

where $\zeta_i = \min\{\dfrac{\min\limits_i\{\lambda_m(-\Omega_{2,i})\}}{\max\limits_i\{\lambda_M(P_i)\}}, 2\beta, \dfrac{1}{2h_M}\}$.

Let $N(t)$ denotes the number of times the system is switched on $[t_0, t)$ such that $\lim\limits_{t\to+\infty} N(t) = +\infty$. Suppose that $\sigma(t_0) = i_0, \sigma(t_1) = i_1, ...$ and $\sigma(t) = i$.

Let $l(t)$ denotes the number of times the unstable subsystems are activated on $[t_0, t)$ and $N(t) - l(t)$ denotes the number of times the stable subsystems are activated on $[t_0, t)$. Suppose that $t_0 < t_1 < t_2 < ...$ and $\lim_{n \to +\infty} t_n = +\infty$.

From (11), (17) and (21), suppose that the j th subsystem of unstable mode is activated on the interval $[t_l, t_{l+1})$,

- if the i th subsystem of unstable mode is activated on the interval $[t_{l-1}, t_l)$, then
$V_j(x_t) \le \psi \parallel V_i(x_{t_{l-1}}) \parallel e^{\xi_i(t_l - t_{l-1})} e^{\xi_j(t-t_l)},\ t \in [t_l, t_{l+1})$.
- if the i th subsystem of stable mode is activated on the interval $[t_{l-1}, t_l)$, then
$V_j(x_t) \le \psi \parallel V_i(x_{t_{l-1}}) \parallel e^{-\zeta_i(t_l - t_{l-1})} e^{\xi_j(t-t_l)},\ t \in [t_l, t_{l+1})$.

Suppose that the j th subsystem of stable mode is activated on the interval $[t_l, t_{l+1})$,

- if the i th subsystem of unstable mode is activated on the interval $[t_{l-1}, t_l)$, then
$V_j(x_t) \le \psi \parallel V_i(x_{t_{l-1}}) \parallel e^{\xi_i(t_l - t_{l-1})} e^{-\zeta_j(t-t_l)},\ t \in [t_l, t_{l+1})$.
- if the i th subsystem of stable mode is activated on the interval $[t_{l-1}, t_l)$, then
$V_j(x_t) \le \psi \parallel V_i(x_{t_{l-1}}) \parallel e^{-\zeta_i(t_l - t_{l-1})} e^{-\zeta_j(t-t_l)},\ t \in [t_l, t_{l+1})$.

In general, we get

$$\begin{aligned} V_i(x_t) &\le \prod_{m=1}^{l(t)} \psi e^{\xi_{i_m}(t_m - t_{m-1})} \times \prod_{n=l(t)+1}^{N(t)-1} \psi e^{\zeta_{i_n} h_M} e^{-\zeta_{i_n}(t_n - t_{n-1})} \times \parallel V_{i_0}(x_{t_0}) \parallel e^{-\zeta_i(t - t_{N(t)-1})} \\ &\le \prod_{m=1}^{l(t)} \psi e^{\lambda^+(t_m - t_{m-1})} \times \prod_{n=l(t)+1}^{N(t)-1} \psi e^{\zeta_{i_n} h_M} e^{-\lambda^-(t_n - t_{n-1})} \times \parallel V_{i_0}(x_{t_0}) \parallel e^{-\lambda^-(t - t_{N(t)-1})}, \end{aligned}$$

$t \ge t_0$. Using (7), we have

$$V_i(x_t) \le \prod_{m=1}^{l(t)} \psi \times \prod_{n=l(t)+1}^{N(t)-1} \psi e^{\zeta_{i_n} h_M} \times \parallel V_{i_0}(x_{t_0}) \parallel e^{-\lambda^*(t-t_0)},\ t \ge t_0.$$

By (8) and (9), we get

$$V_i(x_t) \le \parallel V_{i_0}(x_{t_0}) \parallel e^{-(\lambda^* - \nu)(t-t_0)},\ t \ge t_0.$$

Thus, by (10), we have

$$\parallel x(t) \parallel \le \sqrt{\frac{\alpha_2}{\alpha_1}} \parallel x_{t_0} \parallel e^{-\frac{1}{2}(\lambda^* - \nu)(t-t_0)},\ t \ge t_0,$$

which concludes the proof of the Theorem 3.1. □

3.2 Robust exponential stability of linear switched system with time-varying delay

In this section, we give conditions for robust exponential stability of the zero solution of system (1) without nonlinear perturbation, namely $f_i(t, x(t), x(t - h(t))) = 0$. The following is the main result.

Theorem 3.2 *The zero solution of system (1) with $f_i(t, x(t), x(t-h(t))) = 0$ is robustly exponentially stable if there exist positive real numbers $\varepsilon_{1i}, \varepsilon_{2i}$, positive definite matrices P_i, Q_i, R_i and $\begin{bmatrix} S_{11,i} & S_{12,i} \\ S_{12,i}^T & S_{22,i} \end{bmatrix}$ such that the following conditions hold:*

A1. (i) For $i \in S_u$,

$$\Xi_i > 0. \tag{22}$$

(ii) For $i \in S_s$,

$$\Xi_i < 0. \tag{23}$$

A2. Assume that, for any t_0 *the switching law guarantees that*

$$\inf_{t\geq t_0} \frac{T^-(t_0,t)}{T^+(t_0,t)} \geq \frac{\lambda^+ + \lambda^*}{\lambda^- - \lambda^*} \tag{24}$$

where $\lambda^* \in (0, \lambda^-)$. *Furthermore, there exists* $0 < \nu < \lambda^*$ *such that*
(i) If the subsystem $i \in S_u$ *is activated in time intervals* $[t_{i_k-1}, t_{i_k}), k = 1, 2, ...,$
then

$$ln\ \psi - \nu(t_{i_k} - t_{i_k-1}) \leq 0,\ k = 1, 2, ..., l(t). \tag{25}$$

(ii) If the subsystem $j \in S_s$ *is activated in time intervals* $[t_{j_k-1}, t_{j_k}), k = 1, 2, ...,$
then

$$ln\ \psi + \zeta_j h_M - \nu(t_{j_k} - t_{j_k-1}) \leq 0,\ k = 1, 2, ..., N(t) - 1. \tag{26}$$

