

Process Capability and Six Sigma Methodology Including Fuzzy and Lean Approaches

Özlem Şenvar and Hakan Tozan
*Marmara University, Turkish Naval Academy
Turkey*

1. Introduction

Process capability analysis (PCA) and Six Sigma methodology occupy important places in quality and process improvement initiatives. As a fundamental technique in any production, quality and process improvement efforts, PCA is used to improve processes, products or services to achieve higher levels of customer satisfaction. In order to measure process capability numerically, process capability indices (PCIs) have been developed. Six Sigma is widely recognized as a systematic methodology that employs statistical and non-statistical tools and techniques for continuous quality and process improvement and for managing operational excellence because it challenges to maximize an organization's return on investment (ROI) through the elimination of nonconforming units or mistakes in the processes (Antony et al., 2005). The application of Six Sigma methodology provides reduction in variance and augmentation in the process capability, which is defined as the proportion of actual process spread to the allowable process spread that is measured by six process standard deviation units. Similar to Six Sigma methodology, in a process capability study, the number of standard deviations between the process mean and the nearest specification limits is given in sigma units. The sigma quality level of a process can be used to express its capability that means how well it performs with respect to specifications.

After Zadeh (1965) introduced the Fuzzy Logic (FL) to the scientific world, this new phenomenon rapidly became an essential systematic used in nearly every field of science. Due to its capability of data processing using partial set membership functions, an enormous literature about FL is developed with full of its applications. In addition, the ability of donating intermediate values between the expressions mathematically turns FL into a strong device for impersonating the ambiguous and uncertain linguistic knowledge (Ross, 2004). But although studies about FL are extremely wide, its application to quality control and especially to PCA is relatively narrow.

The aim of this chapter is to carry out a literature review of PCA, fuzzy PCA, PCIs, to make comparisons between PCIs, to introduce ppm and Taguchi Loss Function, to discuss the effects of estimation on PCIs as well as to provide general discussion about sample size determination for estimating PCIs. Another objective of this chapter is to provide the investigation of the relationship between Process Capability and Six Sigma along with the examination of Six Sigma methodology, and a relatively new approach called Lean Six Sigma methodology, and to identify the key factors that influence the success of Six Sigma project implementation for improving overall management process.

2. Process capability

2.1 Process

Process is defined as a combination of materials, methods, equipments and people engaged in producing a measurable output. As a matter of fact, all processes have inherent statistical variability, which can be identified, evaluated and reduced by statistical methods.

The source and amount of variability should always be considered by organizations. In order to satisfy customer requirements, organizations must improve the quality by reducing variance in production processes. The less variation the system has, the better quality it provides. Thereby, the variability of critical-to-quality characteristics (CTQs) is a measure of the uniformity of outputs. When the variation is large, the numbers of products that are nonconforming are large. Nonconforming (NC) is the failure of meeting specification limits whereas specifications are the desired measurements for a quality characteristic.

2.2 Process capability

In particular, process capability deals with the uniformity of the process. Variability of CTQs in the process is a measure of the uniformity of outputs. Here, variability can be thought in two ways: one is inherent variability in a CTQ at a specified time, and the other is variability in a CTQ over time. It should be considered that process capability study frequently measures functional parameters or CTQs on the product. It does not measure the process itself (Montgomery, 2009). Process capability compares inherent variability in a process with the specifications that are determined according to the customer requirements. In other words, process capability is the proportion of actual process spread to the allowable process spread, which is measured by six process standard deviation units. Process capability compares the output of a process that is an in-control state to the specification limits by using PCIs. To sum up, a capable process is the one where almost all the measurements fall inside the specification limits and process capability study can be conducted to indicate the extent to which the process can meet these specifications.

In a true process capability study, when there is direct observation of the process, inferences can be made about the stability of the process over time by directly controlling or monitoring data collection activity and understanding the time sequence of the data. However, when there is no direct observation of the process, only sample units of product are known, in this case, the study is called product characterization. In a product characterization study, distribution of the product quality characteristic or the fraction that conforms to specifications, which is referred to as process yield, can only be estimated, notably information about stability or dynamic behavior of the process cannot be given (Montgomery, 2009).

2.3 Process Capability Analysis (PCA)

PCA involves statistical techniques, which are useful throughout the product cycle. Generally, PCA is used in development activities prior to manufacturing process, in quantification of process variability, in analysis of this variability relative to specifications and in elimination or reduction of the process variability (Montgomery, 2009).

As a fundamental technique in any production, quality and process improvement efforts, PCA is used to improve processes, products or services to achieve higher levels of customer satisfaction. PCA has become widely adopted as the measure of performance to evaluate the ability of a process to satisfy customer requirements in terms of specification limits (English

& Taylor, 1993). The output of a process is expected to meet specifications, which can be determined according to the customer requirements. PCA is a prominent technique that is used to determine how well a process meets to these specification limits. PCA is based on a sample of data taken from a process and often produces: an estimate of the dpmo (defects per million opportunities), one or more capability indices, an estimate of the sigma quality level at which the process operates. The sigma quality level of a process can be used to express its capability that means how well it performs with respect to specifications.

As a measure of process capability, it is customary to take six sigma spread in the distribution of product quality characteristic. For a process whose quality characteristic has a normal distribution with process mean μ and process standard deviation σ ; the lower natural tolerance limit of the process is $LNTL = \mu - 3\sigma$, and the upper natural tolerance limit of the process is $UNTL = \mu + 3\sigma$. It should be considered that natural tolerance limits include 99.73% of the variable and 0.27% of the process output falls outside the natural tolerance limits.

PCA is often used to estimate the process capability. The estimate of process capability can be in the form of a distribution that has parameters of shape, center (mean) and spread (standard deviation). In this case, PCA can be performed without regard to specifications of the quality characteristic. Here, process capability can be expressed as a percentage outside of specifications (Montgomery, 2009). For PCA, the following techniques can be used:

- **Histograms:** In statistics, histograms are defined as graphical displays of frequencies. In the quality applications, histograms are well-known as one of the seven basic tools of quality control. Histograms are very useful in estimating process capability and for visualizing process performance. Hence, histograms can be used to determine the reason for poor process performance, instantaneously. As quality characteristics are often assumed to have normal distribution, histogram along with the sample mean and sample standard deviation can provide information about process capability as it is possible to estimate the process capability independent of the specifications. Here, normality assumption can be investigated by looking at the shape of the histogram. If the histogram is fairly skewed, then the normality assumption might be a concern and estimate of the process capability is unlikely to be correct. On the other hand, there are some drawbacks of using histograms. Fundamentally, it is necessary to divide the range of a variable into classes. Also, histograms cannot be used for small samples, for this reason, at least hundred observations are needed. Essentially, in order to have reliable estimate of process capability, these observations must be moderately stable (Montgomery, 2009).
- **Probability Plots:** Probability plots are very useful in estimating the process capability. Also, probability plots can be used to determine distribution's parameters, which are shape, center and spread. Furthermore, it is unnecessary to divide the range of a variable into class intervals. Probability plots can be used for moderately small samples, as well. However, if the data of quality characteristic do not come from the assumed distribution, inferences about process capability may be seriously in error. That can be shown as drawback of probability plots as they are not objective procedures. Practically, normal probability plots are very useful in process capability studies (Montgomery, 2009). Here, the fat pencil test is preferred to be used for testing the adequacy of the normality assumption. The fat pencil test is performed like that: when the data are plotted against a theoretical normal distribution, the points should form an approximate straight line. Departures from this straight line indicate departures from normality.

- **Design of Experiments (DOE):** DOE is very useful for identifying critical parameters associated with a process and determining optimal settings for these process parameters for enhanced capability and performance of the process. In other words, DOE is a systematic approach that is carried out to vary the input controllable variables in the process and analyze the effects of these process variables on the output, which is referred to as response in the DOE terminology. DOE is used to discover which set of process variables is influential on the output, and at what levels these variables should be held to optimize process performance. One of the major uses of DOE is discriminating and estimating the sources of variability in a process (Montgomery, 2009). Literally, DOE has been widely accepted in manufacturing processes and is useful in more general problems rather than merely estimating the process capability.
- **Control Charts:** Control charts are very useful for establishing a baseline of the process capability or process performance. Control charts can be used as monitoring devices to show effects of changes in the process on process performance. Basically, control charts can determine whether a manufacturing or business process is in a state of statistical control or not. They show systematic patterns in process output, as well. In particular, before using PCIs, there is a need for establishment of a state of statistical control. That is, if a control chart indicates that the process is currently under control, then it can be used with confidence to predict the future performance of the process. In the contrary, if a control chart indicates that the process being monitored is not in control, the pattern it reveals can help to determine the source of variation to be eliminated in order to bring the process back into control. Concisely, the control chart allows significant change to be differentiated from the natural variability of the process. This is shown to be the key for effective process control and improvement. Control charts are effective in displaying potential capability of the process by performing the issue of statistical control, for this reason, they should be regarded as the primary technique of PCA. In PCA, both variables and attributes control charts can be used (Montgomery, 2009).

