

# Fuzzy frequency response for stochastic linear parameter varying dynamic systems

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## 1. Introduction

The design of control systems is currently managed by a large number of requirements such as: increasing competition, environmental requirements, energy and material costs, and demand for robust and fault-tolerant systems which require considerations for effective process control techniques. In this context, the analysis and synthesis of compensators are completely related to each other. In the analysis, the characteristics or dynamic behaviour of the control system are determined. In the synthesis, the compensators are obtained to attend to desired characteristics of the control system based on certain performance criteria. Generally, these criteria may involve disturbance rejection, steady-state errors, transient response characteristics and sensitivity to parameter changes in the plant (Franklin et al. (1986); Ioannou & Sun (1996); Phillips & Harbor (1996)).

Test input signals is one way to analyse the dynamic behaviour of real world system. Many test signals are available, but a simple and useful signal is the sinusoidal wave form because the system output with a sinusoidal wave input is also a sinusoidal wave, but with a different amplitude and phase for a given frequency. This frequency response analysis describes how a dynamic system responds to sinusoidal inputs in a range of frequencies and it has been widely used in academic field and industry, as well as been considered essential for robust control theory (Serra & Bottura (2006; 2009); Serra et al. (2009); Tanaka et al. (1998)).

The frequency response methods were developed during the period 1930 – 1940 by Harry Nyquist (1889 – 1976) (Nyquist (1932)), Hendrik Bode (1905 – 1982) (Bode (1940)), Nathaniel B. Nichols (1914 – 1997) (James et al. (1947)), and many others. Since then, frequency response methods are among the most useful techniques being available to analyse and synthesise the compensators. In (Jr. (1973)), the U.S. Navy obtains frequency responses for aircraft by applying sinusoidal inputs to the autopilots and measuring its resulting position in flight. In (Lascu et al. (2009)), four current controllers for selective harmonic compensation in parallel with Active Power Filters (APFs) have been analytically compared in terms of frequency response characteristics and maximum operational frequency.

Most real systems, such as circuit components (inductor, resistor, operational amplifier, etc.) are often formulated using differential/integral equations with stochastic parameters (Kolev (1993)). These random variations are most often quantified in terms of boundaries. The classical methods of frequency response do not explore these boundaries for Stochastic Linear Parameter Varying (SLPV) dynamic systems. To overcome this limitation, this chapter pro-

poses the definition of Fuzzy Frequency Response (FFR) and its application for analysis of stochastic linear parameter varying dynamic systems.

## 2. Formulation Problem

This section presents some essential concepts for the formulation and development of this chapter.

### 2.1 Stochastic Linear Parameter Varying Dynamic Systems

In terms of transfer function, the general form of SLPV dynamic system is given by Eq. (1), as depicted in Fig.1.

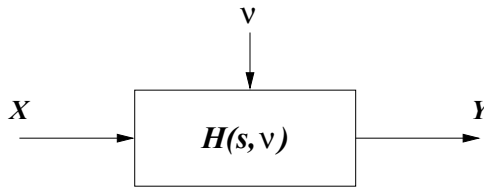


Fig. 1. SLPV dynamic system.

$$H(s, \nu) = \frac{Y(s, \nu)}{X(s)} = \frac{b_\alpha(\nu)s^\alpha + b_{\alpha-1}(\nu)s^{\alpha-1} + \dots + b_1(\nu)s + b_0(\nu)}{s^\beta + a_{\beta-1}(\nu)s^{\beta-1} + \dots + a_1(\nu)s + a_0(\nu)}, \tag{1}$$

where:

- $H(s, \nu)$  is the transfer function of the SLPV dynamic system;
- $X(s)$  and  $Y(s, \nu)$  represent the input and output of SLPV dynamic system, respectively;
- $a_*(\nu)$  and  $b_*(\nu)$  are the varying parameters;
- $\nu(t)$  is the time varying scheduling variable;
- $s$  is the Laplace operator and
- $\alpha$  and  $\beta$  are the orders of the numerator and denominator of the transfer function, respectively (with  $\beta \geq \alpha$ ).

The scheduling variable  $\nu$  belongs to a compact set  $\nu \in V$ , with its variation limited by  $|\dot{\nu}| \leq d^{\max}$ , with  $d^{\max} \geq 0$ .

