Plasma stabilization system design on the base of model predictive control

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1. Introduction

Tokamaks, as future nuclear power plants, currently present exceptionally significant research area. The basic problems are electromagnetic control of the plasma current, shape and position. High-performance plasma control in a modern tokamak is the complex problem (Belyakov et al., 1999). This is mainly connected with the design requirements imposed on magnetic control system and power supply physical constraints. Besides that, plasma is an extremely complicated dynamical object from the modeling point of view and usually control system design is based on simplified linear system, representing plasma dynamics in the vicinity of the operating point (Ovsyannikov et al., 2005). This chapter is focused on the control systems design on the base of Model Predictive Control (MPC) (Camacho & Bordons, 1999; Morari et al., 1994). Such systems provide high-performance control in the case when accurate mathematical model of the plant to be controlled is unknown. In addition, these systems allow to take into account constraints, imposed both on the controlled and manipulated variables (Maciejowski, 2002). Furthermore, MPC algorithms can base on both linear and nonlinear mathematical models of the plant. So MPC control scheme is quite suitable for plasma stabilization problems.

In this chapter two different approaches to the plasma stabilization system design on the base of model predictive control are considered. First of them is based on the traditional MPC scheme. The most significant drawback of this variant is that it does not guarantee stability of the closed-loop control circuit. In order to eliminate this problem, a new control algorithm is proposed. This algorithm allows to stabilize control plant in neighborhood of the plasma equilibrium position. Proposed approach is based on the ideas of MPC and modal parametric optimization. Within the suggested framework linear closed-loop system eigenvalues are placed in the specific desired areas on the complex plane for each sample instant. Such areas are located inside the unit circle and reflect specific requirements and constraints imposed on closed-loop system stability and oscillations.

It is well known that the MPC algorithms are very time-consuming, since they require the repeated on-line solution of the optimization problem at each sampling instant. In order to reduce computational load, algorithms parameters tuning are performed and a special method is proposed in the case of modal parametric optimization based MPC algorithms.

The working capacity and effectiveness of the MPC algorithms is demonstrated by the example of ITER-FEAT plasma vertical stabilization problem. The comparison of the approaches is done.

2. Control Problem Formulation

2.1 Mathematical model of the plasma vertical stabilization process in ITER-FEAT tokamak

The dynamics of plasma control process can be commonly described by the system of ordinary differential equations (Misenov, 2000; Ovsyannikov et al., 2006)

$$\frac{d\Psi}{dt} + RI = V, \tag{1}$$

where Ψ is the poloidal flux vector, R is a diagonal resistance matrix, I is a vector of active and passive currents, V is a vector of voltages applied to coils. The vector Ψ is given by nonlinear relation

$$\Psi = \Psi(I,\, I_p), \tag{2}$$

where I_p is the plasma current. The vector of output variables is given by

$$y = y(I, I_p). \tag{3}$$

Linearizing equations (1)–(3) in the vicinity of the operating point, we obtain a linear model of the process in the state space form. In particular, the linear model describing plasma vertical control in ITER-FEAT tokamak is presented below.
ITER-FEAT tokamak (Gribov et al., 2000) has a separate fast feedback loop for plasma vertical stabilization. The Vertical Stabilization (VS) converter is applied in this loop. Its voltage is evaluated in the feedback controller, which uses the vertical velocity of plasma current centroid as an input. So the linear model can be written as follows

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{A}\mathbf{x} + \mathbf{b}u, \\ y &= \mathbf{c}\mathbf{x} + du, \end{aligned} \tag{4}$$

where $\mathbf{x} \in \mathbf{E}^{58}$ is a state space vector, $u \in E^1$ is the voltage of the VS converter, $y \in E^1$ is the vertical velocity of the plasma current centroid.
Since the order of this linear model is very high, an order reduction is desirable to simplify the controller synthesis problem. The standard Matlab function ***schmr*** was used to perform model reduction **from 58th to 3rd** order. As a result, we obtain a transfer function of the reduced SISO model (from input u to output y)

$$P(s) = \frac{1.732 \cdot 10^{-6}(s - 121.1)(s + 158.2)(s + 9.641)}{(s + 29.21)(s + 8.348)(s - 12.21)}. \tag{5}$$

This transfer function has poles which dominate the dynamics of the initial plant. The unstable pole corresponds to vertical instability. It is natural to assume that two other poles are determined by the virtual circuit dynamic related to the most significant elements in the tokamak vessel construction. The quality of the model reduction can be illustrated by the comparison of the Bode diagram for both initial and reduced models. Fig. 1 shows the Bode diagrams for initial and reduced 3^{rd} order models on the left and for initial and reduced 2^{nd} order model on the right. It is easy to see that the curves for initial model and reduced 3^{rd} order model are actually indistinguishable, contrary to the 2^{nd} order model.

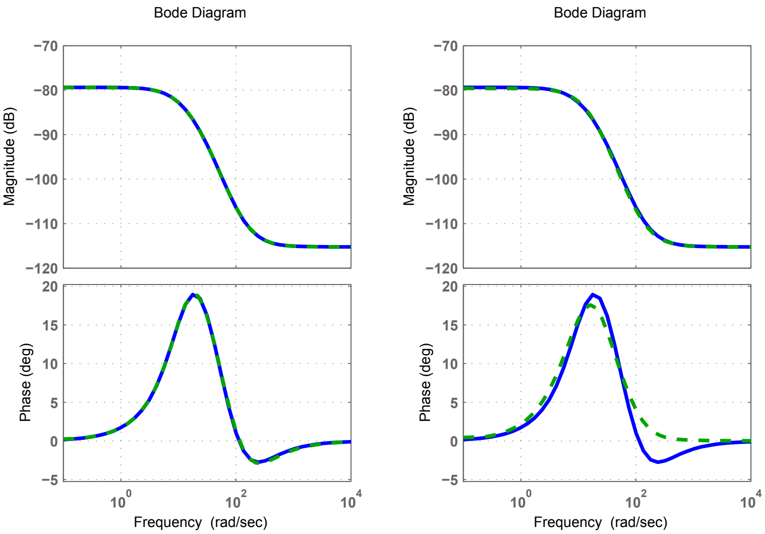


Fig. 1. Bode diagrams for initial (solid lines) and reduced (dotted lines) models.

In addition to plant model (5), we must take into account the following limits that are imposed on the power supply system

$$V_{\max}^{VS} = 0.6kV,\ I_{\max}^{VS} = 20.7kA, \tag{6}$$

where $V_{\max}^{VS}$ is the maximum voltage, $I_{\max}^{VS}$ is the maximum current in the VS converter. So, the linear model (5) together with constraints (6) is considered in the following as the basis for controller synthesis.

2.2 Optimal control problem formulation

The desired controller must stabilize vertical velocity of the plasma current centroid. One of the approaches to control synthesis is based on the optimal control theory. In this framework, plasma vertical stabilization problem can be stated as follows. One needs to find a feedback control algorithm $u = u(t, y)$ that provides a minimum of the quadratic cost functional

$$J = J(u) = \int_0^\infty (y^2(t) + \lambda u^2(t))dt, \tag{7}$$

subject to plant model (5) and constraints (6), and guarantees closed-loop stability. Here λ is a constant multiplier setting the trade-off between controller's performance and control energy costs.

Specifically, in order to find an optimal controller, LQG-synthesis can be performed. Such a controller has high stabilization performance in the unconstrained case. However, it is perhaps not the best choice in the presence of constraints.

Contrary to this, the MPC synthesis allows to take into account constraints. Its basic scheme implies on-line optimization of the cost functional (7) over a finite horizon subject to plant model (5) and imposed constraints (6).

3. Model Predictive Control Algorithms

3.1 MPC Basic Principles

Suppose we have a mathematical model, which approximately describes control process dynamics

$$\dot{\tilde{\mathbf{x}}}(\tau) = \mathbf{f}(\tau, \tilde{\mathbf{x}}(\tau), \tilde{\mathbf{u}}(\tau)), \quad \tilde{\mathbf{x}}|_{\tau=t} = \mathbf{x}(t). \tag{8}$$

Here $\tilde{\mathbf{x}}(\tau) \in \mathbf{E}^n$ is a state vector, $\tilde{\mathbf{u}}(\tau) \in \mathbf{E}^m$ is a control vector, $\tau \in [t, \infty)$, $\mathbf{x}(t)$ is the actual state of the plant at the instant t or its estimation based on measurement output.

This model is used to predict future outputs of the process given the programmed control $\tilde{\mathbf{u}}(\tau)$ over a finite time interval $\tau \in [t, t + T_p]$. Such a model is called **prediction model** and the parameter T_p is named **prediction horizon**. Integrating system (8) we obtain $\tilde{\mathbf{x}}(\tau) = \tilde{\mathbf{x}}(\tau, \mathbf{x}(t), \tilde{\mathbf{u}}(\tau))$—predicted process evolution over time interval $\tau \in [t, t + T_p]$.

