

# Decentralized Adaptive Stabilization for Large-Scale Systems with Unknown Time-Delay

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## 1. Introduction

In dealing with a large-scale system, one usually does not have adequate knowledge of the plant parameters and interactions among subsystems. The decentralized adaptive technique, designed independently for local subsystems and using locally available signals for feedback propose, is an appropriate strategy to be employed. In the context of decentralized adaptive control, a number of results have been obtained, see for examples Ioannou (1986); Narendra & Oleng (2002); Ortega (1996); Wen (1994). Since backstepping technique was proposed, it has been widely used to design adaptive controllers for uncertain systems Krstic et al. (1995). This technique has a number of advantages over the conventional approaches such as providing a promising way to improve the transient performance of adaptive systems by tuning design parameters. Because of such advantages, research on decentralized adaptive control using backstepping technique has also received great attention. In Wen & Soh (1997), decentralized adaptive tracking for linear systems was considered. In Jiang (2000), decentralized adaptive tracking of nonlinear systems was addressed, where the interaction functions satisfy global Lipschitz condition and the proposed controllers are partially decentralized. In Wen & Zhou (2007); Zhou & Wen (2008a;b), systems with higher order nonlinear interactions were considered by using backstepping technique.

Stabilization and control problem for time-delay systems have received much attention, see for examples, Jankovic (2001); Luo et al. (1997); Wu (1999), etc. The Lyapunov-Krasovskii method and Lyapunov-Razumikhin method are always employed. The results are often obtained via linear matrix inequalities. Some fruitful results have been achieved in the past when dealing with stabilizing problem for time-delay systems using backstepping technique. In Ge et al. (2003), neural network control cooperating with iterative backstepping was constructed for a class of nonlinear system with unknown but constant time delays. Jiao & Shen (2005) and Wu (2002) considered the control problem of the class of time-invariant large-scale interconnected systems subject to constant delays. In Chou & Cheng (2003), a decentralized model reference adaptive variable structure controller was proposed for a large-scale time-delay system, where the time-delay function is known and linear. In Hua et al. (2005), the robust output feedback control problem was considered for a class of nonlinear time-varying delay systems, where the nonlinear time-delay functions are bounded by known functions. In Shyu et al. (2005), a decentralized state-feedback variable structure controller was proposed for large-scale systems with time delay and dead-zone nonlinearity. However, in Shyu et al. (2005), the time delay is constant and the parameters of the dead-zone are

known. Due to state feedback, no filter is required for state estimation. Furthermore, only the stabilization problem was considered. A decentralized feedback control approach for a class of large scale stochastic systems with time delay was proposed in Wu et al. (2006). In Hua et al. (2007) a result of backstepping adaptive tracking in the presence of time delay was established. In Zhou (2008), we develop a totally decentralized controller for large scale time-delays systems with dead-zone input. In Zhou et al. (2009), adaptive backstepping control is developed for uncertain systems with unknown input time-delay.

In fact, the existence of time-delay phenomenon usually deteriorates the system performance. The stabilization and control problem for time-delay systems is a topic of great importance and has received increasing attention. Due to the difficulties on considering the effects of interconnections and time delays, extension of single-loop results to multi-loop interconnected systems is still a challenging task, especially for decentralized tracking. In this chapter, the decentralized adaptive stabilization is addressed for a class of interconnected systems with subsystems having arbitrary relative degrees, with unknown time-varying delays, and with unknown parameter uncertainties. The nonlinear time-delay functions are unknown and are allowed to satisfy a nonlinear bound. Also, the interactions between subsystems satisfy a nonlinear bound by nonlinear models. As system output feedback is employed, a state observer is required. Practical control is carried out in the backstepping design to compensate the effects of unknown interactions and unknown time-delays. In our design, the term multiplying the control effort and the system parameters are not assumed to be within known intervals. Besides showing stability of the system, the transient performance, in terms of  $L_2$  norm of the system output, is shown to be an explicit function of design parameters and thus our scheme allows designers to obtain closed-loop behavior by tuning design parameters in an explicit way.

The main contributions of the chapter include: (i) the development of adaptive compensation to accommodate the effects of time-delays and interactions; (ii) the use of new Lyapunov-Krasovskii function in eliminating the unknown time-varying delays.

## 2. Problem formulation

Considered a system consisting of  $N$  interconnected subsystems modelled as follows:

$$\dot{x}_i = \mathbf{A}_i x_i + \Phi_i(y_i) \theta_i + \begin{bmatrix} 0 \\ \mathbf{b}_i \end{bmatrix} u_i + \sum_{j=1}^N \mathbf{h}_{ij}(y_j(t - \tau_j(t))) + \sum_{j=1}^N \mathbf{f}_{ij}(t, y_j), \quad (1)$$

$$y_i = \mathbf{c}_i^T x_i, \text{ for } i = 1, \dots, N, \quad (2)$$

$$\mathbf{A}_i = \begin{bmatrix} 0 \\ \vdots \\ \mathbf{I}_{(n_i-1) \times (n_i-1)} \\ 0 \\ \vdots \\ 0 \quad \dots \quad 0 \end{bmatrix}, \mathbf{b}_i = \begin{bmatrix} b_{i,m_i} \\ \vdots \\ b_{i,0} \end{bmatrix}, \Phi_i(y_i) = \begin{bmatrix} \Phi_{i,1}(y_i) \\ \vdots \\ \Phi_{i,n_i}(y_i) \end{bmatrix}, \quad (3)$$

$$\mathbf{c}_i = [1, 0, \dots, 0]^T.$$

where  $x_i \in \mathbb{R}^{n_i}$ ,  $u_i \in \mathbb{R}^1$  and  $y_i \in \mathbb{R}^1$  are the states, input and output of the  $i$ th subsystem, respectively,  $\theta_i \in \mathbb{R}^{r_i}$  and  $\mathbf{b}_i \in \mathbb{R}^{m_i+1}$  are unknown constant vectors,  $\Phi_i(y_i) \in \mathbb{R}^{n_i \times r_i}$  is a known smooth function,  $\mathbf{f}_{ij}(t, y_j) = [f_{ij}^1(t, y_j), \dots, f_{ij}^{m_i}(t, y_j)]^T \in \mathbb{R}^{m_i}$  denotes the nonlinear interactions from the  $j$ th subsystem to the  $i$ th subsystem for  $j \neq i$ , or a nonlinear un-modelled part of the  $i$ th subsystem for  $j = i$ ,  $\mathbf{h}_{ij} = [h_{ij}^1, \dots, h_{ij}^{m_i}]^T \in \mathbb{R}^{m_i}$  is an unknown function,

the unknown scalar function  $\tau_j(t)$  denotes any nonnegative, continuous and bounded time-varying delay satisfying

$$\dot{\tau}_j(t) \leq \bar{\tau}_j < 1, \quad (4)$$

where  $\bar{\tau}_j$  are known constants. For each decoupled local system, we make the following assumptions.

**Assumption 1:** The triple  $(A_i, b_i, c_i)$  are completely controllable and observable.

**Assumption 2:** For every  $1 \leq i \leq N$ , the polynomial  $b_{i,m_i}s^{m_i} + \dots + b_{i,1}s + b_{i,0}$  is Hurwitz. The sign of  $b_{i,m_i}$  and the relative degree  $\rho_i (= n_i - m_i)$  are known.