2.4 Process Capability Indices (PCIs)

In the literature, process capability indices (PCIs) are also called process capability ratios (PCRs). PCIs are used as tools for characterizing the process quality. In order to measure the process capability numerically, PCIs have been developed. PCIs use process specifications as well as process variability, in this regard, the use of PCIs is important as they are statistical indicators of the process capability. PCIs are also defined as the quantitative indicators that compare the behavior of process or product characteristic to the specifications. In other words, PCIs are used to determine how well the process performs with respect to specifications and they express the ability of the process to meet these specifications, as a unique value quantitatively.

There are several statistics that can be used to measure the capability of a process. Frequently used measures of performance are the PCIs, which relate the natural tolerance limits of a process to the specification limits (English & Taylor, 1993). In practice, C_p , C_{pk} (C_{pl} , C_{pu}), C_{pm} are some of the widely used PCIs. In next sections, process capability indices: C_p , C_{pk} (C_{pl} , C_{pu}), C_{pm} , C_{pmk} will be explained.

2.4.1 Process Capability index C_p

In the literature, C_p index is also called process potential index, or process capability ratio, or inherent capability index, and two-sided PCI for two-sided specifications, that is, process

is having both lower and upper specification limits. C_p is frequently used in industrial environment in order to express process capability in a simple quantitative way. When the parameters are known, that is, in that case, when process standard deviation σ is known, PCI C_p is computed as follows:

$$C_p = \frac{USL - LSL}{6\sigma} \quad (1)$$

where LSL and USL are lower and upper specification limits, respectively. The percentage of the specification band used up by the process can be calculated in the following way:

$$P = \left(\frac{1}{C_p} \right) * 100 \quad (2)$$

In practice, it is often impossible to know parameters. Generally, it is suitable to use sample standard deviation s to estimate process standard deviation σ . Thus, when the parameters are unknown, that is, in that case, when process standard deviation σ is unknown, by replacing sample standard deviation s to estimate process standard deviation σ , the formula used for estimating C_p is given below:

$$\widehat{C}_p = \frac{USL - LSL}{6s} \quad (3)$$

where LSL and USL are lower and upper specification limits, respectively.

A C_p value less than 1 indicates that the process variation exceeds the specifications and a significant number of defects are made. A C_p value equal to 1 indicates that the process is exactly meeting the specifications. At least 3% defects would be made. However, if the process is not centered on the target value (off-center), more defects are expected to be made. A C_p value greater than 1 indicates that the process variation is less than the specifications. However, if the process is not centered on the target value (off-center), more defects are expected to be made. A C_p value greater than 1.67 indicates that the process is highly capable.

2.4.2 Process Capability index C_{pk}

In the literature, for one-sided specifications, C_{pk} is defined as one-sided PCI for specification limit nearest to the process mean. When the parameters are known, that is, in that case, when process mean μ and process standard deviation σ are known, PCI C_{pk} is computed as follows:

$$C_{pk} = \frac{1}{3\sigma} \min(USL - \mu, \mu - LSL) = \min(C_{pu}, C_{pl}) \quad (4)$$

where LSL and USL are lower and upper specification limits, respectively. In practice, it is often impossible to know parameters. Generally, it is suitable to use sample mean \bar{x} to estimate process mean μ and sample standard deviation s to estimate process standard deviation σ . When the parameters are unknown, that is, in that case, when process mean μ and process standard deviation σ are unknown, by replacing sample mean \bar{x} and sample standard deviation s to estimate process mean μ and process standard deviation σ , respectively, the formula used for estimating C_{pk} is given below:

$$\widehat{Cpk} = \frac{1}{3s} * \min(USL - \bar{x}, \bar{x} - LSL) = \min(Cpu, Cpl) \quad (5)$$

where LSL and USL are lower and upper specification limits, respectively.

Montgomery (2009) defined C_p as the measurement of the potential capability in the process. As a matter of fact, C_p does not consider where the process mean is located relative to the specification limits. C_p only measures the spread of the specifications relative to the six sigma spread in the process. C_p does not deal with the case of a process with mean μ that is not centered between the specification limits. On the other hand, he defined C_{pk} as the measurement of the actual capability in the process. C_{pk} takes process centering into account. In other words, C_{pk} deals with the case of a process with mean μ that is not centered between the specification limits. The magnitude of C_{pk} relative to C_p is the direct measure of how off-center the process is operating. Montgomery (2009) examined several cases, which can explain the relationship between C_p and C_{pk} , are given below:

- If $C_p = C_{pk}$, the process is centered at the midpoint of the specification limits.
- If $C_{pk} < C_p$, the process is off-centered. This can be accepted as lower capability than the case that the process is centered. The reason is that it is not operating at the midpoint of the interval between the specification limits.
- If $C_{pk} = 0$, the process mean is exactly equal to one of the specification limits.
- If $C_{pk} < 0$, the process mean lies outside the specification limits, that is for $\mu > USL$ or $\mu < LSL$, $C_{pk} < 0$.
- If $C_{pk} < -1$, the entire process lies outside the specification limits. It should be noted that some authors define C_{pk} to be nonnegative so that values less than zero are defined as zero.

$1 < C_{pk} < 1.33$ means that the process is barely capable. Automotive industry uses $C_{pk} = 1.33$ as a benchmark in accessing the capability of a process (AIAG, 2002).

2.4.3 Process Capability index C_{pm}

In the literature, C_{pm} is referred to as Taguchi index. Simply, C_{pm} is defined as the ability of the process to be clustered around the target or nominal value, which is the measurement that meets to exact desired value for the quality characteristic. Actually, C_{pm} was developed because C_{pk} is observed to be inadequate measure of process centering although C_{pk} was developed to deal with the case of a process with mean μ that is not centered between the specification limits whereas C_p is inadequate in process centering. As a matter of fact, when μ is in the interval of the specification limits, LSL and USL, C_{pk} depends inversely on process standard deviation σ and becomes large as process standard deviation σ gets closer to zero. Keeping these features in mind, it is possible to say that C_{pk} is not convenient as a measure of centering. This means a large value of C_{pk} does not actually give any information about the location of the mean in the interval of the specification limits, LSL and USL. In that case, process capability index C_{pm} , which is a better indicator of process centering, would be much more convenient (Montgomery, 2009). Consequently, the PCI C_{pm} is intended to account for variability from the process mean and deviation from the target value T and C_{pm} is shown to be useful in process centering. When the parameters are known, that is, in that case, parameters of process mean μ and process standard deviation σ are known, PCI C_{pm} is computed as follows:

$$C_{pm} = \frac{USL - LSL}{6\tau} \quad (6)$$

where τ is the square root of expected squared deviation from target T . The target value T , which is the measurement that meets to exact desired value for the quality characteristic, is known to be the midpoint of the specification interval. Target T is evaluated as follows:

$$T = \frac{1}{2}(LSL + USL) \tag{7}$$

The formula for process variation around desired process target is given below:

$$\tau^2 = E[(x - T)^2] = E[(x - \mu)^2] + (\mu - T)^2 = \sigma^2 + (\mu - T)^2 \tag{8}$$

Computation of C_{pm} can also be performed with the following way:

$$C_{pm} = \frac{USL - LSL}{6\sqrt{\sigma^2 + (\mu - T)^2}} = \frac{C_p}{\sqrt{1 + \left(\frac{\mu - T}{\sigma}\right)^2}} \tag{9}$$

C_{pm} approaches zero asymptotically as $|\mu - T| \rightarrow \infty$. When the parameters are unknown, that is, in that case, when process mean μ and process standard deviation σ are unknown, by replacing sample mean \bar{x} and sample standard deviation s to estimate process mean μ and process standard deviation σ , respectively, the formulas used for estimating PCI C_{pm} is given below:

$$\widehat{C_{pm}} = \frac{\widehat{C_p}}{\sqrt{1 + V^2}} \tag{10}$$

where; $V = \frac{\bar{x} - T}{s}$.

2.4.4 Process Capability index C_{pk}m

The motivation of C_{pkm} is increased sensitivity to departures of the process mean μ from the desired target value T . C_{pkm} is known as a third generation PCI, since it is derived from the second generation PCIs C_{pk} and C_{pm} , in the same way that the PCIs, C_{pk} and C_{pm} are derived from the first generation PCI C_p . Computation of C_{pkm} is as follows:

$$C_{pkm} = \frac{C_{pk}}{\sqrt{1 + \left(\frac{\mu - T}{\sigma}\right)^2}} \tag{11}$$

At the end of this section, it has to be emphasized that PCIs can measure expected future performance. Industrial use of PCIs concentrates on evaluating and interpreting the point estimates of the desired quantities of PCIs, which are utilized to measure the ability of a process to meet the specification limits. It must be noted that point estimates of PCIs are simply point estimates and they are subject to statistical fluctuation. In other words, since point estimates of PCIs are subject to variability, alternatively, researchers recommend practitioners to use confidence intervals for estimating PCIs. There is a recent focus on

hypothesis testing and confidence intervals on PCIs that are used as the basis for establishing the process capability (English & Taylor, 1993). For details about hypothesis testing and confidence intervals on PCIs, interested readers are referred to Montgomery, 2009.

