

# A Unified Approach to Robust Control of Flexible Mechanical Systems Using $\mathcal{H}_\infty$ Control Powered by PD Control

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## 1. Introduction

This chapter presents a unified approach to robust control of a variety of flexible mechanical systems, which are not only systems having flexible structure themselves such as a robotic manipulator with a flexible structure and a crane system, but also systems not having flexible structure but handling flexible objects such as a liquid container system and a fishery robot. So far, a lot of research efforts have been devoted to solve control problems of such flexible systems, one of the most typical problems among which is the problem of flexible robotic manipulators, e.g., [Sharon & Hardt (1984); Spong (1987); Wang & Vidyasagar (1990); Torres et al., (1994); Magee & Book (1995); Nenchev et al., (1996); Nenchev et al., (1997)]. As other types of applications, the problems of a crane system [Kang et al. (1999)] and of a liquid container system [Yano & Terashima (2001); Yano et al., (2001)] have been investigated. The common control problem for flexible systems can be stated as “*how to achieve required motion control with suppressing undesirable oscillation due to its flexibility*”.

From the control methodology point of view, let us review those previous works. For so-called micro-macro manipulators associated with large flexible space robots, [Torres et al (1994)] and [Nenchev et al., (1996); Nenchev et al., (1997)] have proposed path-planning based control methods using a coupling map and a reaction null-space respectively, which utilize the geometric redundancy. The control methods in [Sharon & Hardt (1984)] for a micro-macro manipulator and in [Kang et al., (1999)] for a crane system rely on the endpoint direct feedback, which require sensors to measure the endpoint. In [Wang & Vidyasagar (1990)], a passivity-based control method has been proposed for a single flexible link, and in [Spong (1987)] an exact-linearization method and an integral manifold method have been presented for a flexible-joint manipulator. The method in [Magee & Book (1995)] is based on input signal filtering where the underlying concept is pole-zero cancellation. [Ueda & Yoshikawa (2004)] has applied a mode-shape compensator based on acceleration feedback to a flexible-base manipulator. For a liquid container system,  $\mathcal{H}_\infty$  control in [Yano & Terashima (2001)] and a notch-type filter based control, that is, equivalent to pole-zero cancellation, in [Yano et al., (2001)] are utilized respectively. In general, most other works have focused on individual systems and hence their control methods are not directly available for various flexible systems. For example, the path-planning methods in [Torres et

al., (1994); Nenchev et al., (1996); Nenchev et al., (1997)] cannot be applied to non-redundant systems. The direct endpoint feedback might be difficult in such a case as of a large space robot where it is difficult to employ sensors to directly measure the endpoint.

In a stark contrast with those works, we have been tackling with a unified control design method which can be applied to various flexible mechanical systems in a uniform and systematic manner. The proposed method exploits a problem setting framework which is referred to as “generic problem setting” in the modeling phase and then, in the control design phase,  $\mathcal{H}_\infty$  control powered by PD control. In the sense of control methodology, the underlying concept is pole-zero cancellation similarly with [Magee & Book (1995); Yano et al., (2001)], however the control design approach is totally different from ones in those works. On the other hand, although [Yano & Terashima (2001)] has employed  $\mathcal{H}_\infty$  control, its usage is different from ours as explained later, and further the pole-zero cancellation is not the case in [Yano & Terashima (2001)]. In our control design method, the point to be emphasized is that PD control plays very important roles in facilitating the generic problem setting and the  $\mathcal{H}_\infty$  control design, and most importantly in enhancing the robustness of the control system. Then, the advantageous features of our control design method are:

1. The method can be applied to various flexible systems in a uniform, systematic, and simple manner where the frequency-domain perspective will be provided;
2. The robustness can easily be enhanced by appropriately choosing the PD control gains;
3. Due to the nature based on pole-zero cancellation, any oscillation sensors will not be required, which is considerably important in the practical sense.

In [Toda (2004)], we have first introduced the fundamental idea and demonstrated control simulations using linear system and weakly nonlinear system examples. Then, in [Toda (2007)], robust control has been explicitly considered and a rather strongly nonlinear system example has been tackled. Now, in this article the control design method and the previous achievements are summarized, moreover a multiple-input-multiple-output (MIMO) system and the optimality with respect to PD control are examined while those points have not been considered in [Toda (2004); Toda (2007)].

The remainder of this chapter is organized as follows. Section 2 presents the generic problem setting and an illustrative MIMO system example. Section 3 introduces the control design method and discusses its features in some detail. Then, Section 4 demonstrates control simulations using the MIMO system example. Finally, Section 5 gives some concluding remarks.

