

# Thermoelastic Stresses in FG-Cylinders

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## 1. Introduction

FGM components are generally constructed to sustain elevated temperatures and severe temperature gradients. Low thermal conductivity, low coefficient of thermal expansion and core ductility have enabled the FGM material to withstand higher temperature gradients for a given heat flux. Examples of structures undergo extremely high temperature gradients are plasma facing materials, propulsion system of planes, cutting tools, engine exhaust liners, aerospace skin structures, incinerator linings, thermal barrier coatings of turbine blades, thermal resistant tiles, and directional heat flux materials. Continuously varying the volume fraction of the mixture in the FGM materials eliminates the interface problems and mitigating thermal stress concentrations and causes a more smooth stress distribution.

Extensive thermal stress studies made by Noda reveal that the weakness of the fiber reinforced laminated composite materials, such as delamination, huge residual stress, and locally large plastic deformations, may be avoided or reduced in FGM materials (Noda, 1991). Tanigawa presented an extensive review that covered a wide range of topics from thermo-elastic to thermo-inelastic problems. He compiled a comprehensive list of papers on the analytical models of thermo-elastic behavior of FGM (Tanigawa, 1995). The analytical solution for the stresses of FGM in the one-dimensional case for spheres and cylinders are given by Lutz and Zimmerman (Lutz & Zimmerman, 1996 & 1999). These authors consider the non-homogeneous material properties as linear functions of radius. Obata presented the solution for thermal stresses of a thick hollow cylinder, under a two-dimensional transient temperature distribution, made of FGM (Obata et al., 1999). Sutradhar presented a Laplace transform Galerkin BEM for 3-D transient heat conduction analysis by using the Green's function approach where an exponential law for the FGMs was used (Sutradhar et al., 2002). Kim and Noda studied the unsteady-state thermal stress of FGM circular hollow cylinders by using of Green's function method (Kim & Noda, 2002). Reddy and co-workers carried out theoretical as well as finite element analyses of the thermo-mechanical behavior of FGM cylinders, plates and shells. Geometric non-linearity and effect of coupling item was considered for different thermal loading conditions (Praveen & Reddy, 1998, Reddy & Chin, 1998, Paraveen et al., 1999, Reddy, 2000, Reddy & Cheng, 2001). Shao and Wang studied the thermo-mechanical stresses of FGM hollow cylinders and cylindrical panels with the assumption that the material properties of FGM followed simple laws, e.g., exponential law,

power law or mixture law in thickness direction. An approximate static solution of FGM hollow cylinders with finite length was obtained by using of multi-layered method; analytical solution of FGM cylindrical panel was carried out by using the Frobinus method; and analytical solution of transient thermo-mechanical stresses of FGM hollow cylinders were derived by using the Laplace transform technique and the power series method, in which effects of material gradient and heat transfer coefficient on time-dependent thermal mechanical stresses were discussed in detail (Shao, 2005, Shao & Wang, 2006, Shao & Wang, 2007). Similarly, Ootao and Tanigawa obtained the analytical solutions of unsteady-state thermal stress of FGM plate and cylindrical panel due to non-uniform heat supply (Ootao & Tanigawa, 1999, 2004, 2005). Using the multi-layered method and through a novel limiting process, Liew obtained the analytical solutions of steady-state thermal stress in FGM hollow circular cylinder (Liew & et al., 2003). Using finite difference method, Awaji and Sivakuman studied the transient thermal stresses of a FGM hollow circular cylinder, which is cooled by surrounding medium (Awaji & Sivakuman, 2001). Ching and Yen evaluated the transient thermoelastic deformations of 2-D functionally graded beams under non-uniformly convective heat supply (Ching & Yen, 2006).

In this paper, by using the Hermitian transfinite element method, nonlinear transient heat transfer and thermoelastic stress analyses is performed for thick-walled FGM cylinder which materials are temperature-dependent. Time variations of the temperature, displacements, and stresses are obtained through a numerical Laplace inversion. Finally, results obtained considering the temperature-dependency of the material properties. Those results are the temperature distribution and the radial and circumferential stresses are investigated versus time, geometrical parameters and index of power law ( $N$ ) and then they are compared with those derived based on temperature independency assumption.

Two main novelties of this research are incorporating the temperature-dependency of the material properties and proposing a numerical transfinite element procedure that may be used in Picard iterative algorithm to update the material properties in a highly nonlinear formulation. In contrast to before researches, second order elements are employed. Therefore, proposed transfinite element method may be adequately used in problems where time integration method is not recommended because of truncation errors (e.g. coupled thermo-elasticity problems with very small relaxation times) or where improper choice of time integration step may lead to loss of the higher frequencies in the dynamic response. Also, accumulated errors that are common in the time integration method and in many cases lead to remarkable errors, numerical oscillations, or instability, do not happen in this technique.

## 2. The governing equations

Geometric parameters of the thick-walled FGM cylinder are shown in Figure (1). The FGM cylinder is assumed to be made of a mixture of two constituent materials so that the inner layer ( $r = r_i$ ) of the cylinder is ceramic-rich, whereas the external surface ( $r = r_o$ ) is metal-rich. The properties can be expressed as follows:

$$P = P_0(P_{-1}T^{-1} + 1 + P_1T + P_2T^2 + P_3T^3) \quad (1)$$

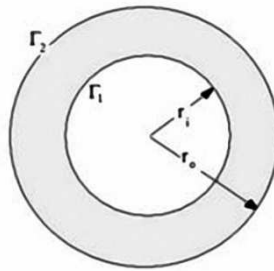


Fig. 1. FG Thick-walled Cylinder

Where  $P_0, P_{-1}, P_1, P_2$  and  $P_3$  are constants in the cubic fit of the materials property. The materials properties are expressed in this way so that higher order effects of the temperature on the material properties can be readily discernible. Volume fraction is a spatial function whereas the properties of the constituents are functions of temperature. The combination of these functions gives rise to the effective material properties of FGM and can be expressed by

