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Review of Numerical Simulation of Microwave Heating Process

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1. Introduction

Since the great experiments were carried out by Dr. Percy Spencer in 1946, microwave heating has been used widely in food industry, medicine, chemical engineering and so on [1][2][3][4][5], for processing many materials ranging from foodstuffs [1][6][7][8] to tobaccos [9][10][11], from wood [12][13][14] to ceramics [15][16][17][18], and from biological objects [19][20] to chemical reactions [21][22][23]. Typical applications include heating and thawing of foods, drying of wood and tobaccos, sintering of ceramics, killing of cancer cells and accelerating of chemical reactions.

Microwaves are electromagnetic (EM) waves with frequencies between 300MHz and 300GHz. Under microwave radiation, dipole rotation occurs in dielectric materials containing polar molecules having an electrical dipole moment. Interactions between the dipole and the EM field result in energy in the form of EM radiation being converted to heat energy in the materials. This is the principle of microwave heating. For each material, permittivity, or dielectric constant, ε is the most essential property relative to the absorption of microwave energy. Permittivity is often treated as a complex number. The real part is a measure of the stored microwave energy within the material, and the imaginary part is a measure of the dissipation (or loss) of microwave energy within the material. The complex permittivity is usually a complicated function of microwave frequency and temperature.

Compared with conventional heating, microwave heating has many advantages such as simultaneous heating of a material in its whole volume, higher temperature homogeneity, and shorter processing time. However, the nonlinear process of interaction between microwave and the heated materials has not been sufficiently and deeply understood. Some phenomena arising from this process, such as hotspots and thermal runaway, decreases the security and efficiency of microwave energy application and prevents its further development.

Hotspot means that where temperature non-uniformity in heated material of local small areas having high temperature increase occurs. For example, some zones in material are overheated, and even burned, while some others have not reached the required minimum temperature. This phenomenon has become one of the major drawbacks for domestic or industrial applications.

"Thermal runaway" refers to temperature varies dramatically with small changes of geometrical sizes of the heated material or microwave power. It may lead to a positive

feedback occurring in the material — the warmer areas are better able to accept more energy than the colder areas. It is potentially dangerous, especially when the heat exchange between the hot spots and the rest of the material is slow. This phenomenon occurs in some ceramics.

These existing problems related to temperature distribution inside materials must be considered. Although experimental studies and qualitative theoretical analysis are important, numerical simulations are indispensable for helping us further the understanding of the complex microwave heating process, predict and control its behaviors. Thanks to rapid hardware and software development of numerical calculation techniques, it has been possible to simulate the microwave heating process on a personal computer. As a result, fully empirical design of heating system and expensive prototype equipment for testing can be avoided.

Numerical simulation of microwave heating process is basically an analysis of multi-physical coupling, so at least two kinds of partial differential equations (PDE) are associated together, the EM field equations (i.e. Maxwell's equation) and the heat transport equation. To describe their interaction relationship, the inhomogeneous term of the heat transport equation representing the heating source is provided by microwave dissipated power; meanwhile, the temperature variation during the heating process can cause the change of complex permittivity, which directly affects the space and time variation of the EM field. Sometimes, more kinds of differential equations may need to be joined, for example, the mass transfer equation may be required when drying the porous materials.

To obtain accurate solution of the above coupled equations, analytical methods are hardly to be used. There are many numerical methods being used to simulate microwave heating. The popular methods include Finite Difference (FD) methods (e.g. Finite Differential Time Domain / FDTD method), Finite Element Method (FEM), the Moment of Method (MoM), Transmission Line Matrix (TLM) method, Finite-Volume Time-Domain (FVTD) method and so on.

In this chapter, we first review all kinds of application backgrounds of numerical simulation of microwave heating process, and then discuss the two most important techniques - numerical modeling and numerical methods. Finally we conclude and outline some desirable future developments.

2. Application backgrounds

According to the statistics of the open publications related to numerical simulation of microwave heating, we find that so many kinds of objects, from common substances such as water [24][25[26], food [27][28][29][30] and wood [31][32][33][34][35][36], to chemical engineering materials such as ceramic material [37][38][39][40][41][42][43][44], minerals[45], SOFC materials[46] and zinc oxide [47], and even human tissues [48][49] and some chemical reactions [50][51][52][53][25], have been investigated.

Water strongly absorbs microwaves, so usually microwave heating works by heating the water in foods in food industry. Furthermore, water is such an important substance whose permittivity shows clear relationship with temperature, that it is natural to regard it as the most typical object in microwave heating and a benchmark example when comparing the results of different numerical methods. At the frequency of 2.45 GHz, the curves plotted on Fig. 1 show the complex permittivity of water varying with temperature.
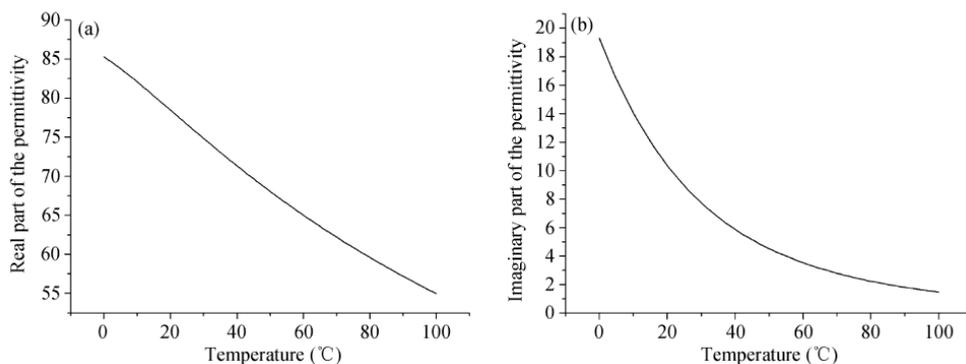
Fig. 1. Variation of the complex permittivity of water with temperature at 2.45 GHz.
(a) Real part; (b) Imaginary part.

When microwave energy being used to dry wood from the green state, some benefits can be offered, such as moisture leveling and increased vapor migration from the interior to the surface of the wood. Perré, et al.,[31] combined a two-dimensional transfer code with a three-dimensional EM computational scheme, obtained the overall behavior of combined microwave and convective drying of heartwood spruce remarkably well and predicted the occurrence of thermal runaway within the material. To provide a detailed algorithm for simulating the nonlinear process of microwave heating, Zhao, et al.,[32] presented a three-dimensional computational model for the microwave heating of wood with low moisture contents. The model coupled a FVTD algorithm for resolving Maxwell's equations, together with an algorithm for determining the thermal distribution within the wood sample. Hansson, et al.,[33] used FEM as a tool to analyze microwave scattering in wood. They used a medical computed tomography scanner to measure distribution of density and moisture content in a piece of Scots pine, calculated dielectric properties from measured values for cross sections from the piece and used in the model. Hansson and his colleagues also studied moisture redistribution in wood during microwave heating process [34]. Rattanadecho [35] used a three dimensional FD scheme to determine EM fields and obtained temperature profiles of wood in a rectangular wave guide. Temperature dependence of wood dielectric properties was simulated by updating dielectric properties in each time step of temperature variation. The influence of irradiation times, working frequencies and sample size were illustrated. To take into consideration of the influence of moisture diffusion during microwave heating of moist wood, Brodie [36] derived of the PDE, which described simultaneous heat and moisture diffusion, yielded two independent values for the combined heat and moisture diffusion coefficient.