2.5 Comparisons between PCIs

In the review paper of Kotz and Johnson (2002), C_p is ascribed to Juran, C_{pk} to Kane, and C_{pm} for the most part to Hsiang and Taguchi. Kotz and Johnson emphasized that it is necessary to distinguish the features of PCIs and the features of their estimators. Apart from this, the relationship between these PCIs are defined as; " $C_p \geq C_{pk}$ and $C_p \geq C_{pm}$ ". Also, researchers realize that C_{pk} and C_{pm} coincide with C_p when $\mu=T$ and decrease as μ moves away from target T , whereas $C_{pk} < 0$ for $\mu < LSL$ or $\mu > USL$.

Spiring et al. (2002) highlighted that both C_p and C_{pk} are related to expected proportion of nonconforming items or defects. In other words, C_p and C_{pk} are related to marginal expected value of ppm (parts per million). On the other hand, C_{pm} does not arise from examining the number of nonconforming product in the process. Therefore, C_{pm} is unreliable if the expected proportion of nonconforming is regarded as the most important feature. Unlike the other PCIs, C_{pm} is not distributionally sensitive.

In industrial practice, it should be noticed that the motivation of C_p , C_{pl} , C_{pu} , C_{pk} are the most extensively used PCIs, while C_{pm} is seldomly being used. According to Bothe (2002), C_{pk} seems to have the greatest degree of acceptability among the PCIs. It is important to emphasize that C_{pk} is not suitable for product features with asymmetric tolerances. Even all the assumptions are satisfied, a higher C_{pk} does not represent a higher level of quality for customers. On the other hand, C_{pm} is related to Taguchi quadratic loss function because C_{pm} is defined as the ability of the process to be clustered around the target. Furthermore, C_p , C_{pl} , C_{pu} , C_{pk} are interpreted as the measure of nonconforming. Any change in the magnitude of these indices, under the constraint of holding customer requirements constant, is due to changes in the distance between the specification limits and the process mean. C_{pk} does not in itself say anything about distance between μ and T and it only measures the process yield (Spiring et al., 2002).

2.6 Taguchi loss function

PCA examines the ability of a process to satisfy customers in terms of specification limits. However, sometimes, it can be more suitable to investigate the costs associated with process variation. For this purpose, Taguchi quadratic loss function can be used in order to examine the costs. In other words, Taguchi loss function is generally preferred to be used in modeling the expected costs. The basis of the Taguchi quadratic loss function is incurred when the quality characteristics of a product deviate from the target value. Taguchi loss function is shown below:

$$L = k \cdot (Y - T)^2 \quad (12)$$

where L symbolizes loss function; k is constant; Y is the observed value of the quality characteristic, and T is the target value of quality characteristic.

English and Taylor (1993) report that the target value for the quality parameter as a design variable can be adjusted easily. Process design engineers can alter the idea of utilization of the specification and utilize more optimal target values supported by known process behavior.

Taguchi used the quadratic loss function for motivating the idea that a product imparts no loss only if it is produced at its target. He maintained that even small deviations from the target result in a loss of quality. Taguchi's philosophy highlights the need to have low variability around the target. As a result of this, the most capable process produces its product at the target.

Taguchi identified that when "nominal is the best" is assumed, the expected value of loss minimized regardless of the distribution. In that case, the target value is adjusted to be equal to the expected value of the underlying process. It should be noticed that, by stabilizing the process and reducing the variation will reduce the cost of the Taguchi loss function. Taguchi loss function strategy emphasizes reducing variability and striving for a process mean that equates to the nominal specification.

In fact, PCIs are based on expected loss. Quality improvement efforts deal with reducing variances and discriminating them as much as possible. For this purpose, there is an increasing importance of clustering around the target rather than conforming to the specification limits. This makes Taguchi loss function to be an alternative to PCIs. Production costs or losses can provide opportunities to access, monitor and compare process capability (Spring et al., 2002). For more details on the topic, interested readers are referred to English & Taylor (1993).

2.7 The effects of estimation on PCIs

Theoretically, PCIs can be computed when the values of the process parameters (process mean μ and process standard deviation σ) are known. However, in practice, these process parameters required for computing PCIs are almost always unknown, for this reason, estimation is used in evaluating the process capability. In order to evaluate the process capability, these unknown process parameters have to be estimated from a sample of observations from the process but this is known to have effects on the estimated values of the PCIs.

Often, after PCIs are computed, these indices are converted into measures such as ppm (parts per million) defective items, also known as nonconforming items. The estimate of expected proportion of the nonconforming items, which is symbolized as p , can be obtained from the tails of the corresponding distribution exceeding the specification limits.

It should be noticed that estimated values are subject to variation and these values are generally different from the actual values. In other words, the estimate is subject to error in estimation as it depends on sample statistics (Montgomery, 2009). As a matter of fact, quality of the estimation is an important issue for the reliability of the calculated statistics such as C_p and ppm. Therefore, quality of the estimation should be considered for the reliability of the estimated PCIs and ppm statistics.

PCIs can only be used when the process is in the state of statistical control. Similarly, in order to have a reliable estimate of process capability, the process should be stable or be in statistical control. Stability or statistical control of the process is really necessary for correct interpretations of the PCIs. If the process is not in statistical control, then its parameters would be unstable. As a result, estimates of these parameters would be uncertain. Thus, predictive aspects of the PCIs regarding process ppm performance are not valid at all (Montgomery, 2009). Unless the process is stable (in control), no index is going to carry useful predictive information about process capability. Ramirez & Runger (2006) pointed out that the fundamental concepts of "in-control" and "capable" are the pillars of the process capability control. According to Kotz & Johnson (2002), the assumption of attaining

state of statistical control of the process is required in order to detect the irregular changes in quality level. Regardless of how robust an estimator may be, if its associated parameter is not stable, then any robustness claims carry little meaning (Spiring et al., 2002). PCA can determine an out-of-control process. In such cases, it is not safe to estimate process capability. When the process is out-of-control at the beginning of the PCA, it is possible to bring the process into an in-control state by eliminating assignable causes. Keeping all these features in mind, it is to be emphasized that for predictive and stable processes, the PCIs can measure the expected future performance.

Apart from the stability assumption, the quality characteristic is assumed to have a normal distribution with a constant mean and variance. Checking the normality assumption of the data is essential for meaningful interpretations of the PCIs. Lack of normality assumption may provide misleading interpretations of the results. When the process output is normally distributed, there is a certain relationship between a given values of C_p and the expected proportion of nonconforming items produced by the process monitored. As a matter of fact, normality assumption is very important because interpretation of process capability and PCIs, especially C_p and C_{pk} , depend on normal distribution of process output. According to English & Taylor (1993), C_p , C_{pk} , C_{pm} statistics assume that the process measurements are independently and identically distributed normal, that is iid $N(\mu, \sigma^2)$. Thus, meaningful interpretation of the indices of C_p , C_{pk} , C_{pm} is based on the normal distribution assumption. If the underlying distribution is non-normal, then expected process fallout attributed to a specific value of C_p , C_{pk} , C_{pm} may be seriously in error.

In essence, when the PCIs are estimated appropriately, they provide important information about how the current process meets customer requirements or the specifications. In contrast to this, incorrect application or interpretation of the PCIs causes unreliable results, which can lead incorrect decision making, waste of resources, money, time, and etc.

It has to be emphasized one more time that PCIs can measure expected future performance. Industrial use of PCIs concentrates on evaluating and interpreting the point estimates of the desired quantities of PCIs, which are utilized to measure the ability of a process to meet the specification limits of the customer requirements. It must be noted that point estimates of PCIs are simply point estimates and they are subject to statistical fluctuation. In other words, since point estimates of PCIs are subject to variability, alternatively, researchers recommend practitioners to use confidence intervals for estimating PCIs. There is a recent focus on hypothesis testing and confidence intervals on PCIs that are used as the basis for establishing the process capability (English & Taylor, 1993).

2.8 General Discussion about Sample Size Determination for Estimating PCIs

The estimate of PCI is always subject to error since it depends on sample statistics. English & Taylor (1993) mentioned that estimating PCI from sample data can cause large errors. As a result of this, the estimate may not be reliable, at all. For convenience, utilization of confidence intervals for estimating PCI should be considered. As point estimators for PCIs are subject to errors, point estimates of PCIs may not be satisfactory, if they are computed from small samples. In other words, point estimates are useless if they are computed from small samples. Similarly, confidence intervals on PCIs with small samples will always be wide, which are not preferable.

Kotz & Johnson (2002) mentioned that there are recent investigations about asymptotic properties of estimators that indicate the importance of determining sample sizes n for which asymptotic results are adequate.

As a matter of fact, point estimates are subject to variability and would change over time, even process remains stable. This can be an advantage for computation of the confidence limits for process control. It should be emphasized that checking normality assumption of the data is necessary for interpretations of PCIs and for the validity of confidence limits.

Considerably, t-distribution with as many as 30 degrees of freedom is symmetric and visually indistinguishable from the normal distribution. Nevertheless, symmetry in the distribution of the process output alone is not sufficient to ensure PCI would provide a reliable estimate of process ppm. Thereby, the longer and heavier tails of t-distribution is making meaningful difference, when ppm is being estimated (Montgomery, 2009).

Notice that, when the process output is normally distributed, there is a certain relationship between a given values of C_p and the expected proportion of nonconforming items produced by the process monitored. Quality of the estimated C_p and ppm values depends on the sample size used in the estimation. As a matter of fact, in practice, the quality of the estimates of PCI, such as C_p , can be changed according to the sample size (Deleryd, 1999). Therefore, large samples are required to be used to obtain reliable estimates.