**Assumption 3:** The nonlinear interaction terms satisfy

$$|f_{ij}(t, y_j)| \leq \bar{\gamma}_{ij} \bar{f}_j(t, y_j) y_j, \quad (5)$$

where  $\bar{\gamma}_{ij}$  are constants denoting the strength of interactions, and  $\bar{f}_j(y_j), j = 1, 2, \dots, N$  are known positive functions and differentiable at least  $\rho_i$  times.

**Assumption 4:** The unknown functions  $h_{ij}(y_j(t))$  satisfy the following properties

$$|h_{ij}(y_j(t))| \leq \bar{v}_{ij} \bar{h}_j(y_j(t)) y_j, \quad (6)$$

where  $\bar{h}_j$  are known positive functions and differentiable at least  $\rho_i$  times, and  $\bar{v}_{ij}^k$  are positive constants.

**Remark 1.** *The effects of the nonlinear interactions  $f_{ij}$  and time-delay functions  $h_{ij}$  from other subsystems to a local subsystem are bounded by functions of the output of this subsystem. With these conditions, it is possible for the designed local controller to stabilize the interconnected systems with arbitrary strong subsystem interactions and time-delays.*

The control objective is to design a decentralized adaptive stabilizer for a large scale system (1) with unknown time-varying delay satisfying Assumptions 1-4 such that the closed-loop system is stable.

### 3. Design of adaptive controllers

#### 3.1 Local state estimation filters

In this section, decentralized filters using only local input and output will be designed to estimate the unmeasured states of each local system. For the  $i$ th subsystem, we design the filters as

$$\dot{\mathbf{v}}_{i,\ell} = \mathbf{A}_{i,0} \mathbf{v}_{i,\ell} + \mathbf{e}_{n_i, (n_i - \ell)} u_i, \quad \ell = 0, \dots, m_i \quad (7)$$

$$\dot{\boldsymbol{\xi}}_{i,0} = \mathbf{A}_{i,0} \boldsymbol{\xi}_{i,0} + \mathbf{k}_i y_i, \quad (8)$$

$$\dot{\boldsymbol{\Xi}}_i = \mathbf{A}_{i,0} \boldsymbol{\Xi}_i + \boldsymbol{\Phi}_i(y_i), \quad (9)$$

where  $\mathbf{v}_{i,\ell} \in \mathbb{R}^{n_i}, \boldsymbol{\xi}_{i,0} \in \mathbb{R}^{n_i}, \boldsymbol{\Xi}_i \in \mathbb{R}^{n_i \times r_i}$ , the vector  $\mathbf{k}_i = [k_{i,1}, \dots, k_{i,n_i}]^T \in \mathbb{R}^{n_i}$  is chosen such that the matrix  $\mathbf{A}_{i,0} = \mathbf{A}_i - \mathbf{k}_i (\mathbf{e}_{n_i,1})^T$  is Hurwitz, and  $\mathbf{e}_{i,k}$  denotes the  $k$ th coordinate vector in  $\mathbb{R}^i$ . There exists a  $\mathbf{P}_i$  such that  $\mathbf{P}_i \mathbf{A}_{i,0} + (\mathbf{A}_{i,0})^T \mathbf{P}_i = -3\mathbf{I}$ ,  $\mathbf{P}_i = \mathbf{P}_i^T > 0$ . With these designed filters, our state estimate is

$$\hat{\mathbf{x}}_i(t) = \boldsymbol{\xi}_{i,0} + \boldsymbol{\Xi}_i \boldsymbol{\theta}_i + \sum_{k=0}^{m_i} b_{i,k} \mathbf{v}_{i,k}, \quad (10)$$

and the state estimation error  $\epsilon_i = \mathbf{x}_i - \hat{\mathbf{x}}_i$  satisfies

$$\dot{\epsilon}_i = \mathbf{A}_{i,0}\epsilon_i + \sum_{j=1}^N \mathbf{f}_{ij}(t, y_j) + \sum_{j=1}^N \mathbf{h}_{ij}(y_j(t - \tau_j(t))). \quad (11)$$

Let  $V_{\epsilon_i} = \epsilon_i^T \mathbf{P}_i \epsilon_i$ . It can be shown that

$$\dot{V}_{\epsilon_i} \leq -\epsilon_i^T \epsilon_i + 2N \|\mathbf{P}_i\|^2 \sum_{j=1}^N \|\mathbf{f}_{ij}(t, y_j)\|^2 + 2N \|\mathbf{P}_i\|^2 \sum_{j=1}^N \|\mathbf{h}_{ij}(y_j(t - \tau_j(t)))\|^2. \quad (12)$$

Now system (1) is expressed as

$$\begin{aligned} \dot{y}_i &= b_{i,m_i} v_{i,(m_i,2)} + \bar{\zeta}_{i,(0,2)} + \bar{\delta}_i^T \Theta_i + \epsilon_{i,2} + \sum_{j=1}^N f_{ij,1}(t, y_j) \\ &\quad + \sum_{j=1}^N h_{ij,1}(y_j(t - \tau_j(t))), \end{aligned} \quad (13)$$

$$\dot{v}_{i,(m_i,q)} = v_{i,(m_i,q+1)} - k_{i,q} v_{i,(m_i,1)}, \quad q = 2, \dots, \rho_i - 1 \quad (14)$$

$$\dot{v}_{i,(m_i,\rho_i)} = v_{i,(m_i,\rho_i+1)} - k_{i,\rho_i} v_{i,(m_i,1)} + u_i, \quad (15)$$

where

$$\bar{\delta}_i = [0, v_{i,(m_i-1,2)}, \dots, v_{i,(0,2)}, \bar{\Xi}_{i,2} + \Phi_{i,1}]^T, \quad \Theta_i = [b_{i,m_i}, \dots, b_{i,0}, \theta_i^T]^T, \quad (16)$$

and  $v_{i,(m_i,2)}, \epsilon_{i,2}, \bar{\zeta}_{i,(0,2)}, \bar{\Xi}_{i,2}$  denote the second entries of  $\mathbf{v}_{i,m_i}, \epsilon_i, \xi_{i,0}, \bar{\Xi}_i$  respectively,  $f_{ij,1}(t, y_j)$  and  $h_{ij,1}(y_j(t - \tau_j(t)))$  are respectively the first elements of vectors  $\mathbf{f}_{ij}(t, y_j)$  and  $\mathbf{h}_{ij}(y_j(t - \tau_j(t)))$ .

**Remark 2.** It is worthy to point out that the inputs to the designed filters (7)-(9) are only the local input  $u_i$  and output  $y_i$  and thus totally decentralized.

**Remark 3.** Even though the estimated state is given in (10), it is still unknown and thus not employed in our controller design. Instead, the outputs  $\mathbf{v}_{i,\rho_i}, \xi_{i,0}$  and  $\bar{\Xi}_i$  from filters (7)-(9) are used to design controllers, while the state estimation error (11) will be considered in system analysis.

### 3.2 Adaptive decentralized controller design

In this section, we develop an adaptive backstepping design scheme for decentralized output tracking. There is no a priori information required from system parameter  $\Theta_i$  and thus they can be allowed totally uncertain. As usual in backstepping approach in Krstic et al. (1995), the following change of coordinates is made.

$$z_{i,1} = y_i, \quad (17)$$

$$z_{i,q} = v_{i,(m_i,q)} - \alpha_{i,q-1}, \quad q = 2, 3, \dots, \rho_i, \quad (18)$$

where  $\alpha_{i,q-1}$  is the virtual control at the  $q$ -th step of the  $i$ th loop and will be determined in later discussion,  $\hat{p}_i$  is the estimate of  $p_i = 1/b_{i,m_i}$ .