Proof. Consider the following Lyapunov functional:

$$V_i(x_t) = V_{1,i}(x(t)) + V_{2,i}(x_t) + V_{3,i}(x_t) + V_{4,i}(x_t)$$

where $x_t \in C([-h_M, 0], \mathbb{R}^n), x_t(s) = x(t+s), s \in [-h_M, 0]$, and $V_{1,i}(x(t)) = x^T(t)P_i x(t)$,

$$V_{2,i}(x_t) = \int_{t-h(t)}^{t} e^{2\beta(s-t)} x^T(s) Q_i x(s) ds,$$

$$V_{3,i}(x_t) = \int_{-h(t)}^{0} \int_{t+s}^{t} e^{2\beta(\xi-t)} x^T(\xi) R_i x(\xi) d\xi ds,$$

$$V_{4,i}(x_t) = \int_{-h(t)}^{0} \int_{t+s}^{t} e^{2\beta(\xi-t)} \begin{bmatrix} x(\xi) \\ x(\xi - h(\xi)) \end{bmatrix}^T \begin{bmatrix} S_{11,i} & S_{12,i} \\ S_{12,i}^T & S_{22,i} \end{bmatrix} \begin{bmatrix} x(\xi) \\ x(\xi - h(\xi)) \end{bmatrix} d\xi ds.$$

It is easy to verify that

$$\alpha_1 \parallel x(t) \parallel^2 \leq V_i(x_t) \leq \alpha_3 \parallel x_t \parallel^2,\ t \geq 0. \tag{27}$$

Similar to (11), we have

$$V_i(x_t) \leq \psi V_j(x_t),\ \forall i, j \in S. \tag{28}$$

Taking derivative of $V_{1,i}(x(t))$ along trajectories of any subsystem ith, we have

$$\begin{aligned} \dot{V}_{1,i}(x(t)) &= \dot{x}^T(t) P_i x(t) + x^T(t) P_i \dot{x}(t) \\ &= \sum_{i=1}^{N} \lambda_i(t) [x^T(t) A_i^T P_i x(t) + x^T(t) \Delta A_i^T(t) P_i x(t) + x^T(t-h(t)) B_i^T P_i x(t) \\ &\quad + x^T(t-h(t)) \Delta B_i^T(t) P_i x(t) + x^T(t) P_i A_i x(t) + x^T(t) P_i \Delta A_i(t) x(t) \\ &\quad + x^T(t) P_i B_i x(t-h(t)) + x^T(t) P_i \Delta B_i(t) x(t-h(t))]. \end{aligned}$$

Applying Lemma 2.1 and from (2) and (3), we get

$$\begin{aligned} 2x^T(t) \Delta A_i^T(t) P_i x(t) &\leq \varepsilon_{1i}^{-1} x^T(t) H_{1i}^T H_{1i} x(t) + \varepsilon_{1i} x^T(t) P_i E_{1i}^T E_{1i} P_i x(t), \\ 2x^T(t-h(t)) \Delta B_i^T(t) P_i x(t) &\leq \varepsilon_{2i}^{-1} x^T(t-h(t)) H_{2i}^T H_{2i} x(t-h(t)) + \varepsilon_{2i} x^T(t) P_i E_{2i}^T E_{2i} P_i x(t). \end{aligned}$$

Next, by taking derivative of $V_{2,i}(x_t), V_{3,i}(x_t)$ and $V_{4,i}(x_t)$, respectively, along the system trajectories yields

$$\dot{V}_{2,i}(x_t) \leq x^T(t)Q_i x(t) - (1-\mu)e^{-2\beta h(t)}x^T(t-h(t))Q_i x(t-h(t)) - 2\beta V_{2,i}(x_t),$$

$$\dot{V}_{3,i}(x_t) \leq h_M x^T(t)R_i x(t) - \int_{t-h(t)}^{t} e^{2\beta(s-t)}x^T(s)R_i x(s)ds - 2\beta V_{3,i}(x_t),$$

$$\begin{aligned}\dot{V}_{4,i}(x_t) \leq\; & h_M \begin{bmatrix} x(t) \\ x(t-h(t)) \end{bmatrix}^T \begin{bmatrix} S_{11,i} & S_{12,i} \\ S_{12,i}^T & S_{22,i} \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-h(t)) \end{bmatrix} \\ & - \int_{t-h(t)}^{t} e^{2\beta(s-t)} \begin{bmatrix} x(s) \\ x(s-h(s)) \end{bmatrix}^T \begin{bmatrix} S_{11,i} & S_{12,i} \\ S_{12,i}^T & S_{22,i} \end{bmatrix} \begin{bmatrix} x(s) \\ x(s-h(s)) \end{bmatrix} ds \\ & -2\beta V_{4,i}(x_t).\end{aligned}$$

Therefore, the estimation of derivative of $V_i(x_t)$ along any trajectory of solution of (1) is given by

$$\begin{aligned}\dot{V}_i(x_t) \leq\; & \sum_{i=1}^{N} \lambda_i(t) \begin{bmatrix} x(t) \\ x(t-h(t)) \end{bmatrix}^T \Xi_i \begin{bmatrix} x(t) \\ x(t-h(t)) \end{bmatrix} - 2\beta V_{2,i}(x_t) \\ & - \int_{t-h(t)}^{t} e^{2\beta(s-t)}x^T(s)R_i x(s)ds - 2\beta V_{3,i}(x_t) \\ & - \int_{t-h(t)}^{t} e^{2\beta(s-t)} \begin{bmatrix} x(s) \\ x(s-h(s)) \end{bmatrix}^T \begin{bmatrix} S_{11,i} & S_{12,i} \\ S_{12,i}^T & S_{22,i} \end{bmatrix} \begin{bmatrix} x(s) \\ x(s-h(s)) \end{bmatrix} ds \\ & -2\beta V_{4,i}(x_t). \end{aligned} \tag{29}$$