## 2. Generic problem setting and an illustrative example

### 2.1 Generic problem setting

For the purpose of accommodating a variety of flexible systems, in the modeling phase, a generic model which can represent such systems in a uniform manner is required. Hence, we consider a cascade chain of linear mass-spring-damper systems as shown in Fig. 1.  $m_i$ ,  $k_i$ ,  $d_i$ ,  $f_i$ , and  $q_i$  denote the mass, stiffness parameter, damping parameter, exerted force, and displacement from the equilibrium of the  $i$ th component respectively. The first component is connected to the stationary base. The number of components depends on systems to be modeled. For example, a single-link flexible-joint manipulator can be modeled as a two-component model, where  $m_1$  denotes the inertia of the actuator,  $f_1$  the actuator torque,  $m_2$  the inertia of the link, and  $f_2$  must be zero, that is, the first component is directly actuated while the second one is not so, thus, is merely an oscillatory component. Applying PD control to the actuator, the corresponding dynamical model can be described as follows,

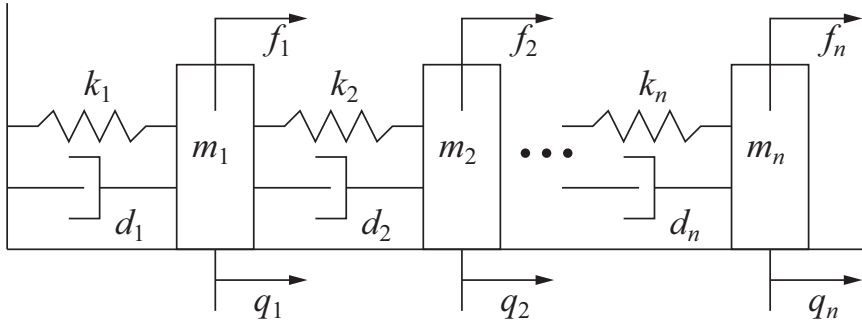


Fig. 1. Schematic diagram of the generic problem setting.

$$\begin{aligned} m_1\ddot{q}_1 + m_2(\ddot{q}_1 + \ddot{q}_2) + d_1\dot{q}_1 + k_1q_1 &= f_1 \\ m_2(\ddot{q}_1 + \ddot{q}_2) + d_2\dot{q}_2 + k_2q_2 &= 0. \end{aligned} \tag{1}$$

On the other hand, let us consider a single-link flexible-base linear manipulator. In this case, conversely, the first component is merely an oscillatory component while the second one is to be directly controlled via the actuator. The dynamical model including PD control to the actuator can be described as follows,

$$\begin{aligned} m_1\ddot{q}_1 + m_2(\ddot{q}_1 + \ddot{q}_2) + d_1\dot{q}_1 + k_1q_1 &= 0 \\ m_2(\ddot{q}_1 + \ddot{q}_2) + d_2\dot{q}_2 + k_2q_2 &= f_2. \end{aligned} \tag{2}$$

As seen from the above discussion, by assigning a component to be directly controlled via the corresponding actuator or an oscillatory component to each mass, this chain model can represent various flexible systems. This problem setting framework based on the chain model is referred to as “generic problem setting”. Then, the control problem is how to control positions of the directly controlled components with suppressing oscillations of the oscillatory components. It should be noted that with the proposed control method any sensors for the oscillatory components will not be required except such cases where, in the steady state, deformation due to the flexibility and the gravity would become a problem. In cases of nonlinear and/or uncertain systems, through some linearization procedures such as nonlinear state feedback and linear approximation around the equilibrium, the system is modeled as a linear model with parametric uncertainties and/or disturbances. Furthermore, by applying PD control to the nonlinear system, one can make the linear dynamics dominant, therefore can facilitate the generic problem setting.

### 2.2 Illustrative example

In [Toda (2004)], as illustrative examples, we have chosen the flexible-joint manipulator and the flexible-base linear one represented by (1) and (2) respectively, and a gantry-crane system which can be represented by the same model as the flexible-joint manipulator one by using linear approximation. Then, in [Toda (2007)], as a strongly nonlinear system example, a single-link revolutionary-joint flexible-base manipulator has been considered. Since all the examples in these previous works are of single-input-single-output (SISO) systems, in this article we choose a two-link flexible-joint manipulator as an MIMO system example as depicted in Fig. 2.

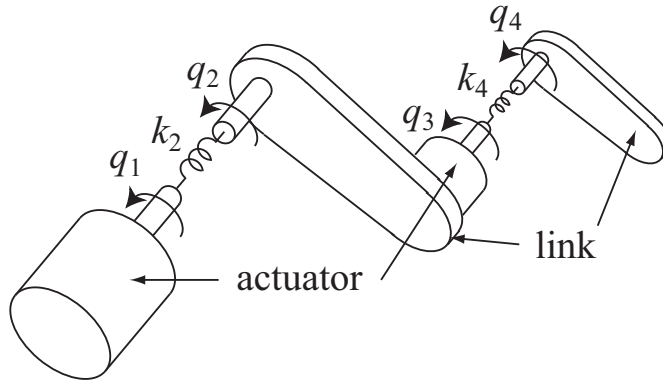


Fig. 2. Two-link flexible-joint manipulator.