To illustrate the controller design procedures, we now give a brief description on the first step.

• *Step 1*: Starting with the equations for the tracking error  $z_{i,1}$  obtained from (13), (17) and (18), we get

$$\begin{aligned} \dot{z}_{i,1} &= b_{i,m_i} v_{i,(m_i,2)} + \zeta_{i,(0,2)} + \bar{\delta}_i^T \Theta_i + \epsilon_{i,2} + \sum_{j=1}^N f_{ij,1}(t, y_j) \\ &\quad + \sum_{j=1}^N h_{ij,1}(t, y_j(t - \tau_j(t))) \\ &= b_{i,m_i} \alpha_{i,1} + b_{i,m_i} z_{i,2} + \zeta_{i,(0,2)} + \bar{\delta}_i^T \Theta_i + \epsilon_{i,2} + \sum_{j=1}^N f_{ij,1}(t, y_j) \\ &\quad + \sum_{j=1}^N h_{ij,1}(t, y_j(t - \tau_j(t))). \end{aligned} \quad (19)$$

The virtual control law  $\alpha_{i,1}$  is designed as

$$\alpha_{i,1} = \hat{p}_i \bar{\alpha}_{i,1}, \quad (20)$$

$$\bar{\alpha}_{i,1} = -(c_{i,1} + l_{i,1}) z_{i,1} - l_i^* z_{i,1} (\bar{f}_i(y_i))^2 - \lambda_i^* z_{i,1} (\bar{h}_i(y_i))^2 - \zeta_{i,(0,2)} - \bar{\delta}_i^T \hat{\Theta}_i, \quad (21)$$

where  $c_{i,1}$ ,  $l_{i,1}$ ,  $l_i^*$  and  $\lambda_i^*$  are positive design parameters,  $\hat{\Theta}_i$  and  $\hat{p}_i$  are the estimates of  $\Theta_i$  and  $p_i$ , respectively. Using  $\tilde{p}_i = p_i - \hat{p}_i$ , we obtain

$$\begin{aligned} b_{i,m_i} \alpha_{i,1} &= b_{i,m_i} \hat{p}_i \bar{\alpha}_{i,1} = \bar{\alpha}_{i,1} - b_{i,m_i} \tilde{p}_i \bar{\alpha}_{i,1}, \\ \bar{\delta}_i^T \tilde{\Theta}_i + b_{i,m_i} z_{i,2} &= \bar{\delta}_i^T \Theta_i + \tilde{b}_{i,m_i} z_{i,2} + \hat{b}_{i,m_i} z_{i,2} \\ &= \bar{\delta}_i^T \tilde{\Theta}_i + (v_{i,(m_i,2)} - \alpha_{i,1}) \mathbf{e}_{(r_i+m_i+1),1}^T \tilde{\Theta}_i + \hat{b}_{i,m_i} z_{i,2} \\ &= (\delta_i - \hat{p}_i \bar{\alpha}_{i,1} \mathbf{e}_{(r_i+m_i+1),1})^T \tilde{\Theta}_i + \hat{b}_{i,m_i} z_{i,2}, \end{aligned} \quad (22)$$

where

$$\delta_i = [v_{i,(m_i,2)}, v_{i,(m_i-1,2)}, \dots, v_{i,(0,2)}, \xi_{i,2} + \Phi_{i,1}]^T. \quad (24)$$

From (20)-(23), (19) can be written as

$$\begin{aligned} \dot{z}_{i,1} &= -c_{i,1} z_{i,1} - l_{i,1} z_{i,1} - l_i^* z_{i,1} (\bar{f}_i(y_i))^2 - \lambda_i^* z_{i,1} (\bar{h}_i(y_i))^2 \\ &\quad + \epsilon_{i,2} + (\delta_i - \hat{p}_i \bar{\alpha}_{i,1} \mathbf{e}_{r_i+m_i+1,1})^T \tilde{\Theta}_i - b_{i,m_i} \bar{\alpha}_{i,1} \tilde{p}_i + \hat{b}_{i,m_i} z_{i,2} \\ &\quad + \sum_{j=1}^N f_{ij,1}(t, y_j) + \sum_{j=1}^N h_{ij,1}(t, y_j(t - \tau_j(t))), \end{aligned} \quad (25)$$

where  $\tilde{\Theta}_i = \Theta_i - \hat{\Theta}_i$ , and  $\mathbf{e}_{(r_i+m_i+1),1} \in \mathcal{R}^{r_i+m_i+1}$ . We now consider the Lyapunov function

$$V_i^1 = \frac{1}{2} (z_{i,1})^2 + \frac{1}{2} \tilde{\Theta}_i^T \Gamma_i^{-1} \tilde{\Theta}_i + \frac{|b_{i,m_i}|}{2\gamma_i} (\tilde{p}_i)^2 + \frac{1}{2l_{i,1}} V_{\epsilon_i}, \quad (26)$$

where  $\Gamma_i$  is a positive definite design matrix and  $\gamma'_i$  is a positive design parameter. Examining the derivative of  $V_i^1$  gives

$$\begin{aligned} \dot{V}_i^1 &= z_{i,1}\dot{z}_{i,1} - \tilde{\Theta}_i^T \Gamma_i^{-1} \dot{\tilde{\Theta}}_i - \frac{|b_{i,m_i}|}{\gamma'_i} \tilde{p}_i \dot{\tilde{p}}_i + \frac{1}{2\bar{l}_{i,1}} \dot{V}\epsilon_i \\ &\leq -c_{i,1}(z_{i,1})^2 - l_{i,1}(z_{i,1})^2 - l_i^*(z_{i,1})^2 (\bar{f}_i(z_{i,1}))^2 - \lambda_i^*(z_{i,1})^2 (\bar{h}_i(y_i))^2 \\ &\quad - \frac{1}{2\bar{l}_{i,1}} \epsilon_i^T \epsilon_i + \hat{b}_{i,m_i} z_{i,1} z_{i,2} - |b_{i,m_i}| \tilde{p}_i \frac{1}{\gamma'_i} [\gamma'_i \text{sgn}(b_{i,m_i}) \bar{\alpha}_{i,1} z_{i,1} + \dot{\tilde{p}}_i] \\ &\quad + \tilde{\Theta}_i^T \Gamma_i^{-1} [\Gamma_i (\delta_i - \hat{p}_i \bar{\alpha}_{i,1} \mathbf{e}_{(r_i+m_i+1),1}) z_{i,1} - \dot{\tilde{\Theta}}_i] \\ &\quad + \left( \sum_{j=1}^N f_{ij,1}(t, y_j) + \sum_{j=1}^N h_{ij,1}(t, y_j(t - \tau_j(t))) \right) + \epsilon_{i,2} z_{i,1} \\ &\quad + \frac{1}{\bar{l}_{i,1}} N \| \mathbf{P}_i \|^2 \left( \left\| \sum_{j=1}^N h_{ij}(t, y_j(t - \tau_j(t))) \right\|^2 + \sum_{j=1}^N \left\| \mathbf{f}_{ij}(t, y_j) \right\|^2 \right). \end{aligned} \tag{27}$$

