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Optimizing Laminated Composites Using Ant Colony Algorithms

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1. Introduction

The high stiffness to weight ratio as well as the possibility of tuning mechanical properties by designing proper fiber orientation and stacking sequence are the key attributes of laminated composites. Hence, they are inevitable candidates against isotropic materials for modern structures especially in aerospace and marine industries. A powerful optimization tool is therefore essential to determine the optimum geometry and lay-up.

1.1 Optimization objectives

Weight, cost, deflection, stress and natural frequencies are general design criteria investigated in laminated structures. Weight and cost minimization, normally being correlated, are the most important design objectives. Decreasing the deflection of structures or maintaining it in safe ranges is also often requested. This goal is generally associated with controlling the maximum amounts of stress or attempting to achieve an optimal pattern of stress distribution. Natural frequency maximization, especially the fundamental one, is of importance in the design of laminates to decrease the risk of resonance caused by external excitations. Frequently, most of the mentioned aspects are tightly connected so multi-objective formulation or considering them as constraints is indispensable.
Besides shape, topology and size optimization applicable to all kinds of structures, number and thickness of layers as well as fiber orientations can be directly considered as design variables to optimize the design objectives in laminated composites. In addition, it is an effective discipline to hybridize the structure by employing high-stiffness and more expensive material in the outer layers and inexpensive low-stiffness material in the inner ones. Hence, Rigidity and material cost remain at reasonable levels. Also, the designer may make use of number of core and surface layers as supplementary optimization variables.

1.2 Optimization levels

Laminated structures optimized in papers are of considerable range of intricacy, nevertheless can be categorized as three main groups. In the first, classical structures such as beams or plates are optimized where generally a closed-form solution to the mechanical analysis is available. As a result, the designer can benefit direct calculation of objective functions. The results reported in these cases are of great value in initial design steps of real-

world applications. Also, when a new optimization method is to be benchmarked, these cases are logically preferred since several iterative evaluations with different parameter settings are necessary. In this group, laminates variables are normally employed and not the geometrical ones.
Structures with average level of complexity are clustered in the second category. Studies on laminated pressure vessels or auto parts such as drive shaft or leaf spring (Shokrieh & Rezaei, 2003) can be referred. The mechanical analysis is regularly performed using FEM packages and the optimization is done using their embedded optimizing tools or independent integrated codes. Both geometrical and laminates variables might be engaged though composites are usually modeled as solid orthotropic structures.
In the final group, assembled systems with multiple design variables are considered. Chassis (Rastogi, 2004), door panels, roof (Botkin, 2000) or hood in automobiles as well as plane wings are typical cases (Venkataraman & Haftka, 2004). Even optimization of fully assembled vehicles or space structures including up to 50000 design variables has been reported (Vanderplaats, 2002). Obviously, analysis in such levels calls for large or very large scale optimization procedures. Additionally, especially developed packages are required to handle the burden of calculations.

1.3 Optimization methods

In the last forty years, nearly all classes of optimization algorithms have been employed for designing laminated composites (Ghiasi et al., 2009). The early attempts have been done using graphical techniques. Gradient-based methods include considerable number of papers mostly earlier ones; while direct methods particularly heuristic algorithms compile the major group of papers. Genetic algorithm (GA) has been the main accepted approach whereas its counterparts such as simulated annealing (SA), Tabu search (TS), scatter search and finally ant colony and bee colony methods have got smaller shares.
In a laminated structure, type and material of each ply are discrete variables while the fiber angles may have any orientations from -90 to 90 degrees. The stacking of plies is also a problem of combinatorial type. In most of papers, fiber angles are considered discrete and hence the whole optimization problem has been tackled using discrete versions of algorithms. It is of course noteworthy to remind that in most real engineering applications, it is reasonable to make use of standard layers with certain thicknesses and limited number of angles. Here, it will be explained later how problems like this with mixed variables can be formulated using ant colony methods. In addition, the results obtained by considering discrete versus mixed variables will be comprehensively discussed for benchmark problems of laminates design.

2. Ant Colony Optimization (ACO)

2.1 Introduction

Inspired by the collective behavior of real ant colonies, Ant System (AS) algorithm was introduced by Marco Dorigo in 1992. The developed metaheuristic named as ant colony system (ACS) was later presented in 1997 by Dorigo and Gambardella for solving the traveling salesman problem (TSP). Up to now, the algorithm has been extensively and successfully applied on many combinatorial problems such as quadratic assignment, vehicle routing and job-shop scheduling (Dorigo & Stutzle, 2004). In the field of structural and mechanical engineering, published papers have been rapidly emerging in the last 5 years.

Trusses (Serra & Venini, 2006), frames (Camp et al., 2005), and manufacturing processes (Vijayakimar et al., 2003) are among the cases optimized using different versions of ant colony algorithms.

Regarding laminated structures, the first study was done by Yang et al (2006). They tried to find optimal orientations of carbon fibers in CFPR composites which were used to reinforce precracked concrete structures. Abachizadeh and Tahani (2007) optimized hybrid laminates for minimum cost and weight using ACS and Pareto techniques. Later, they performed multi-objective optimization of hybrid laminates for maximum fundamental frequency and minimum cost using ACS (2009). Lay-up design of laminated panels for maximization of buckling load with strength constraints was studied by Aymerich and Serra (2008) and the results demonstrated improvement over GA and TS results. A simply supported composite laminate was investigated for optimal stacking sequence under strength and buckling constraints by Bloomfield et al. (2010). They compared the performance of GA, ACS and particle swarm optimization (PSO) methods assuming both discrete and continuous ply orientations. It was claimed that for discrete design spaces, ACS performs better but when it comes to continuous design spaces, PSO outperforms the two other techniques.