For $i \in S_u$, we have

$$\dot{V}_i(x_t) \leq \sum_{i=1}^{N} \lambda_i(t) \begin{bmatrix} x(t) \\ x(t-h(t)) \end{bmatrix}^T \Xi_i \begin{bmatrix} x(t) \\ x(t-h(t)) \end{bmatrix}.$$

Similar to Theorem 3.1, from (22) and (29), we get

$$V_i(x_t) \leq \sum_{i=1}^{N} \lambda_i(t) \parallel V_i(x_{t_0}) \parallel e^{\xi_i(t-t_0)},\ t \geq t_0, \tag{30}$$

where $\xi_i = \dfrac{2\max_i\{\lambda_M(\Xi_i)\}}{\min_i\{\lambda_m(P_i)\}}$.

For $i \in S_s$, from (13), (14) and (29), we have

$$\begin{aligned}\dot{V}_i(x_t) \leq\; & \sum_{i=1}^{N} \lambda_i(t) \begin{bmatrix} x(t) \\ x(t-h(t)) \end{bmatrix}^T \Xi_i \begin{bmatrix} x(t) \\ x(t-h(t)) \end{bmatrix} - 2\beta V_{2,i}(x_t) \\ & -(2\beta + \frac{1}{h_M})(V_{3,i}(x_t) + V_{4,i}(x_t)) \end{aligned} \tag{31}$$

Similar to Theorem 3.1, from (23) and (31), we get

$$V_i(x_t) \leq \sum_{i=1}^{N} \lambda_i(t) \parallel V_i(x_{t_0}) \parallel e^{-\zeta_i(t-t_0)},\ t \geq t_0. \tag{32}$$

where $\zeta_i = \min\{\frac{\min_i\{\lambda_m(-\Xi_i)\}}{\max_i\{\lambda_M(P_i)\}}, 2\beta\}$.

In general, from (28), (30) and (32), with the same argument as in the proof of Theorem 3.1, we get

$$V_i(x_t) \leq \prod_{m=1}^{l(t)} \psi e^{\lambda^+(t_m - t_{m-1})} \times \prod_{n=l(t)+1}^{N(t)-1} \psi e^{\zeta_{i_n} h_M} e^{-\lambda^-(t_n - t_{n-1})} \times \parallel V_{i_0}(x_{t_0}) \parallel e^{-\lambda^-(t - t_{N(t)-1})},$$

$t \geq t_0$. Using (24), we have

$$V_i(x_t) \leq \prod_{m=1}^{l(t)} \psi \times \prod_{n=l(t)+1}^{N(t)-1} \psi e^{\zeta_{i_n} h_M} \times \parallel V_{i_0}(x_{t_0}) \parallel e^{-\lambda^*(t-t_0)},\ t \geq t_0.$$

By (25) and (26), we get

$$V_i(x_t) \leq \parallel V_{i_0}(x_{t_0}) \parallel e^{-(\lambda^* - \nu)(t-t_0)},\ t \geq t_0.$$

Thus, by (27), we have

$$\parallel x(t) \parallel \leq \sqrt{\frac{\alpha_3}{\alpha_1}} \parallel x_{t_0} \parallel e^{-\frac{1}{2}(\lambda^* - \nu)(t-t_0)},\ t \geq t_0,$$

which concludes the proof of the Theorem 3.2. □

3.3 Robust exponential stability of switched system with time-varying delay and nonlinear perturbation

In this section, we deal with the problem for robust exponential stability of the zero solution of system (1).

Theorem 3.3 *The zero solution of system (1) is robust exponentially stable if there exist positive real numbers* $\varepsilon_{3i}, \varepsilon_{4i}, \varepsilon_{5i}$, *positive definite matrices* P_i, Q_i, R_i *and* $\begin{bmatrix} S_{11,i} & S_{12,i} \\ S_{12,i}^T & S_{22,i} \end{bmatrix}$ *such that the following conditions hold:*

*A*1. (i) *For* $i \in S_u$,

$$\Theta_i > 0. \tag{33}$$

(ii) *For* $i \in S_s$,

$$\Theta_i < 0. \tag{34}$$

*A*2. *Assume that, for any* t_0 *the switching law guarantees that*

$$\inf_{t \geq t_0} \frac{T^-(t_0, t)}{T^+(t_0, t)} \geq \frac{\lambda^+ + \lambda^*}{\lambda^- - \lambda^*} \tag{35}$$

where $\lambda^* \in (0, \lambda^-)$. *Furthermore, there exists* $0 < \nu < \lambda^*$ *such that*
(i) *If the subsystem* $i \in S_u$ *is activated in time intervals* $[t_{i_k-1}, t_{i_k}), k = 1, 2, ...,$ *then*

$$ln\ \psi - \nu(t_{i_k} - t_{i_k-1}) \leq 0,\ k = 1, 2, ..., l(t). \quad (36)$$

(ii) *If the subsystem* $j \in S_s$ *is activated in time intervals* $[t_{j_k-1}, t_{j_k}), k = 1, 2, ...,$ *then*

$$ln\ \psi + \zeta_j h_M - \nu(t_{j_k} - t_{j_k-1}) \leq 0,\ k = 1, 2, ..., N(t) - 1. \quad (37)$$