Microwave sintering of ceramic material has also been studied widely. It is a promising technique for the densification of ceramics. Compared to conventional heating, microwave sintering can obtain higher densities with a finer grain structure. Thermal runaway is a common phenomenon in microwave sintering. Numerical simulation may help to understand the mechanism of the thermal runaway, and then avoid it. Iskander, et al., [39] developed a FDTD code that was used to model some of the many factors that influence a realistic microwave sintering process in multimode cavities. The factors included the conductivity of the insulation surrounding the sample and the role of the Sic rods in

modifying the EM field distribution pattern in the sample. Chatterjee, et al., [40] analyzed the microwave heating of ceramic materials by solving the equations for grain growth and porosity[54] during the late stages of sintering, coupled with the heat conduction equation and electric field equations for 1-D slabs. Gupta, et al., [41] analyzed the steady-state behavior of a ceramic slab under microwave heating by transverse magnetic illumination. They observed that for a certain set of parameters, there are periodically recurring ranges of slab thickness for which thermal runaway may be avoided. The runaway dependence on other parameters critical to the operation of the process was also studied. Riedel, et al., [42]modeled microwave sintering by the Maxwell's equation, the heat conduction equation and a sintering model describing the evolution of density and the grain size, so as to predict runaway instabilities and to find process parameters leading to a homogeneously sintered product with a uniform, fine grain structure. Liu, et al., [43] solved Maxwell's equations (by using FDTD) coupled with a heat transfer equation (by using FD), and obtained the temperature variation in a ceramic slab during microwave heating. Various ceramic parameters and applied microwave powers were simulated so as to analyze the condition under which thermal runaway is introduced. Furthermore, they presented a microwave power control method based on a single temperature threshold and dual applied microwave powers, which can improve microwave heating efficiency and controls thermal runaway.

Although reports of the potential of microwave heating in chemical modification can be traced back to the 1950s, microwave heating began to gain wide acceptance with the publication of several papers in 1986 [55]. Presently, microwave has been widely used in various chemical domains from inorganic reaction to organic reaction, from medicine chemical industry to food chemical industry and from simple molecules to complex life process. However, most of studies on the mechanism of interaction between microwave and chemical reaction are only related with the experimental studies of some concrete reactions and qualitative theoretical analysis. The main reason is that the complex effective permittivities of many chemical reactions are still unknown. This fact brings difficulty to implement numerical simulation. Wang, et al., [50] presented a numerical model of microwave heating of a chemical reaction, used FDTD technology to solve Maxwell's equation, and the explicit FD scheme to solve heat equation. Huang, et al., [51] presented a numerical model to study the microwave heating on saponification reaction in test tube, where the reactant was considered as a mixture of dilute solution. They solved the coupled EM field equations, reaction equation (RE) and heat transport equation by using FDTD method, and employed dual-stage leapfrog scheme and method of time scaling factor to overcome the difficulty of long time calculation with FDTD. Yamada, et al., [52] simulated microwave plasmas inside a reactor for thick diamond syntheses. In a model reactor used in the simulation, a diamond substrate with finite thickness and area is taken into account. Distributions of electric field, density of microwave power absorbed by the plasma, temperatures and flow field of gas were studied not only in a bulk region inside a reactor but also a local region around the substrate surface. Numerical results implied that the adopted arrangement of the substrate is not desirable for continuous growth of large diamond crystals. Huang, et al., [53] studied the precise condition of thermal runaway in microwave heating on chemical reaction with a feedback control system. They derived the condition of thermal runaway according to the principle of the feedback control system, simulated the temperature rising rates for 4 different kinds of reaction systems placed in the waveguide and irradiated by microwave with different input powers, and discussed the

relationship between the thermal runaway and the sensor's response time. Zhao, et al., [25] carried out preliminary study on numerical simulation of microwave heating process for chemical reaction. They considered calculation of 2D multi-physical coupling, solved the integral equation of EM field by using MoM, and the heat transport equation by using a semi-analysis method. Moreover, a method to determine the equivalent complex permittivity of reactant under the heating was presented in order to perform the calculation. The numerical results for water and a dummy chemical reaction showed some features of the hotspot and the thermal runaway phenomenon.

Besides the above-mentioned typical applications of numerical simulation, there are more application cases used in other materials [56][57][58]. Furthermore, numerical simulation was used to analyze and design a variety of microwave heating applicators [59][60][61].

3. Numerical simulation technology

In essence, numerical simulation of microwave heating process is an analysis of multi-physical coupling. The mathematic model consists of at least EM field equation (i.e. Maxwell's equation) and the heat transport equation. There is an interaction relationship between them (Fig. 1). The inhomogeneous term of heat transport equation, i.e., the heating source is provided by microwave dissipated power. And meanwhile, the temperature variation during the heating process can cause the change of complex permittivity, which also affects the space and time variation of the EM field. Consequently, to perform the simulation, one must model the two equations together with their coupling relation at first, and then solve them by using some numerical methods.

*The microwave dissipated power is
the heat source*

| Electromagnetic Field (Maxwell's equation) | Temperature Field (Heat transport equation) |

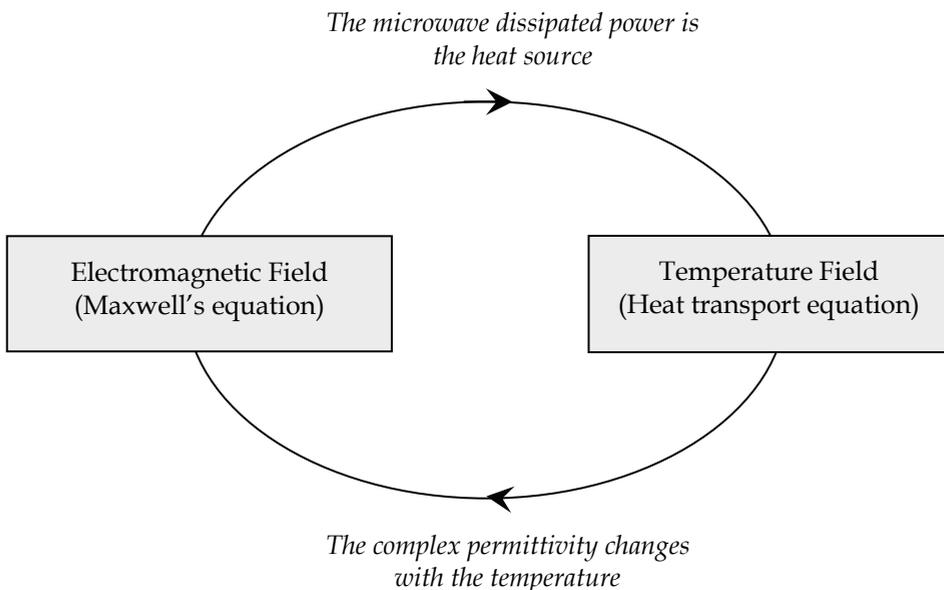*The complex permittivity changes
with the temperature*

Fig. 1. Multi-physical coupling in microwave heating process

In some cases of more complex application scenarios, more kinds of differential equations may need to be joined, for example, the mass transfer equation may be required when drying the porous materials [62][63][64].

As the key techniques, the numerical modeling and the numerical method will be discussed below.

A. Numerical modeling

- **Maxwell's equation**

The governing equation of the EM field vectors is based on the well-known Maxwell's equation. The differential and integral form of Maxwell's equation can be expressed as

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \ , \quad \oint_C \vec{E} \cdot d\vec{l} = -\int_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S} \ , \tag{1}$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \ , \quad \oint_C \vec{H} \cdot d\vec{l} = \int_S \vec{J} \cdot d\vec{S} + \int_S \frac{\partial \vec{D}}{\partial t} \cdot d\vec{S} \ , \tag{2}$$

$$\nabla \cdot \vec{D} = \rho \ , \quad \oint_S \vec{D} \cdot d\vec{S} = \int_V \rho \, dV \ , \tag{3}$$

$$\nabla \cdot \vec{B} = 0 \ , \quad \oint_S \vec{B} \cdot d\vec{S} = 0 \ , \tag{4}$$

where $\vec{E} = \vec{E}(x,y,z,t)$ is the fields electric, $\vec{H} = \vec{H}(x,y,z,t)$ the magnetic fields, $\vec{D} = \vec{D}(x,y,z,t)$ the flux density, $\vec{B} = \vec{B}(x,y,z,t)$ the magnetic flux density, $\vec{J} = \vec{J}(x,y,z,t)$ the current density and $\rho = \rho(x,y,z,t)$ the charge density. The constraint relations of them are

$$\vec{D} = \varepsilon \vec{E} \ , \quad \vec{B} = \mu \vec{H} \ , \quad \vec{J} = \sigma \vec{E} \ , \tag{5}$$

and

$$\nabla \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0 \ , \quad \oint_S \vec{J} \cdot d\vec{S} = -\frac{d}{dt} \int_V \rho \, dV \ , \text{ (Continuity of electric current)} \tag{6}$$

where ε is the dielectric constant (or electrical permittivity), μ the magnetic permeability and σ the electric conductivity.