For a hybrid laminated plate with mixed domain of solution including fiber angles as continuous and number of surface (or core) layers as integer discrete variables, Abachizadeh et al. (2010) showed that an extension of ACO for continuous domains called ACO_R method proposed by Socha and Dorigo (2008) results in improved designs against GA and ACS. In an industrial application, Hudson et al. (2010) described the appliance of ACO algorithm to the multiple objective optimization of a rail vehicle floor sandwich panel to reduce material as well as cost. A modified ant colony algorithm with novel operators called multi-city-layer ant colony algorithm (MCLACA) is also presented by Wang et al. (2010) exhibiting more robust and efficient comparing with GA for buckling load maximization of a rectangular laminate.

2.2 Solving problems with continuous or mixed design space

While most of the heuristic methods have been initially proposed to tackle combinatorial optimization problems, many real-world engineering problems include either continuous or mixed variables. Hence, there has been a considerable amount of research to suggest new metaheuristics or adapt the existing ones. The same account for ACO, methods were proposed to handle continuous variables. Although taking inspiration from original ACO and expressing relatively acceptable results, they did not follow its original concept exactly (Abachizadeh and Kolahan, 2007). In the original approach of ant algorithms, each ant constructs the solution incrementally using the set of available solution components defined by the problem formulation. This selection is done with help of probabilistic sampling from a discrete probability distribution. For tackling continuous problems using this method, the continuous domain should be discretized into finite ranges. This is not always an appropriate technique especially if the initial range is wide or the resolution required is high. In such cases, methods which can natively handle continuous variables usually perform better.

Besides minor operators which are different in various versions of ant algorithms, there are two distinct differences about ACO_R in comparison with preceding ones. The first is the shift from using a discrete probability distribution to a continuous one called Probability Distribution Function (PDF). Therefore, each ant instead of selecting from available finite sets, samples a PDF biased toward high quality solutions.

The other variation to the original ACO is the revision of the way pheromone information is stored and updated. The continuous nature of design variables prevents a tabular discrete formation and hence, an archive of solutions similar to what is constructed in Tabu search method is employed.

For problems with mixed domain, two common approaches are available. The first is the discretization of continuous variables and as the problem is modified to a totally discrete one; all direct and heuristic methods are applicable with their original procedures. The other approach is relaxation of discrete variables. First, an index is assigned to any element of discrete set of variables. Henceforward, the discrete variable is treated as a continuous variable. Only before evaluation of objective functions, the obtained values for discrete variables are rounded to the nearest index number. It is remarkable that discrete variables are of different types. Integers, zero-one values, ordered standard values (e.g. cross sections of reinforcements round bars) and categorical variables (e.g. different material candidates for a structure) are the major types. Socha (2008) claims that employing relaxation method for mixed problems by ACO_R when no categorical variables exist results in reasonable outcomes. However, facing categorical variables, he expects poor performance and suggests a new method exclusively developed for mixed domains called ACO_{MV} (Socha, 2008).

2.3 ACO_R procedure

Given an n-dimensional continuous problem, an archive of dimension k is constructed. As shown in Fig. 1, s_j^i denotes the value of the ith variable of the jth solution. The two additional columns are considered for the amount of objective function and a weighting parameter associated with each solution.

At the beginning, the rows of this archive are constructed using randomly generated solutions. The solutions in the archive are always sorted according to their quality i.e., the value of the objective function; hence the position of a solution in the archive always corresponds to its rank and the best solution will be on top. At each iteration and with employing m ants, m new solutions are generated and the best k solutions among $m+k$ solutions are kept i.e. the archive is updated with best solutions found so far. For generating new solutions, each ant chooses probabilistically one of the solutions in the archive:

$$p_j = \frac{\omega_j}{\sum_{r=1}^{k} \omega_r} \tag{1}$$

where ω_j is the weight associated with solution j. As proposed by Socha and Dorigo (2008), the flexible and nonlinear Gaussian function is employed to define this weight:

$$\omega_j = \frac{1}{qk\sqrt{2\pi}} e^{\frac{-(j-1)^2}{2q^2k^2}} \tag{2}$$

where q is a parameter of the algorithm. The ant then takes the value s_j^i and samples its neighborhood for a new value for variable i and repeats this procedure for all variables i=1, …, n using the same jth solution. This is done using a probability density function (PDF). There are different choices to be selected for demonstrating this distribution however Gaussian function is again chosen (Socha, 2008):

$$P(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} \tag{3}$$

where μ and σ are two other algorithm parameters and should be properly defined. For μ, s_j^i is easily assigned. For σ, we have

$$\sigma = \xi \sum_{r=1}^{k} \frac{\left|s_r^i - s_j^i\right|}{k-1} \tag{4}$$

which is the average distance between the ith variable of the solution s_j and the ith variable of the other solution in the archive multiplied by a parameter ξ. This parameter has an effect similar to the evaporation rate in original ACO i.e. with higher values of ξ, the new solutions are positioned closer to existing solutions of higher ranks.