Proof. Consider the following Lyapunov functional:

$$V_i(x_t) = V_{1,i}(x(t)) + V_{2,i}(x_t) + V_{3,i}(x_t) + V_{4,i}(x_t)$$

where $x_t \in C([-h_M, 0], \mathbb{R}^n), x_t(s) = x(t+s), s \in [-h_M, 0]$ and
$V_{1,i}(x(t)) = x^T(t)P_i x(t),$
$V_{2,i}(x_t) = \int_{t-h(t)}^{t} e^{2\beta(s-t)} x^T(s) Q_i x(s) ds,$
$V_{3,i}(x_t) = \int_{-h(t)}^{0} \int_{t+s}^{t} e^{2\beta(\xi-t)} x^T(\xi) R_i x(\xi) d\xi ds,$
$V_{4,i}(x_t) = \int_{-h(t)}^{0} \int_{t+s}^{t} e^{2\beta(\xi-t)} \begin{bmatrix} x(\xi) \\ x(\xi - h(\xi)) \end{bmatrix}^T \begin{bmatrix} S_{11,i} & S_{12,i} \\ S_{12,i}^T & S_{22,i} \end{bmatrix} \begin{bmatrix} x(\xi) \\ x(\xi - h(\xi)) \end{bmatrix} d\xi ds.$
It is easy to verify that

$$\alpha_1 \parallel x(t) \parallel^2 \leq V_i(x_t) \leq \alpha_3 \parallel x_t \parallel^2,\ t \geq 0. \quad (38)$$

Similar to (11), we have

$$V_i(x_t) \leq \psi V_j(x_t),\ \forall i, j \in S. \quad (39)$$

Taking derivative of $V_{1,i}(x(t))$ along trajectories of any subsystem ith we have

$$\begin{aligned}
\dot{V}_{1,i}(x(t)) &= \dot{x}^T(t)P_i x(t) + x^T(t)P_i \dot{x}(t) \\
&= \sum_{i=1}^{N} \lambda_i(t)[x^T(t)A_i^T P_i x(t) + x^T(t)\Delta A_i^T(t) P_i x(t) + x^T(t - h(t))B_i^T P_i x(t) \\
&\quad + x^T(t - h(t))\Delta B_i^T(t) P_i x(t) + f_i^T(t, x(t), x(t - h(t)))P_i x(t) + x^T(t)P_i A_i x(t) \\
&\quad + x^T(t)P_i \Delta A_i(t) x(t) + x^T(t)P_i B_i x(t - h(t)) + x^T(t)P_i \Delta B_i(t) x(t - h(t)) \\
&\quad + x^T(t)P_i f_i(t, x(t), x(t - h(t)))].
\end{aligned}$$

From lemma 2.1, we have

$$\begin{aligned}
2f_i^T(t, x(t), x(t-h(t)))P_i x(t) &\leq f_i^T(t, x(t), x(t-h(t)))W_i^{-1} f_i(t, x(t), x(t-h(t))) \\
&\quad + x^T(t)P_i W_i P_i x(t).
\end{aligned}$$

By choosing $W_i = \varepsilon_{3i} I_i$ and from (4), we have

$$\begin{aligned}
2f_i^T(t, x(t), x(t-h(t)))P_i x(t) &\leq \varepsilon_{3i}^{-1} f_i^T(t, x(t), x(t-h(t))) f_i(t, x(t), x(t-h(t))) \\
&\quad + \varepsilon_{3i} x^T(t) P_i P_i x(t) \\
&\leq \varepsilon_{3i}^{-1}[\gamma_i x^T(t)x(t) + \delta_i x^T(t-h(t))x(t-h(t))] \\
&\quad + \varepsilon_{3i} x^T(t) P_i P_i x(t).
\end{aligned}$$

Applying Lemma 2.1 and from (2) and (3), we get

$$2x^T(t)\Delta A_i^T(t)P_ix(t) \le \varepsilon_{4i}^{-1}x^T(t)H_{4i}^TH_{4i}x(t) + \varepsilon_{4i}x^T(t)P_iE_{4i}^TE_{4i}P_ix(t),$$
$$2x^T(t-h(t))\Delta B_i^T(t)P_ix(t) \le \varepsilon_{5i}^{-1}x^T(t-h(t))H_{5i}^TH_{5i}x(t-h(t)) + \varepsilon_{5i}x^T(t)P_iE_{5i}^TE_{5i}P_ix(t).$$

Next, by taking derivative of $V_{2,i}(x_t), V_{3,i}(x_t)$ and $V_{4,i}(x_t)$, respectively, along the system trajectories yields

$$\begin{aligned}
\dot V_{2,i}(x_t) &\le x^T(t)Q_ix(t) - (1-\mu)e^{-2\beta h(t)}x^T(t-h(t))Q_ix(t-h(t)) - 2\beta V_{2,i}(x_t),\\
\dot V_{3,i}(x_t) &\le h_Mx^T(t)R_ix(t) - \int_{t-h(t)}^t e^{2\beta(s-t)}x^T(s)R_ix(s)ds - 2\beta V_{3,i}(x_t),\\
\dot V_{4,i}(x_t) &\le h_M\begin{bmatrix} x(t)\\ x(t-h(t))\end{bmatrix}^T\begin{bmatrix} S_{11,i} & S_{12,i}\\ S_{12,i}^T & S_{22,i}\end{bmatrix}\begin{bmatrix} x(t)\\ x(t-h(t))\end{bmatrix}\\
&\quad - \int_{t-h(t)}^t e^{2\beta(s-t)}\begin{bmatrix} x(s)\\ x(s-h(s))\end{bmatrix}^T\begin{bmatrix} S_{11,i} & S_{12,i}\\ S_{12,i}^T & S_{22,i}\end{bmatrix}\begin{bmatrix} x(s)\\ x(s-h(s))\end{bmatrix}ds\\
&\quad -2\beta V_{4,i}(x_t).
\end{aligned}$$