When there exists discontinuity of the parameters of medium (ε, μ and σ), e.g., at the surface of the medium, the boundary conditions are required to specify the behavior of EM field at the boundary. For example, at the boundary of a perfect conductor, boundary conditions are given as

$$E_t = 0 \ , \quad H_n = 0 \ , \tag{7}$$

where E_t is the tangential component of $\vec{E}$, H_n the normal component of $\vec{H}$. And the boundary conditions along the interface between two different dielectric materials are given as

$$E_{t1} = E_{t2} \ , \quad H_{t1} = H_{t2} \ , \quad D_{n1} = D_{n2} \ , \quad B_{n1} = B_{n2} \ . \tag{8}$$

There are some equivalent or derivative forms of Maxwell's equation, such as wave equation, Helmholtz equation (the time-harmonic form of wave equation), scalar potential equation, vector potential equation, integral equation based on Green's theorem and so on. All of these equations can be used to model the behavior of EM field.

- **Heat transport equation**

Heat transport equation is also called heat transfer equation or thermal conduction equation, which describes the space and time behavior of the temperature field, that is

$$\rho_m C_m \frac{\partial T}{\partial t} - \nabla \bullet (k_t \nabla T) = P \text{ ,} \tag{9}$$

where ρ_m, C_m and k_t denote material density, specific heat capacity and thermal conductivity, respectively. $T = T(x,y,z,t)$ is the absolute temperature , $P = P(x,y,z,t)$ the EM power dissipated per unit volume. For simplicity in solving this equation, the parameters ρ_m, C_m and k_t are usually taken as constants that are independent of position, time and temperature.

According to Newton's law of cooling, the convective boundary condition at the material surfaces is used such as

$$h(T_a - T) = k_t \frac{\partial T}{\partial n} \text{ ,} \tag{10}$$

and the adiabatic boundary condition as

$$\frac{\partial T}{\partial n} = 0 \text{ ,} \tag{11}$$

where h is the convective heat transfer coefficient, T_a the temperature of the surrounding air. Also, the initial condition is needed to determine the unique solution of (9), which can be given as

$$T(x,y,z,t)\big|_{t=0} = T_{ini}(x,y,z) \text{ .} \tag{12}$$

In some cases, the heat radiation of the material can be considered by subtracting it from the right-hand side of (9)[65][43]. Its ideal case is a blackbody radiation, which is described by the Stefan-Boltzmann law.

- **Coupling relationship**

The inhomogeneous item P in (9) is determined by the EM power density that is dissipated in the nonmagnetic and nonconductive material due to its dielectric losses, and can be expressed by [24]

$$P = \frac{1}{2} \left(\vec{E} \bullet \frac{\partial \vec{D}}{\partial t} - \vec{D} \bullet \frac{\partial \vec{E}}{\partial t} \right) \text{ .} \tag{13}$$

For the case of steady-state time harmonic EM fields, it yields

$$\overline{P} = \frac{1}{2} \omega \varepsilon_0 \varepsilon'' E^2 \tag{14}$$

where ω is the angular frequency, ε_0 the vacuum permittivity, ε'' the imaginary part of the permittivity of the material, E the amplitudes of $\vec{E}$.

On the other hand, the space and time variation of the temperature can change the ε, μ and σ of the material. For nonmagnetic and nonconductive material, we can only discuss ε.

It is well known that the response delay of the materials to external EM fields generally depends on the frequency of the field. For this reason permittivity is often treated as a complex function of the (angular) frequency ω. The complex permittivity $\varepsilon^* = \varepsilon^*(\omega)$ is defined by

$$\mathbf{D} = \varepsilon^* \mathbf{E} = (\varepsilon' - j\varepsilon'')\mathbf{E}, \tag{15}$$

where $\mathbf{D} = \mathbf{D}(x,y,z)$ and $\mathbf{E} = \mathbf{E}(x,y,z)$ are the phasor representation of time harmonic field $\vec{D}$ and $\vec{E}$ respectively, which satisfy the following

$$\vec{D}(x,y,z,t) = \mathrm{Re}\left[\mathbf{D}(x,y,z)e^{j\omega t}\right] \text{ and } \vec{E}(x,y,z,t) = \mathrm{Re}\left[\mathbf{E}(x,y,z)e^{j\omega t}\right]. \tag{16}$$

And j is the imaginary unit, $j^2 = -1$. Note that the sign of the imaginary part of ε^* is constantly negative when the time harmonic factor is $e^{j\omega t}$.

For conductive material, one can consider ohmic loss by taking σ into the imaginary part of ε^* as follows

$$\varepsilon^* = \varepsilon' - j\left(\varepsilon'' + \frac{\sigma}{\omega}\right). \tag{17}$$

Debye (1926) deduced the well known equation for the complex relative permittivity ε_r^* ($\varepsilon_r^* = \varepsilon^* / \varepsilon_0$) as

$$\varepsilon_r^* = \varepsilon_\infty + \frac{\varepsilon_s - \varepsilon_\infty}{1 + j\omega\tau}, \tag{18}$$

where ε_∞ is the permittivity at the high frequency limit, ε_s is the static, low frequency permittivity, and τ is the relaxation time. The variation of temperature can affect the parameters of ε_∞, ε_s and τ, and consequently affect ε_r^*. So ε_r^* is a function of the temperature, that is

$$\varepsilon_r^* = \varepsilon_r^*(\omega, T). \tag{19}$$

Particularly, for a chemical reaction, it is also the function of the reaction duration time.

Although in some cases, thermal and dielectric properties may be assumed to be temperature-independent [78], the knowledge of the function $\varepsilon_r^*(\omega, T)$ of a material is a necessary and key condition to implement the numerical simulation of interaction between microwave and the material. There is a lot of basic work that has been done for this area [66][67][68][69][70][26][71]. Besides, the complex permittivities of many specific materials are studied [72][73][74][75][76][77][78]. Obviously, it is more difficult for the chemical

reaction to determine the complex permittivity of the reactant, because it varies not only with temperature, but also with reaction duration time.

- **Geometric model**

As far as the geometric model is concerned, one-dimensional (1D)[43][40][79][80] and two-dimensional (2D)[94][81][25] models are more generally used for theoretical studies, and three-dimensional (3D)[24][82][83][90][84][85] model (e.g. the heated object is placed in a microwave oven) is more close to practical applications.

Besides, Plaza-Gonzalez, et al,[86] studied the case of mode stirrers which are widely used by industrial applicators.

B. Numerical methods

Typically, four iterative steps are used to solve the coupling between the EM and thermal processes [87][88][89][90][91][92][93][94][95] (See Fig. 2).

Start

Solve the Maxwell's equation to obtain the EM field distribution

Calculate the dissipated EM power

Calculate the temperature

Update the complex permittivity according to temperature

The heating time reaches the preset value?