s_1^1	s_1^2	$\cdots$	s_1^i	$\cdots$	s_1^n	$f(s_1)$	ω_1
s_2^1	s_2^2	$\cdots$	s_2^i	$\cdots$	s_2^n	$f(s_2)$	ω_2
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
s_j^1	s_j^2		s_j^i		s_j^n	$f(s_j)$	ω_j
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
s_k^1	s_k^2	$\cdots$	s_k^i	$\cdots$	s_k^n	$f(s_k)$	ω_k

Fig. 1. The structure of solution archive in ACO_R (Socha, 2008)

3. Benchmark problems

A simply supported symmetric hybrid laminated plate with length *a*, width *b* and thickness *h* in the *x*, *y* and *z* direction, respectively, is considered. Each of the material layers is of equal thickness *t* and idealized as a homogeneous orthotropic material. The total thickness of the laminate is equal to $h = N \times t$ with *N* being the total number of the layers. In the analysis presented here, the total thickness of the laminate is kept constant which allows for comparing the performance of designs with equal thicknesses.

The optimal design problem involves selection of optimal stacking sequence of hybrid laminated composite plates to obtain maximum fundamental frequency and minimum cost in a multi-objective process. The design variables engaged are the fiber orientation in each layer and the number of core (or surface) layers while the total number of plies is constant in each problem case. Three benchmark problems with different levels of complexity are optimized and the results of ACO_R (Abachizadeh et al., 2010), ACS (Abachizadeh & Tahani, 2009), GA (Tahani et al., 2005) and SA (Kolahan et al.) are reported. It is tried to evaluate the performance of ant algorithms against GA and SA. The improvements obtained by employing continuous operators are also discussed.

3.1 Analysis of vibration in laminates

The hybrid laminate is made up of N_i inner and N_o outer layers so that $N = N_i + N_o$. The governing equation of motion within the classical laminated plate theory for the described symmetric laminate is given below (Reddy, 2004):

$$\begin{aligned} &D_{11}\frac{\partial^4 w}{\partial x^4} + 4D_{16}\frac{\partial^4 w}{\partial x^3 \partial y} + 2(D_{12} + 2D_{66})\frac{\partial^4 w}{\partial x^2 \partial y^2} \\ &+4D_{26}\frac{\partial^4 w}{\partial x \partial y^3} + D_{22}\frac{\partial^4 w}{\partial y^4} = \rho h \frac{\partial^2 w}{\partial t^2} \end{aligned} \tag{5}$$

where w is the deflection in the z direction, h is the total thickness of the laminate and ρ is the mass density averaged in the thickness direction which is given by:

$$\rho = h^{-1}\int_{-h/2}^{h/2} \rho^{(k)} dz = \frac{1}{N}\sum_{k=1}^{N} \rho^{(k)} \tag{6}$$

where $\rho^{(k)}$ represents the mass density of material in the kth layer. The bending stiffnesses D_{ij} in Eq. (5) are defined as:

$$D_{ij} = \sum_{k=1}^{N}\int_{z_k}^{z_{k+1}} \bar{Q}_{ij}^{(k)} z^2 dz \tag{7}$$

where $\bar{Q}_{ij}^{(k)}$ is the transformed reduced stiffness of the kth layer. The boundary conditions are given by:

$$\begin{aligned} w = 0,\ M_x = 0 \quad \text{at} \quad x = 0, a \\ w = 0,\ M_y = 0 \quad \text{at} \quad y = 0, b \end{aligned} \tag{8}$$

where the moment resultants are defined as:

$$(M_x, M_y) = \int_{-h/2}^{h/2} (\sigma_x, \sigma_y)\ dz \tag{9}$$

It is shown by Nemeth (1986) that in buckling problems, the terms D_{16} and D_{26} which demonstrate the bending-twisting interactions in composite laminates, can be safely neglected if the non-dimensional parameters:

$$\gamma = D_{16}(D_{11}^3 D_{22})^{-1/4}, \quad \delta = D_{26}(D_{11} D_{22}^3)^{-1/4} \tag{10}$$

satisfy the constraints:

$$\gamma \le 0.2, \quad \delta \le 0.2 \tag{11}$$

Due to the analogy between buckling and free vibration analysis, the same constraints are used to reduce the complexity of the problem. Taking into account the governing equation and the boundary conditions in Eq. (8), a general form of solution for w in the natural vibration mode (m,n) is presented as:

$$w(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}A_{mn}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}e^{i\omega_{mn}t} \tag{12}$$

where ω_{mn} is the natural frequency of the vibration mode (m,n) and $i=\sqrt{-1}$.
Substituting (12) into (5) yields:

$$\omega_{mn}^2=\frac{\pi^4}{\rho h}\left[D_{11}\left(\frac{m}{a}\right)^4+2(D_{12}+2D_{66})\left(\frac{m}{a}\right)^2\left(\frac{n}{b}\right)^2+D_{22}\left(\frac{n}{b}\right)^4\right] \tag{13}$$

Different mode shapes are obtained by inserting different values of m and n. For computing the fundamental frequency, both are obviously put equal to one.