$q = [q_1, q_2, q_3, q_4]^t$  denotes the position vector of the manipulator,  $k_2, k_4$ , and  $d_2, d_4$  denote the joint stiffness and damping parameters respectively.  $[\cdot]^t$  denotes the transpose. Additionally, by introducing PD control to the actuators with the P gains  $k_1, k_3$  and the D gains  $d_1, d_3$ , the dynamical model is as in the following.

$$M(q)\ddot{q} + C(q, \dot{q}) + D\dot{q} + Kq = f \tag{3}$$

where  $M(q)$  is the inertia matrix,  $C(q, \dot{q})$  is the centripetal and Coriolis term,  $D = \text{diag}[d_1, d_2, d_3, d_4]$  is the damping diagonal matrix,  $K = \text{diag}[k_1, k_2, k_3, k_4]$  is the stiffness diagonal matrix, and  $f = [f_1, 0, f_3, 0]^t$  is the control torque vector excluding the PD control scheme. Specifically, each element of  $M(q)$ ,  $M_{ij}$  is as follows:

$$\begin{aligned} M_{11} &= m_1 + m_2 + m_3 + m_4 + 2R\cos(q_3 + q_4) \\ M_{12} &= M_{21} = M_{22} = m_2 + m_3 + m_4 + 2R\cos(q_3 + q_4) \\ M_{13} &= M_{23} = M_{31} = M_{32} = m_3 + m_4 + R\cos(q_3 + q_4) \\ M_{14} &= M_{24} = M_{41} = M_{42} = m_4 + R\cos(q_3 + q_4) \\ M_{33} &= m_3 + m_4 \\ M_{34} &= M_{43} = M_{44} = m_4 \end{aligned} \tag{4}$$

where  $m_i$  and  $R$  are the inertia parameters. And  $C(q, \dot{q})$  is formulated as

$$C(q, \dot{q}) = \begin{bmatrix} -\{2(\dot{q}_1 + \dot{q}_2) + (\dot{q}_3 + \dot{q}_4)\}(\dot{q}_3 + \dot{q}_4)R\sin(q_3 + q_4) \\ -\{2(\dot{q}_1 + \dot{q}_2) + (\dot{q}_3 + \dot{q}_4)\}(\dot{q}_3 + \dot{q}_4)R\sin(q_3 + q_4) \\ (\dot{q}_1 + \dot{q}_2)^2 R\sin(q_3 + q_4) \\ (\dot{q}_1 + \dot{q}_2)^2 R\sin(q_3 + q_4) \end{bmatrix}. \tag{5}$$

As seen from Equations (3)–(5), it is confirmed that except the nonlinear terms the dynamical model can completely be represented in the generic problem setting with four components. Moreover, assuming that the dynamics due to the PD control scheme is more dominant than  $C(q, \dot{q})$  and that  $M(q)$  with  $q_3 = \pi/3$  and  $q_4 = 0$  is a nominal constant matrix, the proposed control design method will be applied to this problem. The physical parameters in the

dynamical model employed for the control design and simulations in the sequel are shown in Table 1, which are set by considering the experimental apparatus at hand.

parameter	value	unit
$m_1$	1.000e-5	kgm <sup>2</sup>
$m_2$	2.027e-3	kgm <sup>2</sup>
$m_3$	1.000e-6	kgm <sup>2</sup>
$m_4$	1.520e-4	kgm <sup>2</sup>
$R$	9.410e-5	kgm <sup>2</sup>
$d_2$	0.000e 0	Nms
$d_4$	0.000e 0	Nms
$k_2$	2.180e-1	Nm
$k_4$	1.520e-2	Nm

Table 1. Physical parameters.

### 3. Control design

Here we introduce our control design method which is applied to the obtained model in the generic problem setting. In the design procedure, first one should determine the PD control gains, then proceed to the  $\mathcal{H}_\infty$  control design aiming to shape the associated sensitivity functions. However, in this section, for ease of exposition we first present the  $\mathcal{H}_\infty$  control design and after that discuss the PD control scheme in some detail.

#### 3.1 Sensitivity function shaping by $\mathcal{H}_\infty$ control

Once the PD control scheme has been determined, the control design procedure is almost automatically processed in the linear  $\mathcal{H}_\infty$  control framework with the aim of shaping the associated sensitivity functions. Fig. 3 depicts the augmented plant for  $\mathcal{H}_\infty$  control design where  $P$  denotes the plant incorporating the PD control scheme which consists of  $P_i$  corresponding to the components to be directly controlled and  $P_j$  to the oscillatory ones,

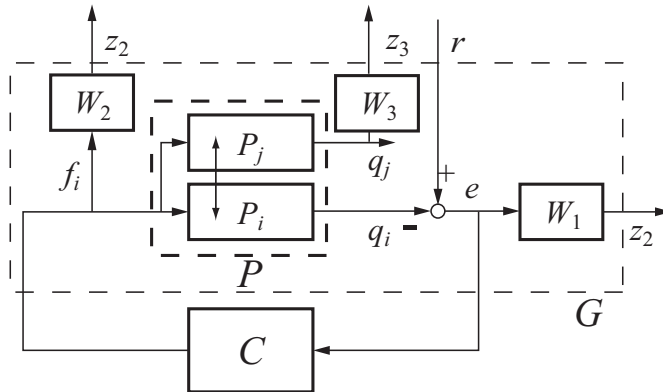


Fig. 3. Augmented plant for  $\mathcal{H}_\infty$  control design.

where  $P_i$  and  $P_j$  are coupled systems each other. The sensitivity functions taken into account are the transfer function  $S_1$  from the reference commands  $r$  to the tracking control errors  $e$ ,  $S_2$  from  $r$  to the control inputs  $f_i$ , and  $S_3$  from  $r$  to the oscillatory component displacements  $q_j$ . In the example given in Section 2.2,  $q_i$  are  $q_1, q_3$ , and  $q_j$  are  $q_2, q_4$  respectively.