### 2.2 Takagi-Sugeno Fuzzy Dynamic Model

The TS inference system, originally proposed in (Takagi (1985)), presents in the consequent a dynamic functional expression of the linguistic variables of the antecedent. The  $i$   $\left[ \begin{smallmatrix} i=1,2,\dots,l \end{smallmatrix} \right]$ -th rule, where  $l$  is the number of rules, is given by:

$$Rule^{(i)} : IF \tilde{x}_1 \text{ is } F_{\{1,2,\dots,p_{\tilde{x}_1}\}}^i \mid_{\tilde{x}_1} \text{ AND } \dots \text{ AND } \tilde{x}_n \text{ is } F_{\{1,2,\dots,p_{\tilde{x}_n}\}}^i \mid_{\tilde{x}_n}$$

$$\text{THEN } y_i = f_i(\tilde{\mathbf{x}}), \tag{2}$$

where the total number of rules is  $l = p_{\tilde{x}_1} \times \dots \times p_{\tilde{x}_n}$ . The vector  $\tilde{\mathbf{x}} = [\tilde{x}_1, \dots, \tilde{x}_n]^T \in \mathfrak{R}^n$  containing the linguistic variables of antecedent, where  $T$  represents the operator for transpose matrix. Each linguistic variable has its own discourse universe  $\mathcal{U}_{\tilde{x}_1}, \dots, \mathcal{U}_{\tilde{x}_n}$ , partitioned by fuzzy sets representing its linguistics terms, respectively. In  $i$ -th rule, the variable  $\tilde{x}_{\{1,2,\dots,n\}}$  belongs to the fuzzy set  $F_{\{\tilde{x}_1, \dots, \tilde{x}_n\}}^i$  with a membership degree  $\mu_{F_{\{\tilde{x}_1, \dots, \tilde{x}_n\}}^i}^i$  defined by a membership function  $\mu_{\{\tilde{x}_1, \dots, \tilde{x}_n\}}^i : \mathfrak{R} \rightarrow [0, 1]$ , with  $\mu_{F_{\{\tilde{x}_1, \dots, \tilde{x}_n\}}^i}^i \in \{\mu_{F_1|\{\tilde{x}_1, \dots, \tilde{x}_n\}}^i}, \mu_{F_2|\{\tilde{x}_1, \dots, \tilde{x}_n\}}^i}, \dots, \mu_{F_p|\{\tilde{x}_1, \dots, \tilde{x}_n\}}^i\}$ , where  $p_{\{\tilde{x}_1, \dots, \tilde{x}_n\}}$  is the partition number of the discourse universe, associated with the linguistic variable  $\tilde{x}_1, \dots, \tilde{x}_n$ . The TS fuzzy dynamic model output is a convex combination of the dynamic functional expressions of consequent  $f_i(\tilde{\mathbf{x}})$ , without loss of generality for the bidimensional case, as illustrated in Fig. 2, given by Eq. (3).