3.2 Problem I

The fundamental frequency for 16-layered glass/epoxy laminates with various aspect ratios is considered to be maximized. The laminate is not hybrid so the problem is single-objective. The benchmark is chosen from Adali and Verijenco (2001) where laminates made up of 0°, $\pm45^\circ$ and 90° plies are optimized by direct enumeration. Here, the fiber angles are allowed to have any value in the range of $[-90^\circ, 90^\circ]$. Hence, the deviation of the fundamental frequency for laminates with the mentioned restricted angles is obtained from the optimal design. Results achieved using ACS is also presented.

3.3 Problem II

The fundamental frequency for 8-layered graphite/epoxy laminates with various aspect ratios is considered to be maximized. The fiber angles are considered continuous in the range of $[-90^\circ, 90^\circ]$ and the results are compared with a solution where the discrete angles have been selected within the same range with 15-degree increments. Although this problem can be considered an independent benchmark, it has been selected so as to be used in the main multi-objective problem.

3.4 Problem III

The fundamental frequencies and material costs of 8-, 16- and 28-layered hybrid laminates with various aspect ratios is considered to be optimized. The fiber angles are continuous as in problems I and II. The results are compared with a solution where discretized fiber angles, the same as problem II, have been employed. The main objective of this reinvestigation is to find the global solution of the problem considering continuous variables in order to verify whether using standard restricted fiber angles is an acceptable approach.
In problems I and II where single-objective optimization is put into operation, the procedure is straightforward. The only point to consider is the penalty added to the solution candidates which violate the constraints in Eq. (11). Obviously, it should be big enough to remove any chance of being selected.

Problem III deals with multi-objective optimization which calls for its own methods. As the objectives in this study are of different dimensions and their relative importance is case-dependent, the Min-Max method is preferred. In this method, the objective functions are normalized and then the deviation from their single-objective optimums is minimized.
The two objective functions engaged here are numbered as 1 for frequency and 2 for material cost. Satisfaction of constraints is imposed by penalty functions just like the first two problems. The general form of the total objective function subjected to minimization is given by:

$$F = k_1 f_1^2 + k_2 f_2^2 + c_1 g_1^2 + c_2 g_2^2 \tag{14}$$

$$f_1 = \left(\frac{\omega_{\max} - \omega}{\omega_{\max}} \right) \tag{15a}$$

$$f_2 = \left(\frac{\text{cost}}{\text{cost}_{\max}} \right) \tag{15b}$$

$$g_1 = \delta - 0.2 \tag{16a}$$

$$g_2 = \gamma - 0.2 \tag{16b}$$

$$k_1 = k_2 = c_1 = c_2 = 1 \tag{16c}$$

In Eqs. (15a,15b), ω and *cost* are the optimization outcomes. The parameter $\omega_{\max}$ denotes the maximum fundamental frequency and $cost_{\max}$ is the cost both obtained by considering all layers being made up of graphite/epoxy. The numerical values of $\omega_{\max}$ are in fact the results of problem II which can be used for the corresponding aspect ratios in problem III.
In (16 a - 16c), g_1 and g_2 are the penalty terms explained earlier. The other parameters, k_1, k_2, c_1 and c_2 are sensitivity coefficients which are all set to one.
The material cost can be easily calculated as:

$$\text{cost} = ab\frac{h}{N} g\left(\alpha_o \rho_o N_o + \rho_i N_i\right) \tag{17}$$

where ρ_o and ρ_i are the mass densities of the outer and inner layers, respectively. Instead of real material prices, α_o is employed as a cost factor expressing the cost ratio of surface against core material.

4. Numerical results

The three problems defined in the previous section have been optimized using a code written in Matlab®. The laminates geometrical dimensions are $h = 0.002$ m and $b = 0.25$ m . As stated earlier, the total thickness h is considered constant for different number of layers while ply thickness may vary. This is a little far from being practical but necessary for comparing the performance of equal-thickness designs.

In all the three problems, designs with different aspect ratios defined as a/b are investigated varying from 0.2 to 2 with a 0.2 increment. The properties of materials taken from Tsai and Hahn (1980) are as follows:

Graphite/Epoxy (T300/5208):
E_1=181 GPa, E_2=10.3 GPa, G_{12}=7.71 GPa, υ_{12}=0.28, ρ=1600 kgm^{-3}

Glass/Epoxy (Scotchply1002):
E_1=38.6 GPa, E_2=8.27 GPa, G_{12}=4.14 GPa, υ_{12}=0.26, ρ=1800 kgm^{-3}

Although the mechanical properties particularly stiffness to weight ratio are considerably higher in graphite/epoxy layers with respect to glass/epoxy ones, they are about 8 times more expensive; this way $\alpha_0 = 8$ is assigned.

There are several parameters in ACO_R which should be tuned to maximize the efficiency of the algorithm. Here, *k*=50 is used as proposed by Socha (2008). The other parameters are set based on some trials for these problems:

$$m = 5 \quad , \quad \xi = 0.9 \quad , \quad q = 10^{-5} \tag{18}$$

Table 1 presents the results obtained by solving problem I using ACS, ACO_R and direct enumeration. Clearly, considering continuous fiber angles, stacking sequences with higher fundamental frequencies are achieved using ACO_R. For aspect ratios of 0.6, 0.8, 1.2, 1.4 and 1.6, angles other than standard angles of 0°, $\pm45^\circ$ and 90° are needed to construct the optimum laminate. However, the greatest difference in frequency (belonging to aspect ratio of 1.4) is equal to 15 rad/s ≡ 2.6%, which is flawlessly negligible and therefore justifies using configurations exclusively made up of either 0°, $\pm45^\circ$ or 90° plies.