The programmed control $\tilde{\mathbf{u}}(\tau)$ is chosen in order to minimize quadratic cost functional over the prediction horizon

$$J = J(\mathbf{x}(t), \tilde{\mathbf{u}}(\cdot), T_p) = \int_t^{t+T_p} ((\tilde{\mathbf{x}} - \mathbf{r}_x)' \mathbf{R}(\tilde{\mathbf{x}})(\tilde{\mathbf{x}} - \mathbf{r}_x) + (\tilde{\mathbf{u}} - \mathbf{r}_u)' \mathbf{Q}(\tilde{\mathbf{x}})(\tilde{\mathbf{u}} - \mathbf{r}_u)) d\tau, \tag{9}$$

where $\mathbf{R}(\tilde{\mathbf{x}})$, $\mathbf{Q}(\tilde{\mathbf{x}})$ are positive definite symmetric weight matrices, $\mathbf{r}_x$, $\mathbf{r}_u$ are state and control input reference signals. In addition, the programmed control $\tilde{\mathbf{u}}(\tau)$ should satisfy all of the constraints imposed on the state and control variables. Therefore, the programmed control $\tilde{\mathbf{u}}(\tau)$ over prediction horizon is chosen to provide minimum of the following optimization problem

$$J(\mathbf{x}(t), \tilde{\mathbf{u}}(\cdot), T_p) \to \min_{\tilde{\mathbf{u}}(\cdot) \in \Omega_u}, \tag{10}$$

where Ω_u is the admissible set given by

$$\Omega_u = \left\{ \tilde{\mathbf{u}}(\cdot) \in \mathbf{K}_n^0[t, t+T_p] : \tilde{\mathbf{u}}(\tau) \in \mathbf{U},\ \tilde{\mathbf{x}}(\tau, \mathbf{x}(t), \tilde{\mathbf{u}}(\tau)) \in \mathbf{X},\ \forall \tau \in [t, t+T_p] \right\}. \tag{11}$$

Here, $\mathbf{K}_n^0[t, t+T_p]$ is the set of piecewise continuous vector functions over the interval $[t, t+T_p]$, $\mathbf{U} \subset \mathbf{E}^m$ is the set of feasible input values, $\mathbf{X} \subset \mathbf{E}^n$ is the set of feasible state values. Denote by $\tilde{\mathbf{u}}^*(\tau)$ the solution of the optimization problem (10), (11). In order to implement feedback loop, the obtained optimal programmed control $\tilde{\mathbf{u}}^*(\tau)$ is used as the input only on the time interval $[t, t+\delta]$, where $\delta << T_p$. So, only a small part of $\tilde{\mathbf{u}}^*(\tau)$ is implemented. At time $t + \delta$ the whole procedure—prediction and optimization—is repeated again to find new optimal programmed control over time interval $[t + \delta,\ t + \delta + T_p]$. Summarizing, the basic MPC scheme works as follows:

1. Obtain the state estimation $\hat{\mathbf{x}}$ on the base of measurements $\mathbf{y}$.
2. Solve the optimization problem (10), (11) subject to prediction model (8) with initial conditions $\tilde{\mathbf{x}}|_{\tau=t} = \hat{\mathbf{x}}(t)$ and cost functional (9).
3. Implement obtained optimal control $\tilde{\mathbf{u}}^*(\tau)$ over time interval $[t, t+\delta]$.
4. Repeat the whole procedure 1–3 at time $t + \delta$.

From the previous discussion, the most significant MPC features can be noted:

- Both linear and nonlinear model of the plant can be used as a prediction model.
- MPC allows taking into account constraints imposed both on the input and output variables.

- MPC is the feedback control with the discrete entering of the measurement information at each sampling instant $0, \delta, 2\delta, \ldots$.
- MPC control algorithms imply the repeated (at each sampling instant with interval δ) on-line solution of the optimization problems. It is especially important from the real-time implementation point of view, because fast calculations are needed.

3.2 MPC real-time implementation

In order for real-time implementation, piece-wise constant functions are used as a programmed control over the prediction horizon. That is, the programmed control $\tilde{\mathbf{u}}(\tau)$ is presented by the sequence$\{\tilde{\mathbf{u}}_k, \tilde{\mathbf{u}}_{k+1}, ..., \tilde{\mathbf{u}}_{k+P-1}\}$, where $\tilde{\mathbf{u}}_i \in \mathbf{E}^m$ is the control input at the time interval $[i\delta, (i+1)\delta]$, δ is the sampling interval. Note that, P is a number of sampling intervals over the prediction horizon, that is $T_p = P\delta$. Likewise, general MPC formulation presented above consider nonlinear prediction model in the discrete form

$$\begin{aligned} &\tilde{\mathbf{x}}_{i+1} = \mathbf{f}(\tilde{\mathbf{x}}_i, \tilde{\mathbf{u}}_i), \quad i = k + j, \quad j = 0, 1, 2, ..., \quad \tilde{\mathbf{x}}_k = \mathbf{x}_k, \\ &\tilde{\mathbf{y}}_i = \mathbf{C}\tilde{\mathbf{x}}_i. \end{aligned} \tag{12}$$

Here $\tilde{\mathbf{y}}_i \in \mathbf{E}^r$ is the vector of output variables, $\mathbf{x}_k \in \mathbf{E}^n$ is the actual state of the plant at time instant k or its estimation on the base of measurement output. We shall say that the sequence of vectors $\{\tilde{\mathbf{y}}_{k+1}, \tilde{\mathbf{y}}_{k+2}, ..., \tilde{\mathbf{y}}_{k+P}\}$ represents the prediction of future plant behavior.
Similar to the cost functional (9), consider also its discrete analog given by

$$\begin{aligned} J_k = J_k(\bar{\mathbf{y}}, \bar{\mathbf{u}}) = \textstyle\sum_{j=1}^{P} \big[&(\tilde{\mathbf{y}}_{k+j} - \mathbf{r}^y_{k+j})^T \mathbf{R}_{k+j} (\tilde{\mathbf{y}}_{k+j} - \mathbf{r}^y_{k+j}) \\ &+ (\tilde{\mathbf{u}}_{k+j-1} - \mathbf{r}^u_{k+j-1})^T \mathbf{Q}_{k+j} (\tilde{\mathbf{u}}_{k+j-1} - \mathbf{r}^u_{k+j-1}) \big] , \end{aligned} \tag{13}$$

where $\mathbf{R}_{k+j}$ and $\mathbf{Q}_{k+j}$ are the weight matrices as in the functional (9), $\mathbf{r}^y_i$ and $\mathbf{r}^u_i$ are the output and input reference signals,

$$\begin{aligned} \bar{\mathbf{y}} &= \begin{pmatrix} \tilde{\mathbf{y}}_{k+1} & \tilde{\mathbf{y}}_{k+2} & \cdots & \tilde{\mathbf{y}}_{k+P} \end{pmatrix}^T \in \mathbf{E}^{rP}, \\ \bar{\mathbf{u}} &= \begin{pmatrix} \tilde{\mathbf{u}}_{k} & \tilde{\mathbf{u}}_{k+1} & \ldots & \tilde{\mathbf{u}}_{k+P-1} \end{pmatrix}^T \in \mathbf{E}^{mP} \end{aligned}$$

are the auxiliary vectors.
The optimization problem (10), (11) can now be stated as follows

$$J_k(\mathbf{x}_k, \tilde{\mathbf{u}}_k, \tilde{\mathbf{u}}_{k+1}, ...\tilde{\mathbf{u}}_{k+P-1}) \to \min_{\{\tilde{\mathbf{u}}_k, \tilde{\mathbf{u}}_{k+1}, ..., \tilde{\mathbf{u}}_{k+P-1}\} \in \Omega \in \mathbf{E}^{mP}}, \tag{14}$$

where $\Omega = \left\{ \bar{\mathbf{u}} \in \mathbf{E}^{mP} : \tilde{\mathbf{u}}_{k+j-1} \in \mathbf{U}, \ \tilde{\mathbf{x}}_{k+j} \in \mathbf{X}, \ j = 1, 2, ..., P \right\}$ is the admissible set.
Generally, the function $J(\mathbf{x}_k, \tilde{\mathbf{u}}_k, \tilde{\mathbf{u}}_{k+1}, ...\tilde{\mathbf{u}}_{k+P-1})$ is a nonlinear function of mP variables and Ω is a non-convex set. Therefore, the optimization task (14) is a nonlinear programming problem.
Now real-time MPC algorithm can be presented as follows:

1. Obtain the state estimation $\hat{\mathbf{x}}_k$ based on measurements $\mathbf{y}_k$ using the observer.
2. Solve the nonlinear programming problem (14) subject to prediction model (12) with initial conditions $\tilde{\mathbf{x}}_k = \hat{\mathbf{x}}_k$ and cost functional (13). It should be noted, that the value of the function $J_k(\mathbf{x}_k, \tilde{\mathbf{u}}_k, \tilde{\mathbf{u}}_{k+1}, ...\tilde{\mathbf{u}}_{k+P-1})$ is obtained by numerically integrating the prediction model (12) and then substituting the predicted behavior $\bar{\mathbf{x}} \in \mathbf{E}^{nP}$ into the cost function (13) given the programmed control $\{\tilde{\mathbf{u}}_k, \tilde{\mathbf{u}}_{k+1}, ..., \tilde{\mathbf{u}}_{k+P-1}\}$ over the prediction horizon and initial conditions $\hat{\mathbf{x}}_k$.

3. Let $\{\tilde{\mathbf{u}}_k^*, \tilde{\mathbf{u}}_{k+1}^*, ..., \tilde{\mathbf{u}}_{k+P-1}^*\}$ be the solution of the problem (14). Implement only the first component $\tilde{\mathbf{u}}_k^*$ of the obtained optimal sequence over time interval $[k\delta, (k+1)\delta]$.
4. Repeat the whole procedure 1–3 at next time instant $(k+1)\delta$.

Note, that the algorithm stated above implies real-time solution of the nonlinear programming problem at each sampling instant. The complexity of such a problem is determined by the number of sampling intervals P.
The simplest way to reduce the optimization problem order is to decrease the prediction horizon. But, it is necessary to keep in mind that the performance of the closed-loop system depends strongly on the number P of samples. The quality of the processes is decreased if the prediction horizon is reduced. Moreover, the system can lose stability if the quantity P is sufficiently small.
So, the following approaches to reduce computational load can be proposed:

1. Using the **control horizon**. The positive integer number $M < P$ is called the **control horizon** if the following condition hold:

$$\tilde{\mathbf{u}}_{k+M-1} = \tilde{\mathbf{u}}_{k+M} = ... = \tilde{\mathbf{u}}_{k+P-1}.$$

   Thus, the number of independent variables is decreased from mP to mM. This approach allows to essentially reduce the optimization problem order. However, if the control horizon M is too small, the closed-loop stability can be compromised and the quality of the processes can decrease.
2. Increasing the sampling interval δ and reducing the number P of samples over the prediction horizon. This also allows to decrease the optimization problem order while preserving the value of the prediction horizon.
3. The computational consumption also depends on the prediction model used. So, one needs to use as simple models as possible. But the prediction model should adequately reflect the dynamics of the plant considered. The simplest case is using the linear prediction model.