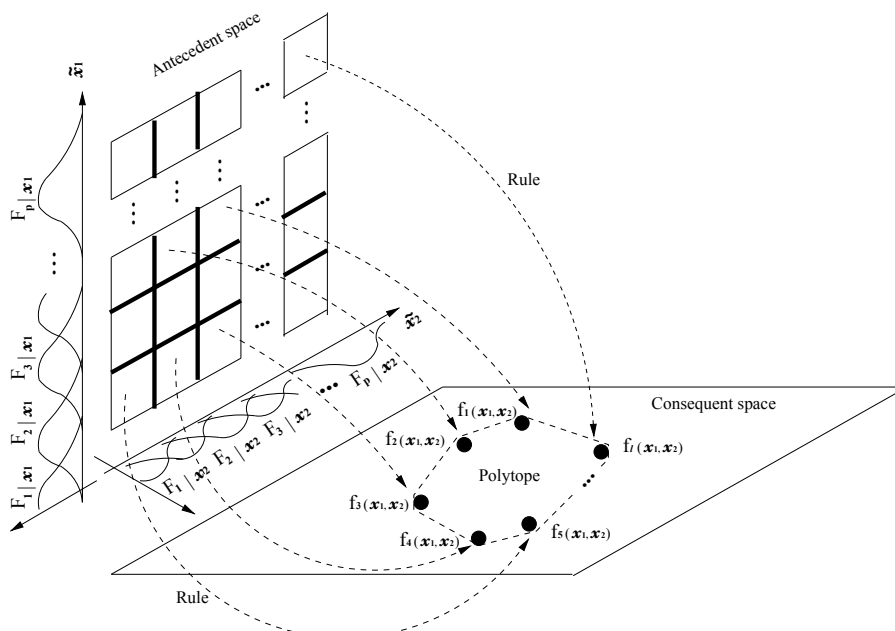


Fig. 2. Fuzzy dynamic model: A TS model can be regarded as a mapping from the antecedent space to the consequent parameter one.

$$y(\tilde{\mathbf{x}}, \gamma) = \sum_{i=1}^l \gamma_i(\tilde{\mathbf{x}}) f_i(\tilde{\mathbf{x}}), \tag{3}$$

where  $\gamma$  is the scheduling variable of the TS fuzzy dynamic model. The scheduling variable, well known as normalized activation degree, is given by:

$$\gamma_i(\tilde{\mathbf{x}}) = \frac{h_i(\tilde{\mathbf{x}})}{\sum_{r=1}^l h_r(\tilde{\mathbf{x}})}. \tag{4}$$

This normalization implies

$$\sum_{k=1}^l \gamma_i(\tilde{\mathbf{x}}) = 1. \tag{5}$$

It can be observed that the TS fuzzy dynamic system, which represents any stochastic dynamic model, may be considered as a class of systems where  $\gamma_i(\tilde{\mathbf{x}})$  denotes a decomposition of linguistic variables  $[\tilde{x}_1, \dots, \tilde{x}_n]^T \in \mathfrak{R}^n$  for a polytopic geometric region in the consequent space based on the functional expressions  $f_i(\tilde{\mathbf{x}})$ .

### 3. Fuzzy Frequency Response (FFR): Definition

This section demonstrates how a TS fuzzy dynamic model responds to sinusoidal inputs, which is proposed as the definition of fuzzy frequency response. The response of a TS fuzzy dynamic model to a sinusoidal input of frequency  $\omega_1$ , in both amplitude and phase, is given by the transfer function evaluated at  $s = j\omega_1$ , as illustrated in Fig. 3.

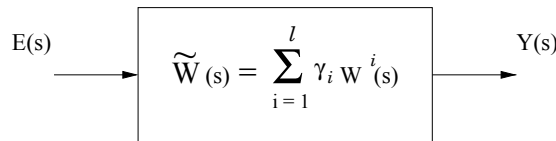


Fig. 3. TS fuzzy transfer function.

For this TS fuzzy dynamic model:

$$Y(s) = \left[ \sum_{i=1}^l \gamma_i W^i(s) \right] E(s). \tag{6}$$

Consider  $\tilde{W}(j\omega) = \sum_{i=1}^l \gamma_i W^i(j\omega)$  as a complex number for a given  $\omega$ :

$$\tilde{W}(j\omega) = \sum_{i=1}^l \gamma_i W^i(j\omega) = \left| \sum_{i=1}^l \gamma_i W^i(j\omega) \right| e^{j\phi(\omega)} = \left| \sum_{i=1}^l \gamma_i W^i(j\omega) \right| \angle \phi(\omega) \tag{7}$$

or

$$\tilde{W}(j\omega) = \sum_{i=1}^l \gamma_i W^i(j\omega) = \left| \sum_{i=1}^l \gamma_i W^i(j\omega) \right| \angle \arctan \left[ \sum_{i=1}^l \gamma_i W^i(j\omega) \right]. \tag{8}$$

Then, for the case that the input signal  $e(t)$  is sinusoidal, that is:

$$e(t) = A \sin \omega_1 t. \tag{9}$$

The output signal  $y_{ss}(t)$ , in the steady state, is given by

$$y_{ss}(t) = A \left| \sum_{i=1}^l \gamma_i W^i(j\omega) \right| \sin [\omega_1 t + \phi(\omega_1)]. \tag{10}$$

As a result of the fuzzy frequency response definition, it is then proposed the following *Theorem*:

**Theorem 3.1.** *Fuzzy frequency response is a region in the frequency domain, defined by the consequent sub-models and based on the operating region of the antecedent space.*