Note that  $S_3$  plays a key role in this problem and, in terms of  $\mathcal{H}_\infty$  control design, makes our method differ from the others such as [Yano & Terashima (2001)] which does not consider  $S_3$  but only the standard mixed sensitivity problem. By explicitly employing  $S_3$ , the resultant  $\mathcal{H}_\infty$  controller will automatically contain the corresponding zeros to the oscillatory poles of the plant and thus pole-zero cancellation will occur in the closed-loop system which leads to suppression of oscillation. Due to this nature of pole-zero cancellation, the control system will not require any sensors to measure the states of the oscillatory components  $q_j$ . The respective weighting functions for the sensitivity functions in the example are

$$W_1(s) = \frac{20}{7} \begin{bmatrix} \frac{1}{s+0.0001} & 0 \\ 0 & \frac{1}{s+0.0001} \end{bmatrix} \quad (6)$$

$$W_2(s) = \frac{3}{7} \begin{bmatrix} \frac{s+0.1}{s+100} & 0 \\ 0 & \frac{s+0.1}{s+100} \end{bmatrix} \quad (7)$$

$$W_3(s) = \frac{20}{7} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \quad (8)$$

$W_1$  is only a quasi-integrator intended for step tracking control.  $W_2$  is a high-pass filter which will be determined by the actuator capability.  $W_3$  for  $S_3$  is only a constant gain. These functions are very simple, and in particular  $W_1$  and  $W_3$  might not depend on problems. Therefore, the designer will only need to care the constants  $20/7, 3/7, 20/7$  to adjust the balance among the functions. This simplicity is one of the important advantages of the proposed method.

Then, by constructing the augmented plant  $G$  as in Fig. 3, an  $\mathcal{H}_\infty$  controller  $C$  will be synthesized such that the  $\mathcal{H}_\infty$  norm of the closed-loop system  $T_{rz}$  from  $r$  to  $z = [z_1, z_2, z_3]^t$ , that is,  $\|T_{rz}\|_\infty$  is minimum. In this example, the resultant  $\|T_{rz}\|_\infty$  was 1.

If one may wish to explicitly consider the model uncertainties in the control design,  $\mu$ -synthesis [Packard & Doyle (1993); Zhou et al., (1995)] can be applied instead of merely  $\mathcal{H}_\infty$  control design. The interested readers may consult [Toda (2007)] for the specific approach in the same framework.

In addition, to improve the transient performance of the obtained control system, a low-pass filter is employed for step reference commands. In this example, the reference command filter is

$$P_r = \begin{bmatrix} \frac{100}{s^2 + 36s + 100} & 0 \\ 0 & \frac{100}{s^2 + 36s + 100} \end{bmatrix}. \quad (9)$$

## 3.2 PD control

### 3.2.1 Roles of PD control

Next, let us discuss the PD control scheme exploited for this problem. One role of the PD control scheme is, as mentioned in Section 2.1, of facilitating the generic problem setting by making the linear dynamics dominant. And as the second role, the scheme serves to facilitate the  $\mathcal{H}_\infty$  control design, that is, by eliminating the poles on the imaginary axis and turning the problem into so-called the standard  $\mathcal{H}_\infty$  control problem [Doyle et al. (1989); Zhou et al., (1995)]. However, a more important role is of enhancing the robustness with respect to the oscillation suppression capability, which is deeply connected with the pole-zero cancellation mechanism of the  $\mathcal{H}_\infty$  controller.

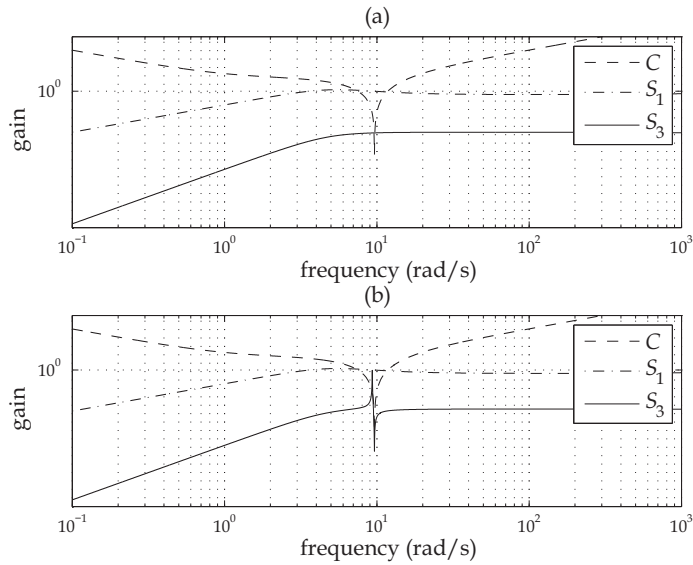
In the case of a completely linear system with neither model uncertainties nor perturbations, the pole-zero cancellation will never fail, and hence the constant oscillation suppression performance can be acquired. However, otherwise, that is, in cases of a nonlinear system and/or with model uncertainties, the pole-zero cancellation will fail since the oscillatory poles of the plant vary. In such a case, the damping property of the plant will become critical. Specifically, when the minimum among the damping factors of the plant poles is too small, the oscillation suppression performance can largely degrade in case of failure of the pole-zero cancellation. Here the damping factor  $\zeta$  of a stable pole  $s$ , whose real part  $\text{Re}(s) \leq 0$ , is defined as