3.3 Linear MPC

In this particular case, MPC is based on the linear prediction model. These algorithms are computationally efficient which is especially important from the real-time implementation point of view.
Generally, linear prediction model is presented by

$$\begin{aligned} &\tilde{\mathbf{x}}_{i+1} = \mathbf{A}\tilde{\mathbf{x}}_i + \mathbf{B}\tilde{\mathbf{u}}_i, \quad i = k+j, \quad j = 0,1,2,..., \quad \tilde{\mathbf{x}}_k = \mathbf{x}_k, \\ &\tilde{\mathbf{y}}_i = \mathbf{C}\tilde{\mathbf{x}}_i. \end{aligned} \tag{15}$$

Suppose $\bar{\mathbf{u}} = \begin{pmatrix} \tilde{\mathbf{u}}_k & \tilde{\mathbf{u}}_{k+1} & ... & \tilde{\mathbf{u}}_{k+P-1} \end{pmatrix}^T$ is the programmed control over the prediction horizon. Then, integrating (15) we obtain future outputs of the plant in the form

$$\bar{\mathbf{y}} = \mathbf{L}\mathbf{x}_k + \mathbf{M}\bar{\mathbf{u}}, \tag{16}$$

where

$$\mathbf{L} = \begin{bmatrix} \mathbf{CA} \\ \mathbf{CA}^2 \\ \vdots \\ \mathbf{CA}^P \end{bmatrix}, \quad \mathbf{M} = \begin{bmatrix} \mathbf{CB} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{CAB} & \ddots & & \\ \vdots & & \ddots & \\ \mathbf{CA}^{P-1}\mathbf{B} & \dots & \mathbf{CAB} & \mathbf{CB} \end{bmatrix}.$$

Substituting (16) into (13) we get

$$J_k = J_k(\mathbf{x}_k, \bar{\mathbf{u}}) = \bar{\mathbf{u}}^T \mathbf{H}\bar{\mathbf{u}} + 2\mathbf{f}^T\bar{\mathbf{u}} + g. \tag{17}$$

Here we assumed that all weight matrices are equal, that is

$$\begin{aligned}\mathbf{R}_{k+1} = \mathbf{R}_{k+2} = ... = \mathbf{R}_{k+P} = \mathbf{R},\\ \mathbf{Q}_{k+1} = \mathbf{Q}_{k+2} = ... = \mathbf{Q}_{k+P} = \mathbf{Q}.\end{aligned}$$

The matrix $\mathbf{H}$ and vector $\mathbf{f}$ in (17) are as follows

$$\mathbf{H} = \mathbf{M}'\mathbf{R}\mathbf{M} + \mathbf{Q}, \quad \mathbf{f} = \mathbf{M}'\mathbf{R}\mathbf{L}\mathbf{x}_k. \tag{18}$$

It can easily be shown that in this case the optimization problem (14) is reduced to the quadratic programming problem of the form

$$J_k(\mathbf{x}_k, \tilde{\mathbf{u}}_k, \tilde{\mathbf{u}}_{k+1}, ..., \tilde{\mathbf{u}}_{k+P-1}) = \bar{\mathbf{u}}^T \mathbf{H}\bar{\mathbf{u}} + 2\mathbf{f}^T\bar{\mathbf{u}} + g \rightarrow \min_{\bar{\mathbf{u}} \in \Omega \subset \mathbf{E}^{mP}}. \tag{19}$$

Here $\mathbf{H}$ is a positive definite matrix and Ω is a convex set defined by the system of linear constraints. On-line solution of the optimization problem (19) at each sampling instant generally leads to nonlinear feedback control law.

Note that the optimization problem (19) can be solved analytically for the unconstrained case. The result is the linear controller

$$\tilde{\mathbf{u}}_k = \mathbf{K}\tilde{\mathbf{x}}_k, \tag{20}$$

which converges to the LQR-optimal one as P is increased. This convergence is obvious, because the discrete LQR controller minimizes the functional (13) with infinity prediction horizon for linear model (15).

4. Model Predictive Control on the base of modal parametrical optimization

In this section a new approach to MPC control algorithm synthesis is considered. The key feature of corresponding algorithms is that they guarantee linear closed-loop system stability at each sampling period. It is necessary to remark that in the case of traditional MPC algorithm implementation, described above, closed-loop system stability can be provided only for the simplest case when we have a linear prediction model, quadratic cost functional and without constraints.

Let us assume that the mathematical model of the plant to be controlled is described by the following system of difference equations

$$\begin{aligned}\hat{\mathbf{x}}_{k+1} = \mathbf{F}(\hat{\mathbf{x}}_k, \hat{\mathbf{u}}_k, \hat{\varphi}_k),\\ \hat{\mathbf{y}}_k = \mathbf{C}\hat{\mathbf{x}}_k.\end{aligned} \tag{21}$$

Here $\hat{\mathbf{y}}_k \in \mathbf{E}^s$ is the vector of output variables, $\hat{\mathbf{x}}_k \in \mathbf{E}^n$ is the state space vector, $\hat{\mathbf{u}}_k \in \mathbf{E}^m$ is the vector of controls, $\hat{\varphi}_k \in \mathbf{E}^l$ is the vector of external disturbances.

Equations (21) can be used as a basis for nonlinear prediction model construction. Suppose that obtained prediction model is given by

$$\begin{aligned}\tilde{\mathbf{x}}_{i+1} = \mathbf{f}(\tilde{\mathbf{x}}_i, \tilde{\mathbf{u}}_i), \quad i = k + j, \quad j = 0, 1, 2, ..., \quad \tilde{\mathbf{x}}_k = \mathbf{x}_k,\\ \tilde{\mathbf{y}}_i = \mathbf{C}\tilde{\mathbf{x}}_i.\end{aligned} \tag{22}$$

Here $\mathbf{x}_k \in \mathbf{E}^n$ is the actual state of the plant at time instant k or its estimation on the base of measurement output.
Let desired object dynamics is presented by the given vector sequences $\{\mathbf{r}_k^x\}$ and $\{\mathbf{r}_k^u\}$, k = 0,1,2,... . The linear mathematical model of the plant, describing its behavior in the neighbourhood of the desired trajectory, can be obtained by performing the equations (21) linearization. As a result of this action, we get the linear system of difference equations

$$\begin{aligned} \bar{\mathbf{x}}_{k+1} &= \mathbf{A}\bar{\mathbf{x}}_k + \mathbf{B}\bar{\mathbf{u}}_k + \mathbf{H}\bar{\varphi}_k, \\ \bar{\mathbf{y}}_k &= \mathbf{C}\bar{\mathbf{x}}_k, \end{aligned} \tag{23}$$

where $\bar{\mathbf{x}}_k \in \mathbf{E}^n$, $\bar{\mathbf{u}}_k \in \mathbf{E}^m$, $\bar{\mathbf{y}}_k \in \mathbf{E}^s$, $\bar{\varphi}_k \in \mathbf{E}^l$ are the vectors of the state, control input, measurements and external disturbances respectively. These vectors represent the deviations from the desired trajectory. Next we shall consider only such situations when all matrices in equations (23) have constant elements. In the framework of proposed approach, the control input over the prediction horizon is generated by the controller of the form

$$\bar{\mathbf{u}}_k = \mathbf{W}(q, \mathbf{h})\bar{\mathbf{y}}_k. \tag{24}$$

Here q is the shift operator, $\mathbf{W}(q, \mathbf{h})$ is the controller transfer function with the fixed structure (that is the degrees of the polynomials in the numerator and denominator of all its components are given and fixed), $\mathbf{h} \in \mathbf{E}^r$ is the vector of tuned parameters, which must be chosen on the stage of control design.
The prediction model equations (22), closed by the feedback (24), can be presented as follows

$$\begin{aligned} &\tilde{\mathbf{x}}_{i+1} = \mathbf{f}(\tilde{\mathbf{x}}_i, \tilde{\mathbf{u}}_i), \quad i = k + j, \quad j = 0, 1, 2, ..., \quad \tilde{\mathbf{x}}_k = \mathbf{x}_k, \\ &\tilde{\mathbf{u}}_i = \mathbf{r}_i^u + \mathbf{W}(q, \mathbf{h})\mathbf{C}(\tilde{\mathbf{x}}_i - \mathbf{r}_i^x). \end{aligned} \tag{25}$$

Let us assume that parameters vector $\mathbf{h}$ is chosen and fixed. Then we can solve system of difference equations (25) with a given initial conditions for the instants $i = k, k+1, ..., k+P-1$. As a result we obtain vectors sequence $\{\tilde{\mathbf{x}}_i\}$, $(i = k+1, ..., k+P)$, which represents the prediction of future plant behavior over the prediction horizon P. It must be noted, that the control sequence $\tilde{\mathbf{u}}_k, \tilde{\mathbf{u}}_{k+1}, ...\tilde{\mathbf{u}}_{k+P-1}$ over this horizon is determined uniquely by the choice of parameter vector $\mathbf{h}$. So, in this case the problem of control is reduced to the problem of parameters vector $\mathbf{h}$ tuning.
The controlled processes quality over the prediction horizon P can be presented by the following cost functional

$$J_k = J_k(\{\tilde{\mathbf{x}}_i\}, \{\tilde{\mathbf{u}}_i\}) = J_k(\mathbf{W}(q, \mathbf{h})) = J_k(\mathbf{h}) \geq 0, \tag{26}$$

where $\{\tilde{\mathbf{x}}_i\}$, $i = k+1, ..., k+P$, $\{\tilde{\mathbf{u}}_i\}$, $i = k, ..., k+P-1$ are the state and control vectors sequences correspondently, which satisfies the system of equations (25). It is easy to see, that the cost functional (26) is reduced to the function of parameter vector $\mathbf{h}$.
Let us consider the following optimization problem

$$J_k = J_k(\mathbf{h}) \to \inf_{\mathbf{h} \in \Omega_H}, \tag{27}$$

where Ω_H is a set of parameter vectors providing that the eigenvalues of the closed-loop system (23), (24) are placed in the desired area C_Δ inside a unit circle.
It is necessary to remark that the problem (27) is a nonlinear programming problem with an extremely complicated definition of the cost function, which, in generall, has no analytical

representation and is given only algorithmically. Besides that, the specific character of the problem (27) is also defined by the complicated constraints imposed, which determines the admissible areas of eigenvalues displacement. It must be noted, that the dimension of the optimization problem (27) is defined only by the dimension of parameter vector **h** and it does not depend on the prediction horizon P value.