Then we choose

$$\dot{\tilde{p}}_i = -\gamma'_i \text{sgn}(b_{i,m_i}) \bar{\alpha}_{i,1} z_{i,1}, \tag{28}$$

$$\tau_{i,1} = \left( \delta_i - \hat{p}_i \bar{\alpha}_{i,1} \mathbf{e}_{(r_i+m_i+1),1} \right) z_{i,1}. \tag{29}$$

Let  $l_{i,1} = 3\bar{l}_{i,1}$  and using Young's inequality we have

$$-\bar{l}_{i,1}(z_{i,1})^2 + \sum_{j=1}^N f_{ij,1}(t, y_j) z_{i,1} \leq \frac{N}{4\bar{l}_{i,1}} \sum_{j=1}^N \left\| f_{ij,1}(t, y_j) \right\|^2, \tag{30}$$

$$-\bar{l}_{i,1}(z_{i,1})^2 + \sum_{j=1}^N h_{ij,1}(t, y_j(t - \tau_j(t))) z_{i,1} \leq \frac{N}{4\bar{l}_{i,1}} \left\| \sum_{j=1}^N h_{ij,1}(t, y_j(t - \tau_j(t))) \right\|^2, \tag{31}$$

$$\begin{aligned} -\bar{l}_{i,1}(z_{i,1})^2 + \epsilon_{i,2} z_{i,1} - \frac{1}{4\bar{l}_{i,1}} \epsilon_i^T \epsilon_i &\leq -\bar{l}_{i,1}(z_{i,1})^2 + \epsilon_{i,2} z_{i,1} - \frac{1}{4\bar{l}_{i,1}} (\epsilon_{i,2})^2 \\ &= -\bar{l}_{i,1}(z_{i,1} - \frac{1}{2\bar{l}_{i,1}} \epsilon_{i,2})^2 \leq 0. \end{aligned} \tag{32}$$

Substituting (28)-(32) into (27) gives

$$\begin{aligned} \dot{V}_i^1 &\leq -c_{i,1}(z_{i,1})^2 - \frac{1}{4\bar{l}_{i,1}} \epsilon_i^T \epsilon_i - l_i^*(z_{i,1})^2 (\bar{f}_i(y_i))^2 - \lambda_i^*(z_{i,1})^2 (\bar{h}_i(y_i))^2 + \hat{b}_{i,m_i} z_{i,1} z_{i,2} \\ &\quad + \tilde{\Theta}_i^T (\tau_{i,1} - \Gamma_i^{-1} \dot{\tilde{\Theta}}_i) + \frac{N}{\bar{l}_{i,1}} \| \mathbf{P}_i \|^2 \sum_{j=1}^N \left\| \mathbf{f}_{ij}(t, y_j) \right\|^2 + \frac{N}{4\bar{l}_{i,1}} \sum_{j=1}^N \left\| f_{ij,1}(t, y_j) \right\|^2 \\ &\quad + \frac{N}{\bar{l}_{i,1}} \| \mathbf{P}_i \|^2 \sum_{j=1}^N \left\| \mathbf{h}_{ij}(t, y_j(t - \tau_j(t))) \right\|^2 + \frac{N}{4\bar{l}_{i,1}} \left\| \sum_{j=1}^N h_{ij,1}(t, y_j(t - \tau_j(t))) \right\|^2. \end{aligned} \tag{33}$$

- Step  $q$  ( $q = 2, \dots, \rho_i, i = 1, \dots, N$ ): Choose virtual control laws

$$\alpha_{i,2} = -\hat{b}_{i,m_i} z_{i,1} - \left( c_{i,2} + l_{i,2} \left( \frac{\partial \alpha_{i,1}}{\partial y_i} \right)^2 \right) z_{i,2} + \bar{B}_{i,2} + \frac{\partial \alpha_{i,1}}{\partial \hat{\Theta}_i} \Gamma_i \tau_{i,2}, \quad (34)$$

$$\alpha_{i,q} = -z_{i,q-1} - \left( c_{i,q} + l_{i,q} \left( \frac{\partial \alpha_{i,q-1}}{\partial y_i} \right)^2 \right) z_{i,q} + \bar{B}_{i,q} + \frac{\partial \alpha_{i,q-1}}{\partial \hat{\Theta}_i} \Gamma_i \tau_{i,q} - \left( \sum_{k=2}^{q-1} z_{i,k} \frac{\partial \alpha_{i,k-1}}{\partial \hat{\Theta}_i} \right) \Gamma_i \frac{\partial \alpha_{i,q-1}}{\partial y_i} \delta_i, \quad (35)$$

$$\tau_{i,q} = \tau_{i,q-1} - \frac{\partial \alpha_{i,q-1}}{\partial y_i} \delta_i z_{i,q}, \quad (36)$$

where  $c_i^q, l_{i,q}, q = 3, \dots, \rho_i$  are positive design parameters, and  $\bar{B}_{i,q}, q = 2, \dots, \rho_i$  denotes some known terms and its detailed structure can be found in Krstic et al. (1995).

Then the local control and parameter update laws are finally given by

$$u_i = \alpha_{i,\rho_i} - v_{i,(m_i,\rho_i+1)}, \quad (37)$$

$$\dot{\hat{\Theta}}_i = \Gamma_i \tau_{i,\rho_i}. \quad (38)$$

**Remark 4.** The crucial terms  $l_i^* z_{i,1} (\bar{f}_i(y_i))^2$  in (21) and  $\lambda_i^* z_{i,1} (\bar{h}_i(y_i))^2$  are proposed in the controller design to compensate for the effects of interactions from other subsystems or the un-modelled part of its own subsystem, and for the effects of time-delay functions, respectively. The detailed analysis will be given in Section 4.

**Remark 5.** When going through the details of the design procedures, we note that in the equations concerning  $\dot{z}_{i,q}, q = 1, 2, \dots, \rho_i$ , just functions  $\sum_{j=1}^N f_{ij,1}(t, y_j)$  from the interactions and  $\sum_{j=1}^N h_{ij,1}(t, y_j(t - \tau_j(t)))$  appear, and they are always together with  $\epsilon_{i,2}$ . This is because only  $\dot{y}_i$  from the plant model (1) was used in the calculation of  $\dot{\alpha}_{i,q}$  for steps  $q = 2, \dots, \rho_i$ .

#### 4. Stability analysis

In this section, the stability of the overall closed-loop system consisting of the interconnected plants and decentralized controllers will be established.

Now we define a Lyapunov function of decentralized adaptive control system as

$$V_i = \sum_{q=1}^{\rho_i} \left( \frac{1}{2} (z_{i,q})^2 + \frac{1}{2\bar{l}_{i,q}} \epsilon_i^T \mathbf{P}_i \epsilon_i \right) + \frac{1}{2} \tilde{\Theta}_i^T \Gamma_i^{-1} \tilde{\Theta}_i + \frac{|b_{i,m_i}|}{2\gamma_i} \bar{p}_i^2. \quad (39)$$