N

Y

End
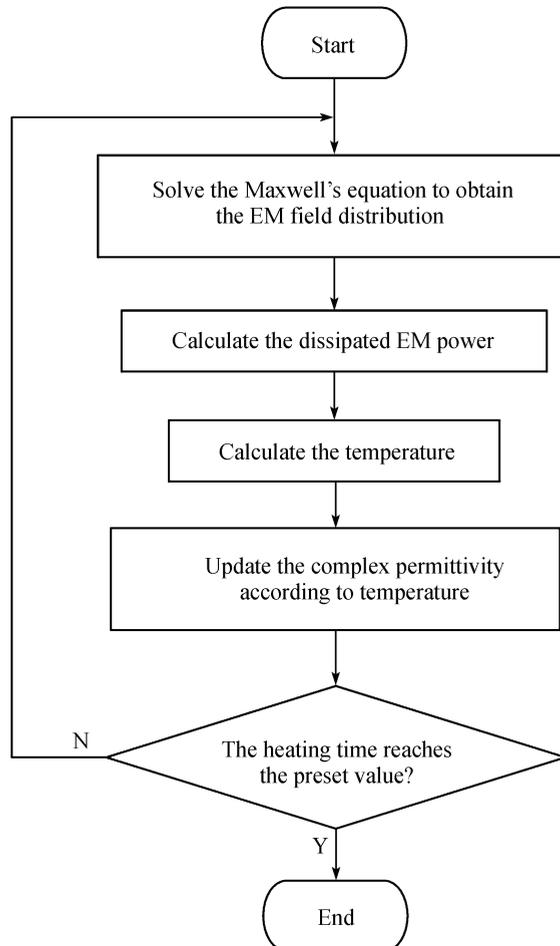
Fig. 2. The general simulation procedure of microwave heating

From the point of view of solving the PDE, the EM field equations are more difficult to solve comparing with the heat transport equation, because the former in fact include two vector equations and the latter is a scalar equation. For this reason, we only discuss below the solving of the EM field equations. There are many numerical methods being used: the Finite Differential Time Domain (FDTD) method, the Finite Element Method (FEM), the Moment of Method (MoM), the Transmission Line Matrix (TLM) method, the Finite Integral Method (FIM)[96] and so on. In some 1D/2D cases, some analytical methods or semi-analytical methods can be applied.

- **FDTD**

The FDTD method [97][98][99] is an application of the FD method for solving EM field equation. It is based on Yee grid as shown in Fig. 3, which is proposed by Kane Yee in 1966 [100].
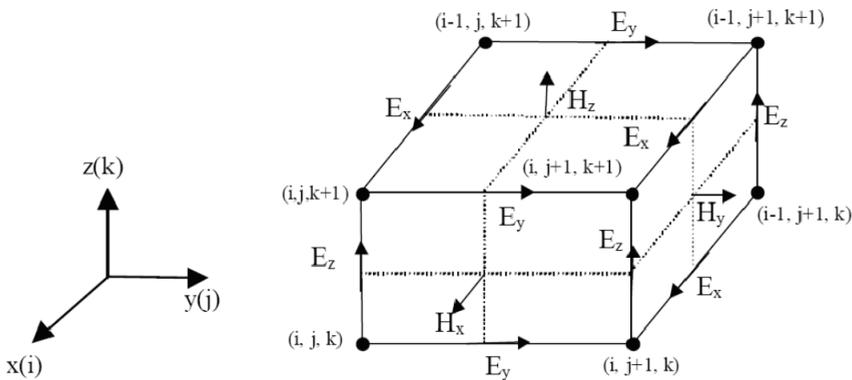
Fig. 3. The Yee grid

The main idea of the FDTD method is that the time-dependent Maxwell's equations (in partial differential form) are discretized using difference approximations to the space and time partial derivatives. It contains of three key elements: differential scheme, stability of solutions and absorbing boundary conditions.

In FDTD method, the differential grid is first set up in space. And at the instant time $n\Delta t$, $F(x,y,z)$ can be written as $F^n(i,j,k) = F(i\Delta x, j\Delta y, k\Delta z)$. Using central difference, the space and time are discretized as follows, respectively

$$\frac{\partial F^n(i,j,k)}{\partial x} = \frac{F^n\left(i+\frac{1}{2},j,k\right) - F^n\left(i-\frac{1}{2},j,k\right)}{\Delta x} + \mathrm{O}\left((\Delta x)^2\right) \tag{20}$$

$$\frac{\partial F^n(i,j,k)}{\partial t} = \frac{F^{n+\frac{1}{2}}(i,j,k) - F^{n-\frac{1}{2}}(i,j,k)}{\Delta t} + \mathrm{O}\left((\Delta t)^2\right) \tag{21}$$

Therefore, the two curl equations in Maxwell's equation (1) and (2) can be converted to six difference equations, for example

$$E_x^{n+1}\left(i+\frac{1}{2},j,k\right) = \frac{1-\dfrac{\sigma\left(i+\frac{1}{2},j,k\right)\Delta t}{2\varepsilon\left(i+\frac{1}{2},j,k\right)}}{1+\dfrac{\sigma\left(i+\frac{1}{2},j,k\right)\Delta t}{2\varepsilon\left(i+\frac{1}{2},j,k\right)}}\cdot E_x^n\left(i+\frac{1}{2},j,k\right)+$$

$$\frac{\Delta t}{\varepsilon\left(i+\frac{1}{2},j,k\right)}\cdot\frac{1}{1+\dfrac{\sigma\left(i+\frac{1}{2},j,k\right)\Delta t}{2\varepsilon\left(i+\frac{1}{2},j,k\right)}}\cdot \tag{22}$$

$$\left[\frac{H_z^{n+\frac{1}{2}}\left(i+\frac{1}{2},j+\frac{1}{2},k\right)-H_z^{n+\frac{1}{2}}\left(i+\frac{1}{2},j-\frac{1}{2},k\right)}{\Delta y}+\frac{H_y^{n+\frac{1}{2}}\left(i+\frac{1}{2},j,k-\frac{1}{2}\right)-H_y^{n+\frac{1}{2}}\left(i+\frac{1}{2},j,k+\frac{1}{2}\right)}{\Delta z}\right]$$

By this way, the electric field vector components in a volume of space are solved at a given instant in time, and then the magnetic field vector components are solved at the next instant in time. The process is repeated until the desired transient or steady-state EM field behavior is fully evolved.

While, in order to ensure the numerical stability condition, the time increment Δt must satisfy with

$$\Delta t \leq \frac{\sqrt{\mu\varepsilon}}{\sqrt{\dfrac{1}{(\Delta x)^2}+\dfrac{1}{(\Delta y)^2}+\dfrac{1}{(\Delta z)^2}}}. \tag{23}$$

Moreover, in order to solve the contradiction between the computing space and the computer memory, the absorbing boundary conditions are introduced, such as PML, Mur and so on.

Nowadays, the FDTD method is the most popular EM simulation method due to its simplicity. Most studies on numerical simulation of microwave heating were carried out by using FDTD method [83][101][102][51][103].

- **FEM**

The Finite Element Method (FEM)[104][105][106][107] is a technique to solve the PDE numerically. According to the variational principle, the PDE is transformed to an equivalent variational. As a result, the solving of the PDE is transformed to the minimization of a certain functional as follows

$$L(u)=f \Leftrightarrow \min_u I(u), \tag{24}$$

where $L(u)=f$ is the PDE, L denotes the differential operator, f a known forcing function, u the unknown solution and I the functional to be minimized with respect to u.

After variational formulation is deduced, the discretization needs to be performed to concrete formulae for a large but finite dimensional linear algebraic equation. One can expand the function u in a series of orthogonal basic functions $\{v_i(\cdot)\}_{i=1,\cdots,n}$ as

$$u(\cdot) = \sum_{i=1}^{n} c_i v_i(\cdot) , \qquad (25)$$

and then, use the minimization condition

$$\partial I / \partial c_i = 0, (i = 1, \cdots, n) , \qquad (26)$$

to concrete a n-dimensional linear algebraic equation with respect to the unknown coefficients $c_i(i=1,\cdots,n)$. As soon as $c_i(i=1,\cdots,n)$ are solved, one can obtain u by substituting them into (25).

One of the main advantages of the FEM, in comparison to the FDTD method, lies in the flexibility concerning the geometry of the solving domain.

Some numerical simulations of microwave heating were studied by using FEM[90][81][85][108] or using a FEM software COMSOL[109][45].

- **MoM**

The Method of Moments (MoM)[110] is an efficient numerical computational method for solving integral or differential equations for the analyses of electromagnetic characteristics of complex objects. Conceptually, it works by constructing a "mesh" over the modeled surface and so can be used to solve open boundary problems without needing to truncate the domain. The MoM solves EM problems mostly in frequency domain.