$$P_{eff}(T, r) = P_m(T)V_m(r) + P_c(T)V_c(r) \tag{2}$$

Where  $P_{eff}$  is the effective material property of FGM, and  $P_m$  and  $P_c$  are the temperature dependent properties metal and ceramic, respectively.  $V_c$  is the volume fraction of the ceramic constituent of the FGM can be written by

$$V_c = \left(\frac{r_o-r}{r_o-r_i}\right)^N, \quad V_m = 1 - V_c \tag{3}$$

Where volume fraction index  $N$  dictates the material variation profile through the beam thickness and may be varied to obtain the optimum distribution of component materials ( $0 \leq N \leq \infty$ ). From above equation, the effective Young's modulus  $E$ , Poisson ratio  $\nu$ , thermal expansion coefficient  $\alpha$  and mass density  $\rho$  of an FGM cylinder can be written by

$$P_{eff} = (P_c - P_m) \left(\frac{r_o-r}{r_o-r_i}\right)^N + P_m \tag{4}$$

In this paper, only the effective Young's modulus and thermal expansion coefficient are dependent of temperature. The related equation is

$$P = P_{0c}(P_{-1c}T^{-1} + 1 + P_{1c}T + P_{2c}T^2 + P_{3c}T^3)V_c + P_{0m}(P_{-1m}T^{-1} + 1 + P_{1m}T + P_{2m}T^2 + P_{3m}T^3)V_m \tag{5}$$

### 3. Finite element method

The equation of heat transfer is

$$-\frac{1}{r} \left[ \frac{\partial T}{\partial r} \frac{\partial (Kr)}{\partial r} + Kr \frac{\partial^2 T}{\partial r^2} \right] + \rho C_v \frac{\partial T}{\partial t} = 0 \tag{6}$$

The boundary conditions are a heat flux in the internal layer and convection condition for external layer of FGM cylinder

$$\begin{cases} 2\pi \kappa r \frac{\partial T}{\partial r} + 2\pi r h (T - T_\infty) = 0 & \text{at } r = r_o \\ -\kappa \frac{\partial T}{\partial r} = q_0 & \text{at } r = r_i \end{cases} \quad (7)$$

The initial condition is  $T(t = 0) = T_\infty$ . Kantorovich approximation is

$$\{T(r, t)\} = [\tilde{N}(r)] \{T^{(e)}(t)\} \quad (8)$$

$[\tilde{N}]$  is the shape function matrix. For second order elements (with 3 nodes) which used in temperature field is

$$[\tilde{N}] = \left[ \frac{1}{2} \xi(\xi - 1) \quad (1 - \xi^2) \quad \frac{1}{2} \xi(\xi + 1) \right] \quad (9)$$

$\xi$ , natural coordinate which changes between -1 and 1 is used because of Gauss-Legendre numerical integration method. The relation between global and natural coordinate is

$$[\tilde{N}]_{,r} = [\tilde{N}]_{,\xi} \xi_{,r} = [\tilde{N}]_{,\xi} \frac{\Delta \xi}{\Delta r} = [\tilde{N}]_{,\xi} \frac{2n}{r_o - r_i} \quad (10)$$

$n$  is number of elements. Residual integration form in Galerkin method is

$$\int_{\Omega} [\tilde{N}]^T R \, d\Omega = 0 \quad (11)$$

For heat transfer problem,  $R$  is

$$R = \rho c [\tilde{N}] \{\dot{T}^{(e)}\} - \frac{1}{r} \left[ [\tilde{N}]_{,r} \frac{\partial(\kappa r)}{\partial r} + \kappa r [\tilde{N}]_{,rr} \right] \{T^{(e)}\} \quad (12)$$

Then the heat transfer problem can be written by

$$[C^{(e)}] \{\dot{T}^{(e)}\} + [A^{(e)}] \{T^{(e)}\} = \{q^{(e)}\} \quad (13)$$

Matrices  $C$ ,  $A$  and  $q$  are damping and stiffness matrices and force vector, regularly.

$$[C^{(e)}] = \int_{\Omega} [\tilde{N}]^T \rho c [\tilde{N}] \, d\Omega \quad (14a)$$

$$\begin{aligned} [A^{(e)}] = & - \int_{\Omega} [\tilde{N}]^T \frac{1}{r} \frac{\partial(\kappa r)}{\partial r} [\tilde{N}]_{,r} \, d\Omega + \int_{\Omega} \left[ [\tilde{N}]^T_{,r} \kappa + [\tilde{N}]^T \kappa_{,r} \right] [\tilde{N}]_{,r} \, d\Omega \\ & + \int_{\Gamma_2} [\tilde{N}]^T h [\tilde{N}] \, d\Gamma_2 \end{aligned} \quad (14b)$$

$$\{q^{(e)}\} = \int_{\Gamma} [\tilde{N}]^T \kappa \frac{\partial T}{\partial r} \cdot n_r \, d\Gamma = \int_{\Gamma_1} [\tilde{N}]^T q_0 \, d\Gamma_1 + \int_{\Gamma_2} [\tilde{N}]^T h T_\infty \, d\Gamma_2 \quad (14c)$$

Strain can be written like below

$$\{\varepsilon\} = [d]\{u\} \quad (15)$$

$$\{\varepsilon\}^T = \langle \varepsilon_r \quad \varepsilon_\theta \rangle, \quad [d]^T = \begin{bmatrix} \frac{\partial}{\partial r} & 1 \\ & r \end{bmatrix} \quad (16)$$