Definition 1. We shall say that the controller (24) has a **full structure** if the degrees of polynomials in the numerators and denominators of the matrix $\mathbf{W}(q, \mathbf{h})$ components and the structure of parameter vector **h** are such that it is possible to assign any given roots for closed-loop system (23),(24) characteristic polynomial $\Delta(z, \mathbf{h})$ by appropriate selection of parameter vector **h**.

In order to get another form of the presented definition, consider the equations of the closed-loop system (23),(24). They can be represented in the normal form as follows

$$\begin{array}{l}\bar{\mathbf{x}}_{k+1} = \mathbf{A}\bar{\mathbf{x}}_k + \mathbf{B}\bar{\mathbf{u}}_k + \mathbf{H}\bar{\varphi}_k,\\ \bar{\mathbf{y}}_k = \mathbf{C}\bar{\mathbf{x}}_k,\\ \xi_{k+1} = \mathbf{A}_c(\mathbf{h})\xi_k + \mathbf{B}_c(\mathbf{h})\bar{\mathbf{y}}_k,\\ \bar{\mathbf{u}}_k = \mathbf{C}_c(\mathbf{h})\xi_k + \mathbf{D}_c(\mathbf{h})\bar{\mathbf{y}}_k,\end{array} \tag{28}$$

where $\xi_k \in \mathbf{E}^\nu$ is a controller (24) state vector. After applying Z-transformation to the system of equations (28) with zero initial conditions, obtain

$$\begin{array}{l}(\mathbf{E}_n z - \mathbf{A})\bar{\mathbf{x}} = \mathbf{B}\bar{\mathbf{u}} + \mathbf{H}\bar{\varphi},\\ (\mathbf{E}_\nu z - \mathbf{A}_c(\mathbf{h}))\xi = \mathbf{B}_c(\mathbf{h})\mathbf{C}\bar{\mathbf{x}},\\ \bar{\mathbf{u}} = \mathbf{C}_c(\mathbf{h})\xi + \mathbf{D}_c(\mathbf{h})\mathbf{C}\bar{\mathbf{x}},\\ \bar{\mathbf{y}} = \mathbf{C}\bar{\mathbf{x}},\end{array}$$

or

$$\begin{pmatrix} \mathbf{E}_n z - \mathbf{A} - \mathbf{B}\mathbf{D}_c(\mathbf{h})\mathbf{C} & -\mathbf{B}\mathbf{C}_c(\mathbf{h}) \\ -\mathbf{B}_c(\mathbf{h})\mathbf{C} & \mathbf{E}_\nu z - \mathbf{A}_c(\mathbf{h}) \end{pmatrix} \begin{pmatrix} \bar{\mathbf{x}} \\ \xi \end{pmatrix} = \begin{pmatrix} \mathbf{H} \\ \mathbf{0} \end{pmatrix} \bar{\varphi}.$$

Therefore, the closed-loop system characteristic polynomial $\Delta(z, \mathbf{h})$ is given by

$$\Delta(z, \mathbf{h}) = det \begin{pmatrix} \mathbf{E}_n z - \mathbf{A} - \mathbf{B}\mathbf{D}_c(\mathbf{h})\mathbf{C} & -\mathbf{B}\mathbf{C}_c(\mathbf{h}) \\ -\mathbf{B}_c(\mathbf{h})\mathbf{C} & \mathbf{E}_\nu z - \mathbf{A}_c(\mathbf{h}) \end{pmatrix}.$$

Let us denote the degree of the polinomial $\Delta(z, \mathbf{h})$ by n_d.
Let find parameter vector **h**, which provide a given roots for the system (28) characteristic polynomial. In other words, it is nesessary to find such parameter vector **h** that provide the following identity

$$\Delta(z, \mathbf{h}) \equiv \tilde{\Delta}(z),$$

where $\tilde{\Delta}(z)$ is a given polynomial with degree n_d, having desired roots. In order to find vector **h**, equate the correspondent coefficients for the same degrees of z-variable. As a result obtain the system of $(n_d + 1)$ nonlinear equations with r unknown components of vector **h** in the form

$$\mathbf{L}(\mathbf{h}) = \gamma. \tag{29}$$

It is evident that the controller (24) has a full structure if and only if the system of equations (33) has a solution for any vector γ.

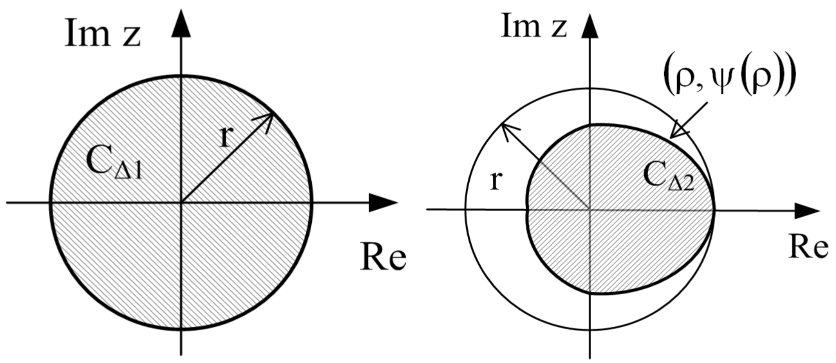
It can be easy shown that if the parameter vector $\mathbf{h}$ consists of the coefficients of numerator and denominator polynomials of matrix $\mathbf{W}(q, \mathbf{h})$, then the system (29) reduced to the linear system of the form

$$\mathbf{Lh} = \gamma, \quad (30)$$

where L is a constant matrix. Note that for any case, the controller (24) has a full structure only if the system (23) is fully controllable and observable.
Let us now refine the optimization problem (27) statement in suppose that the controller (23) has a full structure and that the following set Ω_H is determined as admissible set of the form

$$\Omega_H = \{\mathbf{h} \in \mathbf{E}^r : \delta_i(\mathbf{h}) \in C_\Delta,\ i = 1, 2, ..., n_d\}. \quad (31)$$

Here δ_i is the roots of the characteristic polynomial $\Delta(z, \mathbf{h})$, $n_d = deg\Delta(z, \mathbf{h})$.
Let consider two different variants of the desired areas C_Δ, depicted in Fig. 2. This areas are located inside a unit circle, i. e. $r < 1$.



(a) area $C_{\Delta 1}$ (b) area $C_{\Delta 2}$

Fig. 2. The areas $C_{\Delta 1}$ and $C_{\Delta 2}$ of the desired root displacement

The formalized description for the desired areas C_Δ are as follows:
$C_\Delta = C_{\Delta 1} = \{z \in \mathbf{C}^1 : |z| \le r\}$, where $r \in (0, 1)$ is a given real number;
$C_\Delta = C_{\Delta 2} = \{z \in \mathbf{C}^1 : z = \rho exp(\pm i\varphi), 0 \le \rho \le r, 0 \le \varphi \le \psi(\rho)\}$, where $r \in (0, 1)$ is a given real number, $\psi(\xi)$ is a real function of variable $\xi \in (0, r]$, which takes the values on the interval $[0, \pi]$ and $\psi(r) = 0$.
The reasons of these areas introduction is obvious. The first area $C_{\Delta 1}$ determines the lower bound for the closed-loop system stability margin and, therefore, the settling time for transient processes. Second area $C_{\Delta 2}$ determines stability bound and, in addition, constraints on the closed-loop system oscillations.
In order to form the algorithm for the problem (27) solution on the admissible set (31), let us firstly perform the parametrization of the considered areas C_Δ with the n-dimensional real vectors on the base of the following statement.

Theorem 1. *For any real vector $\gamma \in \mathbf{E}^{n_d}$ the roots of the polynomial $\Delta^*(z, \gamma)$, given by the formulas presented below, are located inside the area $C_{\Delta 1}$ or on its bound. And reversly, if the roots of the some* *polynomial $\Delta(z)$ are located inside the area $C_{\Delta 1}$ and, in addition, all its real roots are positive, then it can be found such a vector $\gamma \in \mathbf{E}^{n_d}$ that the following identity holds $\Delta(z) \equiv \Delta^*(z, \gamma)$. Here*

$$\Delta^*(z, \gamma) = \prod_{i=1}^{d} (z^2 + a_i^1(\gamma, r) z + a_i^0(\gamma, r)), \tag{32}$$

if n_d is even, $d = n_d/2$;

$$\Delta^*(z, \gamma) = (z - a_{d+1}(\gamma, r)) \prod_{i=1}^{d} (z^2 + a_i^1(\gamma, r) z + a_i^0(\gamma, r)), \tag{33}$$

if n_d is odd, $d = [n_d/2]$;

$$\begin{gathered} a_i^1(\gamma, r) = -r \left(exp\left(-\tfrac{\gamma_{i1}^2}{2} + \sqrt{\tfrac{\gamma_{i1}^4}{4} - \gamma_{i2}^2} \right) + exp\left(-\tfrac{\gamma_{i1}^2}{2} - \sqrt{\tfrac{\gamma_{i1}^4}{4} - \gamma_{i2}^2} \right) \right), \\ a_i^0(\gamma, r) = r^2 exp\left(-\gamma_{i1}^2\right), \; i = 1, ..., d, \quad a_{d+1}(\gamma, r) = r\, exp\left(-\gamma_{d0}^2\right), \end{gathered} \tag{34}$$

$$\gamma = \{\gamma_{11}, \gamma_{12}, \gamma_{21}, \gamma_{22}, ..., \gamma_{d1}, \gamma_{d2}, \gamma_{d0}\}. \tag{35}$$

Proof If the n_d is even, then the proof of the direct and reverse propositions arises from the elementary properties of the quadratic trinomials in the formula (32). Really, for any given pair of the real numbers γ_{i1}, γ_{i2} the roots of the trinomial $\Delta_i^*(z)$ in (32) are presented by the expression

$$z_{1,2}^i = r \cdot exp\left(-\frac{\gamma_{i1}^2}{2} \pm \sqrt{\frac{\gamma_{i1}^4}{4} - \gamma_{i2}^2} \right).$$

From this expression it follows that $|z_{1,2}^i| \leq r$ and, therefore, the roots $z_{1,2}^i$ of the trinomial are located inside the area $C_{\Delta 1}$ or on its bound, and this proves the direct proposition.