For an EM problem, u in the operator function (24) represents the unknown function (e.g. the induced charge or current) and f the known excitation source (e.g. the incident field).

Just like FEM, let us expand the function u in a series of orthogonal basic functions $\{v_i(\cdot)\}_{i=1,\cdots,n}$ as (25), and use the so-called weight functions $\{w_j(\cdot)\}_{j=1,\cdots,n}$ to take the inner product at both sides of (24), which yields

$$\sum_{i=1}^{n} c_i \langle L(v_i), w_j \rangle = \langle f, w_j \rangle, j = 1, \cdots, n , \qquad (27)$$

or in matrix form,

$$\left[L_{ij} \right] \left[c_i \right] = \left[F_j \right], \qquad (28)$$

where $L_{ij} = \langle L(v_i), w_j \rangle$ and $F_j = \langle f, w_j \rangle$.

So we can get $c_i(i=1,\cdots,n)$ by solving the n-dimensional linear algebraic equation (27), and then get u by substituting $c_i(i=1,\cdots,n)$ into (25).

Usually, the weight functions are taken same as the basic functions. The basis functions (including *local* and *global* basis functions) are chosen to model the expected behavior of the unknown function throughout its domain.

The MoM is also used to the numerical simulations of microwave heating[25].

- **TLM**

The transmission-line matrix (TLM) method[111][112][113] is a time-domain, differential method for solving EM field problems using circuit equivalent. Based on the analogy between the wave propagation in space and the voltage and current propagation in a transmission line, it can be regarded as a discrete version of Huygens's continuous wave model.

When using this method, the computational domain is modeled by a transmission line network organized in nodes of a grid (or mesh). The value of electric and magnetic fields at each node is related to the voltages and the currents at corresponding node of the mesh.

Take a 2D case as an example. As can be seen in Fig. 4, a voltage pulse of amplitude 1V is launched on the central node in the space modeled by a Cartesian rectangular mesh. This pulse is partially reflected and transmitted according to the transmission line theory. Assume that each line has a characteristic impedance Z, then the reflection coefficient and the transmission coefficient are given as follows

$$R = \frac{Z/3 - Z}{Z/3 + Z} = -\frac{1}{2}, \; T = 1 + R = \frac{1}{2}. \tag{29}$$
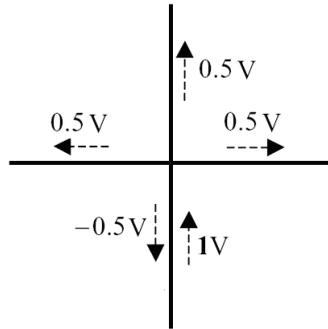
Fig. 4. The scattering of incident pulse of 1V in TLM mesh
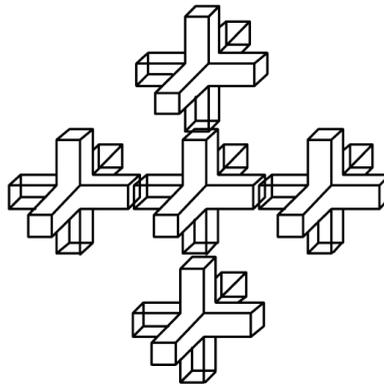

Fig. 5. The typical 3D structure of TLM model

Therefore, we can get values of reflected and transmitted voltage shown in Fig. 4. Obviously, the energy conservation law is fulfilled by the model. And then, the reflected and transmitted voltages excite the neighboring nodes again. Fig. 5 shows the typical 3D structure of TLM model – Symmetrical Condensed Node. One can see [114] for the relative formulae and more details of TLM method.

Some numerical simulations of microwave heating were studied by using the TLM method[115][116].

When using the time domain methods such as the FDTD method, one has to consider the compatibility of the two time steps in the EM equation and the heat transport equation. Because the variation speed of the EM field (e.g. time step is 10^{-12} s) is much faster than the temperature field (e.g. time step is 1s). There are two techniques – the leapfrog scheme [51] and the time scaling factor method[24] which have been used to deal with this difficulty.

4. Conclusions

Remarkable progress has been achieved in recent years in the numerical simulation of microwave heating, which range from numerical modeling and computing to many applications in processing kinds of materials. We believe that the development will continue based on more refined models, faster numerical methods and more complete description of the complex permittivity of more materials.

With the helping of the numerical simulation, the complex process of interaction between microwave and materials will be further understood, the critical parameters influencing the process will be identified, and a rapid design of optimized and controllable applicators will be provided.

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6. References

[1] A. J. H. Sale, "A review of microwaves for food processing," *International Journal of Food Science & Technology*, vol. 11, no. 4, pp.319–329, 1976.

[2] A. C. Metaxas, and R. J. Meredith, *Industry Microwave Heating*. London: Peter Peregrinus Ltd, 1983.

[3] D.A. Copson, *Microwave heating*, *Second Edition*. Westport: AVI Publishing Co. Inc., 1975.

[4] J. M. Osepchuk, "A History of Microwave Heating Applications", *IEEE Transactions on Microwave Theory and Techniques*, vol. 32, no. 9, pp.1200 - 1224, 1984.

[5] Materials Research Society (MRS) Proceedings on "*Microwave Processing of Materials*", vol. 269, 1992; vol. 347, 1994, and vol. 430, 1996.

[6] M. E. C. Oliveira, A. S. Franca, "Microwave heating of foodstuffs," *Journal of Food Engineering*, vol. 53, no. 4, pp. 347-359, 2002

[7] Y. Wang, T. D. Wig, and J. Tang, etc., "Sterilization of foodstuffs using radio frequency heating," *Journal of Food Science*, vol. 68, no. 2, pp. 539-544, 2003.

[8] F. Marra, J. Lyng, and V. Romano, etc., "Radio-frequency heating of foodstuff: Solution and validation of a mathematical model," *Journal of Food Engineering*, vol. 79, no. 3, pp. 998-1006, 2007.

[9] G. S. Michael, and T. P. Gaines, "Microwave drying as a rapid means of sample preparation," *Tobacco Science*, vol. 6, pp. 67-68, 1975.

[10] X. Zhu, Q. Su, and J. Cai, etc., "Optimization of microwave-assisted solvent extraction for volatile organic acids in tobacco and its comparison with conventional extraction methods," *Analytica Chimica Acta*, vol. 579, no.1, pp. 88-94, 2006.

[11] H. Y. Zhou, and C. Z. Liu, "Microwave-assisted extraction of solanesol from tobacco leaves," *Journal of Chromatography A*, vol. 1129, no. 1, pp. 135-139, 2006.

[12] A. Oloyede, and P. Groombridge, "The influence of microwave heating on the mechanical properties of wood," *Journal of Materials Processing Technology*, vol. 100, no.1-3, pp. 67-73, 2000.

[13] M. Miura, H. Kaga, and A. Sakurai, etc., "Rapid pyrolysis of wood block by microwave heating", *Journal of Analytical and Applied Pyrolysis*, vol. 71, no. 1, pp. 187-199, 2004.

[14] P. Rattanadecho, "The simulation of microwave heating of wood using a rectangular wave guide: Influence of frequency and sample size", *Chemical Engineering Science*, vol. 61, no. 14, pp. 4798-4811, 2006.

[15] M. L. Li, G. O. Beale, Y. L. Tian, etc., "Modeling and control for microwave heating of ceramics," in *Proc.1995 American Control Conference*, Jun 1995, vol. 2, pp. 1235-1239.

[16] A.W. Fliflet, R.W. Bruce, and A.K. Kinkead, etc., "Application of microwave heating to ceramic processing: Design and initial operation of a 2.45-GHz single-mode furnace", *IEEE Transactions on Plasma Science,* vol. 24, no. 3, pp. 1041-1049, 1996.

[17] S. Aravindan, and R. Krishnamurthy, "Joining of ceramic composites by microwave heating," *Materials Letters*, vol. 38, no. 4, pp. 245-249, 1999.