Table 2 with a similar pattern shows that ACO_R has been again successful in improving designs obtained by ACS. From practical point of view, the maximum deviation belonging to aspect ratio of 0.8 is about 19 rad/s ≡ 1%, which is yet again negligible.

Tables 3-5 show the best results found by ACO_R for 8-, 16- and 28-layered laminates. In addition, the related results obtained by ACS, GA and SA are reported. Since Tahani et al. (2005) and Kolahan et al. (2005) have reported nearly identical solutions for GA and SA; the results are presented as only one alternative against ACS and ACO_R. Considering the values of objective functions, it is clear that ACO_R has outperformed both ACS and GA (as well as SA) or in any case equalized in performance. It could be estimated from the results of Table 2 that for aspect ratios in which ACO_R have been able to find designs with higher fundamental frequencies, improvements in the multi-objective problem can be anticipated. This is further verified observing that all three involved methods could find the optimal number of glass and graphite plies i.e. the cost part of the objective function, nearly for all laminates designs.

Regarding the similarity of fiber angles in Tables 3-5 obtained by ACO_R and in single-objective versions of problem reported in Tables 1-2, we can conservatively claim that ACO_R has found global optima for all aspect ratios. The significance is furthermore highlighted if it is reminded that the problem has been tackled considering continuous domain for design variables which is naturally of higher complexity in comparison to the combinatorial problem of constructing laminates from finite available fiber angles.

In analogous to the results of problems I and II, similar deductions can be presented about the reliability of the approach of utilizing standard fiber angles. The optimum designs obtained by ACO_R do not demonstrate being more than 3% better than the corresponding designs obtained using discretized fiber angles.

Finally, It is noteworthy to mention that for designs with N=28 in problem III which are the most complex problem cases in this paper, the results of ACS and ACO_R are considerably better than GA and SA in most cases which can indicate the efficiency and robustness of ant

$\frac{a}{b}$	θ_{best} by ACO_R	$\omega_{\max}$ by ACO_R (rad/sec)	θ_{best} by ACS	θ_{best} by enum.	$\omega_{\max}$ by ACS (rad/sec)
0.2	$[0]_{8s}$	10747	$[0]_{8s}$	$[0]_{8s}$	10747
0.4	$[0]_{8s}$	2776	$[0]_{8s}$	$[0]_{8s}$	2776
0.6	$[6.28]_{8s}$	1305.0	$[0]_{8s}$	$[0]_{8s}$	1304.9
0.8	$[\pm 36.85]_{4s}$	852	$[\pm 45]_{4s}$	$[\pm 45]_{4s}$	843
1	$[\pm 45]_{4s}$	663	$[\pm 45]_{4s}$	$[\pm 45]_{4s}$	663
1.2	$[\pm 51.44]_{4s}$	563	$[\pm 45]_{4s}$	$[\pm 45]_{4s}$	559
1.4	$[\pm 59.16]_{4s}$	506	$[\pm 45]_{4s}$	$[\pm 45]_{4s}$	493
1.6	$[\pm 72.64]_{4s}$	475	$[90]_{8s}$	$[90]_{8s}$	474
1.8	$[90]_{8s}$	463	$[90]_{8s}$	$[90]_{8s}$	463
2	$[90]_{8s}$	455	$[90]_{8s}$	$[90]_{8s}$	455

Table 1. Design for maximum fundamental frequency with N=16

$\frac{a}{b}$	θ_{best} by ACO_R	$\omega_{\max}$ by ACO_R (rad/sec)	θ_{best} by ACS	θ_{best} by enum.	$\omega_{\max}$ by ACS (rad/sec)
0.2	$[0]_{4s}$	24390	$[0]_{4s}$	$[0]_{4s}$	24390
0.4	$[0]_{4s}$	6170	$[0]_{4s}$	$[0]_{4s}$	6170
0.6	$[\pm 11.64]_{2s}$	2802	$[\pm 15]_{2s}$	$[\pm 15]_{2s}$	2801
0.8	$[\pm 37.34]_{2s}$	1816	$[\pm 30]_{2s}$	$[\pm 30]_{2s}$	1797
1	$[\pm 45]_{2s}$	1413	$[\pm 45]_{2s}$	$[\pm 45]_{2s}$	1413
1.2	$[\pm 51.05]_{2s}$	1199	$[\pm 45]_{2s}$	$[\pm 45]_{2s}$	1189
1.4	$[\pm 58.26]_{2s}$	1078.4	$[\pm 60]_{2s}$	$[\pm 60]_{2s}$	1078.2
1.6	$[\pm 70.34]_{2s}$	1017	$[\pm 75]_{2s}$	$[\pm 75]_{2s}$	1016
1.8	$[90]_{4s}$	1003	$[90]_{4s}$	$[90]_{4s}$	1003
2	$[90]_{4s}$	996	$[90]_{4s}$	$[90]_{4s}$	996

Table 2. Design for maximum fundamental frequency with N=8

algorithms against these two known methods in engineering optimization. This is of higher value for ACO_R that has been recently developed and for which few benchmark problems has been presented in the literature.