In order to prove reverse one, let consider some quadratic trinomial of the form $\Delta_i(z) = z^2 + \beta_1 z + \beta_0$. By the conditions of the reverse proposition, the roots $z_{1,2}$ of this trinomial are located inside the area $C_{\Delta 1}$ and, if the roots are real numbers, then they are positive. In order to locate the roots $z_{1,2}$ inside the area $C_{\Delta 1}$, it is necessary and sufficient that the following relations holds

$$1 - \frac{\beta_1}{r} + \frac{\beta_0}{r^2} \geq 0, \; 1 - \frac{\beta_0}{r^2} \geq 0, \; 1 + \frac{\beta_1}{r} + \frac{\beta_0}{r^2} \geq 0. \tag{36}$$

Besides that, the roots product $z_1 z_2$ is positive in anycase if they are being complex conjugated pair or positive real numbers. Therefore, the following inequality is true

$$\beta_0 > 0. \tag{37}$$

Let find such numbers γ_{i1} and γ_{i2} that the identity $\Delta_i^*(z) \equiv \Delta_i(z)$ is satisfied. By equating the correspondent coefficients for the same degrees of z-variable, obtain

$$-r \left(exp\left(-\frac{\gamma_{i1}^2}{2} + \sqrt{\frac{\gamma_{i1}^4}{4} - \gamma_{i2}^2} \right) + exp\left(-\frac{\gamma_{i1}^2}{2} - \sqrt{\frac{\gamma_{i1}^4}{4} - \gamma_{i2}^2} \right) \right) = \beta_1,$$

$$r^2 exp(-\gamma_{i1}^2) = \beta_0,$$

and consequently

$$\gamma_{i1} = \sqrt{-ln\left(\beta_0/r^2\right)},$$
$$\gamma_{i2} = \sqrt{-\tfrac{1}{4} ln\left(w\tfrac{r^2}{\beta_0}\right) ln\left(w\tfrac{\beta_0}{r^2}\right)}, \quad where \quad w = \tfrac{\beta_1^2}{2\beta_0} - 1 + \sqrt{\left(\tfrac{\beta_1^2}{2\beta_0} - 1\right)^2 - 1}. \tag{38}$$

Now let verify that the γ_{i1} and γ_{i2}, given by the formulas (38), are the real numbers.
Really, from the inequalities (36), (37) it follows that $0 < \beta_0/r^2 \leq 1$, therefore $-ln\left(\beta_0/r^2\right) \geq 0$ and γ_{i1} is a real number.
Let show that the expression under radical in the formula for γ_{i2} is nonnegative. For the first, consider the case when the trinomial $\Delta_i(z)$ has two real positive roots $z_{1,2}$. Then his coefficients must satisfies to the condition $\beta_1^2 - 4\beta_0 \geq 0$, whence it follows that $w \geq 1$ – is a real number. As a result, taking into account (36), we obtain

$$ln\left(w \cdot r^2/\beta_0\right) \geq 0. \tag{39}$$

It could be noted that the inequalities (36) implies also the satisfaction of the inequality $\beta_1^2 - 2\beta_0 \leq r^2 + \beta_0/r^2$. Hence, we have

$$w\beta_0 \leq r^2, \ \ and \ \ -ln\left(w\beta_0/r^2\right) \geq 0. \tag{40}$$

Thus from the inequalities (39) and (40) it is easy to see that the expression under radical in the formula for γ_{i2} is nonnegative and γ_{i2} is a real number.
Consider now a case, when the trinomial $\Delta_i(z)$ has a pair of complex-conjugate roots $z_{1,2}$. Then the following inequality is hold $\beta_1^2 - 4\beta_0 < 0$, and therefore w is a complex number, which can be presented in the form $w = \beta_1^2/2\beta_0 - 1 + i\sqrt{1 - \left(\beta_1^2/2\beta_0 - 1\right)^2}$. It is not difficult to see that $|w| = 1$, hence, the expression under the radical for γ_{i2} has a form

$$\gamma_{i2} = \sqrt{-\frac{1}{4}\left(ln\left(\frac{r^2}{\beta_0}\right) + i \cdot argw\right)\left(ln\left(\frac{\beta_0}{r^2}\right) + i \cdot argw\right)} = \sqrt{\frac{1}{4}\left(ln^2\left(\frac{r^2}{\beta_0}\right) + arg^2 w\right)},$$

i.e. it is nonnegative and γ_{i2} is a real number.
If the n_d is odd, the polynomial Δ^* has, in according to (33), an additional linear binomial, for which the propositions of the theorem are evident.■
Now consider more difficult second variant of the admissible set C_Δ. Let us prove the analogous theorem, which allows to perform parametrization of this area.

Theorem 2. *For any real vector $\gamma \in \mathbf{E}^{n_d}$ the roots of the polynomial $\Delta^*(z,\gamma)$ (32),(33) are located inside the area $C_{\Delta 2}$, and reversly, if the roots of the some polynomial $\Delta(z)$ are located inside the area $C_{\Delta 2}$ and, in addition, all its real roots are positive, then it can be found such a vector $\gamma \in \mathbf{E}^{n_d}$ that the following identity holds $\Delta(z) \equiv \Delta^*(z,\gamma)$. Here*

$$a_i^1(\gamma,r) = -r\left(exp\left(-\gamma_{i1}^2 + \nu_i\right) + exp\left(-\gamma_{i1}^2 - \nu_i\right)\right),$$
$$a_i^0(\gamma,r) = r^2 exp\left(-2\gamma_{i1}^2\right), \ i = 1,...,d, \ \ a_{d+1}(\gamma,r) = r \cdot exp(-\gamma_{d0}^2), \tag{41}$$

where $\nu_i = \sqrt{\gamma_{i1}^4 - f\left(\gamma_{i2}\right)\left(\psi^2\left(r \cdot exp\left(-\gamma_{i1}^2\right)\right) + \gamma_{i1}^4\right)}, i = 1,2,...,d; \ \gamma = \{\gamma_{11}, \gamma_{12}, \gamma_{21}, \gamma_{22}, ..., \gamma_{d1}, \gamma_{d2}, \gamma_{d0}\}$.
The function f is such that $f(\cdot) : (-\infty, +\infty) \to (0,1)$ and its inverse function exists in the whole region of the definition; the function $\psi(\xi)$ is a real function from the variable $\xi \in (0,r]$, which takes the values in the interval $[0,\pi]$ and $\psi(r) = 0$.

Proof Similar to theorem 1, consider the properties of the quadratic trinomials in (32). Firstly, let prove a direct proposition.
For any given pair of the real numbers γ_{i1}, γ_{i2} the roots of the trinomial $\Delta_i^*(z)$ in (32) is given by the expression $z^i_{1,2} = r \cdot exp(-\gamma_{i1}^2 \pm \nu_i)$. Here two different variants are possible. If ν_i is a real number, then the roots $z^i_{1,2}$ are also real. Besides that, taking into account the properties of the function f, the following inequality holds $\gamma_{i1}^4 - f(\gamma_{i2})\left(\psi^2\left(r \cdot exp(-\gamma_{i1}^2)\right) + \gamma_{i1}^4\right) \leq \gamma_{i1}^4$. Hence the roots are positive and $|z^i_{1,2}| \leq r$, that is $z^i_{1,2} \in C_{\Delta 2}$.
If ν_i is a complex number, then $z^i_{1,2}$ is the pair of complex-conjugated roots and $|z^i_{1,2}| = \rho = r \cdot exp(-\gamma_{i1}^2) \leq r$. Taking into account the properties of the function f, the following inequality is valid

$$\varphi = \sqrt{f(\gamma_{i2})\left(\psi^2\left(r \cdot exp(-\gamma_{i1}^2)\right) + \gamma_{i1}^4\right) - \gamma_{i1}^4} \leq \sqrt{\psi^2\left(r \cdot exp(-\gamma_{i1}^2)\right)} = \psi(\rho). \qquad (42)$$

Since the $arg\, z^i_{1,2} = \pm\varphi$ and, accordingly to (42), $0 \leq \varphi \leq \psi(\rho)$, then the roots $z^i_{1,2}$ are located inside the area $C_{\Delta 2}$, so the direct proposition is proven.
Let consider the reverse proposition. The roots $z_{1,2}$ of some trinomial $\Delta_i(z) = z^2 + \beta_1 z + \beta_0$ are located inside the area $C_{\Delta 2}$ in accordance with the reverse proposition if these roots are positive real numbers. Notice that the coefficients of this trinomial must satisfy to the inequalities (36),(37), because $|z_{1,2}| \leq r$ and the roots product $z_1 z_2$ is positive in any way.
Let find such numbers γ_{i1}, γ_{i2} that the identity $\Delta_i^*(z) \equiv \Delta_i(z)$ holds. By equating the correspondent coefficients for the same degrees of z-variable, obtain

$$-r\left(exp\left(-\gamma_{i1}^2 + \nu_i\right) + exp\left(-\gamma_{i1}^2 - \nu_i\right)\right) = \beta_i, \;\; r^2 exp(-2\gamma_{i1}^2) = \beta_0,$$

hence

$$\gamma_{i1} = \sqrt{-0.5 \cdot ln(\beta_0/r^2)},$$

$$f(\gamma_{i2}) = \frac{1}{4\left(\psi^2\left(r \cdot exp(-\gamma_{i1}^2)\right) + \gamma_{i1}^4\right)}\left(ln^2\left(\frac{\beta_0}{r^2}\right) - ln^2\left(\frac{\beta_1^2}{2\beta_0} - 1 + \sqrt{\left(\frac{\beta_1^2}{2\beta_0} - 1\right)^2 - 1}\right)\right).$$

Let us show that the γ_{i2} is a real number. For γ_{i1} the proof is equivalent to the such one in the first theorem.
It is evident that the equation with respect to γ_{i2} has a solution, if the expression in the right part of it takes the values inside the interval (0,1). Let denote this expression by h. Notice that the denominator for h is equal to zero only if $z_1 = z_2 = r$, but in this case γ_{i2} can be chosen as any real number. In general case, taking into account the proof of the theorem 1, it is not difficult to see that $h > 0$. Besides that the following inequality holds

$$h < 1 - ln^2\left(\beta_1^2/2\beta_0 - 1 + \sqrt{\left(\beta_1^2/2\beta_0 - 1\right)^2 - 1}\right)/ln^2\left(\beta_0/r^2\right),$$

hence the real number γ_{i2} exists and this one is determined as a solution of the equation $f(\gamma_{i2}) = h$.
If the n_d is odd, the polynomial Δ^* has, in accordance with (33), an additional linear binomial, for which the propositions of the theorem are evident.■

Now let us show how introced areas $C_{\Delta 1}$ and $C_{\Delta 2}$ are related to the standart areas on the complex plane, which are commonly used in the analysis and synthesis of the continuos time systems.
Primarily, it may be noticed that the eigenvalues of the continues linear model and the discrete linear model are connected by the following rule (Hendricks et al., 2008): if s is the eigenvalue of the continuos time system matrix, then $z = e^{sT}$ is the correspondent eigenvalue of the discrete time system matrix, where T is the sampling period. Taking into account this relation, let consider the examples of the mapping of some standart areas for continuous systems to the areas for discrete systems.
Example 1 Let we have given area $C = \{s = x \pm yj \in \mathbf{C}^1 : x \leq -\alpha\}$, depicted in Fig. 3. It is evident that the points of the line $x = -\alpha$ are mapped to the points of the circle $|z| = e^{-\alpha T}$. The area C itself is mapped on the disc $|z| \leq e^{-\alpha T}$, as shown in Fig.3. This disc corresponds to the area $C_{\Delta 1}$, which defines the degree of stability for discrete system.