From (12), (20), (33), (35)-(38), and (49), the derivative of  $V_i$  in (39) satisfies

$$\begin{aligned} \dot{V}_i \leq & - \sum_{q=1}^{\rho_i} c_{i,q} (z_{i,q})^2 - l_i^* (z_{i,1})^2 (\bar{f}_i(y_i))^2 - \lambda_i^* (z_{i,1})^2 (\bar{h}_i(y_i))^2 \\ & + \sum_{q=1}^{\rho_i} \frac{1}{\bar{l}_{i,q}} N \|\mathbf{P}_i\|^2 \left( \sum_{j=1}^N \|\mathbf{h}_{ij}(t, y_j(t - \tau_j))\|^2 + \sum_{j=1}^N \|\mathbf{f}_{ij}(t, y_j)\|^2 \right) \end{aligned}$$

$$\begin{aligned}
 & + \frac{1}{4\bar{l}_{i,1}} \left( N \sum_{j=1}^N \| f_{ij,1}(t, y_j) \|^2 + N \sum_{j=1}^N \| h_{ij,1}(t, y_j(t - \tau_j)) \|^2 \right) \\
 & - \frac{1}{4\bar{l}_{i,1}} \epsilon_i^T \epsilon_i + \sum_{q=2}^{\rho_i} \left[ -l_{i,q} \left( \frac{\partial \alpha_{i,q-1}}{\partial y_i} \right)^2 (z_{i,q})^2 - \frac{1}{2\bar{l}_{i,q}} \epsilon_i^T \epsilon_i \right. \\
 & \left. + \frac{\partial \alpha_{i,q-1}}{\partial y_i} \left( \sum_{j=1}^N f_{ij,1}(t, y_j) + \sum_{j=1}^N h_{ij,1}(t, y_j(t - \tau_j)) + \epsilon_{i,2} \right) z_{i,q} \right]. \tag{40}
 \end{aligned}$$

Using Young’s inequality and let  $l_{i,q} = 3\bar{l}_{i,q}$ , we have

$$-\bar{l}_{i,q} \left( \frac{\partial \alpha_{i,q-1}}{\partial y_i} \right)^2 (z_{i,q})^2 + \frac{\partial \alpha_{i,q-1}}{\partial y_i} \sum_{j=1}^N f_{ij,1}(t, y_j) z_{i,q} \leq \frac{N}{4\bar{l}_{i,q}} \sum_{j=1}^N \| f_{ij,1}(t, y_j) \|^2, \tag{41}$$

$$-\bar{l}_{i,q} \left( \frac{\partial \alpha_{i,q-1}}{\partial y_i} \right)^2 (z_{i,q})^2 + \frac{\partial \alpha_{i,q-1}}{\partial y_i} \epsilon_{i,2} z_{i,q} - \frac{1}{4\bar{l}_{i,q}} \epsilon_i^T \epsilon_i \leq 0, \tag{42}$$

$$\begin{aligned}
 & -\bar{l}_{i,q} \left( \frac{\partial \alpha_{i,q-1}}{\partial y_i} \right)^2 (z_{i,q})^2 + \frac{\partial \alpha_{i,q-1}}{\partial y_i} \sum_{j=1}^N h_{ij,1}(t, y_j(t - \tau_j)) z_{i,q} \\
 & \leq \frac{N}{4\bar{l}_{i,q}} \sum_{j=1}^N \| h_{ij,1}(t, y_j(t - \tau_j)) \|^2. \tag{43}
 \end{aligned}$$

Then from (40),

$$\begin{aligned}
 \dot{V}_i \leq & - \sum_{q=1}^{\rho_i} c_{i,q} (z_{i,q})^2 - \sum_{q=1}^{\rho_i} \frac{1}{4\bar{l}_{i,q}} \epsilon_i^T \epsilon_i - l_i^* (z_{i,1})^2 (\bar{f}_i(y_i))^2 - \lambda_i^* (z_{i,1})^2 (\bar{h}_i(y_i(t)))^2 \\
 & + \sum_{q=1}^{\rho_i} \frac{N}{4\bar{l}_{i,q}} \left( 4 \| \mathbf{P}_i \|^2 \sum_{j=1}^N \| \mathbf{f}_{ij}(t, y_j) \|^2 + \sum_{j=1}^N \| f_{ij,1}(t, y_j) \|^2 \right) \\
 & + \sum_{q=1}^{\rho_i} \frac{N}{4\bar{l}_{i,q}} \left( 4 \| \mathbf{P}_i \|^2 \sum_{j=1}^N \| \mathbf{h}_{ij}(t, y_j(t - \tau_j)) \|^2 + \sum_{j=1}^N \| h_{ij,1}(t, y_j(t - \tau_j)) \|^2 \right). \tag{44}
 \end{aligned}$$

From Assumptions 3 and 4, we can show that

$$\sum_{q=1}^{\rho_i} \frac{N}{4\bar{l}_{i,q}} \left( 4 \| \mathbf{P}_i \|^2 \sum_{j=1}^N \| \mathbf{f}_{ij}(t, y_j) \|^2 + \sum_{j=1}^N \| f_{ij,1}(t, y_j) \|^2 \right) \leq \sum_{j=1}^N \gamma_{ij} (\bar{f}_j(y_j))^2 (y_j)^2, \tag{45}$$

$$\begin{aligned}
 & \sum_{q=1}^{\rho_i} \frac{N}{4\bar{l}_{i,q}} \left( 4 \| \mathbf{P}_i \|^2 \sum_{j=1}^N \| \mathbf{h}_{ij}(t, y_j(t - \tau_j)) \|^2 + \sum_{j=1}^N \| h_{ij,1}(t, y_j(t - \tau_j)) \|^2 \right) \\
 & \leq \sum_{j=1}^N l_{ij} (\bar{h}_j(y_j)(t - \tau_j))^2 (y_j(t - \tau_j))^2, \tag{46}
 \end{aligned}$$



where  $\gamma_{ij} = O(\bar{\gamma}_{ij}^2)$  indicates the coupling strength from the  $j$ th subsystem to the  $i$ th subsystem depending on  $\bar{l}_{i,q}, \| \mathbf{P}_i \|$  and  $O(\bar{\gamma}_{ij}^2)$  denotes that  $\gamma_{ij}$  and  $O(\bar{\gamma}_{ij}^2)$  are in the same order mathematically, and  $\iota_{ij} = O(\bar{l}_{ij}^2)$ .

Then the derivative of  $V_i$  is given as

$$\begin{aligned} \dot{V}_i \leq & - \sum_{q=1}^{\rho_i} c_{i,q}(z_{i,q})^2 - \sum_{q=1}^{\rho_i} \frac{1}{4\bar{l}_{i,q}} \boldsymbol{\epsilon}_i^T \boldsymbol{\epsilon}_i - l_i^*(z_{i,1})^2 (\bar{f}_i(y_i))^2 - \lambda_i^*(z_{i,1})^2 (\bar{h}_i(y_i(t)))^2 \\ & + \sum_{j=1}^N \gamma_{ij} (\bar{f}_j(y_j)y_j)^2 + \sum_{j=1}^N \iota_{ij} (\bar{h}_j(y_j)(t - \tau_j)y_j(t - \tau_j))^2. \end{aligned} \tag{47}$$

To tackle the unknown time-delay problem, we introduce the following Lyapunov-Krasovskii function

$$W_i = \sum_{j=1}^N \frac{\iota_{ij}}{1 - \bar{\tau}_j} \int_{t-\tau_j(t)}^t (\bar{h}_j^1(y_j(s))y_j(s))^2 ds. \tag{48}$$

The time derivative of  $W_i$  is given by

$$\dot{W}_i \leq \sum_{j=1}^N \left( \frac{\iota_{ij}}{1 - \bar{\tau}_j} [\bar{h}_j(y_j(t))y_j(t)]^2 - \iota_{ij} [\bar{h}_j(y_j(t - \tau_j(t)))y_j(t - \tau_j(t))]^2 \right). \tag{49}$$