Energy function is

$$\pi = \frac{1}{2} \int_{\Omega} \{\varepsilon\}^T \{\sigma\} d\Omega + \int_{\Omega} \rho \{u\}^T \{\ddot{u}\} d\Omega - \int_{\Gamma} \{u\}^T \{\hat{p}\} d\Gamma \quad (17)$$

And so

$$\delta\pi = \int_{\Omega} \delta(\{\varepsilon\}^T) \cdot \{\sigma\} d\Omega + \int_{\Omega} \rho \cdot \delta(\{u\}^T) \cdot \{\ddot{u}\} d\Omega - \int_{\Gamma} \delta(\{u\}^T) \cdot \{\hat{p}\} d\Gamma = 0 \quad (18)$$

Differential of strain is

$$\delta(\{\varepsilon\}^T) = \delta(\{u\}^T) \cdot [d]^T \quad (19)$$

Displacements in order to shape functions are

$$\{u\} = [N]\{U^{(e)}\} \quad (20)$$

[N] for displacement is a Hermitian shape function.

$$[N] = [N_1 \quad \bar{N}_1 \quad N_2 \quad \bar{N}_2] \quad (21a)$$

$$N_1 = \frac{1}{4}(\xi - 1)^2(2 + \xi) \quad (21b)$$

$$\bar{N}_1 = \frac{1}{4}(1 - \xi)^2(\xi + 1) \quad (21c)$$

$$N_2 = \frac{1}{4}(\xi + 1)^2(2 - \xi) \quad (21d)$$

$$\bar{N}_2 = \frac{1}{4}(1 + \xi)^2(\xi - 1) \quad (21e)$$

And displacement vector is

$$\{U^{(e)}\}^T = \langle u_0^{(1)} \quad \dot{u}_0^{(1)} \quad u_0^{(2)} \quad \dot{u}_0^{(2)} \rangle \quad (22)$$

By defining of

$$[B] = [d][N] \quad (23)$$

In which

$$[B] = \begin{bmatrix} N_{1,r} & \bar{N}_{1,r} & N_{2,r} & \bar{N}_{2,r} \\ \frac{N_1}{r} & \frac{\bar{N}_1}{r} & \frac{N_2}{r} & \frac{\bar{N}_2}{r} \end{bmatrix} \quad (24)$$

And then, equation (18) is changed to

$$\delta\pi = \int_{\Omega} \delta(\{U^{(e)}\}^T) [B]^T \cdot \{\sigma\} d\Omega + \int_{\Omega} \rho \cdot \delta(\{U^{(e)}\}^T) [N]^T \cdot \{\dot{u}\} d\Omega - \int_{\Gamma} \delta(\{U^{(e)}\}^T) [N]^T \cdot \{\hat{p}\} d\Gamma \quad (25)$$

And it's simplified to

$$\int_{\Omega} [B]^T \cdot \{\sigma\} d\Omega + \int_{\Omega} \rho [N]^T \{\dot{u}\} d\Omega - \int_{\Gamma} [N]^T \cdot \{\hat{p}\} d\Gamma = 0 \quad (26)$$

The relation between stress and strain is

$$\{\sigma\} = [D]\{\{\varepsilon\} - \{\varepsilon_T\}\} \quad (27)$$

In which

$$\{\varepsilon_T\}^T = \langle \alpha \Delta T \quad \alpha \Delta T \rangle \quad (28a)$$

$$[D] = \frac{E(r)}{1 - \nu(r)^2} \begin{bmatrix} 1 & \nu(r) \\ \nu(r) & 1 \end{bmatrix} \quad (28a)$$

Then equation (26) becomes

$$\int_A ([B]^T [D] ([B]\{U^{(e)}\} - \{\varepsilon_T\})) dA + \int_A \rho [N]^T [N] \{\dot{U}^{(e)}\} dA - \int_s [N]^T \cdot \{\hat{p}\} ds = 0 \quad (29)$$

Thermo-elastic stress problem can be written like below

$$[M^{(e)}]\{\dot{\Phi}^{(e)}\} + [K^{(e)}]\{\Phi^{(e)}\} = \{f^{(e)}\} \quad (30)$$

Matrices M, K and f are mass and stiffness matrices and force vector, regularly.

$$[M^{(e)}] = \int_A \rho [N]^T [N] dA \quad (31a)$$

$$[K^{(e)}] = \int_A [B]^T [D] [B] dA \quad (31b)$$

$$\{f^{(e)}\} = \int_A [N]^T \cdot \{\hat{p}\} dA + \int_A [B]^T [D] \{\varepsilon_T\} dA \quad (31c)$$

The general integral is

$$\int_{\Omega} L(r, \xi) d\Omega = \int 2\pi r L(r, \xi) dr = 2\pi \int_{-1}^1 r L(r, \xi) \frac{\Delta r}{\Delta \xi} dz = \pi \int_{-1}^1 r L(r, \xi) \frac{r_o - r_i}{n} dz \quad (32)$$

To solve the above equations, a program which writes in MATLAB is used.

The geometrical characteristics and coefficients of properties of FGM cylinder are listed in Tables (1) and (2), regularly.