3. Fuzzy Process Capabilities Analysis (Fuzzy PCA)

Since pioneer work of Zadeh (1965), fuzzy logic (FL) has been successfully applied to many fields of science and engineering. Studies in quality and PCAs domain have also effected from researches that involves the application of FL; especially from ones which have been applied to statistical methods such as quality control (Wang et al., 1990; Faraz et al., 2006; Gulbay et al., 2006 and 2007). Thought after 2000, studies about PCAs and its integral part PCIs from the FL point of view began to grab attention and stepped up; beside of the enormous crisp literature of PCAs, they are relatively in the minority.

The elementary idea of using FL approach for PCAs and PCIs can simply be express as, to overcome infirmity of PCIs arisen from the sharp crisp nature that restricts the flexibility, applicability and sensitivity which; both, individually and together directly affect the performance of PCAs. In this section, after a shallow mention to this brilliant subject, a brief summary on the studies about fuzzy PCA and PCIs will be given.

3.1 Fuzzy logic

FL can simply be defined as “a form of mathematical logic in which truth can assume a continuum of values between 0 and 1” (<http://wordnetweb.princeton.edu/>, 2009). On the contrary to many cases that involves human judgement, crisp (discrete) sets divide the given universe of discourse in to basic two groups; members, which are certainly belonging the set and nonmembers, which certainly are not. This delimitation which arises from their mutually exclusive structure enforces the decision maker to set a clear-cut boundary between the decision variables and alternatives. The basic difference of FL is its capability of data processing using partial set membership functions. This characteristic; including the ability of donating intermediate values between the expressions mathematically, turn FL into a strong device for impersonating the ambiguous and uncertain linguistic knowledge. But the main advantage of fuzzy system theory is its ability “to approximate system behavior where analytic functions or numerical relations do not exist” (Ross, 2004, pg.7). Palit et al. (2005) give a basic definition of FL from mathematical perspective as a nonlinear mapping of an input feature vector into a scalar output. As fuzzy set theory became an important problem modeling and solution technique due to its ability of modeling problems

quantitatively and qualitatively those involve vagueness and imprecision (Kahraman, 2006, pg.2), it has been successfully applied many disciplines such as control systems, decision making, pattern recognition, system modeling and etc. in fields of scientific researches as well as industrial and military applications (Tozan et al., 2008, 2009).

As stated before, differently from the classical sets that can be defined by characteristic functions with crisp boundaries, fuzzy sets can be characterized by membership which provides expressing belongings with gradually smoothed boundaries (Tanaka, 1997). Let A be a set on the on universe X with the objects denoted by x in the classical set theory. Then the binary characteristic function of subset A of X is defined as follow;

$$\mu_A(x) : X \rightarrow \{0,1\} \quad (13)$$

such that

$$\mu_A(x) = \begin{cases} 1 & x \in X \\ 0 & x \notin X \end{cases} \quad (14)$$

But fuzzy sets the characteristic functions; differently from the crisp sets whose characteristic function is defined binary (i.e., 0 or 1), are defined in the interval of $[0,1]$ (Zadeh, 1965). From this point, fuzzy set \tilde{A} in the universe set X with the objects x and membership function $\mu_{\tilde{A}}$ is defined as follow;

$$\tilde{A} = \{ (x, \mu_{\tilde{A}}(x)) \quad \forall x \in X \} \quad (15)$$

where $\mu_{\tilde{A}}(x) : X \rightarrow [0,1]$.

If the fuzzy set is discrete then it can be represented as;

$$\tilde{A} = \sum_k^n \frac{\mu_{\tilde{A}}(x_k)}{x_k}, \quad \forall x_k \in X, \quad k = 1, 2, \dots, n \quad (16)$$

And if the fuzzy set is continuous then it can be denoted as;

$$\tilde{A} = \int_X \frac{\mu_{\tilde{A}}(x_k)}{x_k}, \quad \forall x_k \in X \quad (17)$$

The two vital factors for building an appropriate fuzzy set gets through the determination of appropriate universe and membership function that fits the system to be defined. The membership functions are the main fact for fuzzy classification. The highest membership grade value 1 represents full membership while the lowest membership value 0 have the meaning that the defined object have no membership to the defined set. Frequently used membership functions in practice are triangular, trapezoidal, Gaussian, sigmoidal and bell curve (the names are given according to the shapes of the functions). For example, the triangular membership function is specified by parameters $\{a,b,c\}$ as:

$$\text{triangular}(x; a, b, c) = \begin{cases} \frac{x-a}{b-a} & ; \quad a \leq x \leq b \\ \frac{c-x}{c-b} & ; \quad b \leq x \leq c \\ 0 & ; \quad x \geq c \text{ or } x \leq a \end{cases} \quad (18)$$

where $a < b < c$. The width of function changes according to the values of a and b .

Characteristic of fuzzy set plays an important role in fuzzy PCA and PCIs studies. For basic concepts of fuzzy sets and related basic definitions see Bellman et al. (1970), Tanaka (1997 pg.5-44), Klir et al. (1995) and Ross (2004, pg.34-44).

3.2 Fuzzy PCA and PCIs

As stated before, though FLs' broad range of application has also effected studies on PCA and PCIs, the fuzzy based perspective on these areas are relatively new. The first spectacular fuzzy PCI studies; to our knowledge, can be traced back to the fuzzy quality and probability study of Yongting (1996); in which, a fuzzy C_{pk} was defined to determine the fuzzy quality. Later Lee et al. (1999) declared a fuzzy based model to maximize PCI via determining upper and lower bounds of PCIs using membership functions. In 2001, Lee proposed an estimation approach for fuzzy C_{pk} using fuzzy observations comprised of fuzzy numbers.

One of the worth mentioning fuzzy based study about process capability evaluation is made by Chen et al. (2003). Chen and his research colleagues proposed a method to interlink PCI with a fuzzy inference system for "bigger-the-best" type evaluation. In the study, the input for the fuzzy inference is the p value is calculated as follow;

$$p - value = p\left\{\hat{C}_{pl}^t \leq V \mid C_{pl} = C_{min}\right\} \quad (19)$$

where \hat{C}_{pl}^t is a uniformly minimum variance unbiased estimator of process capability index (C_{pl}) for a normal distribution, C_{min} is the minimum process capability required for the "bigger the best" type. The proposed steps of Chen et al.'s study for fuzzy evaluation to specify process capability are:

- i. Assigning C_{min} and lower specification limit;
- ii. Deciding the manufacturing allowance and the test of α -risk;
- iii. Calculating the mean (\bar{x}) and the standard deviation s from the selected n sample data set;
- iv. Computing the required parameters for obtaining p value using equation (19) through the cumulative distribution function with a non-central t distribution.
- v. Defining the membership functions for input and output and inferring the score value by difuzzification. Here, the membership functions for the input and output variables are defined by linguistic variables. The triangular type membership function is used for the input whereas Gaussian type is used for the output.

After inferring the score with difuzzification, authors used a conscience score concept to represent the grade of process capability. Later in their study about multi process capability plot, they proposed a fuzzy inference system approach which is effective for the assessment of multi process capability (2008).

Parchami and his research colleagues (2005, 2006a, 2006b, 2007, 2008, 2010a, 2010b) have also made several important studies related to fuzzy PCIs and fuzzy quality control. In 2005 they introduced fuzzy PCIs determining the relations governing between PCIs when lower specification limits are fuzzy numbers. Moeti et al. (2006) using lower specification limits as L-R intervals also discussed a generalized version of PCIs introduced in Parchami et al.(2005)'s study. Later for a new PCI; \tilde{C}_p , they obtained a fuzzy confidence interval and analyzed process capability based on the introduced fuzzy index. (2006a, 2006b, 2008). In

2007, they proposed a Buckley (2004, 2005a, 2005b) approach based algorithm to determine fuzzy estimates (which contains both point and interval estimates) for PCIs providing more information for the practitioners. Parchami and research colleagues declared that when lower specification limits are fuzzy rather than crisp, traditional PCIs does not suit for process capability measurement and they introduced new indices for the cases in which engineering lower specification limits are fuzzy (2010a, 2010b).

From 2007 till 2010, Kahraman & Kaya have made remarkable studies; which indeed, dynamise the researches on quality control and PCA in fuzzy domain. In 2007, they proposed a methodology for air pollution control by using fuzzy and traditional PCIs (Kaya et al., 2007). Later, they used fuzzy PCIs for controlling pH value of dam's water (Kahraman et al., 2008). They also applied fuzzy PCA to learning processes for faculty courses (Kaya et al., 2008a). Using fuzzy PCIs, they have analyzed the risk assessment of air pollution in largest city of Turkey via measuring air pollutants from different stations deployed in different parts of the city (Kaya et al., 2008b). In 2009, they used fuzzy process accuracy index to evaluate risk assessments of draught effects (Kahraman et al., 2009) and; the same year, with their study on air pollution control, they used fuzzy PCIs in six-sigma approach to prevent air pollution (Kaya, 2009).