$$\zeta := -\frac{\text{Re}(s)}{|s|} \quad (10)$$

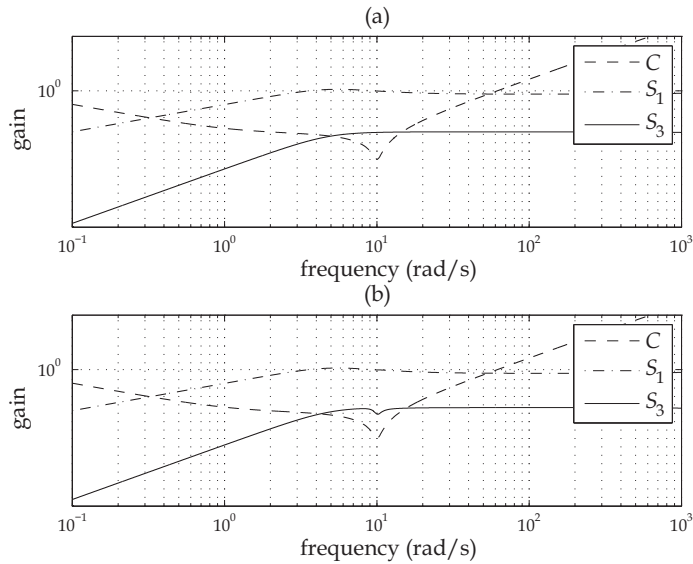
where  $\zeta$  of a real  $s$  is the maximum of 1.

However, by choosing the PD control gains, this damping property can be appropriately modified. We illustrate this fact by using a nonlinear SISO system example, i.e., a single-link revolutionary-joint flexible-base manipulator, investigated in [Toda (2007)]. Fig. 4 shows the frequency responses of the  $\mathcal{H}_\infty$  controller  $C$ , sensitivity functions  $S_1$ ,  $S_3$  of the two control systems with the different PD gains respectively. The upper figure shows the case with the minimal damping factor of  $8 \times 10^{-4}$ , and the lower one does the case with the factor of  $6 \times 10^{-2}$ . Further, in each figure, the nominal and perturbed cases are compared. As seen from the figures, in the upper case, the controller has a very stark notch compared to that in the lower case. Then, considering the sensitivity function  $S_1$  corresponding to the tracking control performance, in both the systems and in both the nominal and perturbed cases, the properties are the same. However, when it comes to  $S_3$  related to the oscillation suppression performance, although in the nominal case their properties are the same in both the system, in the perturbed case they are totally different. In the upper case, the stark oscillatory property has appeared due to the pole-zero cancellation failure while in the lower case it is not the case despite of such a failure. This difference stems from the difference in the minimal damping factors. Therefore, all the above discussions have been demonstrated, and it has been proved that the PD control scheme plays an important role of enhancing the robustness with respect to the oscillation suppression capability.

Additionally, note that considering the fact that the obtained  $\mathcal{H}_\infty$  controller is strictly proper, employing PD control obviously extends the class of controllers.



(a) Frequency responses of  $C$ ,  $S_1$ , and  $S_3$  with the minimal damping factor of  $8 \times 10^{-4}$ .  
 (a) nominal case (b) perturbed case.



(b) Frequency responses of  $C$ ,  $S_1$ , and  $S_3$  with the minimal damping factor of  $6 \times 10^{-2}$ .  
 (a) nominal case (b) perturbed case.

Fig. 4. Pole-zero cancellation failure examples from [Toda (2007)].



### 3.2.2 Optimality with respect to the PD gains

Here, one question may arise, “when is it optimal in choosing the PD control gains and/or the minimal damping factor?”. To seek the answer to this question, by using the illustrative example, we have examined various PD gains, the resultant minimal damping factors and control simulation results in a trial and error manner. Then, we have found the following points:

- P1 A too small minimal damping factor leads to poor oscillation suppression performance;
- P2 The maximum of minimal damping factor however does not necessarily reveal the optimal control performance;
- P3 even if with the same minimal damping factor, the control performance varies according to the P gain.

Accordingly, in this example, we have employed the following cost function  $\eta_1$  to be minimized in choosing the PD gains;

$$\eta_1(d_1, d_3, k_1, k_3) := (\zeta_{min} - 0.4)^2 + 100(k_1 + k_3) \tag{11}$$

where  $d_i$ 's and  $k_i$ 's are bounded as  $2.18e-5 \leq d_1 \leq 2.18e1$ ,  $1.52e-6 \leq d_3 \leq 1.52$ ,  $2.18e-6 \leq k_1 \leq 2.18e2$ ,  $1.52e-7 \leq k_3 \leq 1.52e1$ , respectively. Further, to demonstrate the above point 3, the other cost function  $\eta_2$  taking only  $\zeta_{min}$  into account

$$\eta_2(d_1, d_3, k_1, k_3) := (\zeta_{min} - 0.4)^2 \tag{12}$$

for similarly bounded  $d_i$ 's and  $k_i$ 's has been also considered. In the next section, these optimization strategies will be discussed based on control simulations.