Now define a new control Lyapunov function for each local subsystem

$$\begin{aligned} V_i^p &= V_i + W_i \\ &= \sum_{q=1}^{\rho_i} \left( \frac{1}{2}(z_{i,q})^2 + \frac{1}{2l_{i,q}} \boldsymbol{\epsilon}_i^T \mathbf{P}_i \boldsymbol{\epsilon}_i \right) + \frac{1}{2} \bar{\Theta}_i^T \boldsymbol{\Gamma}_i^{-1} \bar{\Theta}_i + \frac{|b_{i,m_i}|}{2\gamma_i} \bar{p}_i^2 \\ &\quad + \sum_{j=1}^N \frac{\iota_{ij}}{1 - \bar{\tau}_j} \int_{t-\tau_j(t)}^t (\bar{h}_j^1(y_j(s))y_j(s))^2 ds. \end{aligned} \tag{50}$$

Therefore, the derivative of  $V_i^p$

$$\begin{aligned} \dot{V}_i^p \leq & - \sum_{q=1}^{\rho_i} c_{i,q}(z_{i,q})^2 - \sum_{q=1}^{\rho_i} \frac{1}{4\bar{l}_{i,q}} \boldsymbol{\epsilon}_i^T \boldsymbol{\epsilon}_i - l_i^*(\bar{f}_i(y_i)z_{i,1})^2 - \lambda_i^*(\bar{h}_i(y_i(t)z_{i,1}))^2 \\ & + \sum_{j=1}^N \gamma_{ij} (\bar{f}_j(y_j)y_j)^2 + \sum_{j=1}^N \frac{\iota_{ij}}{1 - \bar{\tau}_j} (\bar{h}_j(y_j)y_j)^2. \end{aligned} \tag{51}$$

Clearly there exists a constant  $\gamma_{ij}^*$  such that for each  $\gamma_{ij}$  satisfying  $\gamma_{ij} \leq \gamma_{ij}^*$  and

$$l_i^* \geq \sum_{j=1}^N \gamma_{ji} \text{ if } l_i^* \geq \sum_{j=1}^N \gamma_{ji}^*. \tag{52}$$

Constant  $\gamma_{ij}^*$  stands for a upper bound of  $\gamma_{ij}$ .

Simialy, there exists a constant  $\iota_{ij}^*$  such that for each  $\iota_{ij}$  satisfying  $\iota_{ij} \leq \iota_{ij}^*$  and

$$\lambda_i^* \geq \sum_{j=1}^N \iota_{ji} \frac{1}{1 - \bar{\tau}_i} \text{ if } \lambda_i^* \geq \sum_{j=1}^N \iota_{ji}^* \frac{1}{1 - \bar{\tau}_i}. \tag{53}$$

Now we define a Lyapunov function of overall system

$$V = \sum_{i=1}^N V_i^p. \tag{54}$$

Now taking the summation of the last four terms in (51) and using (52) and (53), we get

$$\begin{aligned} & \sum_{i=1}^N \left[ -l_i^* (\bar{f}_i(y_i)z_{i,1})^2 - \lambda_i^* (\bar{h}_i(y_i(t)z_{i,1})^2 + \sum_{j=1}^N \gamma_{ij} (\bar{f}_j(y_j)y_j)^2 + \sum_{j=1}^N \frac{t_{ij}}{1 - \bar{\tau}_j} (\bar{h}_j(y_j)y_j)^2 \right] \\ & = \sum_{i=1}^N \left[ - \left( l_i^* - \sum_{j=1}^N \gamma_{ji} \right) (\bar{f}_i(y_i)y_i)^2 - \left( \lambda_i^* - \sum_{j=1}^N \frac{t_{ji}}{1 - \bar{\tau}_i} \right) (\bar{h}_i(y_i)y_i)^2 \right] \leq 0. \end{aligned} \tag{55}$$

Therefore,

$$\dot{V} \leq - \sum_{i=1}^N \sum_{q=1}^{\rho_i} c_{i,q} (z_{i,q})^2 - \sum_{i=1}^N \sum_{q=1}^{\rho_i} \frac{1}{4l_{i,q}} \epsilon_i^T \epsilon_i \leq 0. \tag{56}$$

This shows that  $V$  is uniformly bounded. Thus  $z_{i,1}, \dots, z_{i,\rho_i}, \hat{p}_i, \hat{\Theta}_i, \epsilon_i$  are bounded. Since  $z_{i,1}$  is bounded,  $y_i$  is also bounded. Because of the boundedness of  $y_i$ , variables  $v_{i,j}, \xi_{i,0}$  and  $\Xi_i$  are bounded as  $A_{i,0}$  is Hurwitz. Following similar analysis to Wen & Zhou (2007), we can show that all the states associated with the zero dynamics of the  $i$ th subsystem are bounded under Assumption 2. In conclusion, boundedness of all signals is ensured as formally stated in the following theorem.

**Theorem 1.** Consider the closed-loop adaptive system consisting of the plant (1) under Assumptions 1-4, the controller (37), the estimator (28) and (38), and the  $\square$ lters (7)-(9). There exist a constant  $\gamma_{ij}^*$  such that for each constant  $\gamma_{ij}$  satisfying  $\gamma_{ij} \leq \gamma_{ij}^*$  and  $t_{ij}$  satisfying  $t_{ij} \leq t_{ij}^*, j = 1, \dots, N$ , all the signals in the system are globally uniformly bounded.

We now derive a bound for the vector  $z_i(t)$  where  $z_i(t) = [z_{i,1}, z_{i,2}, \dots, z_{i,\rho_i}]^T$ . Firstly, the following definitions are made.

$$c_i^0 = \min_{1 \leq q \leq \rho_i} c_{i,q} \tag{57}$$

$$\|z_i\|_2 = \sqrt{\int_0^\infty \|z_i(t)\|^2 dt}. \tag{58}$$

From (56), the derivative of  $V$  can be given as

$$\dot{V} \leq -c_i^0 \|z_i\|^2. \tag{59}$$

Since  $V$  is nonincreasing, we obtain

$$\|z_i\|_2^2 = \int_0^\infty \|z_i(t)\|^2 dt \leq \frac{1}{c_i^0} (V(0) - V(\infty)) \leq \frac{1}{c_i^0} V(0). \tag{60}$$

Similarly, the output  $y_i$  is bounded by

$$\|y_i\|_2^2 = \int_0^\infty (y_i(t))^2 dt \leq \frac{1}{c_{i,1}} V(0). \tag{61}$$

**Theorem 2.** *The  $L_2$  norm of the state  $z_i$  is bounded by*

$$\|z_i(t)\|_2 \leq \frac{1}{\sqrt{c_i^0}} \sqrt{V(0)}, \quad (62)$$

$$\|y_i\|_2^2 \leq \frac{1}{\sqrt{c_{i,1}}} \sqrt{V(0)}. \quad (63)$$

**Remark 6.** *Regarding the output bound in (63), the following conclusions can be drawn by noting that  $\hat{\Theta}_i(0)$ ,  $\hat{p}_i(0)$ ,  $\epsilon_i(0)$  and  $y_i(0)$  are independent of  $c_{i,1}$ ,  $\Gamma_i$ ,  $\gamma_i$ .*

• *The transient output performance in the sense of truncated norm given in (62) depends on the initial estimation errors  $\hat{\Theta}_i(0)$ ,  $\hat{p}_i(0)$  and  $\epsilon_i(0)$ . The closer the initial estimates to the true values, the better the transient output performance.*