In 2010, by defining specification limits and standard deviations with fuzzy numbers, Kaya and Kahraman increased PCIs' flexibility and obtained robust PCIs for a piton manufacturing company (Kaya et al., 2010a). They concluded that, for the cases in which crisp numbers can not be appropriate for defining specification limits, fuzzy numbers can be applied to represent specification limits via which, results gathered from the measurement can be analyzed more flexibly. In the study fuzzy PCIs are obtained using triangular fuzzy numbers (TFN) for defining upper and lower specification limits in addition to fuzzy variances follow.

Let the fuzzy upper specification limit be $US\tilde{L} = (a_1, a_2, a_3)$ and the fuzzy lower specification limit be $LS\tilde{L} = (b_1, b_2, b_3)$. Then, α -cuts for $US\tilde{L}$ and $LS\tilde{L}$ are;

$$\begin{aligned} US\tilde{L}_\alpha &= [(a_2 - a_1)\alpha + a_1, (a_2 - a_3)\alpha + a_3] \\ LS\tilde{L}_\alpha &= [(b_2 - b_1)\alpha + b_1, (b_2 - b_3)\alpha + b_3] \end{aligned} \quad (20)$$

Assuming parameters analyzed by PCIs have coloration, fuzzy robust PCIs are derivated in the study with the following formulas.

$$\tilde{C}_{pc} = \left(\frac{[(a_2 - a_1) + (b_3 - b_2)]x\alpha + (a_1 - b_3)}{6x\sqrt{\Phi_1}} \right) x \left(\frac{[(a_2 - a_3) - (b_2 - b_1)]x\alpha + (a_3 - b_1)}{6x\sqrt{\Phi_2}} \right) \quad (21)$$

$$(\tilde{C}_{puc})_\alpha = \left(\frac{[(a_2 - a_1)x\alpha + a_1] - \mu}{3x\sqrt{\Phi_1}}, \frac{[(a_2 - a_3)x\alpha + a_3] - \mu}{3x\sqrt{\Phi_2}} \right) \quad (22)$$

$$(\tilde{C}_{plc})_\alpha = \left(\frac{\mu - [(b_2 - b_3)x\alpha + b_3]}{3x\sqrt{\Phi_1}}, \frac{\mu - [(b_2 - b_1)x\alpha + b_1]}{3x\sqrt{\Phi_2}} \right) \quad (23)$$

$$\tilde{C}_{pkc} = \min\{\tilde{C}_{puc}, \tilde{C}_{plc}\} \quad (23)$$

where; $\Phi_1 = \frac{n\tilde{\sigma}}{(1-\alpha)X_{L,0.005}^2 + (\alpha x n)}$; $\Phi_2 = \frac{n\tilde{\sigma}}{(1-\alpha)X_{R,0.005}^2 + (\alpha x n)}$; X_R^2 and X_L^2 are the points on the right and left sides of X^2 density (see Buckley, 2004) and \tilde{C}_{pc} , \tilde{C}_{puc} , \tilde{C}_{plc} and \tilde{C}_{pkc} are fuzzy robust PCIs.

In the study authors also define upper and lower specification limits using trapezoidal fuzzy number. And using a ranking method a comparison of fuzzy PCIs is also performed. After the implementation of the proposed system illustrating obtained results, Kaya and Kahraman concluded that compared to crisp ones, fuzzy analyses for robust PCIs have some advantages as they are more sensitive and include more information than crisp robust PCIs. The literature exposes that, fuzzy quality control and fuzzy PCA (including fuzzy PCIs) have considerable amount of advantages and remarkable capabilities than their crisp types. They provide more information, they are more sensitive and flexible; and also, more appropriate for implementation to real life cases as they successfully can illustrate human judgment. For these reasons, strongly claiming that "the studies on quality control and PCA in fuzzy domain will rapidly increase" will not be a wrong proposition.

4. Six Sigma methodology

4.1 Relationship between process capability and Six Sigma

The technical elaboration of Six Sigma can be achieved through the use of normal distribution and PCIs. Historically, the creators of Six Sigma employed Cp, as it was accepted as a standard quality measure. Six Sigma was developed for solving the complexity of products and observing the failure of the products in order to achieve the predictive performances (Ramberg, 2002). Similar to Six Sigma methodology, in a process capability study, the number of standard deviations between the process mean and the nearest specification limits is given in sigma units. The sigma quality level of a process can be used to express its capability that means how well it performs with respect to the specification limits. By the way, in terminology of statistics, sigma represents the variation about the process mean. The application of Six Sigma methodology provides reduction in variance and augmentation in the process capability.

As it is mentioned above, a Six Sigma process can be interpreted in terms of process capability, which is associated with process variation by using PCI, such as Cpk. Nowadays, most of the manufacturers are required to produce a product with a specified Cpk value. As the market competition is getting tougher and tougher, organizations are under pressure to sustain world class competition so that they need to meet or exceed this specified Cpk value or quality level. It should be noticed that Cpk values are related to sigma quality levels. Higher value of Cpk indicates a better process. For instance; a process capability, that is, Cpk of 1.00 is roughly equivalent to three sigma capability. That is, the mean plus and the mean minus three standard deviations should be the points at which the nearest specification limits lie. With three sigma capability or Cpk = 1.00, a process will produce approximately 99.73% good product or 0.27% bad product. This represents an unacceptably high level of poor products. On the other hand, nowadays high quality standards dictate reducing variation by four standard deviations between the process mean and the nearest specifications. This corresponds to the value of Cpk = 1.33. At this level, the process will produce approximately 99.9937% good product or 0.0063% bad product. This

represents a better figure than the figure of three sigma capability (or $C_{pk} = 1.00$), but it is still having high level of poor products.

Process capability measures have been used to provide number of nonconforming product. As it is mentioned in earlier sections, ppm is used in this regard. At ± 3 sigma level, the probability of producing a product within specification limits is 0.9973. This implies 2700 ppm. Therefore, at a six sigma capability level, a process will produce very few defects. This level represents a C_{pk} value of 2.0 which is more commonly referred to as six sigma capability.

4.2 Statistical interpretation of Six Sigma

In Six Sigma process, as its name implies, there are six standard deviations between the process mean and specification limits, when the process is centered. The objective of using Six Sigma approach is to reduce process variation, and thereby defects. The six sigma metric uses dpmo, which is the abbreviation for defects per million opportunities. Here, opportunities represent the number of potential chances within a unit for a defect to occur. It is essential to be consistent about the definition of the opportunities because by increasing the number of opportunities over time, a process may be artificially improved (Montgomery, 2009). Computation of dpmo is given below:

$$dpmo = \frac{\text{total number of defects}}{\text{number of units} \times \text{number of opportunities}} \quad (24)$$

Equivalently, dpmo can also be computed like that:

$$dpmo = \frac{dpu \times 10^6}{\text{opportunities for error}} \quad (25)$$

where dpu stands for defects per unit. Computation of dpu is given below:

$$dpu = \frac{\text{total number of defects}}{\text{total number of units}} \quad (26)$$

When dealing with defects or nonconformities, dpu statistic can also be used as a measure of capability. From sample data, quantities of dpu can be estimated, too. Larger samples provide more reliable estimates. Notice that, measure of dpu does not directly take the complexity of the unit into account whereas measure of dpmo does (Montgomery, 2009).

Six Sigma represents a quality level of at most 3.4 dpmo in the long term. Unavoidable assignable causes lead processes to shift 1.5 standard deviations from process mean toward either specification limit that would provide the maximum of 3.4 defects per million. That means Six Sigma measure of process capability allows process mean to shift by up to 1.5 sigma over the long term basis. In order to achieve this goal in the long term, the process capability has to reach the Six Sigma level in the short term, that is, the range between the process mean and the specification limits contains six process standard deviations on both sides of the process mean. In this way, the defect rate of a Six Sigma process is only about 0.002 dpmo. However, if the process mean shifts 1.5 process standard deviations over time, the defect rate will increase from 0.002 dpmo to 3.4 dpmo (Feng, 2008). For Six Sigma process, 3.4 dpmo value is the area under the normal curve beyond $6-1.5=4.5$ sigma. Same

logic is valid for three sigma process, that is, 66,807 dpmo value is the area under the normal curve beyond $3-1.5=1.5$ sigma (Antony et al., 2005).

As a matter of fact that, Six Sigma has been accepted to mean a 4.5 sigma process, not true Six Sigma process, just because of Six Sigma professionals have allowed for the process to drift by up to 1.5 standard deviations from the process mean. Actually, a process that operates true Six Sigma performance takes up 50% of the specification if the process is centered. This gives $C_p = C_{pk} = 2.00$. A process such as this will produce defects at a rate of only approximately 2 parts per billion. From this standpoint, a process with a $C_p = 2.00$ can have 1.5 sigma drift that is equivalent to 4.5 sigma process. That is, the mean will be 4.5 sigma from the specification limit at the edges of the drift. A 4.5 sigma process yields a 3.4 ppm defect level.

For a process that has a lower quality level than Six Sigma, the success rate will decrease significantly when the process shifts. In this point of view, if an organization is operating at Six Sigma level, it is defined as having less than 3.4 dpmo. This corresponds to a success rate of 99.9997%. On the other hand, if an organization is operating at three sigma level, it is defined as having 66,807 dpmo. This corresponds to a success rate of 93%. (McClusky, 2000). In other words, the fraction outside of the specifications for the three sigma process increases dramatically compared to the fraction for a Six Sigma process and may cause serious quality problems over time. Therefore, three sigma level cannot be regarded as having good quality performance as it is not good enough for many products or processes that attempt to avoid quality problems in the long run.