## 4. Control simulations

In this article, to prove that the proposed control method can be applied to even MIMO systems, and to demonstrate the above discussions on the optimality with respect to the PD gains, we here present control simulations. According to the last section, four cases of PD gains are considered, which includes the cases of the respective optimal gains due to  $\eta_1$  and  $\eta_2$ , and additional two non-optimal cases. The respective  $\zeta_{min}$  and PD gains are shown in Table 2. Comparing Cases 1 and 2 in Table 2, it is noticed that the same  $\zeta_{min}$  and similar D gains can be obtained, however that the P gains in Case 2 are considerably larger than those in Case 1, which indeed reflects the cost functions in (11) and (12).

Case	$\zeta_{min}$	$d_1$ (Nms)	$d_3$ (Nms)	$k_1$ (Nm)	$k_3$ (Nm)
Case 1 ( $\eta_1$ )	0.40	2.25e-2	1.60e-3	2.18e-6	1.52e-7
Case 2 ( $\eta_2$ )	0.40	2.20e-2	1.46e-3	8.49e-2	6.22e-3
Case 3	0.06	1.02e-1	1,76e-2	4.68e-5	7.60e-7
Case 4	1.00	3.3e-3	5.67e-4	9.35e-4	1.52e-5

Table 2.  $\zeta_{min}$  and PD gains.

For these cases, step tracking control simulations have been conducted. The conditions are:

1. the simulation period is 10 s;
2. all the initial states are zeros;

3. two types of references  $0 \rightarrow \pi/3$  rad and  $0 \rightarrow \pi/2$  rad for both  $r_1$  and  $r_3$ , with the step time of 1 s are applied.

The simulation results are shown in Figs. 5–7 respectively. First we shall see the two optimal cases. In Figs. 5 and 6, the upper figures show each displacement on large scale graphs while the lower ones do each tracking control error to the final goal on fine scale ones. Comparing Case 1 of  $\eta_1$  and Case 2 of  $\eta_2$ , that is, with the same  $\zeta_{min}$  of 0.40, on large scale graphs those results are almost the same and reveal the good performances for both tracking control and oscillation suppression. On fine scale graphs, they are still very similar, however the oscillations of the oscillatory components  $e_2$  and  $e_4$  in Case 2 are slightly larger than those in Case 1, and slight overshoots of  $e_3$  can be seen at around 3 s in Case 2, which might be due to the largenesses of  $k_1$  and  $k_3$ .

Next, let us see the non-optimal cases in Fig. 7. In the figure, the upper figure shows the results of Case 3 with the small  $\zeta_{min}$  of 0.06, while the lower one does those of Case 4 with the large, in fact, maximal  $\zeta_{min}$  of 1.00 on fine scale graphs respectively. As seen from the figures, as pointed out before, the results of Case 3 reveal poor oscillation suppression performances, while the results of Case 4 reveal a slightly slow response in  $e_3$  and a slight steady error in  $e_1$ , which thus has demonstrated P1 and P2 in the last section.

Consequently, the main goal of extending our proposed method to MIMO systems has successfully been achieved, that is, it has been confirmed that the proposed method is effective and feasible for even MIMO systems. Additionally, discussions on the optimality with respect to the PD control gains have been given in some detail. The obtained control system based on the cost function  $\eta_1$  has revealed good performances in both tracking control and oscillation suppression, which therefore can be one of the promising candidates for the optimality, although it has not yet been conclusive that  $\eta_1$  can be useful for other examples.

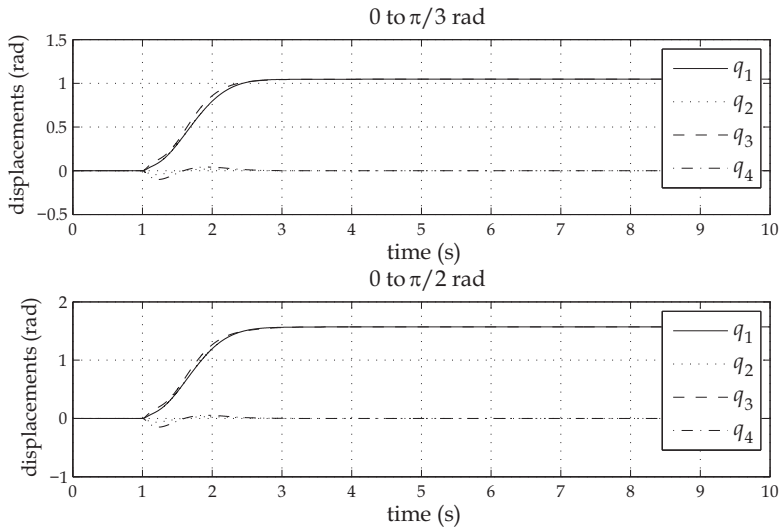
## 5. Conclusions

In this article, we have presented the control design method based on  $\mathcal{H}_\infty$  control and PD control aiming at a uniform approach to motion control of various flexible mechanical systems. In particular, with a special emphasis on MIMO systems and the optimal PD gains, we have introduced and demonstrated the concept of the generic problem setting in the modeling phase, the physics behind our control method, that is, how the PD control scheme elaborately powers the  $\mathcal{H}_\infty$  control system, the promising candidate of cost function for the optimal PD gains, and the control simulations which have supported all the discussions.