$\frac{a}{b}$	θ_{best} by ACO_R	θ_{best} by ACS	θ_{best} by GA	N_i by ACO_R, ACS & GA
0.2	$[0]_{4s}$	$[0]_{4s}$	$[0]_{4s}$	6
0.4	$[0]_{4s}$	$[0]_{4s}$	$[0]_{4s}$	6
0.6	$[11.64/6.28_3]_s$	$[15/0/0/0]_s$	$[0/30/-30_2]_s$	6
0.8	$[-37.34/36.85_3]_s$	$[\pm 30]_{2s}$	$[\pm 45_2/-45_2]_s$	6
1	$[\pm 45]_{2s}$	$[\pm 45]_{2s}$	$[\pm 45]_{2s}$	6
1.2	$[-51.05/51.44_3]_s$	$[\pm 45]_{2s}$	$[90/45/\pm 45]_s$	6
1.4	$[-58.26/59.16_3]_s$	$[\pm 60]_{2s}$	$[90/60/-60_2]_s$	6
1.6	$[-70.34/72.64_3]_s$	$[\pm 75]_{2s}$	$[90/-75/60_2]_s$	6
1.8	$[90]_{4s}$	$[90]_{4s}$	$[90]_{4s}$	6
2	$[90]_{4s}$	$[90]_{4s}$	$[90]_{4s}$	6

Table 3a. Optimum stacking sequence for maximum frequency and minimum cost with N=8

$\frac{a}{b}$	ω by ACO_R (rad/sec)	ω by ACS (rad/sec)	ω by GA (rad/sec)	Cost by ACO_R, ACS & GA	F by ACO_R	F by ACS	F by GA
0.2	19093	19093	19093	0.1138	0.1735	0.1735	0.1735
0.4	4844	4844	4844	0.2275	0.1725	0.1725	0.1725
0.6	2210.9	2210.4	2208	0.3413	0.17071	0.17076	0.1716
0.8	1435	1421	1306	0.4550	0.1705	0.1708	0.1710
1	1116	1116	1116	0.5687	0.1705	0.1705	0.1705
1.2	947	940	845	0.6825	0.1705	0.1706	0.2103
1.4	851.8	851.4	816	0.7963	0.17051	0.17054	0.1854
1.6	803.2	802.4	799	0.9100	0.17075	0.17086	0.1724
1.8	790	790	790	1.0238	0.1714	0.1714	0.1714
2	784	784	784	1.1375	0.1720	0.1720	0.1720

Table 3b. Optimum stacking sequence for maximum frequency and minimum cost with N=8

$\frac{a}{b}$	θ_{best} by ACO_R	θ_{best} by ACS	θ_{best} by GA	N_i by ACO_R, ACS & GA
0.2	$[0]_{8s}$	$[0]_{8s}$	$[0]_{8s}$	12
0.4	$[0]_{8s}$	$[0]_{8s}$	$[0/0/\pm15/15/0/-15/0]_s$	12
0.6	$[11.64_2/6.28_6]_s$	$[\pm15/0_6]_s$	$[\pm30]_{4s}$	12
0.8	$[-37.34_2/36.85_6]_s$	$[\pm30]_{4s}$	$[-45/30/\pm45_3]_s$	12
1	$[\pm45]_{4s}$	$[\pm45]_{4s}$	$[\pm45]_{4s}$	12
1.2	$[-51.05_2/51.44_6]_s$	$[\pm45]_{4s}$	$[\pm45/-60/-45/-60_2/-45_2]_s$	12
1.4	$[-58.26_2/59.16_6]_s$	$[\pm60]_{4s}$	$[\pm60]_{4s}$	12
1.6	$[-70.34_2/72.64_6]_s$	$[\pm75]_{4s}$	$[\pm60_3/\pm75]_s$	12
1.8	$[90]_{8s}$	$[90]_{8s}$	$[90]_{8s}$	12
2	$[90]_{8s}$	$[90]_{8s}$	$[90]_{8s}$	12

Table 4a. Optimum stacking sequence for maximum frequency and minimum cost with N=16

$\frac{a}{b}$	ω by ACO_R (rad/sec)	ω by ACS (rad/sec)	ω by GA (rad/sec)	Cost by ACO_R, ACS & GA	F by ACO_R	F by ACS	F by GA
0.2	19093	19093	19093	0.1138	0.1735	0.1735	0.1735
0.4	4844	4844	4829	0.2275	0.1725	0.1725	0.1736
0.6	2210.9	2210.4	2171	0.3413	0.17071	0.17076	0.1768
0.8	1435	1421	1418	0.4550	0.1705	0.1708	0.1714
1	1116	1116	1116	0.5687	0.1705	0.1705	0.1705
1.2	947	940	939	0.6825	0.1705	0.1706	0.1707
1.4	851.8	851.4	851.4	0.7963	0.17051	0.17054	0.17054
1.6	803.2	802.4	796	0.9100	0.17056	0.17086	0.1735
1.8	790	790	790	1.0238	0.1714	0.1714	0.1714
2	784	784	784	1.1375	0.1720	0.1720	0.1720

Table 4b. Optimum stacking sequence for maximum frequency and minimum cost with N=16