[18] A.W. Fliflet, S.H. Gold, R.W. Bruce, etc. , "Temperature Profiles in Ceramic Cylinders Produced by Microwave and Millimeter-wave Heating," in *Vacuum Electronics Conference, 2006 held Jointly with 2006 IEEE International Vacuum Electron Sources., IEEE International*, 2006, pp. 385-386.

[19] A. Sekkak, V. N. Kanellopoulos, and L. Pichon, etc., "A thermal and electromagnetic analysis in biological objects using 3D finite elements and absorbing boundary conditions", *IEEE Transactions on Magnetics*, vol. 31, no. 3, pp. 1865-1868, 1995.

[20] R. I. Kovach , Calculation of the temperature distribution in a biological object heated by microwaves, Biomedical Engineering Volume 5, Number 6 , 336-339.

[21] J. Berlan, "Microwaves in chemistry: Another way of heating reaction mixtures," *Radiation Physics and Chemistry*, vol. 45, no. 4, pp. 581-589, 1995.

[22] Y. Wang, Y. Jing, A. Wei, and W. Song, "Numerical model of microwave heating to organic reaction," in *Proc. 1999 International Conference on Computational Electromagnetics and Its Applications*, 1999, pp. 505-508.

[23] K. Huang, and L. Bo, "The precise condition of thermal runaway in microwave heating on chemical reaction," *Science in China Series E: Technological Sciences*, vol. 52, no. 2, pp. 491-496, 2009.

[24] F. Torres, and B. Jecko, "Complete FDTD analysis of microwave heating processes in frequency-dependent and temperature dependent media," *IEEE Trans. on Microwave Theory Tech.*, vol. 45, pp. 108-116, 1997.

[25] X. Zhao, K. Huang, L. Yan, and Y. Yao, "A preliminary study on numerical simulation of microwave heating process for chemical reaction and discussion of hotspot and thermal runaway phenomenon," *Science in China Series G-Physics Mechanics Astron*, vol. 52, no. 4, pp. 551-562, 2009.

[26] R. W. P. King, and G. S. Smith, *Antennas in Matter: Fundamentals, theory, and application*. Cambridge, NASA: MIT Press, 1981.

[27] S. Ryynänen, "Microwave heating uniformity of multicomponent prepared foods," *Academic Dissertation*. University of Helsinki, 2002.

[28] R. Malheiro, I. Oliveira, M. Vilas-Boas, S. Falcão, A. Bento, and J. A. Pereira, "Effect of microwave heating with different exposure times on physical and chemical parameters of olive oil," *Food and Chemical Toxicology*, vol. 47 pp. 92–97, 2009.

[29] L. A. Campañone, C.A. Paola, and R.H. Mascheroni, "Modeling and Simulation of Microwave Heating of Foods Under Different Process Schedules," *Food and Bioprocess Technology*, DOI: 10.1007/s11947-010-0378-5, 2010.

[30] H. Zhang, and A. K. Datta, "Coupled electromagnetic and thermal modeling of microwave oven heating of foods," *Journal of Microwave Power & Electromagnetic Energy*, vol. 35, no. 2, pp. 71-86, 2000.

[31] P. Perré, and I. W. Turner, "The use of numerical simulation as a cognitive tool for studying the microwave drying of softwood in an over-sized waveguide," *Wood Science and Technology*, vol. 33, pp. 445-464, 1999.

[32] H. Zhao, and I. W. Turner, "The use of a coupled computational model for studying the microwave heating of wood," *Applied Mathematical Modelling*, vol. 24, no. 3, pp. 183-197, Mar. 2000.

[33] L. Hansson, N. Lundgren, A.L. Antti, and O. Hagman, "Finite element modeling (FEM) simulation of interactions between wood and microwaves," *Journal of Wood Science*, vol. 52, no. 5, pp. 406-410, 2006.

[34] L. Hansson, and L. Antti, "Modeling Microwave Heating and Moisture Redistribution in Wood," *Drying Technology*, vol. 26, no. 5, pp. 552-559, May 2008.

[35] P. Rattanadecho, "The simulation of microwave heating of wood using a rectangular wave guide: Influence of frequency and sample size," *Chemical Engineering Science*, vol. 61, pp. 4798- 4811, 2006.

[36] G. Brodie, "Simultaneous heat and moisture diffusion during microwave heating of moist wood," *Applied Engineering in Agriculture*, vol. 23, no. 2, pp. 179-187, 2007.

[37] A. Birnboim, and Y. Carmel, "Simulation of microwave sintering of ceramic bodies with complex geometry," *J. Am. Ceram. Soc.*, vol. 82, pp. 3024-3030, 1999.

[38] D. Bouvard, S. Charmond, and C.P. Carry, "Finite element modeling of microwave sintering," *Advances in Sintering Science and Technology, Ceram. Trans.*, vol. 209, pp. 173-180, 2010.

[39] M. D. Iskander, and A. O. N. M. Andrade, "FDTD simulation of microwave sintering of ceramics in multimode cavities", *IEEE Trans. Microwave Theory and Techniques*, vol. 42, pp. 793-799, 1994.

[40] A. Chatterjee, T. Basak, and K.G. Ayappa, "Analysis of microwave sintering of ceramics", *AIChE Journal*, vol. 44, no. 10, pp. 2302–2311, Oct. 1998.

[41] N. Gupta, V. Midha, and V. Balakotaiah, etc, "Bifurcation Analysis of Thermal Runaway in Microwave Heating of Ceramics", *Journal of the Electrochemical Society*, vol.146, no.12, pp.4659-4665, 1999.

[42] R. Riedel, and J. Svoboda, "Simulation of microwave sintering with advances sintering models", *Advances in Microwave and Radio Frequency Processing*. Springer Press, 2006.

[43] C. Liu, and D. Sheen, "Analysis and control of the thermal runaway of ceramic slab under microwave heating", *Science in China Series E-Technological Sciences,* vol.51, no.12, pp. 2233-2241, 2008.

[44] P. V. Kozlov, I. R. Rafatov, and E. B. Kulumbaev, et al, "On modeling of microwave heating of a ceramic material", *J Phys D: Appl Phys*, vol.40, pp. 2927-2935, 2007.

[45] M. Lovás, M. Kováčová, G. Dimitrakis, and S. Čuvanová, "Modeling of microwave heating of andesite and minerals", *International Journal of Heat and Mass Transfer*, vol. 53, no. 17-18, pp. 3387-3393, 2010.

[46] K. Darcovich, P. S. Whitfield, G. Amow, K. Shinagawa, and R.Y. Miyahara, "A microstructure based numerical simulation of microwave sintering of specialized SOFC materials", *Journal of the European Ceramic Society*, vol. 25, no. 12, pp. 2235-2240, 2005.

[47] P. Fischer, "Numerical Simulation of Microwave Sintering of Zinc Oxide," Master's Thesis, Virginia Technologic Institute, 1997.

[48] H. S. Ho, A. W. Guy, R. A. Sigelmann, and J. F. Lehmann, "Microwave heating of simulated human limbs by aperture sources," *IEEE Transactions on Microwave Theory and Techniques*, vol. 19, no. 2, pp. 224-226, 1971.

[49] A. Sekkak, V. N. Kanellopoulos, L. Pichon, and A. Razek, "A thermal and electromagnetic analysis in biological objects using 3D finite elements and absorbing boundary conditions," *IEEE Transactions on Magnetics*, vol.31, no.3, pp.1865-1868, 1995.

[50] Y. Wang, Y. Jing, A. Wei, and W. Song, "Numerical model of microwave heating to organic reaction," in Proc. 1999 International Conference on Computational Electromagnetics and Its Applications, 1999, pp. 505-508.

[51] K. Huang, Z. Lin, and X. Yang, "Numerical simulation of microwave heating on chemical reaction in dilute solution", Progress In Electromagnetics Research, vol. 49, pp. 273-289, 2004.