*Proof.* Considering that  $\tilde{v}$  is stochastic and can be represented by linguistic terms, once known its discourse universe, as shown in Fig. 4, the activation degrees,  $h_i(\tilde{v})|^{i=1,2,\dots,l}$  are also stochastic, since it depends of the dynamic system:

$$h_i(\tilde{v}) = \mu_{F_{\tilde{v}_1^*}}^i \star \mu_{F_{\tilde{v}_2^*}}^i \star \dots \star \mu_{F_{\tilde{v}_n^*}}^i, \tag{11}$$

where  $\tilde{v}_{\{1,2,\dots,n\}}^* \in \mathcal{U}_{\tilde{v}_{\{1,2,\dots,n\}}}$ , respectively, and  $\star$  is a fuzzy logic operator.

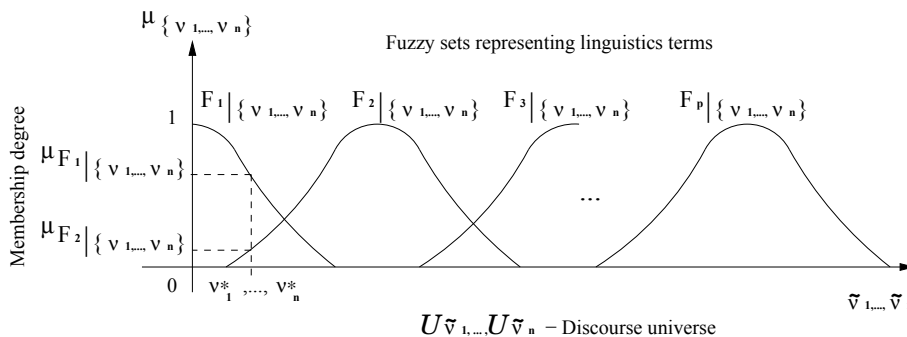


Fig. 4. Functional description of the linguistic variable  $\tilde{v}$ : linguistic terms, discourse universe and membership degrees.

So, the normalized activation degrees  $\gamma_i(\tilde{v})|^{i=1,2,\dots,l}$ , are also stochastics:

$$\gamma_i(\tilde{v}) = \frac{h_i(\tilde{v})}{\sum_{r=1}^l h_r(\tilde{v})}. \tag{12}$$

This normalization implies

$$\sum_{k=1}^l \gamma_i(\tilde{v}) = 1. \quad (13)$$

Let  $F(s)$  be a vectorial space with degree  $l$  and  $f^1(s), f^2(s), \dots, f^l(s)$  be transfer functions which belong to this vectorial space. A transfer function  $f(s) \in F(s)$  must be a linear convex combination of the vectors  $f^1(s), f^2(s), \dots, f^l(s)$ :

$$f(s) = \xi_1 f^1(s) + \xi_2 f^2(s) + \dots + \xi_l f^l(s), \quad (14)$$

where  $\xi_{1,2,\dots,l}$  are the linear convex combination coefficients. If they are normalized  $\left(\sum_{i=1}^l \xi_i = 1\right)$ , the vectorial space presents a decomposition of the transfer functions  $[f^1(s), f^2(s), \dots, f^l(s)]$  in a polytopic geometric shape of the vectorial space  $F(s)$ . The points of the polytopic geometric shape are defined by the transfer functions  $[f^1(s), f^2(s), \dots, f^l(s)]$ .

The TS fuzzy dynamic model attends to this polytopic property. The sum of the normalized activation degrees is equal to 1, as demonstrated in Eq. (5). To define the points of this fuzzy polytopic geometric shape, each rule of the TS fuzzy dynamic model must be individually activated. This condition is called boundary condition. Thus, the following results are obtained for the Fuzzy Frequency Response (FFR) of the TS fuzzy transfer function:

- *If only the rule 1 is activated, it has  $(\gamma_1 = 1, \gamma_2 = 0, \gamma_3 = 0, \dots, \gamma_l = 0)$ . Hence,*

$$\tilde{W}(j\omega, \tilde{v}) = \left| \sum_{i=1}^l \gamma_i(\tilde{v}) W^i(j\omega) \right| \angle \arctan \left[ \sum_{i=1}^l \gamma_i(\tilde{v}) W^i(j\omega) \right], \quad (15)$$

$$\tilde{W}(j\omega, \tilde{v}) = \left| 1W^1(j\omega) + 0W^2(j\omega) + \dots + 0W^l(j\omega) \right| \angle \arctan \left[ 1W^1(j\omega) + 0W^2(j\omega) + \dots + 0W^l(j\omega) \right], \quad (16)$$

$$\tilde{W}(j\omega, \tilde{v}) = \left| W^1(j\omega) \right| \angle \arctan \left[ W^1(j\omega) \right]. \quad (17)$$

- *If only the rule 2 is activated, it has  $(\gamma_1 = 0, \gamma_2 = 1, \gamma_3 = 0, \dots, \gamma_l = 0)$ . Hence,*

$$\tilde{W}(j\omega, \tilde{v}) = \left| \sum_{i=1}^l \gamma_i(\tilde{v}) W^i(j\omega) \right| \angle \arctan \left[ \sum_{i=1}^l \gamma_i(\tilde{v}) W^i(j\omega) \right], \quad (18)$$

$$\tilde{W}(j\omega, \tilde{v}) = \left| 0W^1(j\omega) + 1W^2(j\omega) + \dots + 0W^l(j\omega) \right| \angle \arctan \left[ 0W^1(j\omega) + 1W^2(j\omega) + \dots + 0W^l(j\omega) \right], \quad (19)$$

$$\tilde{W}(j\omega, \tilde{v}) = \left| W^2(j\omega) \right| \angle \arctan \left[ W^2(j\omega) \right]. \tag{20}$$

- If only the rule  $l$  is activated, it has  $(\gamma_1 = 0, \gamma_2 = 0, \gamma_3 = 0, \dots, \gamma_l = 1)$ . Hence,

$$\tilde{W}(j\omega, \tilde{v}) = \left| \sum_{i=1}^l \gamma_i(\tilde{v}) W^i(j\omega) \right| \angle \arctan \left[ \sum_{i=1}^l \gamma_i(\tilde{v}) W^i(j\omega) \right], \tag{21}$$

$$\tilde{W}(j\omega, \tilde{v}) = \left| 0W^1(j\omega) + 0W^2(j\omega) + \dots + 1W^l(j\omega) \right| \angle \arctan \left[ 0W^1(j\omega) + 0W^2(j\omega) + \dots + 1W^l(j\omega) \right], \tag{22}$$

$$\tilde{W}(j\omega, \tilde{v}) = \left| W^l(j\omega) \right| \angle \arctan \left[ W^l(j\omega) \right]. \tag{23}$$

Where  $W^1(j\omega), W^2(j\omega), \dots, W^l(j\omega)$  are the linear sub-models of the uncertain dynamic system.

Note that  $\left| W^1(j\omega) \right| \angle \arctan \left[ W^1(j\omega) \right]$  and  $\left| W^l(j\omega) \right| \angle \arctan \left[ W^l(j\omega) \right]$  define a boundary region. Under such circumstances, the fuzzy frequency response converges to a boundary in the frequency domain defined by a surface based on membership degrees. Figure 5 shows the fuzzy frequency response for the bidimensional case, without loss of generality.

□

#### 4. Fuzzy Frequency Response (FFR): Analysis

In this section, the behaviour of the fuzzy frequency response is analysed at low and high frequencies. The idea is to study the magnitude and phase behaviour of the TS fuzzy dynamic model, when  $\omega$  varies from zero to infinity.

##### 4.1 Low Frequency Analysis

Low frequency analysis of the TS fuzzy dynamic model  $\tilde{W}(s)$  can be obtained by:

$$\lim_{\omega \rightarrow 0} \sum_{i=1}^l \gamma_i W^i(j\omega). \tag{24}$$

The magnitude and phase behaviours at low frequencies, are given by

$$\lim_{\omega \rightarrow 0} \left| \sum_{i=1}^l \gamma_i W^i(j\omega) \right| \angle \arctan \left[ \sum_{i=1}^l \gamma_i W^i(j\omega) \right]. \tag{25}$$