Literally, Six Sigma is achieved when the process width is half of the specification band. Six Sigma requires process mean is being in control. Inevitably, process mean would not be closer than six standard deviations from the nearest specification limit. That is, Six Sigma needs process specifications are at six standard deviations beyond the process mean. For a Six Sigma process, process potential index C_p and process actual index C_{pk} would be necessarily 2.00, when process is centered. For a Six Sigma process, actual process performance, that is, C_{pk} would be 1.5, when there is 1.5 Sigma shift in the process mean.

In general conclusion, Six Sigma is a business approach that drives defects produced by all processes down into parts per million levels of performance as it is accepted as a measure of process performance and the process operating at Six Sigma quality has a defect rate of 3.4 parts per million opportunities (Harry, 1998). In other words, 3.4 dpmo is challenged to be obtained in Six Sigma process. For this reason, Six Sigma is represented by 3.4 defective parts per million. This means it is about improving the process capability for all CTQs from all processes in the organization. The goal in a Six Sigma organization is to achieve defect levels of less than 3.4 ppm for every process in the organization and for every CTQ characteristic produced by those processes.

4.3 Six Sigma methodology

In order to sustain world-class competition, organizations should attain Six Sigma activities by integrating their knowledge of the process with statistics, engineering and project management (Anbari, 2002). Six Sigma is a quality management philosophy as well as a methodology that focuses on reducing variation, eliminating defects and improving the quality of processes, products and services. In other words, Six Sigma Methodology is defined as a data-driven, statistics-based approach and a project-driven management that improves processes, products and services of organization by continuously reducing both

nonconforming items or mistakes and variation as well as costs in the organization. In the literature, Six Sigma has also proven to be a customer-focused and a robust methodology.

In practice, organizations should give importance to improve overall performance instead of detecting and counting defects. The application of Six Sigma methodology provides reduction in variance and augmentation in the process capability, and process performance, simultaneously. Significant improvement in process capability and process performance can be achieved after a successful implementation of Six Sigma methodology that is accepted as a rigorous concept of quality control with this feature.

One of the advantages of the Six Sigma methodology over the other process improvement initiatives is that the use of data analysis tools in Six Sigma projects, which enables to identify process hindering problems and demonstrate the improvements using objective data, accurately. In the literature, several researchers or authors classified the tools and perspectives of Six Sigma methodology in several different ways. For instance; Kwak and Anbari (2006) categorized Six Sigma methodology into two major perspectives, which are statistical and business perspectives. The statistical perspectives of Six Sigma must complement business perspectives and challenge to the organization for a successful implementation of Six Sigma projects. Originally, statisticians created the Six Sigma concept. From the statistical point of view, Six Sigma is defined as having less than 3.4 dpmo. Equivalently, this corresponds to a success rate of 99.9997%. By using statistical tools and techniques, organizations improve sigma quality level as well as process capability, and process performance simultaneously. Feng (2008) highlighted that the requirement of 3.4 dpmo or Cpk of 1.5 is not the ultimate goal of Six Sigma. According to Feng, the attitude is to establish the right business strategy toward organizational excellence. From this standpoint, for the business perspectives of Six Sigma, it is accepted as a business strategy in the business environment that concentrates on improving the effectiveness, efficiency of all operations to meet or exceed customer requirements as well as productivity, business profitability and financial performance (Kwak & Anbari, 2006; Antony & Banuelas, 2001). Beneficial contributions can be expressed in terms of financial returns for organization as Six Sigma increases return on investment (ROI) by process improvement through cost savings as it reduces defects and improves efficiency. Consequently, it results in enhanced customer satisfaction as it fulfills quality requirements. As a result of this, increase in market share can be achieved in the competitive global market.

According to Antony et al. (2005), Six Sigma is a systematic methodology that employs statistical and non-statistical tools and techniques for continuous quality and process improvement and for managing operational excellence. While implementing project-by-project, Six Sigma provides an overall process improvement that clearly shows how to link and sequence individual tools (Feng, 2008). Six Sigma is a strategy for achieving significant financial savings to the bottom-line of the organization. As a matter of fact, organization's ROI can be maximized through the elimination of defects in the processes. Thence, Six Sigma approach is starting with a business strategy and ending with top-down implementation and is having a significant impact on profit by continuously reducing defects throughout the processes of organization and thereby improving customer satisfaction. It must be taken into account that Six Sigma quality level of performance or Six Sigma process capability should not be the primary objective for all the processes. A lower sigma quality level of performance can be acceptable for some processes except the vital ones that are related with zero tolerance for mistakes such as healthcare, safety, reliability, and so on.

According to Allen (2006), tools that are used in Six Sigma methodology can be categorized as tools of statistical methods and quality management, which are very useful in identifying and eliminating causes of defects in business processes by examining the inputs, the outputs, and the relationship between the inputs and outputs.

Tools of statistical methods	Tools of quality management
Statistical Hypothesis Testing	Process Mapping
Regression Analysis	Cause-and-effect diagrams
SPC	Pareto charts
DOE	QFD
ANOVA	FMEA

Table 1. Tools of statistical methods and quality management

In addition to all these tools and techniques, researchers and practitioners observed that Six Sigma has its inherent limitations and cannot be used as a universal solution for any process in any organization. In order to enhance the effectiveness of Six Sigma, additional tools and techniques should be integrated. There is a recent technical development in the fields of management science as well as statistics and engineering which provide more effective tools for enhancing the efficiency and the productivity of organizations such as queuing systems, heuristics, and data envelopment analysis (DEA) (Tang et al., 2007).

Six Sigma builds on improvement methods that have been proved to be effective and integrates the human and process elements of process improvement. The human elements of process improvement consist of teamwork, customer focus and organization's culture change. On the other hand, the process elements of process improvement consist of understanding the types of process variation, process stability, PCA, and DOE for identifying, reducing or eliminating process variation, and thence improve process performance and process capability at the same time (Antony et al., 2005).

Feng (2008) defined Six Sigma as a systematic approach for structured and process-oriented quality or performance improvement and classified two road maps, which are provided by Six Sigma methodology in order to achieve optimum business performance benchmarks for organizations. One is known as the road map for Six Sigma process improvement that is called DMAIC Procedure that consists of five phases, which are Define (D), Measure (M), Analyze (A), Improve (I), and Control (C). DMAIC Procedure involves the improvement of existing processes by removing defects, without changing the fundamental structure of the processes. The other road map is known as Design for Six Sigma (DFSS). DFSS is a Six Sigma approach that involves changing and redesigning the process at the early stages of product or process life cycle. DFSS also consists of five phases, which are Define (D), Measure (M), Analyze (A), Design (D) and Verify (V).

4.4 Key factors for a success of Six Sigma project implementation

In this section, key factors that influence the success of Six Sigma project implementation for improving overall management process would try to be identified. The success of Six Sigma

is related to a set of cross-functional metrics which lead to significant improvements in customer satisfaction and bottom-line benefits (Antony et al., 2005). Generally, wider applications of Six Sigma principles to the organization are achieved through sustained and visible management commitment and involvement as well as whole organizational commitment and organizational infrastructure; organizational cultural refinement; effective project management; continuous education and training, and etc. (Kwak & Anbari, 2006). It should be noticed that, these issues are basically performed with the help of statistics, quality and process improvement tools and techniques.

There can be positive impact on application of Six Sigma when there is continuous managerial support for implementation process. According to Haikonen et al. (2004), managers should adopt as well as internalize Six Sigma philosophy throughout the organization. Top-management involvement and provision of resources and training activities are inevitable for a successful implementation of Six Sigma (Halliday, 2001). Management involvement and organizational commitment are influential to restructure the business and change the attitudes of the organization toward Six Sigma (Hendricks & Kelbaugh, 1998). Commitment of resources, time, money and effort from entire the organization is essential for Six Sigma project implementation. Organizational infrastructure needs to be established with well trained individuals. Before introducing Six Sigma concepts and tools, SWOT analysis can be performed in order to identify strengths and weaknesses of organization to ensure long term sustainability of Six Sigma Methodology (Kwak & Anbari, 2006).

Refining the organizational culture continuously is also compulsory for a successful implementation of Six Sigma. Leadership are necessary for a change in organizational culture. The attitudes of the employees or all of the participants should also be changed towards the Six Sigma philosophy. More concisely, implementation of a Six Sigma program needs the right mindset and attitude in the people working at all levels within the organization (Antony & Banuelas, 2001). For this purpose, clear communication plan needs to be developed. Motivation and education for Six Sigma are influential factors for refining the organizational culture, too. It should be taken into consideration that organizational cultural changes require time and commitment. Effective Six Sigma principles as well as practices can be more likely achieved by refining the organizational culture continuously (Kwak & Anbari, 2006).

Six Sigma project selection, review and tracking are fundamental parts of effective project management. Effective project management includes careful consideration of projects to be feasible, organizationally and financially beneficial and conformation of appropriate set of measures and metrics to satisfy customer requirements. Periodic review of project evaluates the state of the project and performance of Six Sigma tools and techniques. Documentation is necessary for tracking of projects within project constraints that are mainly cost, time and quality (Kwak & Anbari, 2006).