Then, the derivative of $V_i(x_t)$ along any trajectory of solution of (1) is estimated by

$$\begin{aligned}
\dot V_i(x_t) &\le \sum_{i=1}^N \lambda_i(t)\begin{bmatrix} x(t)\\ x(t-h(t))\end{bmatrix}^T\Theta_i\begin{bmatrix} x(t)\\ x(t-h(t))\end{bmatrix} - 2\beta V_{2,i}(x_t)\\
&\quad - \int_{t-h(t)}^t e^{2\beta(s-t)}x^T(s)R_ix(s)ds - 2\beta V_{3,i}(x_t)\\
&\quad - \int_{t-h(t)}^t e^{2\beta(s-t)}\begin{bmatrix} x(s)\\ x(s-h(s))\end{bmatrix}^T\begin{bmatrix} S_{11,i} & S_{12,i}\\ S_{12,i}^T & S_{22,i}\end{bmatrix}\begin{bmatrix} x(s)\\ x(s-h(s))\end{bmatrix}ds\\
&\quad -2\beta V_{4,i}(x_t). \qquad (40)
\end{aligned}$$

For $i \in S_u$, it follows from (40) that

$$\dot V_i(x_t) \le \sum_{i=1}^N \lambda_i(t)\begin{bmatrix} x(t)\\ x(t-h(t))\end{bmatrix}^T\Theta_i\begin{bmatrix} x(t)\\ x(t-h(t))\end{bmatrix}. \qquad (41)$$

Similar to Theorem 3.1, from (33) and (41), we get

$$V_i(x_t) \le \sum_{i=1}^N \lambda_i(t) \parallel V_i(x_{t_0}) \parallel e^{\xi_i(t-t_0)},\ t \ge t_0. \qquad (42)$$

where $\xi_i = \dfrac{2\max\limits_i\{\lambda_M(\Theta_i)\}}{\min\limits_i\{\lambda_m(P_i)\}}$.

For $i \in S_s$, from (13), (14) and (40), we have

$$\dot{V}_i(x_t) \le \sum_{i=1}^{N} \lambda_i(t) \begin{bmatrix} x(t) \\ x(t-h(t)) \end{bmatrix}^T \Theta_i \begin{bmatrix} x(t) \\ x(t-h(t)) \end{bmatrix} - 2\beta V_{2,i}(x_t)$$
$$-(2\beta + \frac{1}{h_M})(V_{3,i}(x_t) + V_{4,i}(x_t)). \tag{43}$$

Similar to Theorem 3.1, from (34) and (43), we get

$$V_i(x_t) \le \sum_{i=1}^{N} \lambda_i(t) \parallel V_i(x_{t_0}) \parallel e^{-\zeta_i(t-t_0)},\ t \ge t_0. \tag{44}$$

where $\zeta_i = \min\{\frac{\min_i\{\lambda_m(-\Theta_i)\}}{\max_i\{\lambda_M(P_i)\}}, 2\beta\}$.

In general, from (39), (42) and (44), with the same argument as in the proof of Theorem 3.1, we get

$$V_i(x_t) \le \prod_{m=1}^{l(t)} \psi e^{\lambda^+(t_m - t_{m-1})} \times \prod_{n=l(t)+1}^{N(t)-1} \psi e^{\zeta_{i_n} h_M} e^{-\lambda^-(t_n - t_{n-1})} \times \parallel V_{i_0}(x_{t_0}) \parallel e^{-\lambda^-(t - t_{N(t)-1})},$$

$t \ge t_0$. Using (35), we have

$$V_i(x_t) \le \prod_{m=1}^{l(t)} \psi \times \prod_{n=l(t)+1}^{N(t)-1} \psi e^{\zeta_{i_n} h_M} \times \parallel V_{i_0}(x_{t_0}) \parallel e^{-\lambda^*(t-t_0)},\ t \ge t_0.$$

By (36) and (37), we get

$$V_i(x_t) \le \parallel V_{i_0}(x_{t_0}) \parallel e^{-(\lambda^* - \nu)(t-t_0)},\ t \ge t_0.$$

Thus, by (38), we have

$$\parallel x(t) \parallel \le \sqrt{\frac{\alpha_3}{\alpha_1}} \parallel x_{t_0} \parallel e^{-\frac{1}{2}(\lambda^* - \nu)(t-t_0)},\ t \ge t_0,$$

which concludes the proof of the Theorem 3.3. □

4. Numerical examples

Example 4.1 Consider linear switched system (1) with time-varying delay but without matrix uncertainties and without nonlinear perturbations. Let $N = 2$, $S_u = \{1\}, S_s = \{2\}$. Let the delay function be $h(t) = 0.51 \sin^2 t$. We have $h_M = 0.51$, $\mu = 1.02, \lambda(A_1 + B_1) = 0.0046, -0.0399, \lambda(A_2) = -0.2156, 0.0007$. Let $\beta = 0.5$.
Since one of the eigenvalues of $A_1 + B_1$ is negative and one of eigenvalues of A_2 is positive, we can't use results in (Alan & Lib, 2008) to consider stability of switched system (1). By using the LMI toolbox in Matlab, we have matrix solutions of (5) for unstable subsystems and (6) for stable subsystems as the following:
For unstable subsystems, we get

$P_1 = \begin{bmatrix} 41.6819 & 0.0001 \\ 0.0001 & 41.5691 \end{bmatrix}, Q_1 = \begin{bmatrix} 24.7813 & -0.0002 \\ -0.0002 & 24.7848 \end{bmatrix}, R_1 = \begin{bmatrix} 33.1027 & -0.0001 \\ -0.0001 & 33.1044 \end{bmatrix},$
$S_{11,1} = \begin{bmatrix} 33.1027 & -0.0001 \\ -0.0001 & 33.1044 \end{bmatrix}, S_{12,1} = \begin{bmatrix} -0.0372 & -0.0023 \\ -0.0023 & 0.7075 \end{bmatrix}, S_{22,1} = \begin{bmatrix} 50.0412 & 0.0001 \\ 0.0001 & 50.0115 \end{bmatrix},$
$T_1 = \begin{bmatrix} 41.7637 & -0.0001 \\ -0.0001 & 41.7920 \end{bmatrix}.$