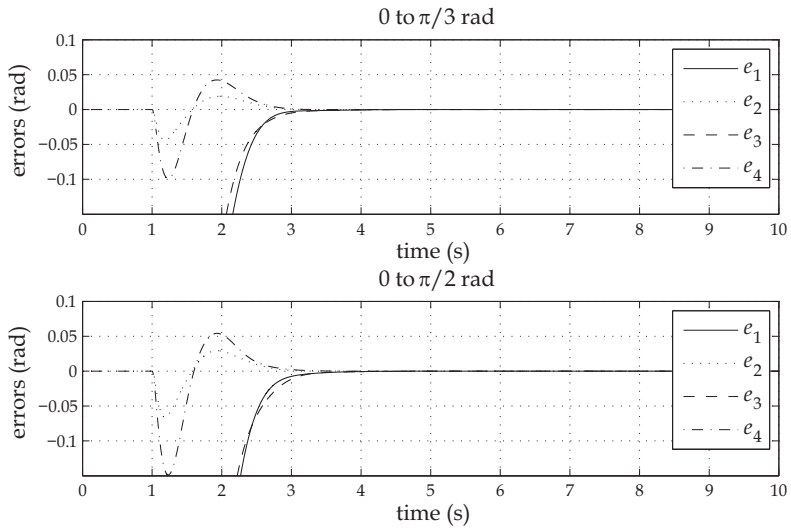
Here, again we emphasize the advantageous features of the proposed approach:

1. A variety of flexible mechanical systems can be systematically dealt with in a uniform and simple manner where the frequency-domain perspective will be provided;
2. The robustness can be easily enhanced by appropriately choosing the PD control gains;
3. Due to the nature based on pole-zero cancellation, any oscillation sensors will not be required, which is considerably important in the practical sense.

Consequently, we have shown that our methodology is easy to use and effective indeed and further will possibly evolve in the sense of optimality.

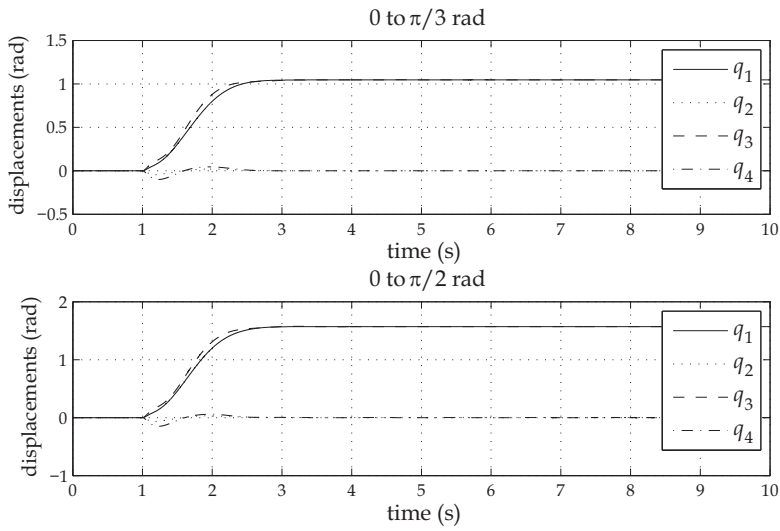


(a) Simulation results (large scale).

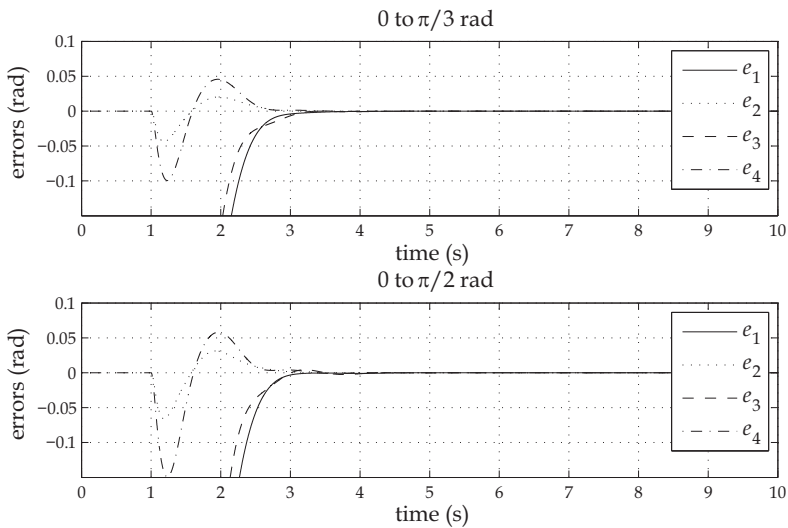


(b) Simulation results (fine scale).