• *This bound can also be systematically reduced down to a lower bound by increasing  $\Gamma_i$ ,  $\gamma_i'$ ,  $c_{i,1}$ .*

## 5. Simulation example

We consider the following interconnected system with two subsystems.

$$\dot{x}_1 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x_1 + \begin{bmatrix} 2y_1 & y_1^2 \\ 0 & y_1 \end{bmatrix} \theta_1 + \begin{bmatrix} 0 \\ b_1 \end{bmatrix} u_1 + f_1 + h_1, \quad y_1 = x_{1,1} \quad (64)$$

$$\dot{x}_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x_2 + \begin{bmatrix} 0 & 0 \\ y_2 & 1 + y_2 \end{bmatrix} \theta_2 + \begin{bmatrix} 0 \\ b_2 \end{bmatrix} u_2 + f_2 + h_2, \quad y_2 = x_{2,1}, \quad (65)$$

where  $\theta_1 = [1, 1]^T$ ,  $\theta_2 = [0.5, 1]^T$ ,  $b_1 = b_2 = 1$ , the nonlinear interaction terms  $f_1 = [0, y_2^2 + \sin(y_1)]^T$ ,  $f_2 = [0.2y_1^2 + y_2, 0]^T$ , the external disturbance  $h_1 = \mathbf{0}$ ,  $h_2 = [y_1(t - \tau_1), y_2(t - \tau_2)]^T$ . The parameters and the interactions are not needed to be known. The objective is to make the outputs  $y_1$  and  $y_2$  converge to zero.

The design parameters are chosen as  $c_{1,1} = c_{1,2} = 2$ ,  $c_{2,1} = c_{2,2} = 3$ ,  $l_{1,1} = l_{1,2} = 1$ ,  $l_{2,1} = l_{2,2} = 2$ ,  $l_1^* = l_2^* = 5$ ,  $\lambda_1^* = \lambda_2^* = 5$ ,  $\gamma_1' = 2$ ,  $\gamma_2' = 2$ ,  $\Gamma_1 = 0.5\mathbf{I}_3$ ,  $\Gamma_2 = \mathbf{I}_3$ ,  $l_{i,p} = l_{i,\Theta} = 1$ ,  $p_{1,0} = p_{2,0} = 1$ ,  $\Theta_{1,0} = [1, 1, 1]^T$ ,  $\Theta_{2,0} = [0.6, 1, 1]^T$ . The initials are set as  $y_1(0) = 0.5$ ,  $y_2(0) = 1$ ,  $\hat{\Theta}_1(0) = [0.5, 0.8, 0.8]^T$ ,  $\hat{\Theta}_2(0) = [0.6, 0.8, 0.8]^T$ . The block diagram in Figure 1 shows the proposed control structure for each subsystem. The input signals to the designed  $i$ th local adaptive controller are  $y_i$ ,  $\xi_{i,0}$ ,  $\Xi_i$ ,  $\mathbf{v}_{i,0}$ . Figures 2-3 show the system outputs  $y_1$  and  $y_2$ . Figures 4-5 show the system inputs  $u_1$  and  $u_2(t)$ . All the simulation results verify that our proposed scheme is effective to cope with nonlinear interactions and time-delay.

## 6. Conclusion

In this chapter, a new scheme is proposed to design totally decentralized adaptive output stabilizer for a class of unknown nonlinear interconnected system in the presence of time-delays. Unknown time-varying delays are compensated by using appropriate Lyapunov-Krasovskii functionals. It is shown that the designed decentralized adaptive controllers can ensure the stability of the overall interconnected systems. An explicit bound in terms of  $L_2$  norms of the output is also derived as a function of design parameters. This implies that the transient the output performance can be adjusted by choosing suitable design parameters.

## 7. References

- Chou, C. H. & Cheng, C. C. (2003). A decentralized model reference adaptive variable structure controller for large-scale time-varying delay systems, *IEEE Transactions on Automatic Control* 48: 1213–1217.
- Ge, S. S., Hang, F. & Lee, T. H. (2003). Adaptive neural network control of nonlinear systems with unknown time delays, *IEEE Transactions on Automatic Control* 48: 4524–4529.
- Hua, C. C., Guan, X. P. & Shi, P. (2005). Robust backstepping control for a class of time delayed systems, *IEEE Transactions on Automatic Control* 50: 894–899.
- Hua, C., Guan, X. & Shi, P. (2007). Robust output feedback tracking control for time-delay nonlinear systems using neural network, *IEEE Transactions on Neural Networks* 18: 495–505.
- Ioannou, P. (1986). Decentralized adaptive control of interconnected systems, *IEEE Transactions on Automatic Control* 31: 291–298.
- Jankovic, M. (2001). Control lyapunov-razumikhin functions and robust stabilization of time delay systems, *IEEE Transactions on Automatic Control* 46: 1048–1060.
- Jiang, Z. P. (2000). Decentralized and adaptive nonlinear tracking of large-scale system via output feedback, *IEEE Transactions on Automatic Control* 45: 2122–2128.
- Jiao, X. & Shen, T. (2005). Adaptive feedback control of nonlinear time-delay systems the lasalle-razumikhin-based approach, *IEEE Transactions on Automatic Control* 50: 1909–1913.
- Krstic, M., Kanellakopoulos, I. & Kokotovic, P. V. (1995). *Nonlinear and Adaptive Control Design*, Wiley, New York.
- Luo, N., Dela, S. M. & Rodellar, J. (1997). Robust stabilization of a class of uncertain time delay systems in sliding mode, *International Journal of Robust and Nonlinear Control* 7: 59–74.
- Narendra, K. S. & Oleng, N. (2002). Exact output tracking in decentralized adaptive control systems, *IEEE Transactions on Automatic Control* 47: 390–395.
- Ortega, R. (1996). An energy amplification condition for decentralized adaptive stabilization, *IEEE Transactions on Automatic Control* 41: 285–288.
- Shyu, K. K., Liu, W. J. & Hsu, K. C. (2005). Design of large-scale time-delayed systems with dead-zone input via variable structure control, *Automatica* 41: 1239–1246.
- Wen, C. (1994). Decentralized adaptive regulation, *IEEE Transactions on Automatic Control* 39: 2163–2166.
- Wen, C. & Soh, Y. C. (1997). Decentralized adaptive control using integrator backstepping, *Automatica* 33: 1719–1724.
- Wen, C. & Zhou, J. (2007). Decentralized adaptive stabilization in the presence of unknown backlash-like hysteresis, *Automatica* 43: 426–440.
- Wu, H. (2002). Decentralized adaptive robust control for a class of large-scale systems including delayed state perturbations in the interconnections, *IEEE Transactions on Automatic Control* 47: 1745–1751.
- Wu, S., Deng, F. & Xiang, S. (2006). Backstepping controller design for large-scale stochastic systems with time delays, *Proceedings of the World Congress on Intelligent Control and Automation (WCICA)*, pp. 1181–1185.
- Wu, W. (1999). Robust linearising controllers for nonlinear time-delay systems, *IEE Proceedings on Control Theory and Applications* 146: 91–97.
- Zhou, J. (2008). Decentralized adaptive control for large-scale time-delay systems with dead-zone input, *Automatica* 44: 1790–1799.