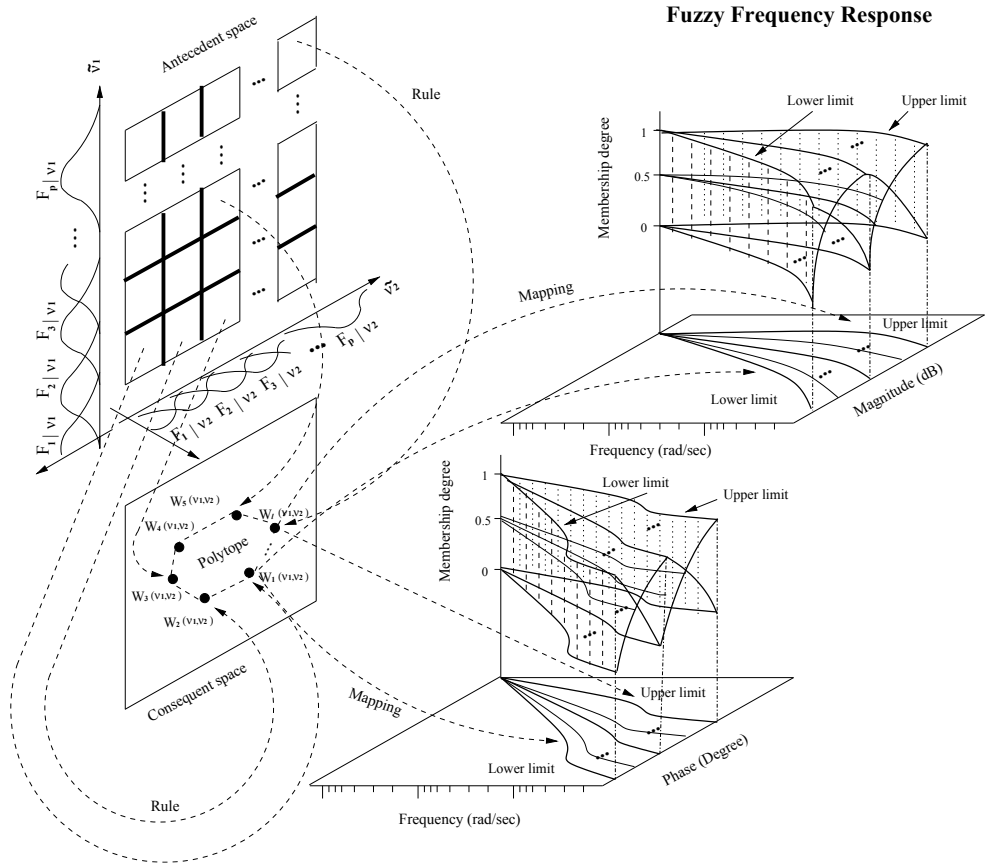


Fig. 5. Fuzzy frequency response: mapping from the consequent space to frequency domain region.

**4.2 High Frequency Analysis**

Likewise, the high frequency analysis of the TS fuzzy dynamic model  $\tilde{W}(s)$  can be obtained by:

$$\lim_{\omega \rightarrow \infty} \sum_{i=1}^l \gamma_i W^i(j\omega). \tag{26}$$

The magnitude and phase behaviours at high frequencies, are given by

$$\lim_{\omega \rightarrow \infty} \left| \sum_{i=1}^l \gamma_i W^i(j\omega) \right| \angle \arctan \left[ \sum_{i=1}^l \gamma_i W^i(j\omega) \right]. \tag{27}$$



### 5. Computational Results

To illustrate the FFR: definition and analysis, as shown in sections 3 and 4, consider the following SLPV dynamic system, given by

$$H(s, \nu) = \frac{Y(s, \nu)}{U(s)} = \frac{2 - \nu}{[(\nu + 1)s + 1] \left[ \left( \frac{\nu}{2} + 0.1 \right) s + 1 \right]}, \tag{28}$$

where the scheduling variable is  $\nu = [0, 1]$ , the gain of the SLPV dynamic system is  $K_p = 2 - \nu$ , the higher time constant is  $\tau = \nu + 1$ , and the lower time constant is  $\tau' = \frac{\nu}{2} + 0.1$ .