Fig. 5. Simulation results using the optimal PD gains due to  $\eta_1$  ( $\zeta_{min}=0.40$ ).

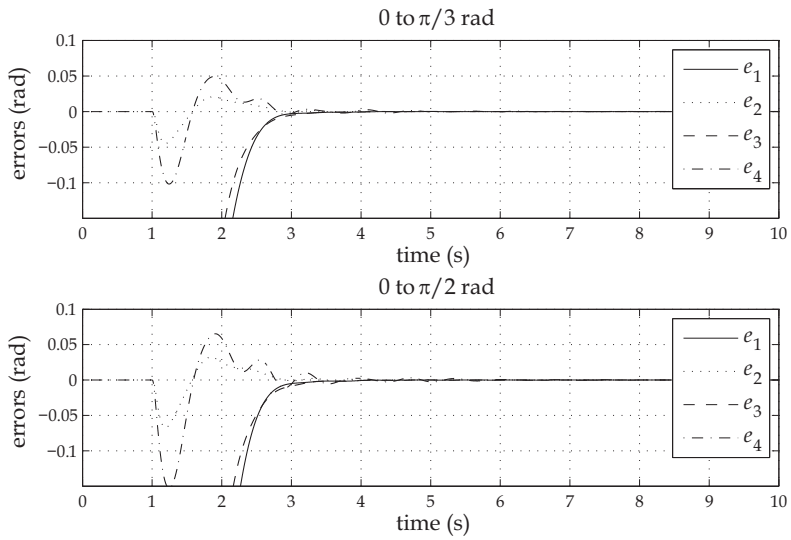


(a) Simulation results (large scale).

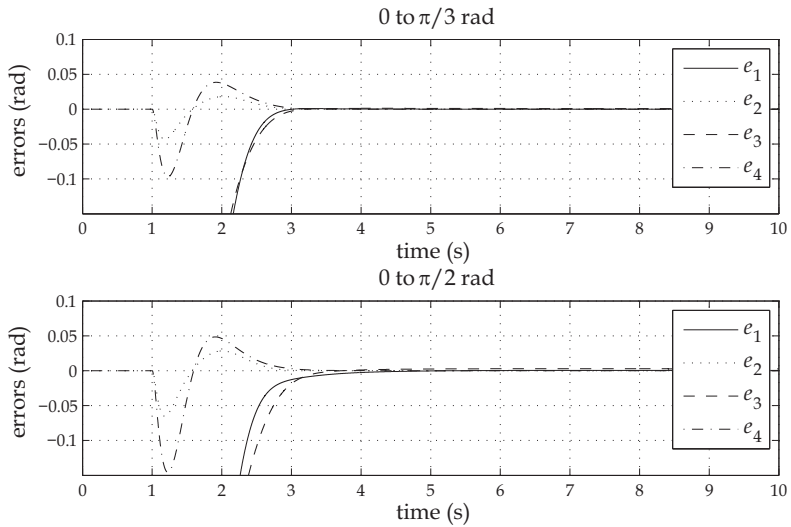


(b) Simulation results (fine scale).

Fig. 6. Simulation results using the optimal PD gains due to  $\eta_2$  ( $\zeta_{min}=0.40$ ).



(a) Simulation results with  $\zeta_{min} = 0.06$ .

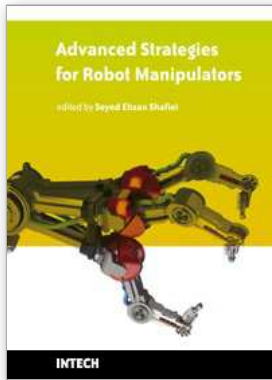


(b) Simulation results with  $\zeta_{min} = 1.00$ .

Fig. 7. Simulation results using the non-optimal PD gains.

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## **Advanced Strategies for Robot Manipulators**

Edited by S. Ehsan Shafiei

ISBN 978-953-307-099-5

Hard cover, 428 pages

**Publisher** Sciyo

**Published online** 12, August, 2010

**Published in print edition** August, 2010

Amongst the robotic systems, robot manipulators have proven themselves to be of increasing importance and are widely adopted to substitute for human in repetitive and/or hazardous tasks. Modern manipulators are designed complicatedly and need to do more precise, crucial and critical tasks. So, the simple traditional control methods cannot be efficient, and advanced control strategies with considering special constraints are needed to establish. In spite of the fact that groundbreaking researches have been carried out in this realm until now, there are still many novel aspects which have to be explored.

### **How to reference**

In order to correctly reference this scholarly work, feel free to copy and paste the following:

Masayoshi Toda (2010). A Unified Approach to Robust Control of Flexible Mechanical Systems Using H-Infinity Control Powered by PD Control, *Advanced Strategies for Robot Manipulators*, S. Ehsan Shafiei (Ed.), ISBN: 978-953-307-099-5, InTech, Available from: <http://www.intechopen.com/books/advanced-strategies-for-robot-manipulators/a-unified-approach-to-robust-control-of-flexible-mechanical-systems-using-h-infinity-control-powered>

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