Characteristic	Amount
Heat Flux ( $q_0$ )	1000 ( $\frac{kW}{m^2}$ )
Coefficient of Conduction (h)	8 ( $\frac{W}{m^2 \cdot K}$ )
Internal Radius ( $r_i$ )	12.7 (mm)
External Radius ( $r_o$ )	25.4 (mm)

Table 1. Geometrical Characteristics

Characteristics	Metal (Ti-6Al-4V)	Ceramic ( $Si_3N_4$ )
$\rho$ ( $\frac{kg}{m^3}$ )	2370	4429
$C_v$ ( $\frac{j}{kg \cdot K}$ )	625.297	555.110
$\kappa$ ( $\frac{W}{m \cdot K}$ )	13.723	1.209
E (GPa)	122.557	348.430
$\nu$	0.29	0.24
$\alpha$ ( $\frac{1}{K}$ )	7.579 e -6	5.872 e -6
$P_{-1}$ (E, $\alpha$ )	0	0
$P_1$ (E)	-4.586 e -4	3.700 e -4
$P_2$ (E)	0	2.160 e -7
$P_3$ (E)	0	-8.948 e -11
$P_1$ ( $\alpha$ )	6.500 e -4	9.095 e -4
$P_2$ ( $\alpha$ )	0.313 e -6	0
$P_3$ ( $\alpha$ )	0	0

Table 2. Coefficients of Properties of FG Material

#### 4. Laplace transform

Making application of the Laplace transform, defined by

$$\Omega(s) = L[\omega(t)] = \int_0^{\infty} e^{-st} \omega(t) dt \quad (33)$$

The governing equations become

$$(s[C^{(e)}] + [A^{(e)}]) \{T(s)^{(e)}\} = \{Q(s)^{(e)}\} \quad (34)$$

$$(s^2[M^{(e)}] + [K^{(e)}]) \{U(s)^{(e)}\} = \{F(s)^{(e)}\} \quad (35)$$

## 5. Numerical Laplace inversion

To obtain the distributions of the displacement and temperature in the physical domain, it is necessary to perform Laplace inversion for the transformed displacement and temperature obtained when a sequence of values of  $s$  is specified. In this paper an accurate and efficient numerical method is used to obtain the inversion of the Laplace transform.

The inversion of the Laplace transform is defined as

$$\omega(t) = L^{-1}[\Omega(s)] = \frac{1}{2\pi i} \int_{v-i\infty}^{v+i\infty} e^{st} \Omega(s) ds \quad (36)$$

The numerical inversion of the Laplace transform can be written

$$\omega_N(t) = \frac{1}{2} \lambda_0 + \sum_{k=1}^p \lambda_k \quad (37)$$

$$\lambda_k = \frac{e^{-vt}}{T} \left\{ \operatorname{Re} \left[ \Omega \left( v + i \frac{k\pi}{T} \right) \right] \cos \frac{k\pi}{T} t \right\} - \frac{e^{-vt}}{T} \left\{ \operatorname{Im} \left[ \Omega \left( v + i \frac{k\pi}{T} \right) \right] \sin \frac{k\pi}{T} t \right\} \quad (38)$$

It should be noted that a good choice of the free parameters  $p$  and  $vT$  is not only important for the accuracy of the results but also for the application of the Korrektur method and the methods for the acceleration of convergence. The values of  $v$  and  $T$  are chosen according to the criteria outlined by Honig and Hirdes (Honig & Hirdes, 1984).

After choosing the optimal  $v$ , any nodal variables in physical domain can be calculated at any specific instant by using the Korrektur method and e-algorithm simultaneously to perform the numerical Laplace inversion (Honig & Hirdes, 1984).

## 6. Numerical results

### 6.1 Results for temperature distribution

Here are the dimensionless parameters which are used in numerical results

$$R = \frac{r - r_i}{r_o - r_i}, \quad \bar{R} = \frac{r_o}{r_i}, \quad \bar{t} = \frac{t}{T} \quad (39)$$

Results obtained in Figure (2) for different number of elements ( $n$ ) shows that the results are convergent in  $\bar{t} = 0.5$ . Hence, seven second order elements are chosen to perform the next analyses.

The distribution of temperature is drawn for  $N=1$  in Figure (3). As it's expected, results of the consecutive times are convergent to each other and then the transient response vanishes and the steady-state response becomes the dominant.



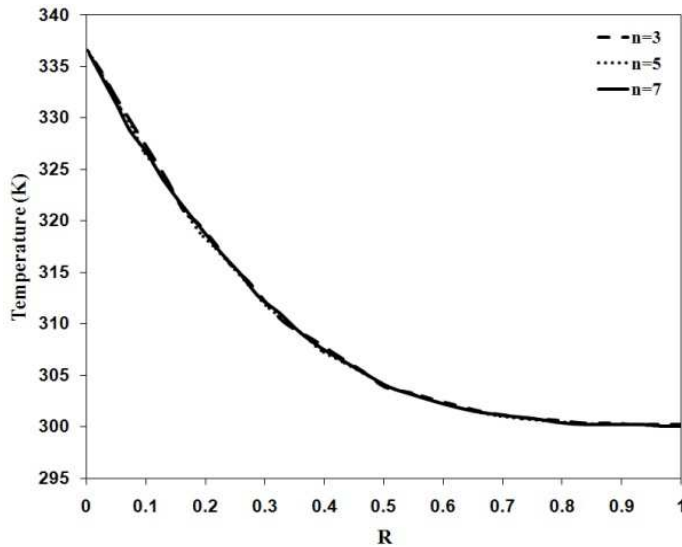


Fig. 2. Effect of element number on response of temperature for  $N=1$  and  $\bar{t} = 0.5$