[52] H. Yamada, A. Chayahara, Y. Mokuno, Y. Horino, and S. Shikata, "Numerical analyses of a microwave plasma chemical vapor deposition reactor for thick diamond syntheses", *Diamond and Related Materials*, vol. 15, no. 9, pp. 1389-1394, 2006.

[53] K. Huang, and B. Lu, "The precise condition of thermal runaway in microwave heating on chemical reaction", *Science in China Series E: Technological Sciences*, vol. 52, no. 2, pp. 491-496, 2009.

[54] J. Svoboda, and H. Riedel, "Pore-Boundary Interactions and Evolution Equations for the Porosity and the Grain Size during Sintering", *Acta Metallurgica et Materialia,* vol. 40, no. 11, pp. 2829-2840, 1992.

[55] R. Gedye, F. Smith, K. Westaway, H. Ali, L. Baldisera, L. Laberge, and J. Rousell, "The use of microwave ovens for rapid organic synthesis," *Tetrahedron Letters*, vol. 27, no. 3, pp. 279-282, 1986.

[56] P. Chen, and P. S. Schmidt, "An integral model for drying hygroscopic and nonhygroscopic materials with dielectric heating", *Drying Technology*, vol. 8, pp. 907-930, 1990.

[57] W. G. Engelhart, "Microwave hydrolysis of peptides and proteins for amino acid analysis", *Am. Biotechnol. Lab*, vol. 8, pp. 30-34, 1990.

[58] T. Basak, and S. Durairaj, "Numerical Studies on Microwave Heating of Thermoplastic-Ceramic Composites Supported on Ceramic Plates", *J. Heat Transfer*, vol. 132, no. 7, 2010.

[59] P. H. Harms, Y. Chen, R. Mittra, and Y. Shimony, "Numerical modeling of microwave heating systems", *Journal of Microwave Power & Electromagnetic Energy*, vol. 31, no. 2, pp. 114-122, 1996.

[60] A. W. Filfet, "Application of microwave heating to ceramic processing: Design and initial operation of a 2.45-GHz single-mode furnace", *IEEE Trans. Plasma Sci*. vol. 24, no. 3, pp. 1041-1049, 1996.

[61] J. M. Tranquilla, M. Feng, and H. M. Al-Rizzo, "A Cartesian-Cylindrical Hybrid FD-TD Analysis of Composite Microwave Applicator Structures," *Journal of Microwave Power and Electromagnetic Energy*, vol.34, no.2, pp.97-105, 1999.

[62] P. PerreÂ, and I.W. Turner, "The use of macroscopic equations to stimulate heat and mass transfer in porous media: some possibilities illustrated by a wide range of configurations that emphasis the role of internal pressure," *Mathematical modeling and Numerical Techniques in Drying Technology*, pp.83-156,(edited by I.W. Turner and A. Mujumdar), Marcel Dekker, Inc., 1996.

[63] H. Feng, J. Tang, R. P. Cavalieri, and O. A. Plumb, "Heat and Mass Transport in Microwave Dying of Hygroscopic Porous Materials in a Spouted Bed," *AIChE J.*, vol.47, no.7, pp.1499-1512, 2001.

[64] F. Marra, M. V. De Bonis, and G. Ruocco, "Combined microwaves and convection heating: A conjugate approach," *Journal of Food Engineering*, vol. 97, pp. 31-39, 2010.

[65] F. Ying, T. Wang, and L. Johan, "Microwave-transmission, heat and temperature properties of electrically conductive adhesive," *IEEE Transactions on CPMT, Part A*, vol.26, no.1, pp.193-198 March, 2003.

[66] J. G. Kirkwood, "The dielectric polarization of polar liquids," *J. Chem. Phys.*, vol. 7, pp. 911-919, Oct. 1939.

[67] K. S. Cole, and R. H. Cole, "Dispersion and absorption in dielectrics," *J. Chem. Phys.*, vol. 9, pp. 341-351, Apr. 1941.

[68] P. Debye, *Polar Molecules*, New York: Dover, 1945.

[69] H. Fröhlich, *Theory of Dielectrics*, Oxford: Clarendon, 1949.

[70] R. H. Cole, "Theories of dielectric polarization and relaxation," in *Progress in Dielectrics*, J. Birks and J. Hart, Eds. London, U.K: Heywood, vol. 3, 1961.

[71] C. Gabriel, S. Gabriel, E. H. Grant, B. S. J. Halstead, and D. M. P. Mingos, "Dielectric parameters relevant to microwave dielectric heating," *Chemical Society Reviews*, vol.27, pp.213-223, 1998.

[72] E. Peyskens, M. De Pourcq, and M. stevens, etc., "Dielectric properties of softwood species at microwave frequencies," *Wood Science and Technology*, vol.18, no.4, pp.267-280, 1984.

[73] B. Zhou, and S. Avramidis, "On the loss factor of wood during radio frequency heating," *Wood Science and Technology*, vol.33, pp.299-310, 1999.

[74] *Ceramic Source*, compilted by American Ceramic Society, Vol. 7, 1991-1992.

[75] R. M. Hutcheon, M. S. De Jong, F. P. Adams, P. G. Lucuta, J. E. McGregor, and L. Bahen, "RF and Microwave Dielectric Measurements to 1400°C and Dielectric Loss Mechanisms," *in: Materials Research Society Symposium Proceedings*, vol.269, pp.541-551, 1992.

[76] G. W. Padua, "Microwave heating of agar gels containing sucrose," *Journal of Food Science*, vol. 58, no. 60, pp. 1426-1428, 1993.

[77] N. Sakai, W. Mao, Y. Koshima, and M. Watanabe, "A method for developing model food system in microwave heating studies", *Journal of Food Engineering*, vol. 66, no.4, pp. 525-531, 2005.

[78] K. G. Ayappa, H. T. Davis, G. Crapiste, E. A. Davis, and J. Gordon , "Microwave heating: an evaluation of power formulations", *Chemical Engineering Science*, vol. 46, no. 4, pp. 1005-1016, 1991.

[79] P. Kopyt, "A one-dimensional semi-analytical model of the microwave heating effect in verification of numerical hybrid modeling software", *EUROCON 2007 The Intl.Conf. on "Computer as a Tool"*, Warsaw, Sept. 2007, pp.65-72.

[80] Y. Alpert, and E. Jerby, "Coupled Thermal-Electromagnetic Model for Microwave Heating of Temperature-Dependent Dielectric Media", *IEEE Trans. on Plasm Science,* vol. 27, no. 2, pp. 555- 562, Apr. 1999.

[81] E. C. M. Sanga, A. S. Mujumdar, and G. S. V. Raghavan, "Simulation of convection-microwave drying for a shrinking material", *Chem. Eng. Process,* vol. 41, pp. 487-499, 2002.

[82] J. Haala, and W. Wiesbeck, "Modeling microwave and hybrid heating processes including heat radiation effects," *IEEE Trans. Microwave Theory Tech.*, vol. 50, no. 5, pp. 1346-1354, 2002.

[83] L. Ma, D.-L. Paul, N. Pothecary, C. Railton, J. Bows, L. Barratt, J. Mullin, and D. Simons, "Experimental validation of a combined electromagnetic and thermal FDTD model of a microwave heating process," *IEEE Trans. Microwave Theory Tech.*, vol. 43, no.11, pp. 2565-2572, 1995.

[84] B. Milovanović, N.Dončov, and J.Joković, "Microwave Heating Cavities: Modelling and Analysis", *Microwave Review*, vol. 10, no. 2, pp. 26-35, Nov. 2004.

[85] A. Sekkak, V. N. Kanellopoulos, L. Pichon, and A. Razek, "A thermal and electromagnetic analysis in biological objects using 3D finite elements and

absorbing boundary conditions", *IEEE Trans. on Magnetics*, vol. 31,no. 3, pp. 1865-1868, May 1995.

[86] P. Plaza-Gonzalez, et al., "New approach for the prediction of the electric field distribution in multimode microwave-heating applicators with mode stirrers", *IEEE Trans. on Magnetics,* vol. 40, no. 3, pp. 1672-1678, May 2004.