Continuous education and training give a clear sense for participants for understanding the tools and techniques and principles of Six Sigma Methodology. On this account, the implementation of Six Sigma should start with the training of a dedicated workforce and the education across the organization. It should be considered that there can be inherent drawback of misapplication of Six Sigma Methodology when personnel are trained inadequately. Although Six Sigma is deployed from top down, people in the organization

need necessary training to realize Six Sigma improvement and its potential benefits to the organization and themselves. In order to implement Six Sigma tools and techniques effectively, communication techniques should be widespread throughout the organization. Participants should be well informed about the Six Sigma tools and techniques and communicate with actual data analysis. Identifying key roles and responsibilities of participants for implementing Six Sigma project should be well defined. Learning the principles behind the Six Sigma methodology requires Six Sigma training activities. Training should also cover quantitative and qualitative measures and metrics along with leadership and project management. Training is a key success factor in implementing Six Sigma projects (Kwak & Anbari, 2006). Understanding of Six Sigma methodology accompanied by tools and techniques is very important for successful Six Sigma applications.

Historically, statisticians created Six Sigma concept, thus, the origin of Six Sigma comes from statistics. In this connection, the success of Six Sigma project implementation for improving overall management process is absolutely related to the appropriate usage of tools and techniques of statistics and quality. By utilizing statistical tools and techniques, Six Sigma methodology enables practitioners to identify process hindering problems accurately. Also, utilization of statistical tools and techniques demonstrate the improvements based on usage of objective data. That's why Six Sigma is accepted as a data-driven approach as it needs to quantify the process by using actual data. Therefore, statistical thinking is vital for Six Sigma methodology, reduction of defects and variation. As Six Sigma originated from the statistical concept for quality improvement, the role of management in statistical thinking is important for quality and process improvement efforts.

5. Lean Six Sigma

Lean Six Sigma is a combination of concepts of two productivity improvement programs, which are Six Sigma and Lean Manufacturing. Particularly, Six Sigma is a quality management philosophy as well as a methodology that focuses on reducing variation, defects and improving the quality of processes, products, and services. Six Sigma cannot reduce waste or reduce cycle time in processes alone. On the other hand, Lean Manufacturing is a methodology that focuses on reducing waste and cycle time in processes. Lean cannot reduce variation alone. To sum up, Lean Six Sigma is an approach that focuses on improving quality by reducing variation and defects as in Six Sigma and eliminating waste along with reducing cycle time in an organization as in Lean Manufacturing.

According to George (2002) Lean Six Sigma is a methodology that maximizes shareholder value by achieving the fastest rate of improvement in customer satisfaction, cost, quality, process speed and invested capital. In order to eliminate waste and reduce variation in any process, Lean Six Sigma can be used.

In fact, Six Sigma differs from Lean Manufacturing because they attack different types of problems. Basically, Six Sigma is concerned with less visible problems in processes such as variation in performance. Six Sigma tools require advanced training and expertise of specialists. However, Lean Manufacturing is concerned with visible problems in processes such as inventory, material flow and safety. Lean tools are more intuitive and easier to apply. Organizations are recommended to start with basic lean principles and evolve toward more sophisticated Six Sigma tools and techniques.

6. Conclusions and recommendations

In today's competitive business environment, the competition power of small and medium size enterprises, companies and even countries (either in private, public or military sectors) in the national and international business area are mainly based on customer (internal or external) relations; understanding the needs of customer, ability and flexibility of immediate response to needs of customer and requirements for providing capability to fulfill those needs. All activities performed to provide capabilities for satisfying customer needs include many sophisticated interrelated functions and processes either directly or indirectly based on what customer wants or more specifically, the customer demand; such as decision making, management, new product development, production, marketing, logistics, finance, quality control, human resources and etc.; which all together compose dynamic, complex and chaotic structures. These of complex structures with all interrelated functions have to be designed and managed perfectly pointing us to two well-known terms supply chain networks (ScNs) and ScN management. In such dynamic and complex systems (i.e., ScN like systems), all processes have to be managed successfully; which in fact, can only be achieved with on time, true and appropriate control mechanisms. As presented through the whole chapter, basic tool for establishing such mechanisms are PCA. PCA and fuzzy PCA together with their integral parts PCIs and fuzzy PCIs occupy a vital place in every field where computational controls are needed. In industrial practice PCA and fuzzy PCA; which is mainly used for predicting how well the process will hold the tolerances, can be used in many segments of the product cycle. They can be used in production and production planning as it reduces the variability in a process and plans the sequence of production processes when there is an interactive effect of processes on tolerances. Also; in process and product design, by assisting designers in selecting or modifying a process and in specifying the performance requirements PCA and fuzzy PCA can successfully be used. Even in the selection of competing suppliers, PCA and fuzzy PCA play important role.

Six Sigma also positively impacts many CTQ features such as timeliness/speed, cost, and quality of product or service as it identifies root causes and eliminates variations and defects. After a successful implementation of Six Sigma project, savings from reduced rework, less waste and decrease in customer returns can be obtained. So, this approach is also one of the indispensable in today business environment. Due to its importance, with every passing time, this approach is developed with researches and ideas, like lean approaches as mentioned before.

As a result, process capability and six sigma methodology; including fuzzy and lean approaches plays an important role in daily and theoretical scientific life. Today it would not be wrong to claim that it is a must for every enterprise in every field to adopt these modalities into their activities without wasting time. It may also be concluded that, studies on these significant subjects on fuzzy domain is still relatively unrefined. In the future, much more effort will and must be expend on these subjects in fuzzy domain.

7. References

- Allen, T. T. (2006). *Introduction to Engineering Statistics and Six Sigma: Statistical Quality Control and Design of Experiments and Systems*, Springer, ISBN: 1852339551, London.

- Anbari, F. T. (2002). Six sigma method and its applications in project management. *Proceedings of the Project Management Institute Annual Seminars and Symposium*, San Antonio, Texas, Oct 3-10, Project Management Institute, Nextown Square, PA.
- Antony, J. & Banuelas, R. (2001). A strategy for survival. *Manufacturing Engineer*, Vol.80, No3, 119-121, ISSN: 0956-9944.
- Antony, J., Kumar, M., & Tiwari, M. K. (2005). An application of six sigma methodology to reduce the engine-overheating problem in an automotive company. *Journal of Engineering Manufacture*, B8, 14, 633-646, ISSN: 2041-2975.
- Automotive Industry Action Group (AIAG) (2002). *Measurement system analysis*, (3rd ed.) Southfield, MI: Author.
- Bellman, R. E., Zadeh L.A., (1970). Decision- making in fuzzy environment, *Management Science*, Vol.17, 141-164.
- Bothe, D. R. (2002). Discussion, *Journal of Quality Technology*, Vol. 34, No.1, ISSN: 0022-4065.
- Buckley, J. J. & Eslami, E. (2004). Uncertain probability II: The continuous case, *Soft Computing*, Vol. 8, 193-199.
- Buckley, J. J. (2005a). Fuzzy statistics: Hypothesis testing, *Soft Computing*, Vol. 9, 512-518.
- Buckley, J. J. (2005b). Fuzzy statistics: Regression and prediction, *Soft Computing*, Vol. 9, 769-775.
- Deleryd, M. (1999). The effect of skewness on estimates of some process capability indices. *International Journal of Applied Quality Management*, Vol. 2, No. 2, 153-186, ISSN: 1096-4738.
- Chen, K. S., Chen, T. W., (2008). Multi-process capability plot and fuzzy inference evaluation, *Int. Journal of Production Economics*, Vol. 111, 70-79.
- Chen, T. W., Chen K. S., Lin J. Y. (2003). Fuzzy evaluation of process capability for bigger-the-best type products, *Int. Journal of Advanced Manufacturing Technology*, Vol. 21, 820-826.
- English, J. R. & Taylor G. D. (1993). Process capability analysis- a robustness study. *International Journal of Production Research*, Vol. 31, No. 7, 1621-1635, ISSN: 0020-7543.
- Evans, J. R. & Lindsay W. M.(2008). *The Management and Control of Quality*, (7th ed.), Thomson, ISBN: 0324382273, OH USA.
- Faraz A. & Moghadam M. B. (2007), Fuzzy control charts a better alternative for shewhart average charts, *Quality and Quality*, Springer, NJ USA.
- Feng, Q.(2008). Six Sigma : Continuous Improvement Toward Excellence (Chapter 3) In: *Collaborative Engineering: Theory and Practice*, A.K. Kamrani, E.S. Abouel Nasr (eds.), 43-60, Springer, ISBN: 978-0-387-47319-2, NY, USA
- Franklin LeRoy A. (1999). Sample size determination for lower confidence limits for estimating process capability indices. *Computers and Industrial Engineering*, Vol.36, 603-614.
- George, M. (2002). *Lean Six Sigma Combining Six Sigma Quality with Lean Speed*, McGraw-Hill, ISBN: 0071385215, USA.