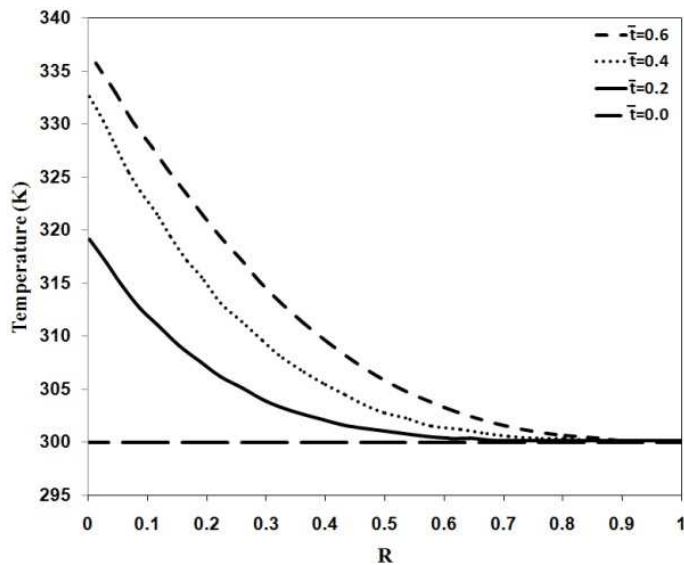


Fig. 3. Temperature distribution vs. dimensionless time for  $N=1$

Effect of index of power law for  $\bar{t} = 0.5$  is conducted in Figure (4). As the volume fraction index increases, the volume fraction of the ceramic material increases in the vicinity of the hot boundary surface (the inner surface). Therefore, higher temperatures, and subsequently higher temperature gradients are achieved in the neighborhood of the inner surface of the FGM cylinder. Furthermore, in this case, the temperature has converged asymptotically to the ambient temperature, in a higher rate.

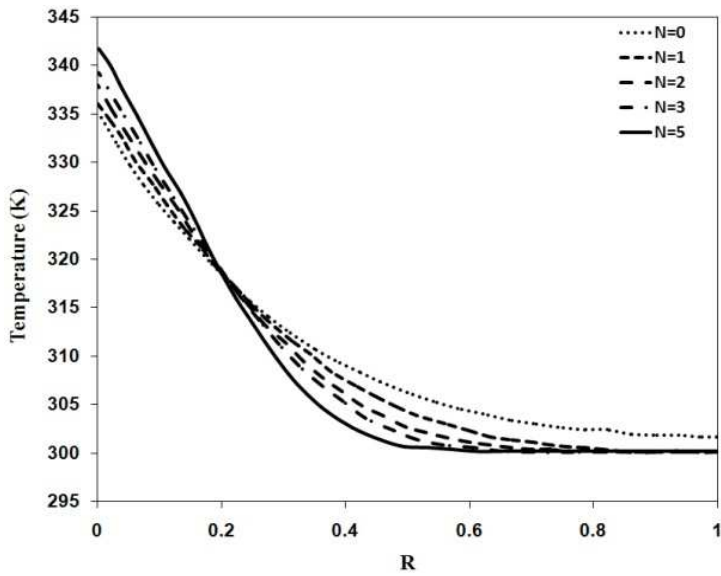


Fig. 4. N effect on temperature distribution for  $\bar{t} = 0.5$

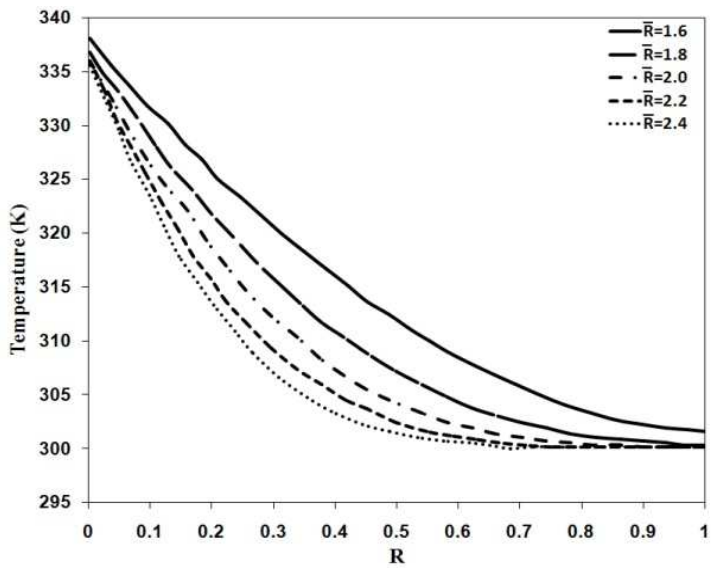


Fig. 5. Geometry effect on temperature distribution for N=1 and  $\bar{t} = 0.5$

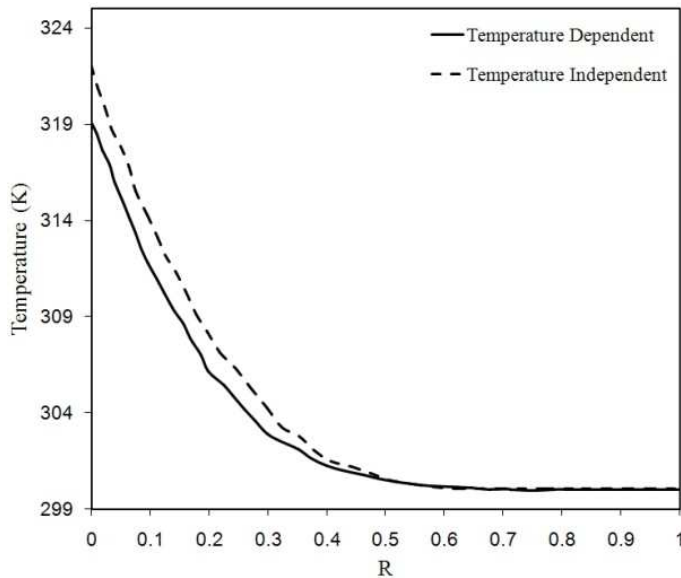


Fig. 6. Temperature distribution for  $N=1$  and  $\bar{t} = 0.5$

Then, Figure (5) shows the effect of change in geometry ( $\bar{R} = r_o/r_i$ ) such as changing in internal and external radius, for  $\bar{t} = 0.5$  and  $N=1$ . Temperatures of the thinner cylinders are generally higher. Therefore, temperature distribution with lower temperature gradient is constructed. In other words, for thinner cylinders, the response is more convergent to the steady-state one.