[87] K. D. Paulsen, D. R. Lynch, and J. W. Strohbehn, "Three dimensional finite boundary and hybrid element solution of the Maxwell equation for lossy dielectric media," *IEEE Trans. Microwave Theory Tech.*, vol. 36, pp. 682-693, Apr. 1988.

[88] C. T. Choi, and A. Konrad, "Finite element modeling of the RF heating process," *IEEE Trans. Magn.*, vol. 27, pp. 4227-4230, Sept. 1991.

[89] W. Li, M. A. Ebadian, T. L. White, R. G. Grubb, and D. Foster, "Heat and mass transfer—In a contaminated porous concrete slab with variable dielectric properties," *Int. J. Heat Mass Transfer*, vol. 37, no. 6, pp. 1013-1027, Apr. 1994.

[90] A. Sekkak, L. Pichon, and A. Razek, "3-D FEM magneto-thermal analysis in microwave ovens," *IEEE Trans. Magn.*, vol. 30, pp. 3347–3350, Sept. 1994.

[91] L. E. Lagos, W. Li, M. A. Ebadian, T. L. White, R. G. Grubb, and D. Foster, "Heat transfer within a concrete slab with a finite microwave heating source," *Int. J. Heat Mass Transfer*, vol. 38, no. 5, pp. 887–897, Mar. 1995.

[92] L. Ma, D. Paul, N. Pothecary, C. Railton, J. Bows, L. Barratt, J. Mullin, and D. Simons, "Experimental validation of a combined electromagnetic and thermal FDTD model of a microwave heating process," *IEEE Trans.Microwave Theory Tech.*, vol. 43, pp. 2565–2572, Nov. 1995.

[93] A. Sekkak, V. N. Kanellopoulos, L. Pichon, and A. Razek, "A thermal and electromagnetic analysis in biological objects using 3D finite elements and absorbing boundary," *IEEE Trans. Magn.*, vol. 31, pp. 1865-1868, May 1995.

[94] J. Clemens, and C. Saltiel, "Numerical modeling of materials processing in microwave furnaces," *Int. J. Heat Mass Transfer*, vol. 39, no. 8, pp. 1665-1675, 1996.

[95] J. Braunstein, K. Connor, S. Salon, and L. Libelo, "Investigation of microwave heating with time varying material properties," *IEEE Trans. Magnetics* , vol. 35, no. 3, pp. 1813-1816, 1999.

[96] M. Clemens, E. G. Jonaj, P. Pinder, and T. Weiland, "Numerical simulation of coupled transient thermal and electromagnetic fields with the finite integration method," *IEEE Trans. Magnetics,* vol. 36, no. 4, pp. 1448-1452, 2000.

[97] A. Taflove and S. C. Hagness, *Computational Electrodynamics: The Finite-Difference Time-Domain Method, 3rd ed.* Artech House Publishers, 2005.

[98] A. Taflove, and M. E. Brodwin, "Numerical solution of steady-state electromagnetic scattering problems using the time-dependent Maxwell's equations", *IEEE Transactions on Microwave Theory and Techniques*, vol. 23, pp. 623-630, 1975.

[99] J. Berenger, "A perfectly matched layer for the absorption of electromagnetic waves", *Journal of Computational Physics*, vol. 114, pp. 185–200, 1994.

[100] K. Yee, "Numerical solution of initial boundary value problems involving Maxwell's equations in isotropic media", *IEEE Transactions on Antennas and Propagation*, vol. 14, pp. 302-307, 1966.

[101] H. M. Al-Rizzo, M. Feng, and J. M. Tranquilla, "Incorporation of waveguide feed and cavity wall losses in a Cartesian/cylindrical hybrid finite-difference time domain (FD-TD) analysis of microwave applicator", *J. Microwave Power & Electromag. Energy*, vol. 35, no. 2, pp. 110-118, 2000.

[102] H. M. Al-Rizzo, J. M. Tranquilla, and M. Feng, "A finite difference thermal model of a cylindrical microwave heating applicator using locally conformal overlapping grids: Part I - Theoretical formulation", *J Microwave Power & Electromag. Energy*, vol. 40, no. 1, pp. 17-29, 2005.

[103] H. Chen, J. Tang, and F. Liu, "Simulation model for moving food packages in microwave heating processes using conformal FDTD method", *Journal of Food Engineering*, vol. 88, no. 3, pp. 294-305, 2008.

[104] G. Strang, and G. Fix, *An Analysis of the Finite Element Method 2nd Edition*, Wellesley-Cambridge, 2008.

[105] I. Babuska, U. Banerjee, and J. E. Osborn, "Generalized Finite Element Methods: Main Ideas, Results, and Perspective," *International Journal of Computational Methods*, vol. 1, no. 1, pp. 67-103, 2004.

[106] G. Pelosi, "The finite-element method, Part I: R. L. Courant [Historical Corner]," *IEEE Antennas and Propagation Magazine*, vol. 49, no. 2, pp.180 – 182, 2007.

[107] R. L. Ferrari, "The Finite-Element Method, Part 2: P. P. Silvester, an Innovator in Electromagnetic Numerical Modeling ," *IEEE Antennas and Propagation Magazine*, vol. 49, no. 3, pp.216-234, 2007.

[108] M. Pauli, T. Kayser, and G. Adamiuk, etc., "Modeling of mutual coupling between electromagnetic and thermal fields in microwave heating," in *Proc. IEEE MTT-S International Microwave Symposium 2007*, Jun. 2007, pp. 1983-1986.

[109] D. Bouvard, S. Charmond, and C. P. Carry, "Multiphysics Simulation of Microwave Sintering in a Monomode Cavity", in *Proc. 12th Seminar Computer Modeling in Microwave Engineering & Applications (Advances in Modeling of Microwave Sintering)*, Grenoble, France, March 8-9, 2010, pp. 21-26.

[110] W. C. Gibson, *The Method of Moments in Electromagnetics, London*: Chapman & Hall/CRC, 2008.

[111] W. J. R. Hoefer, "The Transmission-Line Matrix Method—Theory and Applications," *IEEE Transactions on Microwave Theory and Techniques*, vol. 33, no. 10, pp.882-893, 1985.

[112] P. B. Johns, and R. L. Beurle, "Numerical solution of two-dimensional scattering problems using a transmission-line matrix," *Proc. IEE*, vol. 118, no. 12, pp.1203-1208, 1971.

[113] P. B. Johns, "A symmetrical condensed node for the TLM method," *IEEE Trans. MTT*, vol. 35, no. 4, pp.370-377, 1987.

[114] C. Christopoulos. *The Transmission-Line Modeling (TLM) Method in Electromagnetics, First Edition*, Morgan & Claypool, 2006.

[115] C. Flockhart, V. Trenkic, and C. Christopoulos, "The simulation of coupled electromagnetic and thermal problems in microwave heating", in *Proc. Second International Conference on Computation in Electromagnetics*, London, Apr. 1994, pp. 267-270.

[116] C. Christopoulos, "The application of time-domain numerical simulation methods to the microwave heating of foods," *IMA Journal of Management Mathematics*, vol. 5, no. 1, pp.385-397, 1993.
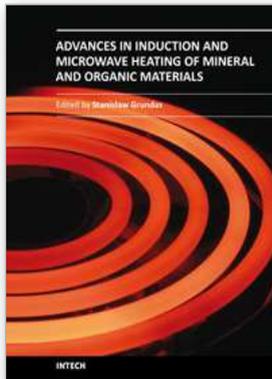
Advances in Induction and Microwave Heating of Mineral and Organic Materials

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The book offers comprehensive coverage of the broad range of scientific knowledge in the fields of advances in induction and microwave heating of mineral and organic materials. Beginning with industry application in many areas of practical application to mineral materials and ending with raw materials of agriculture origin the authors, specialists in different scientific area, present their results in the two sections: Section 1-Induction and Microwave Heating of Mineral Materials, and Section 2-Microwave Heating of Organic Materials.