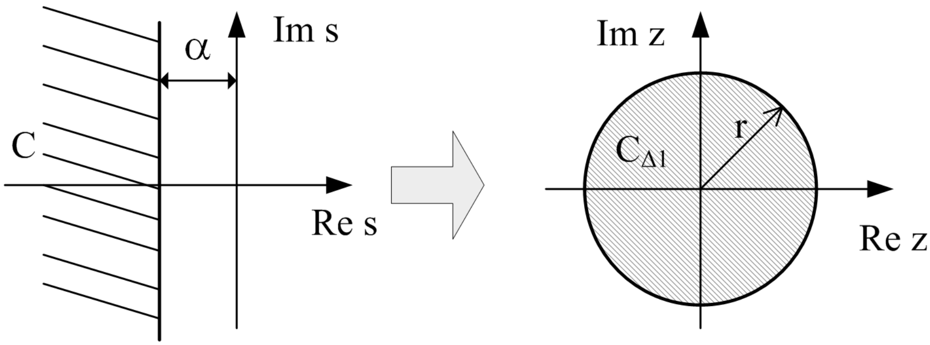


Fig. 3. The correspondence of the areas for continuous and discrete system

Example 2 Consider the area

$$C = \{s = x \pm yj \in \mathbf{C}^1 : x \leq -\alpha, 0 \leq y \leq (-x - \alpha)tg\beta\},$$

depicted in Fig. 4, where $0 \leq \beta < \frac{\pi}{2}$ and $\alpha > 0$ is a given real numbers.
Let perform the mapping of the area C on the z-plane. It is evident that the vertex of the angle $(-\alpha, 0)$ is mapped to the point with polar coordinates $r = e^{-\alpha T}, \varphi = 0$ on the plane z. Let now map each segment from the set

$$L_\gamma = \{s = x \pm yj \in \mathbf{C}^1 : x = \gamma, \gamma \leq -\alpha, 0 \leq y \leq (-\gamma - \alpha)tg\beta\}$$

to the z-plane. Each point $s = \gamma \pm yj$ of the segment L_γ is mapped to the point $z = e^{sT} = e^{\gamma T \pm jyT}$ on the plane z. Therefore, the points of the segment L_γ are mapped to the arc of the circle with radius $e^{\gamma T}$ if the following condition holds $-\alpha - \pi/(Ttg\beta) < \gamma \leq -\alpha$, and to the whole circle if $\gamma \leq -\alpha - \pi/(Ttg\beta)$. Therefore, the maximum radius of the circle, which is fullfilled by the points of the segment, is equal to $r' = e^{-\alpha T - \pi/tg\beta}$, corresponding with the equality $\gamma_0 = -\alpha - \pi/(Ttg\beta)$. Notice that the rays, which constitutes the angle, mapped to the logarithmic spirals. Moreover, the bound of the area on the plane z is formed by the arcs of these spirals in accordace with the x varying from $-\alpha$ to γ_0.

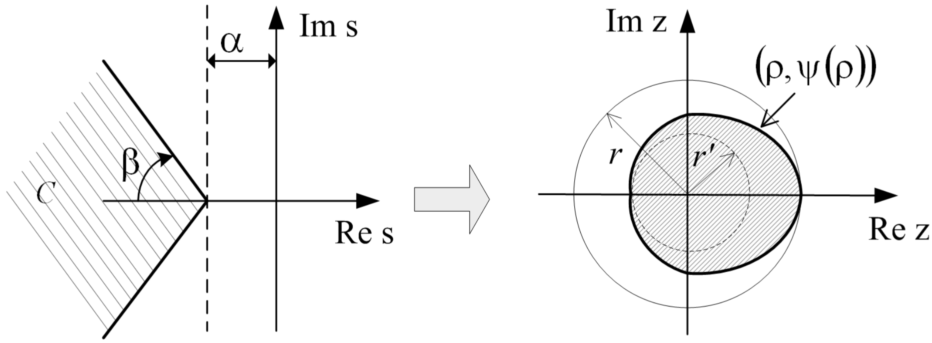


Fig. 4. The correspondence of the areas for continuous and discrete time systems

Let introduce the notation $\rho = e^{\alpha T}$, and define the function $\psi(\rho)$, which represents the constraints on the argument values while the radius ρ of the circle is fixed:

$$\psi(\rho) = \begin{cases} (-ln\rho - \alpha T)tg\beta, & if\ \rho \in [r', r], \\ \pi, & if\ \rho \in [0, r']. \end{cases}$$

The result of the mapping is shown on the Fig. 4. It can be noted that the obtained area reflects the desired degree of the discrete time system stability and oscillations.
Let us use the results of the theorem 2 in order to formulate the computational algoritm for the optimization problem (27) solution on the admissible set Ω_H taking into account the condition $C_\Delta = C_{\Delta 2}$. It is evident that the first case, where $C_\Delta = C_{\Delta 2}$, is a particular case of the second one.
Consider a real vector $\gamma \in \mathbf{E}^{n_d}$ and form the polynomial $\Delta^*(z,\gamma)$ with the help of formulas (32),(33),(41). Let require that the tuned parameters of the controller (24), defined by the vector $\mathbf{h} \in \mathbf{E}^r$, provides the identity

$$\Delta(z, \mathbf{h}) \equiv \Delta^*(z, \gamma), \tag{43}$$

where $\Delta(z, \mathbf{h})$ is the characteristic polynomial of the closed-loop system with the degree n_d. By equating the correspondent coefficients for the same degrees of z-variable, we obtain the following system of nonlinear equations

$$\mathbf{L}(\mathbf{h}) = \chi(\gamma) \tag{44}$$

with respect to unknown components of the parameters vector $\mathbf{h}$. The last system has a solution for any given $\gamma \in \mathbf{E}^{n_d}$ due to the controller (24) has a full structure. Let consider that, in general case, the system (44) has a nonunique solution. Then the vector $\mathbf{h}$ can be presented as a set of two vectors $\mathbf{h} = \{\bar{\mathbf{h}}, \mathbf{h}_c\}$, where $\mathbf{h}_c \in \mathbf{E}^{n_c}$ is a free component, $\bar{\mathbf{h}}$ is the vector that is uniquely defined by the solution of the system (44) for the given vector $\mathbf{h}_c$.
Let introduce the following notation for the general solution of the system (44)

$$\mathbf{h} = \mathbf{h}^* = \{\bar{\mathbf{h}}^*(\mathbf{h}_c, \gamma), \mathbf{h}_c\} = \mathbf{h}^*(\gamma, \mathbf{h}_c) = \mathbf{h}^*(\epsilon),$$

where $\epsilon = \{\gamma, \mathbf{h}_c\}$ is a vector of the independent parameters with the dimension λ given by

$$\lambda = dim\,\epsilon = dim\,\gamma + dim\,\mathbf{h}_c = n_d + n_c.$$

Let form the equations of the prediction model, closed by the controller (24) with the obtained parameter vector $\mathbf{h}^*$

$$\begin{aligned}&\tilde{\mathbf{x}}_{i+1} = \mathbf{f}(\tilde{\mathbf{x}}_i, \tilde{\mathbf{u}}_i), \quad i = k + j, \quad j = 0, 1, 2, ..., \quad \tilde{\mathbf{x}}_k = \mathbf{x}_k,\\ &\tilde{\mathbf{u}}_i = \mathbf{r}_i^u + \mathbf{W}(q, \mathbf{h}^*(\epsilon))\mathbf{C}(\tilde{\mathbf{x}}_i - \mathbf{r}_i^x).\end{aligned} \tag{45}$$

Now the functional J_k, which is given by (26) and computed on the solutions of the system (45), becomes the function of the vector ϵ:

$$J_k = J_k\left(\{\tilde{\mathbf{x}}_i\}, \{\tilde{\mathbf{u}}_i\}\right) = J_k^*\left(\mathbf{W}\left(q, \mathbf{h}^*(\epsilon)\right)\right) = J_k^*(\epsilon). \tag{46}$$

Theorem 3. *Consider the optimization problem (27), where Ω_H is the admissible set, given by (31), and the desired area $C_\Delta = C_{\Delta 2}$. If the extremum of this problem is achieved at the some point $\mathbf{h}_{k0} \in \Omega_H$, then there exists a vector $\epsilon \in \mathbf{E}^\lambda$ such that*

$$\mathbf{h}_{k0} = \mathbf{h}^*(\epsilon_{k0}), \text{ with } \epsilon_{k0} = arg \min_{\epsilon \in \mathbf{E}^\lambda} J_k^*(\epsilon). \tag{47}$$

And reversly, if there exists such a vector $\epsilon_{k0} \in \mathbf{E}^\lambda$, that satisfies to the condition (47), then the following vector $\mathbf{h}_{k0} = \mathbf{h}^*(\epsilon_{k0})$ is the solution of the optimization problem (27). In other words, **the problem (27) is equivalent to the unconstrained optimization problem of the form**

$$J_k^* = J_k^*(\epsilon) \to \inf_{\epsilon \in \mathbf{E}^\lambda}. \tag{48}$$

Proof Assume that the following condition is hold

$$\mathbf{h}_{k0} = arg \min_{\mathbf{h} \in \Omega_H} J_k(\mathbf{h}), J_{k0} = J_k(\mathbf{h}_{k0}). \tag{49}$$