Influence of temperature-dependency on temperature distribution of FGM cylinder is drawn in Figure (6) for  $N=1$ ,  $\bar{t} = 0.2$  and  $\bar{R} = 2$ . This result shows that higher temperatures and temperature gradients are resulted when the temperature-dependency of the material properties is ignored. A difference up to 15 percent in temperature is observed. Since according to Figure (3), temperature values increase with the time, this difference is more remarkable for greater values of the dimensionless time ( $\bar{t}$ ).

## 6.2 Results for thermo-elastic stresses

In Figures (7) and (8), radial and hoop stresses versus  $R$  and time for  $N=1$ . This result shows that radial and hoop stresses are increasing by time. In the inner layers of cylinder, stresses are higher because of higher temperature gradient. At both ends of cylinder (inner and outer layers) radial stress is to some extent zero (according to the numerical errors in FEM) due to free surface and having no pressure.

For various index of power law ( $N$ ), radial and hoop stresses are drawn in Figures (9) and (10) for  $\bar{t} = 0.4$ . By increasing of  $N$ , as FGM material is become softer, both radial and hoop stresses reduced.

Figures (11) and (12) are shown for differences between dependency and independency of properties in temperature and then the changes in radial and hoop stresses for  $\bar{t} = 0.4$  and  $N=1$ . This result shows that higher stresses are resulted when the temperature-dependency of material properties is ignored. A difference up to 15 percent in stresses is observed. Since

temperature values increase with the time (Figure 3), stresses increase too (Figure 7) and therefore this difference will be more remarkable for greater values of the dimensionless time ( $\bar{t}$ ), consequently.

Geometry effect is shown in Figures (13) and (14) for both radial and hoop stress. By increasing of thickness, stresses decrease due to decreasing of temperature gradient.