For stable subsystems, we get
$P_2 = \begin{bmatrix} 71.8776 & 2.3932 \\ 2.3932 & 110.8889 \end{bmatrix}, Q_2 = \begin{bmatrix} 7.2590 & -0.3265 \\ -0.3265 & 0.8745 \end{bmatrix}, R_2 = \begin{bmatrix} 10.4001 & -0.4667 \\ -0.4667 & 1.2806 \end{bmatrix},$
$S_{11,2} = \begin{bmatrix} 12.7990 & -0.4854 \\ -0.4854 & 3.5031 \end{bmatrix}, S_{12,2} = \begin{bmatrix} -3.1787 & 0.0240 \\ 0.0240 & -2.8307 \end{bmatrix}, S_{22,2} = \begin{bmatrix} 4.6346 & -0.0289 \\ -0.0289 & 4.0835 \end{bmatrix},$
$T_2 = \begin{bmatrix} 16.9964 & 0.0394 \\ 0.0394 & 17.7152 \end{bmatrix}, X_{11,2} = \begin{bmatrix} 17.2639 & -0.1536 \\ -0.1536 & 14.2310 \end{bmatrix}, X_{12,2} = \begin{bmatrix} -9.6485 & -0.1466 \\ -0.1466 & -12.5573 \end{bmatrix},$
$X_{22,2} = \begin{bmatrix} 16.9716 & -0.1635 \\ -0.1635 & 13.8095 \end{bmatrix}, Y_2 = \begin{bmatrix} -3.4666 & -0.1525 \\ -0.1525 & -6.3485 \end{bmatrix}, Z_2 = \begin{bmatrix} 6.8776 & -0.0574 \\ -0.0574 & 5.7924 \end{bmatrix}.$

By straight forward calculation, the growth rate is $\lambda^+ = \xi = 2.8291$, the decay rate is $\lambda^- = \zeta = 0.0063$, $\lambda(\Omega_{1,1}) = 25.8187, 25.8188, 58.7463, 58.8011$, $\lambda(\Omega_{2,2}) = -10.1108, -3.7678, -2.0403, -0.7032$ and $\lambda(\Omega_{3,2}) = 1.4217, 4.2448, 5.4006, 9.1514, 29.3526, 30.0607$. Thus, we may take $\lambda^* = 0.0001$ and $\nu = 0.00001$. Thus, from inequality (7), we have $T^- \geq 456.3226\, T^+$. By choosing $T^+ = 0.1$, we get $T^- \geq 45.63226$. We choose the following switching rules:
(i) for $t \in [0, 0.1) \cup [50, 50.1) \cup [100, 100.1) \cup [150, 150.1) \cup ...$, subsystem $i = 1$ is activated.
(ii) for $t \in [0.1, 50) \cup [50.1, 100) \cup [100.1, 150) \cup [150.1, 200) \cup ...$, subsystem $i = 2$ is activated.
Then, by Theorem 3.1, the switching system (1) is exponentially stable. Moreover, the solution $x(t)$ of the system satisfies

$$\| x(t) \| \leq 11.8915 e^{-0.000045t},\ t \in [0, \infty).$$

The trajectories of solution of switched system switching between the subsystems $i = 1$ and $i = 2$ are shown in Figure 1, Figure 2 and Figure 3, respectively.

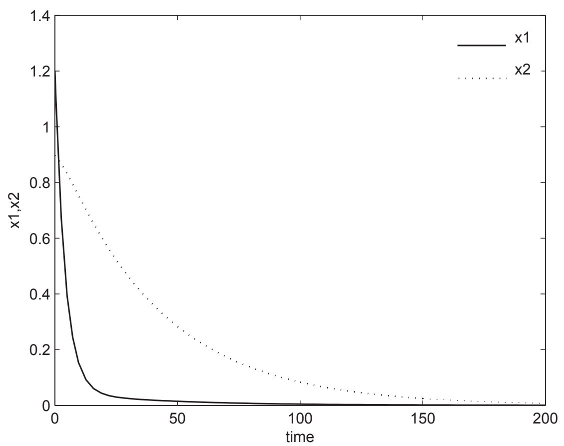

Fig. 1. The trajectories of solution of linear switched system.

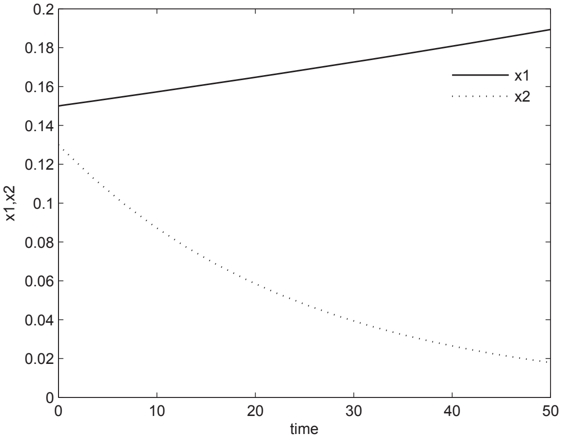

Fig. 2. The trajectories of solution of subsystem $i = 1$.