In this case, the characteristic polynomial $\Delta(z, \mathbf{h}_{k0})$ of the closed-loop system (28) has the roots that are located inside the area $C_{\Delta 2}$. Then, accordingly to the theorem 2, it can be found such a vector $\gamma = \gamma_{k0} \in \mathbf{E}^{n_d}$, that $\Delta(z, \mathbf{h}_{k0}) \equiv \Delta^*(z, \gamma_{k0})$, where Δ^* is a polynomial formed by the formulas (32), (33). Hence, there exists such a vector $\epsilon = \{\gamma_{k0}, \mathbf{h}_{k0c}\}$, for which the following conditions is hold $\mathbf{h}_{k0} = \mathbf{h}^*(\epsilon_{k0}), J_k^*(\epsilon_{k0}) = J_{k0}$. Here $\mathbf{h}_{k0c}$ is the correspondent constituent part of the vector $\mathbf{h}_{k0}$.
Now it is only remain to show that there no exists a vector $\epsilon_{01} \in \mathbf{E}^\lambda$ that the condition $J_k^*(\epsilon_{01}) < J_{k0}$ is valid. Really, let suppose that such vector exists. But then for the vector $\mathbf{h}^*(\epsilon_{01})$ the following inequality takes place $J_k(\mathbf{h}^*(\epsilon_{01}) = J_k^*(\epsilon_{01}) < J_{k0}$. But this is not possible due to the condition (49). The reverse proposition is proved analogously.■
Let formulate the computational algorithm in order to get the solution of the optimization problem (27) on the base of the theorems proved above.
The algorithm consists of the following operations:

1. Set any vector $\gamma \in \mathbf{E}^{n_d}$ and construct the polynomial $\Delta^*(z, \gamma)$ by formulas (32),(33), (41).
2. In accordance with the identity $\Delta(z, \mathbf{h}) \equiv \Delta^*(z, \gamma)$, form the system of nonlinear equations
$$\mathbf{L}(\mathbf{h}) = \chi(\gamma), \tag{50}$$
which has a solution for any vector γ. If the system (50) has a nonunique solution, assign the vector of the free parameters $\mathbf{h}_c \in \mathbf{E}^{n_c}$.
3. For a given vector $\epsilon = \{\gamma, \mathbf{h}_c\} \in \mathbf{E}^\lambda$ solve the system of equations (50). As a result, obtain vector $\mathbf{h}^*(\epsilon)$.

4. Form the equations of the prediction model closed by the controller (24) with the parameter vector $\mathbf{h}^*(\epsilon)$ and compute the value of the cost function $J_k^*(\epsilon)$ (46).
5. Solve the problem (48) by using any numerical method for unconstrained minimization and repeating the steps 3–5.
6. When the optimal solution $\epsilon_{k0} = arg \min\limits_{\epsilon \in \mathbf{E}^\lambda} J_k^*(\epsilon)$ is found, compute the parameter vector $\mathbf{h}_{k0} = \mathbf{h}^*(\epsilon_{k0})$ and accept them as a solution.

Now real-time MPC algorithm, which is based on the on-line solution of the problem (27), can be formulated. This algorithm consists of the following steps:

- Obtain the state estimation $\hat{\mathbf{x}}_k$ on the base of measurements $\mathbf{y}_k$.
- Solve the optimization problem (27), using the algorithm stated above, subject to the prediction model (22) with initial conditions $\tilde{\mathbf{x}}_k = \hat{\mathbf{x}}_k$.
- Let $\mathbf{h}_{k0}$ be the solution of the problem (27). Implement controller (24) with the parameter vector $\mathbf{h}_{k0}$ over time interval $[k\delta, (k+1)\delta]$, where δ is the sampling period.
- Repeat the whole procedure 1–3 at next time instant $(k+1)\delta$.

As a result, let notice the following important features of the proposed MPC-algorithm. For the first, the linear closed-loop system stability is provided at each sampling interval. Secondly, the control is realised in the feedback loop. Thirdly, the dimension of the unconstrained optimization problem is fixed and does not depend on the length of prediction horizon P.

5. Plasma Vertical Stabilization Based on the Model Predictive Control

Let us remember that SISO model (5) represents plasma dynamics in the vertical stabilization process and limits (6) are imposed on the power supply system. It is necessary to transform the system (5) to the state-space form for MPC algorithms implementation. Besides that, in order to take into account the constraint imposed on the current, one more equation should be added to the model (5). Finally, the linear model of the stabilization process is given by

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{A}\mathbf{x} + \mathbf{b}u, \\ \mathbf{y} &= \mathbf{c}\mathbf{x} + \mathbf{d}u, \end{aligned} \tag{51}$$

where $\mathbf{x} \in \mathbf{E}^4$ and the last component of $\mathbf{x}$ corresponds to VS converter current, $\mathbf{y} = (y_1, y_2) \in \mathbf{E}^2$, y_1 is the vertical velocity and y_2 is the current in the VS-converter. We shall assume that the model (51) describes the process accurately.

We can obtain a linear prediction model in the form (15) by the system (51) discretization. As a result, we get

$$\begin{aligned} \tilde{\mathbf{x}}_{i+1} &= \mathbf{A}_d\tilde{\mathbf{x}}_i + \mathbf{b}_d\tilde{u}_i, \qquad \tilde{\mathbf{x}}_k = \mathbf{x}_k, \\ \tilde{\mathbf{y}}_i &= \mathbf{C}_d\tilde{\mathbf{x}}_i. \end{aligned} \tag{52}$$

The constraints (6) form the system of linear inequalities given by

$$\begin{aligned} \tilde{u}_i &\le V_{max}^{VS}, \quad i = k, ..., k+P-1; \\ \tilde{y}_{i2} &\le I_{max}^{VS}, \quad i = k+1, ..., k+P. \end{aligned} \tag{53}$$

These constraints define the admissible convex set Ω. The discrete analog of the cost functional (7) with $\lambda = 1$ is given by

$$J_k = J_k(\bar{\mathbf{y}}, \bar{\mathbf{u}}) = \sum_{j=1}^{P} \left(\tilde{y}_{k+j,1}^2 + \tilde{u}_{k+j-1}^2 \right). \tag{54}$$

So, in this case MPC algorithm leads to real-time solution of the quadratic programming problem (19) with respect to the prediction model (52), constraints (53) and the cost functional (54). From the experiments the following values for the sampling time and number of sampling intervals over the horizon were obtained

$$\delta = 0.004 \text{ sec}, \quad P = 250.$$

Hence, we have the following prediction horizon

$$T_p = P\delta = 1 \text{ sec}.$$

Let us consider the MPC controller synthesis without taking into account the constraints imposed. Remember that in this case we obtain a linear controller (20) that is practically the same as the LQR-optimal one. The transient response of the system closed by the controller is presented in Fig. 5. The initial state vector $\mathbf{x}(0) = \mathbf{h}$ is used, where $\mathbf{h}$ is a scaled eigenvector of the matrix $\mathbf{A}$ corresponding to the only unstable eigenvalue. The eigenvector $\mathbf{h}$ is scaled to provide the initial vertical velocity $y_1 = 0.03$ m/sec. It can be seen from the figure that the constraints (6) imposed on the voltage and current are violated.

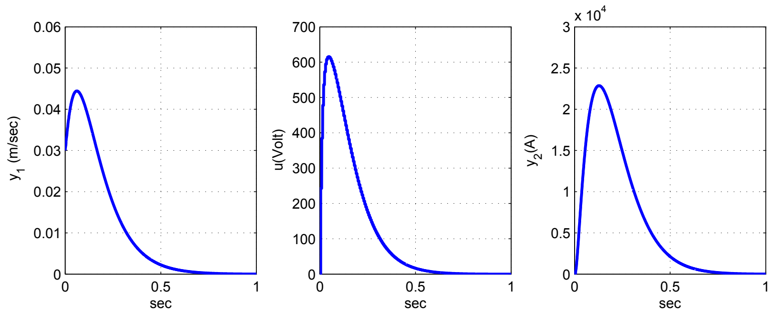


Fig. 5. Transient response of the closed-loop system with unconstrained MPC-controller

Now consider the MPC algorithm synthesis with constraints. Fig. 6 shows transient response of the closed-loop system with constrained MPC-controller. It is not difficult to see that all constraints imposed are satisfied. In order to reduce computational consumptions, the approaches proposed above in Section 3.2 can be implemented.

1. Experiments with using the control horizon were carried out. This experiments show that the quality of stabilization remains approximately the same with control horizon $M = 50$ and prediction horizon $P = 250$. So, optimization problem order can be significantly reduced.
2. Another approach is to increase the sampling interval up to $\delta = 0.005$ sec and reduce the number of samples down to $P = 200$. Hence, prediction horizon has the same value $T_p = P\delta = 1$ sec. The optimization problem order is also reduced in this case and consequently time consumptions at each sampling instant is decreased. However, further increase of δ tends to compromise closed-loop system stability.

Now consider the processes of the plasma vertical stabilization on the base of new MPC-scheme.