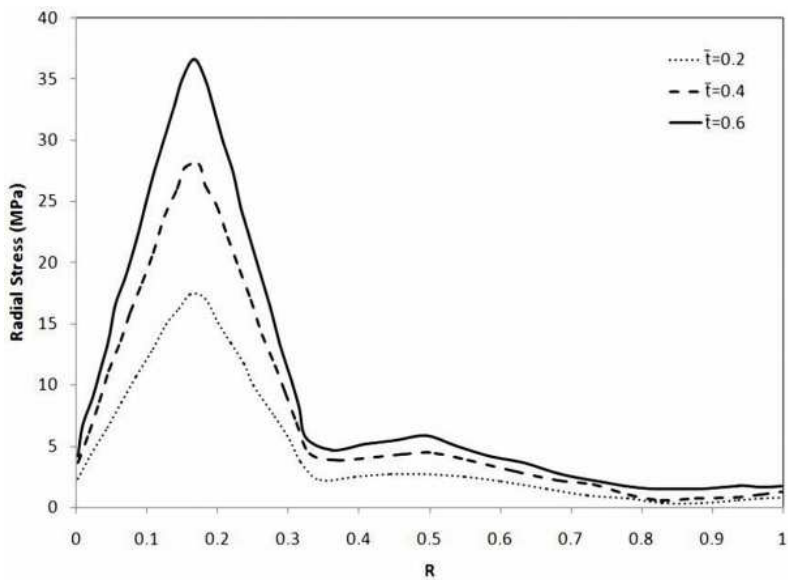


Fig. 7. Radial stress for  $N=1$  versus  $R$  and dimensionless time

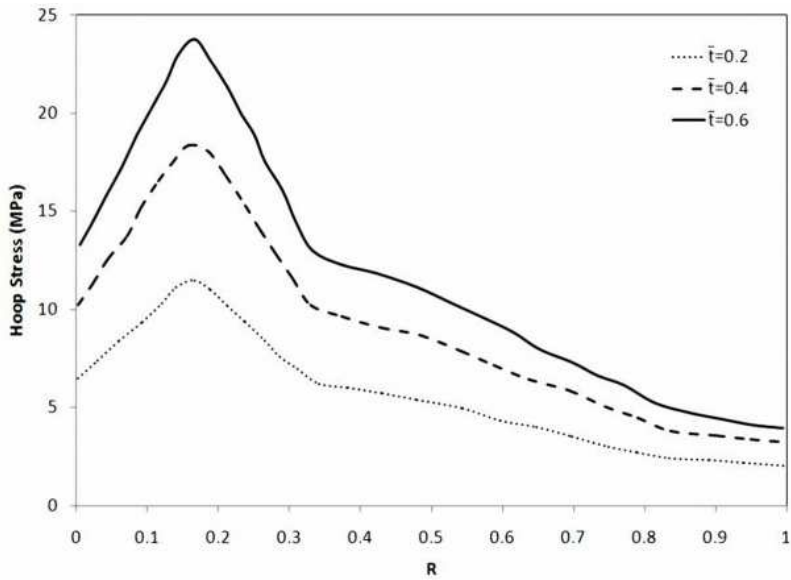


Fig. 8. Hoop stress for  $N=1$  versus  $R$  and dimensionless time

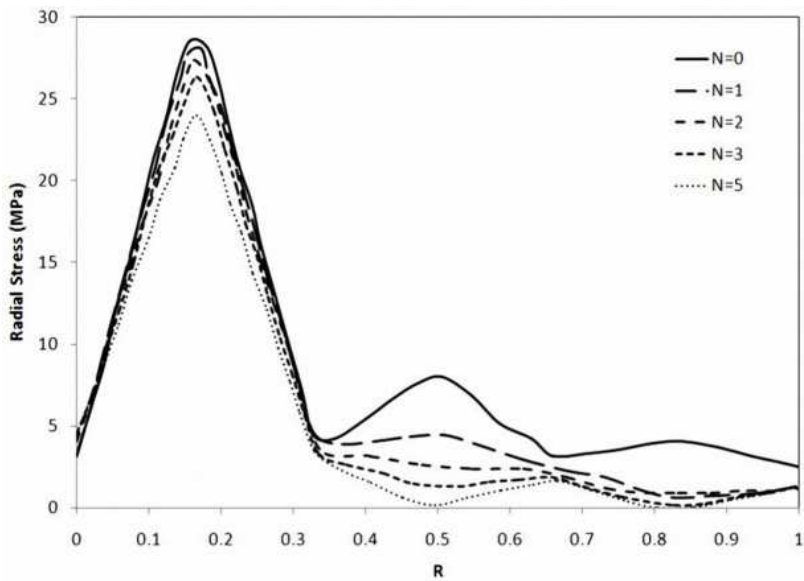


Fig. 9. Radial stress for  $\bar{t} = 0.4$  versus  $R$  and  $N$

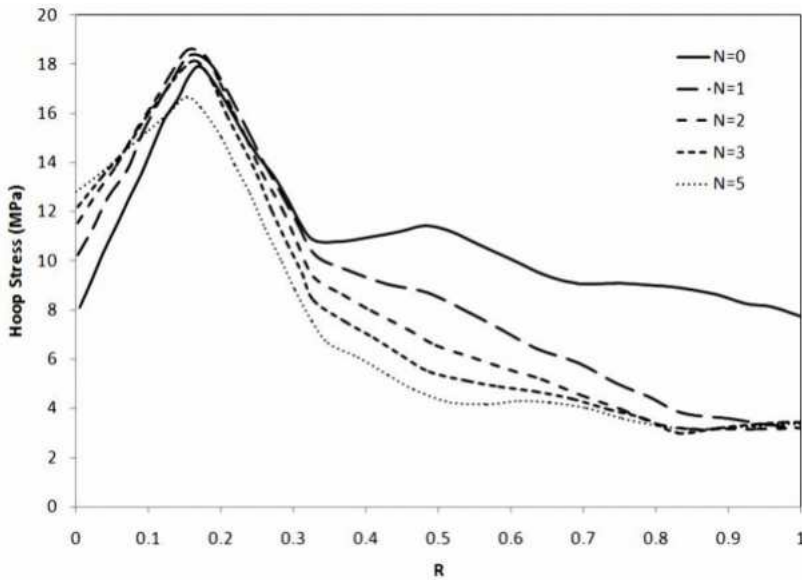


Fig. 10. Hoop Stress for  $\bar{t} = 0.4$  versus R and N

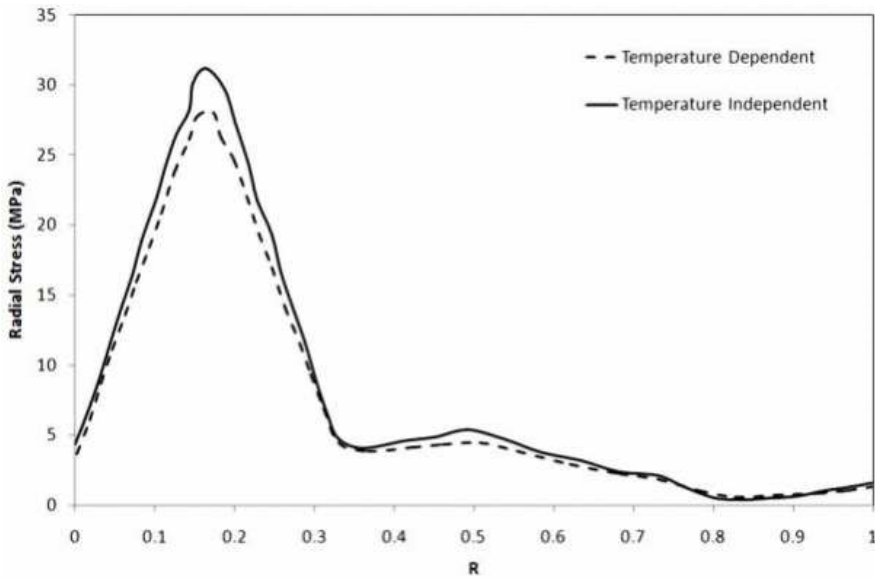


Fig. 11. Radial stress versus R in  $\bar{t} = 0.4$  and N=1

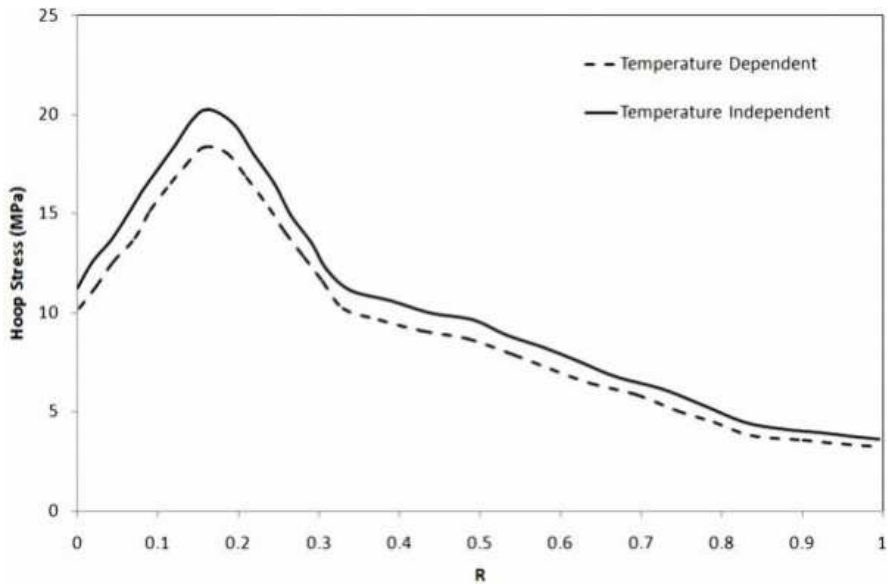


Fig. 12. Hoop stress versus R in  $\bar{t} = 0.4$  and  $N=1$

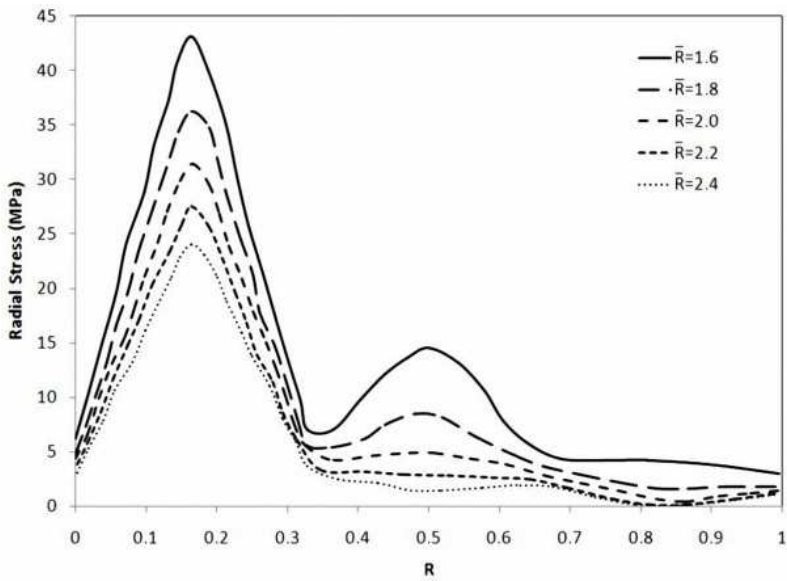


Fig. 13. Geometry effect on radial stress for  $N=1$  and  $\bar{t} = 0.4$

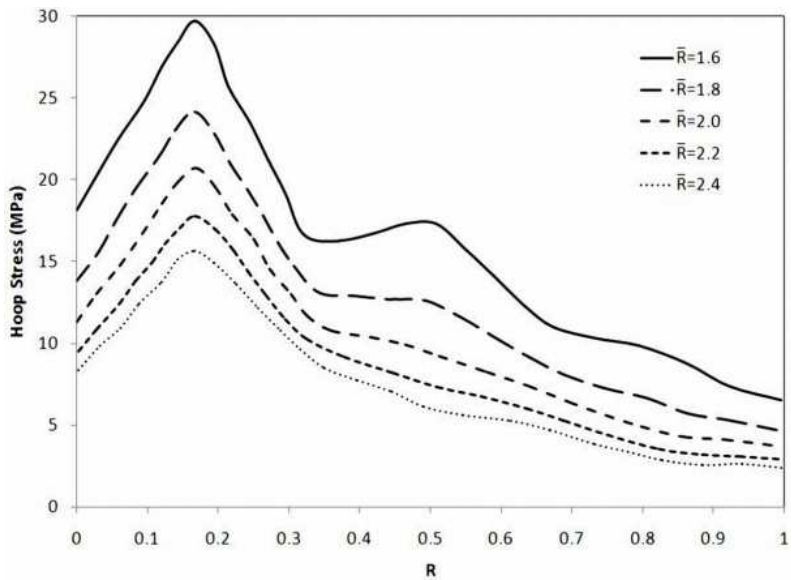


Fig. 14. Geometry effect on hoop stress for  $N=1$  and  $\bar{t} = 0.4$

## 7. Conclusion

In this paper, nonlinear transient heat transfer and thermo-elastic analysis of a thick-walled FGM cylinder is analyzed by using a transfinite element method that can be used in an updating and iterative solution scheme. Results also show that the temperature-dependency of the material properties may have significant influence (up to 15 percent) on the temperature distribution and gradient and also radial and hoop stresses that have remarkable effect on some critical behaviors such as thermal buckling or dynamic response, crack and wave propagation. Some other parameters such as index of power law ( $N$ ) and geometrical parameter have also important effects on those mentioned results.

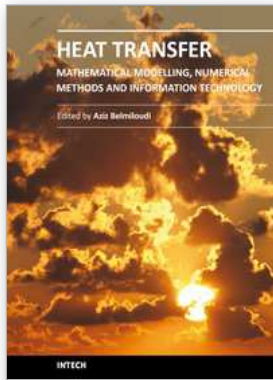
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## **Heat Transfer - Mathematical Modelling, Numerical Methods and Information Technology**

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Over the past few decades there has been a prolific increase in research and development in area of heat transfer, heat exchangers and their associated technologies. This book is a collection of current research in the above mentioned areas and describes modelling, numerical methods, simulation and information technology with modern ideas and methods to analyse and enhance heat transfer for single and multiphase systems. The topics considered include various basic concepts of heat transfer, the fundamental modes of heat transfer (namely conduction, convection and radiation), thermophysical properties, computational methodologies, control, stabilization and optimization problems, condensation, boiling and freezing, with many real-world problems and important modern applications. The book is divided in four sections : "Inverse, Stabilization and Optimization Problems", "Numerical Methods and Calculations", "Heat Transfer in Mini/Micro Systems", "Energy Transfer and Solid Materials", and each section discusses various issues, methods and applications in accordance with the subjects. The combination of fundamental approach with many important practical applications of current interest will make this book of interest to researchers, scientists, engineers and graduate students in many disciplines, who make use of mathematical modelling, inverse problems, implementation of recently developed numerical methods in this multidisciplinary field as well as to experimental and theoretical researchers in the field of heat and mass transfer.

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