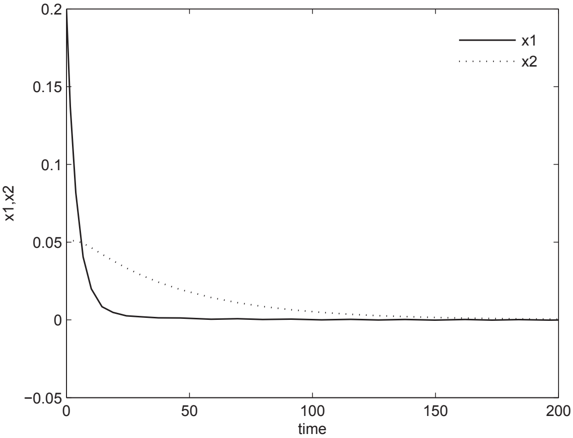

Fig. 3. The trajectories of solution of subsystem $i = 2$.

Example 4.2 Consider uncertain switched system (1) with time-varying delay and nonlinear perturbation. Let $N = 2$, $S_u = \{1\}, S_s = \{2\}$ where

$$A_1 = \begin{bmatrix} 0.1130 & 0.00013 \\ 0.00015 & -0.0033 \end{bmatrix}, B_1 = \begin{bmatrix} 0.0002 & 0.0012 \\ 0.0014 & -0.5002 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -5.5200 & 1.0002 \\ 1.0003 & -6.5500 \end{bmatrix}, B_2 = \begin{bmatrix} 0.0245 & 0.0001 \\ 0.0001 & 0.0237 \end{bmatrix},$$

$$E_{1i} = E_{2i} = \begin{bmatrix} 0.2000 & 0.0000 \\ 0.0000 & 0.2000 \end{bmatrix}, H_{1i} = H_{2i} = \begin{bmatrix} 0.1000 & 0.0000 \\ 0.0000 & 0.1000 \end{bmatrix}, i = 1, 2,$$

$$F_{1i} = F_{2i} = \begin{bmatrix} \sin t & 0 \\ 0 & \sin t \end{bmatrix}, i = 1, 2,$$

$$f_1(t, x(t), x(t-h(t))) = \begin{bmatrix} 0.1x_1(t)\sin(x_1(t)) \\ 0.1x_2(t-h(t))\cos(x_2(t)) \end{bmatrix},$$
$$f_2(t, x(t), x(t-h(t))) = \begin{bmatrix} 0.5x_1(t)\sin(x_1(t)) \\ 0.5x_2(t-h(t))\cos(x_2(t)) \end{bmatrix}.$$

From

$$\begin{aligned} \| f_1(t, x(t), x(t-h(t))) \|^2 &= [0.1x_1(t)\sin(x_1(t))]^2 + [0.1x_2(t-h(t))\cos(x_2(t))]^2 \\ &\le 0.01x_1^2(t) + 0.01x_2^2(t-h(t)) \\ &\le 0.01 \| x(t) \|^2 + 0.01 \| x(t-h(t)) \|^2 \\ &\le 0.01[\| x(t) \| + \| x(t-h(t)) \|]^2, \end{aligned}$$

we obtain

$$\| f_1(t, x(t), x(t-h(t))) \| \le 0.1 \| x(t) \| + 0.1 \| x(t-h(t)) \| .$$

The delay function is chosen as $h(t) = 0.25\sin^2 t$. From

$$\begin{aligned} \| f_2(t, x(t), x(t-h(t))) \|^2 &= [0.5x_1(t)\sin(x_1(t))]^2 + [0.5x_2(t-h(t))\cos(x_2(t))]^2 \\ &\le 0.25x_1^2(t) + 0.25x_2^2(t-h(t)) \\ &\le 0.25 \| x(t) \|^2 + 0.25 \| x(t-h(t)) \|^2 \\ &\le 0.25[\| x(t) \| + \| x(t-h(t)) \|]^2, \end{aligned}$$

we obtain

$$\| f_2(t, x(t), x(t-h(t))) \| \le 0.5 \| x(t) \| + 0.5 \| x(t-h(t)) \| .$$

We may take $h_M = 0.25$, and from (4), we take $\gamma_1 = 0.1, \delta_1 = 0.1, \gamma_2 = 0.5, \delta_2 = 0.5$. Note that $\lambda(A_1) = 0.11300016, -0.00330016$. Let $\beta = 0.5, \mu = 0.5$. Since one of the eigenvalues of A_1 is negative, we can't use results in (Alan & Lib, 2008) to consider stability of switched system (1). From Lemma 2.4 , we have the matrix solutions of (33) for unstable subsystems and of (34) for stable subsystems by using the LMI toolbox in Matlab as the following:
For unstable subsystems, we get
$\varepsilon_{31} = 0.8901,\ \varepsilon_{41} = 0.8901,\ \varepsilon_{51} = 0.8901,$

$$P_1 = \begin{bmatrix} 0.2745 & -0.0000 \\ -0.0000 & 0.2818 \end{bmatrix}, Q_1 = \begin{bmatrix} 0.4818 & -0.0000 \\ -0.0000 & 0.5097 \end{bmatrix}, R_1 = \begin{bmatrix} 0.8649 & -0.0000 \\ -0.0000 & 0.8729 \end{bmatrix},$$
$$S_{11,1} = \begin{bmatrix} 0.8649 & -0.0000 \\ -0.0000 & 0.8729 \end{bmatrix}, S_{12,1} = 10^{-4} \times \begin{bmatrix} -0.1291 & -0.8517 \\ -0.8517 & 0.1326 \end{bmatrix},$$
$$S_{22,1} = \begin{bmatrix} 1.0877 & -0.0000 \\ -0.0000 & 1.0902 \end{bmatrix}.$$

For stable subsystems, we get
$\varepsilon_{32} = 2.0180,\ \varepsilon_{42} = 2.0180,\ \varepsilon_{52} = 2.0180,$

$$P_2 = \begin{bmatrix} 0.2741 & 0.0407 \\ 0.0407 & 0.2323 \end{bmatrix}, Q_2 = \begin{bmatrix} 1.3330 & -0.0069 \\ -0.0069 & 1.3330 \end{bmatrix}, R_2 = \begin{bmatrix} 1.0210 & -0.0002 \\ -0.0002 & 1.0210 \end{bmatrix},$$
$$S_{11,2} = \begin{bmatrix} 1.0210 & -0.0002 \\ -0.0002 & 1.0210 \end{bmatrix}, S_{12,2} = \begin{bmatrix} -0.0016 & -0.0002 \\ -0.0002 & -0.0016 \end{bmatrix},$$
$$S_{22,2} = \begin{bmatrix} 0.8236 & -0.0006 \\ -0.0006 & 0.8236 \end{bmatrix}.$$