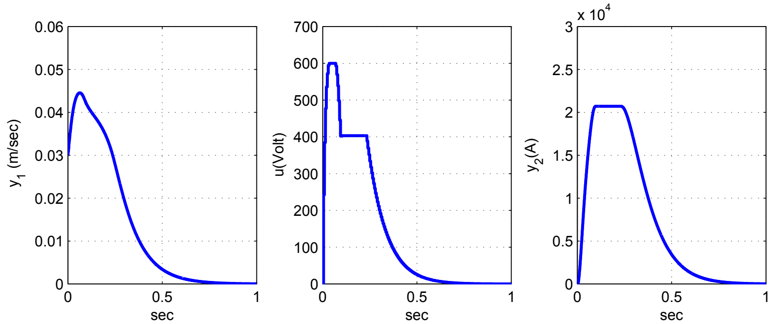


Fig. 6. Transient response of the closed-loop system with constrained MPC-controller

Let us, for the first, transform system (5) into the state space form. As a result, we get

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{A}\mathbf{x} + \mathbf{b}u, \\ y &= \mathbf{c}\mathbf{x} + \mathbf{d}u, \end{aligned} \tag{55}$$

where $\mathbf{x} \in \mathbf{E}^3$, y is the vertical velocity, u is the voltage in the VS-converter. We shall assume that this model describes the process accurately.
As early, we can obtain linear prediction model by the system (55) discretization. So, we have the following prediction model

$$\begin{aligned} \tilde{\mathbf{x}}_{i+1} &= \mathbf{A}_d\tilde{\mathbf{x}}_i + \mathbf{b}_d\tilde{u}_i, \qquad \tilde{\mathbf{x}}_k = \mathbf{x}_k, \\ \tilde{y}_i &= \mathbf{C}_d\tilde{\mathbf{x}}_i. \end{aligned} \tag{56}$$

Let also form the discrete linear model of the process, describing its behavior in the neighbourhood of the zero equilibrium position. Such a model is obtained by the system (55) discretization and can be presented as follows

$$\begin{aligned} \bar{\mathbf{x}}_{k+1} &= \mathbf{A}_d\bar{\mathbf{x}}_k + \mathbf{b}_d\bar{u}_k, \\ \bar{y}_k &= \mathbf{C}_d\bar{\mathbf{x}}_k, \end{aligned} \tag{57}$$

where $\bar{\mathbf{x}}_k \in \mathbf{E}^3$, $\bar{u}_k \in \mathbf{E}^1$, $\bar{y}_k \in \mathbf{E}^1$. We shall form the control over the prediction horizon by the linear proportional controller, that is given by

$$\bar{u}_k = \mathbf{K}\bar{x}_k, \tag{58}$$

where $\mathbf{K} \in \mathbf{E}^3$ is the parameter vector of the controller. In the real processes control input (58) is computed on the base of the state estimation, obtained with the help of asymptotic observer. It must be noted that the controller (58) has a full structure, because the matrices of the controllability and observability for the system (57) have a full rank.
Now consider the equations of the prediction model (56), closed by the controller (58). As a result, we get

$$\begin{aligned} \tilde{\mathbf{x}}_{i+1} &= (\mathbf{A}_d + \mathbf{b}_d\mathbf{K})\tilde{\mathbf{x}}_i, \qquad \tilde{\mathbf{x}}_k = \mathbf{x}_k, \\ \tilde{y}_i &= \mathbf{C}_d\tilde{\mathbf{x}}_i. \end{aligned} \tag{59}$$

The controlled processes quality over the prediction horizon P is presented by the cost functional

$$J_k = J_k(\mathbf{K}) = \sum_{j=1}^{P} \left(\tilde{y}_{k+j}^2 + \tilde{u}_{k+j-1}^2 \right). \tag{60}$$

It is easy to see that the cost functional (60) becomes the function of three variables, which are the components of the parameter vector **K**. It is important to note that the cost function remains essentialy nonlinear for this variant of the MPC approach even in the case when the prediction model is linear. It is a price for providing stability of the closed-loop linear system. Consider the optimization problem (27) statement for the particular case of plasma vertical stabilization processes

$$J_k = J_k(\mathbf{K}) \to \min_{\mathbf{K}\in\Omega_K}, \ \textit{where} \ \ \Omega_K = \{\mathbf{K} \in \mathbf{E}^3 : \ \delta_i(\mathbf{K}) \in C_\Delta, i = 1, 2, 3\}. \tag{61}$$

Here δ_i are the roots of the closed-loop system (57), (58) characteristic polynomial $\Delta(z, \mathbf{K})$ with the degree $n_d = 3$. Let given desirable area be $C_\Delta = C_{\Delta 2}$, where $r = 0.97$ and the function $\psi(\rho)$ is presented by the formula

$$\psi(\rho) = \begin{cases} ln\left(\frac{r}{\rho}\right) tg\beta, & re^{-\pi/tg\beta} \le \rho \le r, \\ \pi, & if \ 0 < \rho \le re^{-\pi/tg\beta}, \end{cases}$$

where $\beta = \pi/10$. This area is presented on the Fig. 7.

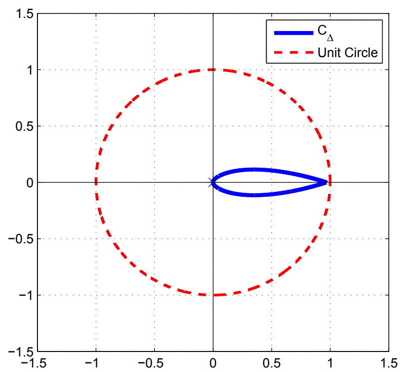


Fig. 7. The area C_Δ of the desired roots location

Let construct now the system of equations in accordance with the identity $\Delta(z, \mathbf{K}) \equiv \Delta^*(z, \gamma)$, where $\gamma \in \mathbf{E}^3$ and the polynomial $\Delta^*(z, \gamma)$ is defined by the formulas (33), (41). As a result, we obtain linear system with respect to unknown parameter vector **K**

$$\mathbf{L}_0 + \mathbf{L}_1\mathbf{K} = \chi(\gamma). \tag{62}$$

Here vector $\mathbf{L}_0$ and square matrix $\mathbf{L}_1$ are constant for any sampling instant k. These are fully defined by the matrices of the system (57). Besides that, the matrix $\mathbf{L}_1$ is nonsingular, hence we can find the unique solution for system (62)

$$\mathbf{K} = \tilde{\mathbf{L}}_0 + \tilde{\mathbf{L}}_1\chi(\gamma), \tag{63}$$

where $\tilde{\mathbf{L}}_1 = \mathbf{L}_1^{-1}$ and $\tilde{\mathbf{L}}_0 = -\mathbf{L}_1^{-1}\mathbf{L}_0$. Substituting (63) into the prediction model (59) and then into the cost functional (60), we get $J_k = J_k(\mathbf{K}) = J_k^*(\gamma)$. That is the functional J_k becomes the function of three indepent variables. Then, accordingly to the theorem 3, optimization problem (61) is equivalent to the unconstrained minimization

$$J_k^* = J_k^*(\gamma) \to \min_{\gamma \in \mathbf{E}^3} . \tag{64}$$

Thus, in conformity with the algorithm of the MPC real-time implementation, presented in the section 4 above, in order to form control input we must solve the unconstrained optimization problem (64) at each sampling instant.
Consider now the processes of the plasma vertical stabilization. For the first, let us consider the unconstrained case. Remember that the structure of the controller (58) is linear. So, if the roots of the characteristic polynomial for the system (57) closed by the LQR-controller are located inside the area C_Δ then parameter vector $\mathbf{K}$ will be practically equivalent to the matrix of the LQR-controller. The roots of the system closed by the discrete LQR are the following $z_1 = 0.9591, z_2 = 0.8661, z_3 = 0.9408$. This roots are located inside the area C_Δ. So, the transient responce of the system closed by the MPC-controller, which is based on the optimization (64), is approximately the same as presented in Fig. 5.

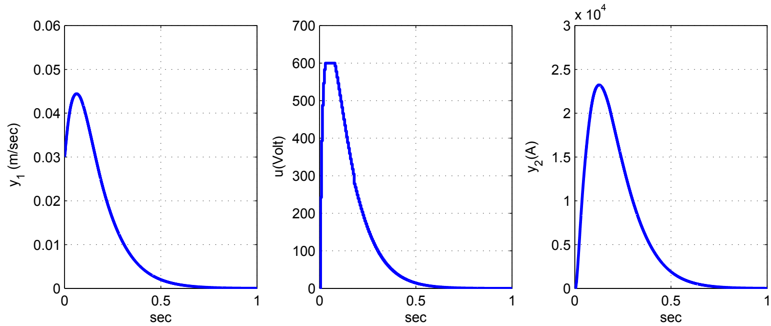


Fig. 8. Transient response of the closed-loop system with constrained MPC-controller

Consider now the processes of plasma stabilization with the constraints (53) imposed. As mentioned above, in order to take into account the constraint imposed on the current, the additional equation should be added. It is necessary to remark that in the presence of the constraints, the optimization problem (64) becomes the nonlinear programming problem. Fig.8 shows transient responce of the closed-loop system with MPC-controller when the only constraint on the VS converter voltage is taked into account. It can be seen from the figure that the constraint imposed on the voltage is satisfied, but the constraint on the current is violated. Fig.9 shows transient responce of the closed-loop system with MPC-controller when both the constraint on the VS converter voltage and current are taken into account. It is not difficult to see that all the imposed constraints are satisfied.

6. Conclusion

The problem of plasma vertical stabilization based on the model predictive control has been considered. It is shown that MPC algorithms are superior compared to the LQR-optimal controller, because they allow taking constraints into account and provide high-performance control. It is also shown that in the case of the traditional MPC-scheme it is possible to reduce

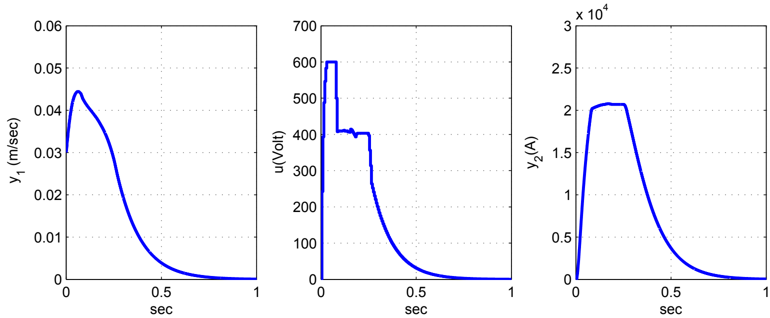


Fig. 9. Transient response of the closed-loop system with constrained MPC-controller

the computational load significantly using relatively small control horizon or by increasing sample interval while preserving the processes quality in the closed-loop system.
New MPC approach was provided. This approach allows us to guarantee linear closed-loop system stability. It's implementation in real-time is connected with the on-line solution of the unconstrained nonlinear optimization problem if there is not constraint imposed and with the nonlinear programming problem in the presence of constraints. The significant feature of this approach is that the dimension of the optimization problem is not depend on the prediction horizon P. The algorithm for the real-time implementation of the suggested approach was described. It allows us to use MPC algorithms to solve plasma vertical stabilization problem.

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