Starting from the SLPV dynamic system in Eq. (28) and assuming the time varying scheduling variable in the range of  $[0, 1]$ , one can obtain the TS fuzzy dynamic model in the following operating points:

**Sub-model 1 ( $\nu = 0$ ):**

$$W^1(s, 0) = \frac{2}{(s + 1)(0.1s + 1)} = \frac{2}{0.1s^2 + 1.1s + 1}. \tag{29}$$

**Sub-model 2 ( $\nu = 0.5$ ):**

$$W^2(s, 0.5) = \frac{1.5}{(1.5s + 1)(0.35s + 1)} = \frac{1.5}{0.525s^2 + 1.85s + 1}. \tag{30}$$

**Sub-model 3 ( $\nu = 1$ ):**

$$W^3(s, 1) = \frac{1}{(2s + 1)(0.6s + 1)} = \frac{1}{1.2s^2 + 2.6s + 1}. \tag{31}$$

The TS fuzzy dynamic model rule base results in:

$$\begin{aligned} Rule^{(1)} : & \text{ IF } \nu \text{ is } 0 \text{ THEN } W^1(s, 0) \\ Rule^{(2)} : & \text{ IF } \nu \text{ is } 0.5 \text{ THEN } W^2(s, 0.5) \\ Rule^{(3)} : & \text{ IF } \nu \text{ is } 1 \text{ THEN } W^3(s, 1), \end{aligned} \tag{32}$$

and the TS fuzzy dynamic model of the SLPV dynamic system is given by

$$\tilde{W}(s, \tilde{\nu}) = \sum_{i=1}^3 \gamma_i(\tilde{\nu}) W^i(s). \tag{33}$$

Again, starting from Eq. (28), one obtains:

$$Y(s, \nu) = \frac{2 - \nu}{\left(\frac{\nu^2}{2} + 0.1\nu + \frac{\nu}{2} + 0.1\right)s^2 + \left(\nu + 1 + 0.1 + \frac{\nu}{2}\right)s + 1} U(s), \quad (34)$$

$$\left(\frac{\nu^2 + 1.2\nu + 0.2}{2}\right)s^2 Y(s, \nu) + \left(\frac{3\nu + 2.2}{2}\right)s Y(s, \nu) + Y(s, \nu) = (2 - \nu) U(s) \quad (35)$$

and taking the inverse Laplace transform, this yields the differential equation of the SLPV dynamic system:

$$\left(\frac{\nu^2 + 1.2\nu + 0.2}{2}\right)\ddot{y}(t) + \left(\frac{3\nu + 2.2}{2}\right)\dot{y}(t) + y(t) = (2 - \nu)u(t). \quad (36)$$

A comparative analysis, via analog simulation between the SLPV dynamic system Eq. (36) and the TS fuzzy dynamic model Eq. (33), can be performed to validate the TS fuzzy dynamic model. A band-limited white noise (normally distributed random signal) was considered as input and the stochastic parameter was based on sinusoidal variation. As shown in Fig. 6, the efficiency of the TS fuzzy dynamic model in order to represent the dynamic behaviour of the SLPV dynamic system in the time domain can be seen.

From Eq. (8) the TS fuzzy dynamic model of the SLPV dynamic system, Eq. (33), can be represented by

$$\tilde{W}(j\omega, \tilde{\nu}) = \left| \sum_{i=1}^3 \gamma_i(\tilde{\nu}) W^i(j\omega) \right| \angle \arctan \left[ \sum_{i=1}^3 \gamma_i(\tilde{\nu}) W^i(j\omega) \right] \quad (37)$$

or

$$\tilde{W}(j\omega, \tilde{\nu}) = \left| \gamma_1 W^1(j\omega, 0) + \gamma_2 W^2(j\omega, 0.5) + \gamma_3 W^3(j\omega, 1) \right| \angle \arctan \left[ \gamma_1 W^1(j\omega, 0) + \gamma_2 W^2(j\omega, 0.5) + \gamma_3 W^3(j\omega, 1) \right]. \quad (38)$$

So,

$$\tilde{W}(j\omega, \tilde{\nu}) = \left| \gamma_1 \frac{2}{0.1s^2 + 1.1s + 1} + \gamma_2 \frac{1.5}{0.525s^2 + 1.85s + 1} + \gamma_3 \frac{1}{1.2s^2 + 2.6s + 1} \right| \angle \arctan \left[ \gamma_1 \frac{2}{0.1s^2 + 1.1s + 1} + \right.$$