- Zhou, J. & Wen, C. (2008a). *Adaptive Backstepping Control of Uncertain Systems: Nonsmooth Nonlinearities, Interactions or Time-Variations*, Lecture Notes in Control and Information Sciences, Springer-Verlag.
- Zhou, J. & Wen, C. (2008b). Decentralized backstepping adaptive output tracking of interconnected nonlinear systems, *IEEE Transactions on Automatic Control* 53: 2378–2384.
- Zhou, J., Wen, C. & Wang, W. (2009). Adaptive backstepping control of uncertain systems with unknown input time-delay, *Automatica* 45: 1415–1422.

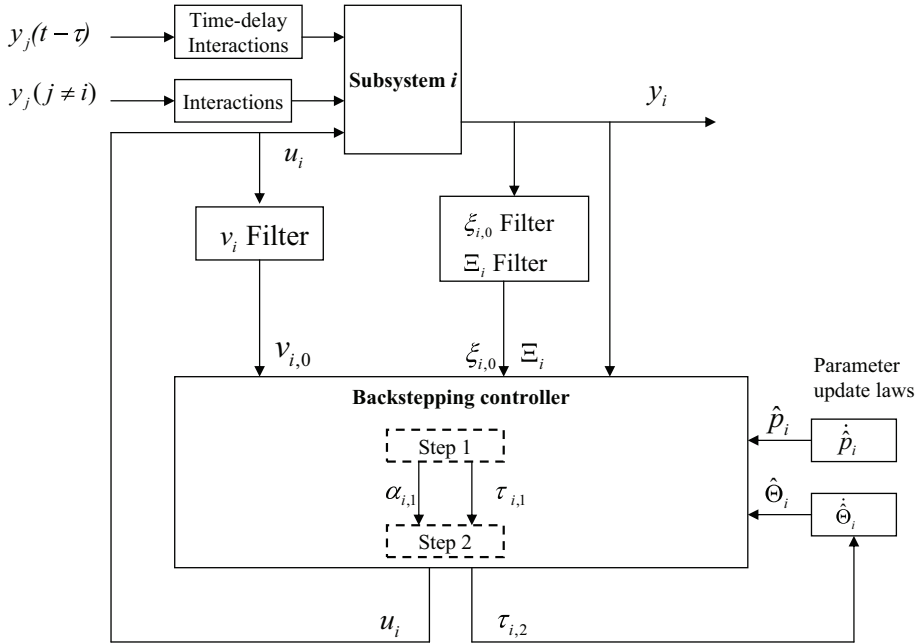


Fig. 1. Control block diagram.

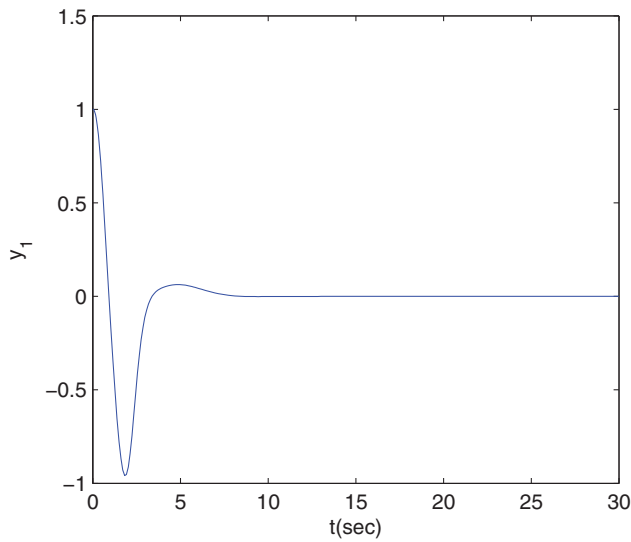
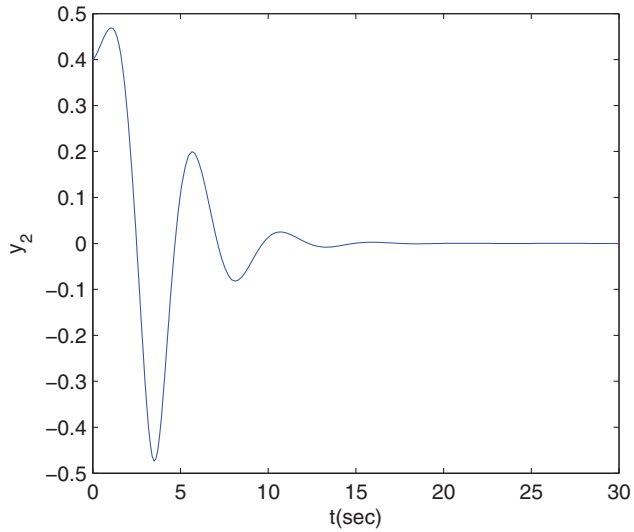
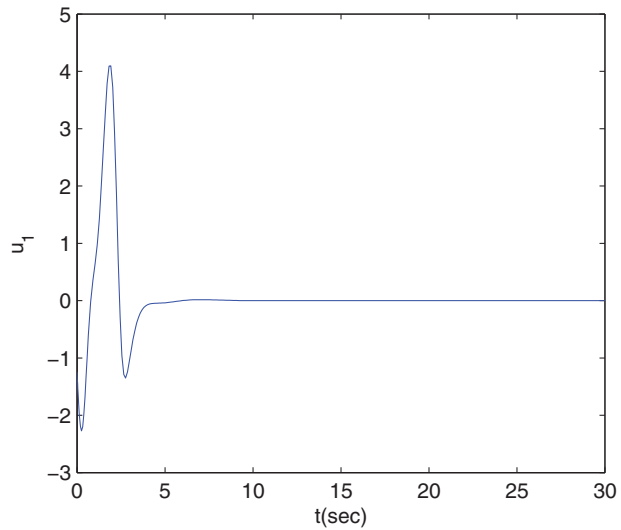


Fig. 2. Output  $y_1$ .

Fig. 3. Output  $y_2$ Fig. 4. Input  $u_1$ .

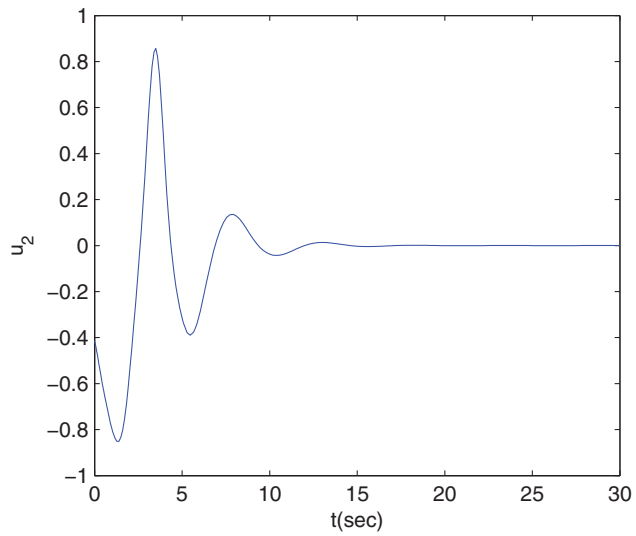


Fig. 5. Input  $u_2$ .





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Time delay is very often encountered in various technical systems, such as electric, pneumatic and hydraulic networks, chemical processes, long transmission lines, robotics, etc. The existence of pure time lag, regardless if it is present in the control or/and the state, may cause undesirable system transient response, or even instability. Consequently, the problem of controllability, observability, robustness, optimization, adaptive control, pole placement and particularly stability and robustness stabilization for this class of systems, has been one of the main interests for many scientists and researchers during the last five decades.

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