$\frac{a}{b}$	θ_{best} by ACO_R	θ_{best} by ACS	θ_{best} by GA	N_i by ACO_R &ACS	N_i by GA
0.2	$[0]_{14s}$	$[0]_{14s}$	$[0_{12}/15_2]_s$	22	24
0.4	$[0]_{8s}$	$[0]_{14s}$	$[0/\pm15/0/15/0_2/-15/0_3/15/0_2]_s$	22	24
0.6	$[11.64_3/6.28_{11}]_s$	$[15_3/0_{11}]_s$	$[30_5/45/30_3/-45/\pm30]_s$	22	24
0.8	$[-37.34_3/36.85_{11}]_s$	$[\pm30]_{7s}$	$[\pm45_5/45/30/\pm45]_s$	22	24
1	$[\pm45]_{7s}$	$[\pm45]_{7s}$	$[\pm45]_{7s}$	22	24
1.2	$[-51.05_3/51.44_{11}]_s$	$[\pm45]_{7s}$	$[\pm45_3/60/45_7]_s$	22	24
1.4	$[-58.26_3/59.16_{11}]_s$	$[\pm60]_{7s}$	$[\pm60_3/-45/\pm60_2/\pm45/60]_s$	22	22
1.6	$[-70.34_3/72.64_{11}]_s$	$[\pm75]_{7s}$	$[\pm60_2/75/\pm60/75/60_3/75/60/75]_s$	22	22
1.8	$[90]_{14s}$	$[90]_{14s}$	$[90]_{14s}$	22	22
2	$[90]_{14s}$	$[90]_{14s}$	$[90]_{14s}$	22	22

Table 5a. Optimum stacking sequence for maximum frequency and minimum cost with N=28

$\frac{a}{b}$	ω by ACO_R (rad/sec)	ω by ACS	ω by GA	Cost by ACO_R &ACS	Cost by GA	F by ACO_R	F by ACS	F by GA
0.2	18339	18339	16518	0.1039	0.0843	0.1670	0.1670	0.1732
0.4	4565	4565	4145	0.2079	0.1686	0.1657	0.1657	0.1771
0.6	2127.7	2127.2	1889	0.3118	0.2529	0.1631	0.1632	0.1768
0.8	1381	1368	1237	0.4157	0.3371	0.1629	0.1631	0.1673
1	1074	1074	974	0.5196	0.4214	0.1629	0.1629	0.1659
1.2	912	905	820	0.6236	0.5057	0.1628	0.1630	0.1660
1.4	819.9	819.6	818.7	0.7275	0.7275	0.1629	0.1630	0.1632
1.6	773	772	767	0.8314	0.8314	0.1630	0.1633	0.1661
1.8	760	760	760	0.9354	0.9354	0.1641	0.1641	0.1641
2	754	754	754	1.0393	1.0393	0.1649	0.1649	0.1649

Table 5b. Optimum stacking sequence for maximum frequency and minimum cost with N=28

5. Conclusion

The results reported here and in many other papers confidently suggest ant colony optimization as a robust and efficient method compared with other known techniques such as genetic algorithm and simulated annealing. Specifically, design and optimization of laminated structures is a field where ant algorithms are expected to outperform other methods. It is also shown here that ACO_R can successfully formulate problems with mixed domains and yield improvements against discretization process. Moreover, it is concluded that using the standard fiber angles which is a practical advantage in laminates production makes negligible variation of optimal designs with respect to the global optima.

6. References

Abachizadeh, M. & Kolahan, F. (2007). Evaluating the discretization of search space in continuous problems for ant colony optimization, *Proceedings of the 37th International Conference on Computers and Industrial Engineering (ICCIE37)*, Alexandria, Egypt

Abachizadeh, M. & Tahani, M. (2007). Ant colony optimization of hybrid laminates for minimum cost and weight, *Proceedings of 6th International Symposium on Advanced Composites (COMP'07)*, Corfu, Greece

Abachizadeh, M. & Tahani, M. (2009). An ant colony optimization approach to multi-objective optimal design of symmetric hybrid laminates for maximum fundamental frequency and minimum cost. *Structural and Multidisciplinary Optimization*, 37, 367-376

Abachizadeh, M.; Shariat Panahi, M.; Yousefi-Koma, A. & Feiz Dizaji, A. (2010). Multi-objective optimal design of hybrid laminates using continuous ant colony method, *Proceeding of 10th ASME Biennial Conference on Engineering Systems Design and Analysis (ESDA 2010)*, Istanbul, Turkey, July 12-14, 2010

Adali, S. & Verijenko, V.E. (2001). Optimum stacking sequence design of symmetric hybrid laminates undergoing free vibrations. *Composite Structures*, 54, 131–138

Aymerich, F. & Serra, M. (2008). Optimization of laminate stacking sequence for maximum buckling load using the ant colony optimization (ACO) metaheuristic. *Composites: Part A*, 39, 262-272

Bloomfield, M.W.; Herencia, J.E. & Weaver, P.M. (2010). Analysis and benchmarking of meta-heuristic techniques for lay-up optimization. *Computers and Structures,* 88, 272-282

Botkin, M.E. (2000). Modeling and optimal design of a fiber reinforced composite automotive roof. *Engineering with Computers*, 16, 16-23