- Gulbayi, M. & Kahraman C. (2006). Development of fuzzy process control charts and fuzzy unnatural pattern analyses, *Computational Statistics and Data Analysis*, Vol. 51, 434-451.
- Gulbay, M. & Kahraman C. (2007). An alternative approach to fuzzy control charts: direct fuzzy approach, *Information Sciences*, Vol. 177, 1463-1480.
- Haikonen, A., Savolainen, T., & Jarvinen, P. (2004). Exploring six sigma and CI capability development: preliminary case study findings on management role. *Journal of Manufacturing Technology Management*, Vol.15, No.4, 369-378.
- Halliday, S. (2001). So what exactly is Six Sigma? *Works Management*, Vol. 15, No.1, 15.
- Harry, M. J. & Lawson, J. R. (1992). *Six sigma producibility analysis and process characterization*, Addison-Wesley Pub., ISBN: 0201634120, USA.
- Harry, M. J. (1998). Six Sigma: a breakthrough strategy for profitability, *Quality Progress*, Vol.31, No. 5, 60-64.
- Hendricks, C.A. & Kelbaugh, R. (1998). Implementing Six Sigma at GE, *The Journal of Quality and Participation*, Vol.21, No.4, 48-53.
- <http://wordnetweb.princeton.edu/>, (2009).
- Kahraman, C. (2006). *Fuzzy Applications in Industrial Engineering*, Studies in Fuzziness and Soft Computing, Vol. 201, Springer Verlag, NJ USA.
- Kahraman, C. & Kaya, I. (2008). Fuzzy process capability indices for quality control of irrigating water, *Stochastic Event Research and Risk Assessment*.
- Kahraman, C. & Kaya, I. (2009). Fuzzy process accuracy index to evaluate risk assessment of drought effects in Turkey, *Human and Ecological Risk Assessment: An International Journal*, Vol. 15, No. 4, 789-910.
- Kaya, I. & Kahraman, C. (2007). Air pollution control using six-sigma approach, *Proceedings of International Conference on Risk Analysis and Crisis Response*, 100-115.
- Kaya, I. & Kahraman, C. (2008a). Fuzzy process capability analyses: An application to teaching process, *Journal of Intelligent and Fuzzy Systems*, Vol. 19, 259-272.
- Kaya, I. & Kahraman, C. (2008b). Fuzzy robust process capability indices for risk assessment of air pollution, *Stochastic Environmental Research and Risk Assessment*.
- Kaya, I. & Kahraman, C. (2009). Air pollution control using fuzzy process capability indices in six-sigma approach, *Human and Ecological Risk Assessments: An international Journal*, Vol. 15, No.4, 689-713.
- Kaya, I. & Kahraman, C. (2010). A new perspective on fuzzy process capability indices: Robustness, *Expert Systems with Applications*, Vol. 37, 4593-4600.
- Klir, G. J., Yuan B.(1995). *Fuzzy sets and fuzzy logic: Theory and applications*, Prentice Hall, ISBN: 0131011715.
- Kotz S. & Johnson N.L. (2002). Process capability indices-A review, 1992-2000 (with subsequent discussions and response), *Journal of Quality Technology*, Vol.34, No.1, 2-53.
- Kwak, Y. H. & Anbari, F. T. (2006). Benefits, obstacles, and future of six sigma approach, *Technovation*, Vol.26, No. 5, 708-715.
- Lee, Y. H., Wei, C.C. & Chang, C.L., (1999). Fuzzy design of process tolerances to maximize process capability, *International Journal of Advanced Manufacturing Technology*, Vol.15, 655-659.

- Lee, H. T. (2001). Cpk index estimation using fuzzy numbers, *European Journal of Operational Research*, Vol. 129, 683-688.
- McClusky, R. (2000). The Rise, fall, and revival of six sigma, *Measuring Business Excellence*, Vol.4, No 2, 6-17.
- Monden, Y. (1993). *Toyota Production System, An Integrated Approach to Just-In-Time*, (2th ed.), Industrial Engineering and Management Institute Press, Norcross GA.
- Montgomery, D. C (2009). *Statistical Quality Control- A Modern Introduction*, Wiley., ISBN: 978047233979, USA.
- Qianmei Feng (2008). Six sigma: Continuous improvement toward excellence. In: *Collaborative Engineering*, A.K. Kamrani, E.S. Abouel Nasr (eds.), 43-60, Springer, ISBN: 9780387473192.
- Parchami, A., Mashinchi, M., Yavari, A. R. & Maleki, H. R. (2005). Process capability indices as fuzzy numbers. *Austrian Journal of Statistics* , Vol. 34 , 391-402.
- Parchami, A. & Mashinchi, M. (September 2006). Making decision to evaluate fuzzy process capability index. *Asian Fuzzy Systems Society International Conference (AFSS 2006)* Boading, China, 28-33.
- Parchami, A. & Mashinchi, M. (2007). Fuzzy estimation for process capability indices. *Information Sciences* , Vol.177 ,1452-1462.
- Parchami, A. & Mashinchi, M. (2008). Testing the capability of fuzzy processes. *Quality Technol. Quant. Management*. – Accepted
- Parchami, A. , Mashinchi, M. & Maleki, H. R. (2006). Fuzzy confidence interval for fuzzy process capability index, *J. Int. Fuzzy Syst.*,Vol.17 , 287-295.
- Parchami A., Mashinchi M. & Sharayei A. (2010a), An effective approach for measuring the capability of manufacturing processes, *Production Planning and Control*, Vol. 21 250-257.
- Parchami A. & Mashinchi M. (2010a), An effective approach for measuring the capability of manufacturing processes, *Production Planning and Control*, Vol. 37, 77-89.
- Palit, A. K., Popovic D., (2005). *Computational intelligence in time series forecasting: Theory and engineering applications*, Advances in Industrial Control, Springer Verlag, NJ USA.
- Ramberg, J. S. (2002). Discussion. *Journal of Quality Technology*, Vol.34, No.1.
- Ramirez B. & Runger G. (2002) Quantitative Techniques to evaluate process stability, *Quality Engineering*, Vol.18, 53-68.
- Ross, T. J. (2004). *Fuzzy Logic with Engineering Applications*, (2th Ed.), Wiley, ISBN: 9780470860748, USA.
- Spiring, F., Cheng, S., Yeung A., Leung, B. (2002). Discussion, *Journal of Quality Technology*, Vol.34, No.1.
- Tanaka, K. (1997). *An introduction to fuzzy logic for practical application*, Springer, ISBN: 9780387948072.
- Tang, L.C., Goh T.N., Lam S.W., Zhang C.W. (2007). Fortification of six sigma: Expanding the DMAIC toolset, *Quality and Reliability Engineering International*, Vol.23, No.1, 3-18.
- Tozan, H. Vayvay, Ö. (2008), Fuzzy forecasting applications on supply chains, *WSEAS Transactions on Systems*, Vol.7, 600-609.

- Tozan, H. Vayvay, Ö. (2009), A hybrid Grey & ANFIS approach to bullwhip effect in supply chain networks, *WSEAS Transactions on Systems*, Vol.7, 600-609.
- Wang, C. H. & Raz, T. (1990). On the Construction of Control Charts Using Linguistic Variable, *International Journal of Production Research*, Vol. 28, No. 3, 477-487.
- Yonting, C. (1996). Fuzzy quality and analysis on fuzzy probability. *Fuzzy Sets and Systems*, Vol. 83, 283-290.
- Zadeh, L. A. (1965). Fuzzy sets, *Information and Control*, Vol. 8, 338-353.



Products and Services; from R&D to Final Solutions

Edited by Igor Fuerstner

ISBN 978-953-307-211-1

Hard cover, 422 pages

Publisher Sciyo

Published online 02, November, 2010

Published in print edition November, 2010

Today's global economy offers more opportunities, but is also more complex and competitive than ever before. This fact leads to a wide range of research activity in different fields of interest, especially in the so-called high-tech sectors. This book is a result of widespread research and development activity from many researchers worldwide, covering the aspects of development activities in general, as well as various aspects of the practical application of knowledge.

How to reference

In order to correctly reference this scholarly work, feel free to copy and paste the following:

Ozlem Senvar and Hakan Tozan (2010). Process Capability and Six Sigma Methodology Including Fuzzy and Lean Approaches, Products and Services; from R&D to Final Solutions, Igor Fuerstner (Ed.), ISBN: 978-953-307-211-1, InTech, Available from: <http://www.intechopen.com/books/products-and-services--from-r-d-to-final-solutions/process-capability-and-six-sigma-methodology-including-fuzzy-and-lean-approaches>

INTECH

open science | open minds

InTech Europe

University Campus STeP Ri
Slavka Krautzeka 83/A
51000 Rijeka, Croatia
Phone: +385 (51) 770 447
Fax: +385 (51) 686 166
www.intechopen.com

InTech China

Unit 405, Office Block, Hotel Equatorial Shanghai
No.65, Yan An Road (West), Shanghai, 200040, China
中国上海市延安西路65号上海国际贵都大饭店办公楼405单元
Phone: +86-21-62489820
Fax: +86-21-62489821

© 2010 The Author(s). Licensee IntechOpen. This chapter is distributed under the terms of the [Creative Commons Attribution-NonCommercial-ShareAlike-3.0 License](#), which permits use, distribution and reproduction for non-commercial purposes, provided the original is properly cited and derivative works building on this content are distributed under the same license.