By straight forward calculation, the growth rate is $\lambda^+ = \xi = 8.5413$, the decay

rate is $\lambda^- = \zeta = 0.1967$, $\lambda(\Theta_1) = 0.1976, 0.2079, 1.1443, 1.1723$ and $\lambda(\Theta_2) = -0.7682, -0.6494, -0.0646, -0.0588$. Thus, we may take $\lambda^* = 0.0001$ and $\nu = 0.00001$.
Thus, from inequality (35), we have $T^- \geq 43.4456\ T^+$. By choosing $T^+ = 0.1$, we get $T^- \geq 4.34456$. We choose the following switching rules:
(i) for $t \in [0, 0.1) \cup [5.0, 5.1) \cup [10.0, 10.1) \cup [15.0, 15.1) \cup ...$, system $i = 1$ is activated.
(ii) for $t \in [0.1, 5.0) \cup [5.1, 10.0) \cup [10.1, 15.0) \cup [15.1, 20.0) \cup ...$, system $i = 2$ is activated.
Then, by theorem 3.3.1, the switched system (1) is exponentially stable. Moreover, the solution $x(t)$ of the system satisfies

$$\| x(t) \| \leq 1.8770e^{-0.000045t},\ t \in [0, \infty).$$

The trajectories of solution of switched system switching between the subsystems $i = 1$ and $i = 2$ are shown in Figure 4, Figure 5 and Figure 6, respectively.

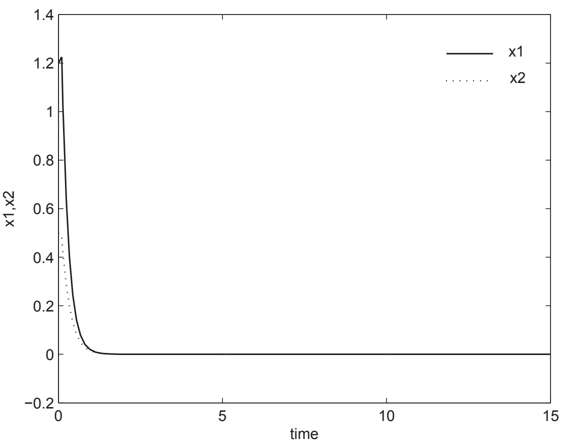

Fig. 4. The trajectories of solution of switched system with nonlinear perturbations

5. Conclusion

In this paper, we have studied the exponential stability of uncertain switched system with time varying delay and nonlinear perturbations. We allow switched system to contain stable and unstable subsystems. By using a new Lyapunov functional, we obtain the conditions for robust exponential stability for switched system in terms of linear matrix inequalities (LMIs) which may be solved by various algorithms. Numerical examples are given to illustrate the effectiveness of our theoretical results.

6. Acknowledgments

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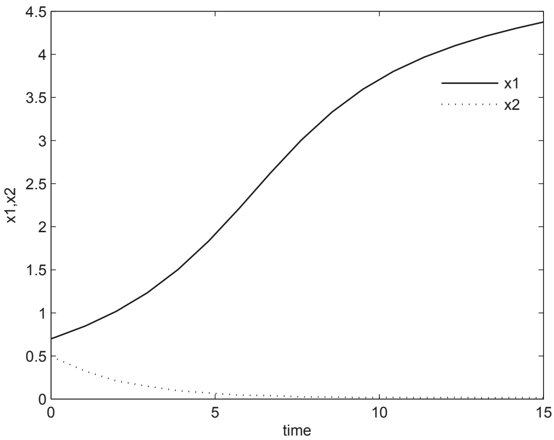

Fig. 5. The trajectories of solution of system $i = 1$

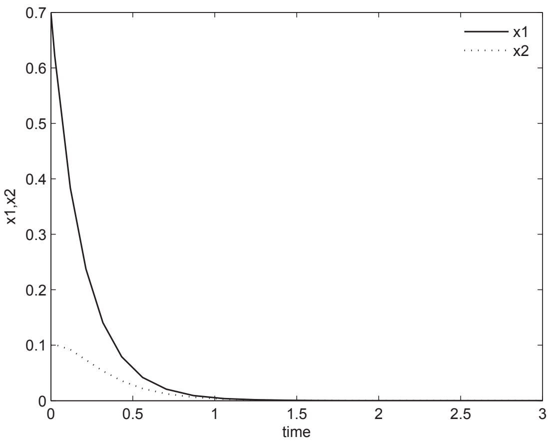

Fig. 6. The trajectories of solution of system $i = 2$

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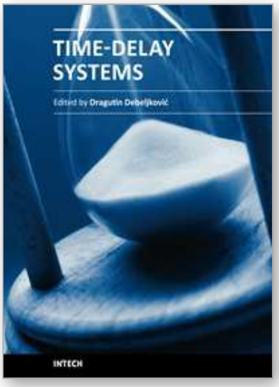

Time-Delay Systems

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Time delay is very often encountered in various technical systems, such as electric, pneumatic and hydraulic networks, chemical processes, long transmission lines, robotics, etc. The existence of pure time lag, regardless if it is present in the control or/and the state, may cause undesirable system transient response, or even instability. Consequently, the problem of controllability, observability, robustness, optimization, adaptive control, pole placement and particularly stability and robustness stabilization for this class of systems, has been one of the main interests for many scientists and researchers during the last five decades.

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