Camp, C.V.; Bichon, B. & Stovall, S.P. (2005). Design of steel frames using ant colony optimization. *Journal of Structural Engineering*, 131, 369-379

Dorigo, M. (1992). *Optimization, Learning and Natural Algorithms*, Ph.D. thesis, DEI, Politecnico di Milano, Italy (In Italian)

Dorigo, M. & Gambardella, L.M. (1997). Ant colony system: a cooperative learning approach to the traveling salesman problem. *IEEE Transactions on Evolutionary Computation*, 1, 1, 53-66

Dorigo, M. & Stutzle, T. (2004). *Ant Colony Optimization*, MIT Press, Cambridge

Ghiasi, H.; Pasini, D. & Lessard, L. (2009). Optimum stacking sequence design of composite materials Part I: Constant stiffness design. *Composite Structures*, 90, 1-11.

Hudson, C.W.; Carruthers, J.J. & Robinson, M. (2010) Multiple objective optimization of composite sandwich structures for rail vehicle floor panels. *Composite Structures,* 92, 9, 2077-2082.

Kolahan, F.; Tahani, M. & Sarhadi, A. (2005). Optimal design of sandwich composite laminates for minimum cost and maximum frequency using simulated annealing, *Proceedings of Tehran International Conference on Manufacturing Engineering (TICME2005)*, Tehran, Iran

Nemeth, M.P. (1986). Importance of anisotropy on buckling of compression-loaded symmetric composite plates. *Journal of AIAA*, 24, 1831-1835

Rastogi, N. (2004). Stress analysis and lay-up optimization of an all-composite pick-up truck chassis structure. *SAE Transactions*, 113, 5, 660-671

Reddy, J.N. (2004). *Mechanics of Laminated Composite Plates and Shells: Theory and Analysis*, 2nd ed., CRC Press, Boca Raton, FL

Serra, M. & Venini, P. (2006). On some applications of ant colony optimization metaheuristic to plane truss optimization. *Structural and Multidisciplinary Optimization*, 32, 6, 499-506.

Shokrieh, M. & Rezaei, D. (2003). Analysis and optimization of a composite leaf spring. *Composite Structures,* 60, 317-325.

Socha, K., (2008). *Ant Colony Optimization for Continuous and Mixed-variable Domains*, Ph.D. thesis, Universite Libre de Bruxelles, Belgium

Socha, K. & Dorigo, M. (2008). Ant colony optimization for continuous domains. *European Journal of Operational Research*, 185, 3, 1155-1173

Tahani, M; Kolahan, F. & Sarhadi, A. (2005). Genetic algorithm for multi-objective optimal design of sandwich composite laminates with minimum cost and maximum frequency, *Proceedings of International Conference on Advances in Materials, Product Design and Manufacturing Systems (ICMPM-2005)*, pp. 741-748, Sathyamangalam, India

Tsai, S.W. & Hahn, H. (1980). *Introduction to Composite Materials*, Wesport Technomic Publishing Company, Connecticut

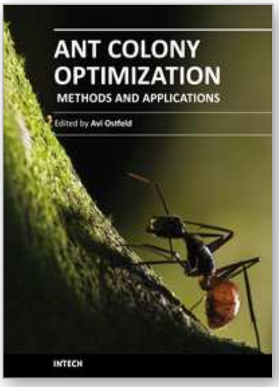
Wang, W.; Guo, S. & Chang, N. (2010). A modified ant colony algorithm for the stacking sequence optimization of a rectangular laminate. *Structural and Multidisciplinary Optimization,* 41, 711-720

Vanderplaats, G.N. (2002). *Very Large Scale Optimization*, NASA contractor report

Venkataraman, S. & Haftka, R.T. (2004). Structural optimization complexity: What has Moore's law done for us? *Structural and Multidisciplinary Optimization,* 28, 375-387

Vijayakumar, K.; Prabhaharan, G.; Asokan, P. & Saravanan, R. (2003). Optimization of multi-pass turning operations using ant colony system. *International Journal of Machine Tools & Manufacture*, 43, 1633–1639

Yang, X.S.; Lees, J.M. & Morley, C.T. (2006). Application of virtual ant algorithms in the optimization of CFRP shear strengthened precracked structures. *LNCS 3991*, 834-837.



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Ants communicate information by leaving pheromone tracks. A moving ant leaves, in varying quantities, some pheromone on the ground to mark its way. While an isolated ant moves essentially at random, an ant encountering a previously laid trail is able to detect it and decide with high probability to follow it, thus reinforcing the track with its own pheromone. The collective behavior that emerges is thus a positive feedback: where the more the ants following a track, the more attractive that track becomes for being followed; thus the probability with which an ant chooses a path increases with the number of ants that previously chose the same path. This elementary ant's behavior inspired the development of ant colony optimization by Marco Dorigo in 1992, constructing a meta-heuristic stochastic combinatorial computational methodology belonging to a family of related meta-heuristic methods such as simulated annealing, Tabu search and genetic algorithms. This book covers in twenty chapters state of the art methods and applications of utilizing ant colony optimization algorithms. New methods and theory such as multi colony ant algorithm based upon a new pheromone arithmetic crossover and a repulsive operator, new findings on ant colony convergence, and a diversity of engineering and science applications from transportation, water resources, electrical and computer science disciplines are presented.

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