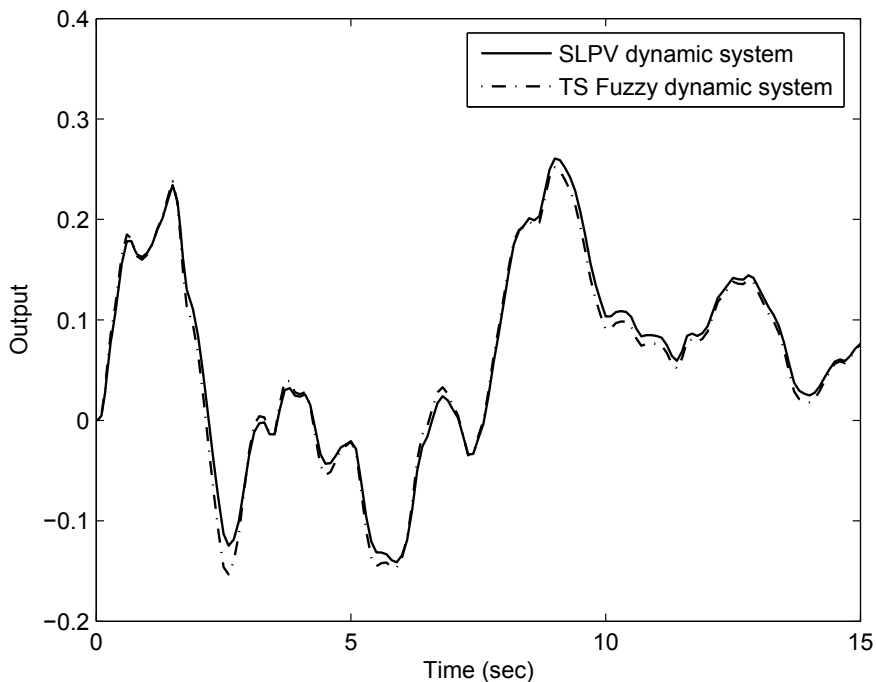


Fig. 6. Validation of the TS fuzzy dynamic model.

$$+ \gamma_2 \frac{1.5}{0.525s^2 + 1.85s + 1} + \gamma_3 \frac{1}{1.2s^2 + 2.6s + 1} \Big], \tag{39}$$

$$\begin{aligned} \tilde{W}(j\omega, \bar{v}) = & \left| 2\gamma_1 \frac{0.6(j\omega)^4 + 3.6(j\omega)^3 + 6.5(j\omega)^2 + 4.5(j\omega) + 1}{Den[\tilde{W}(j\omega, \bar{v})]} + \right. \\ & 1.5\gamma_2 \frac{0.1(j\omega)^4 + 1.6(j\omega)^3 + 4.2(j\omega)^2 + 3.7(j\omega) + 1}{Den[\tilde{W}(j\omega, \bar{v})]} + \\ & \left. \gamma_3 \frac{0.1(j\omega)^4 + 0.8(j\omega)^3 + 2.7(j\omega)^2 + 3(j\omega) + 1}{Den[\tilde{W}(j\omega, \bar{v})]} \right| \angle \arctan \\ & \left[ 2\gamma_1 \frac{0.6(j\omega)^4 + 3.6(j\omega)^3 + 6.5(j\omega)^2 + 4.5(j\omega) + 1}{Den[\tilde{W}(j\omega, \bar{v})]} + \right. \end{aligned}$$

$$1.5\gamma_2 \frac{0.1(j\omega)^4 + 1.6(j\omega)^3 + 4.2(j\omega)^2 + 3.7(j\omega) + 1}{Den_{[\tilde{W}(j\omega, \tilde{v})]}} + \left. \gamma_3 \frac{0.1(j\omega)^4 + 0.8(j\omega)^3 + 2.7(j\omega)^2 + 3(j\omega) + 1}{Den_{[\tilde{W}(j\omega, \tilde{v})]}} \right], \quad (40)$$

where:

$$Den_{[\tilde{W}(j\omega, \tilde{v})]} = 0.1(j\omega)^6 + 1.1(j\omega)^5 + 5.2(j\omega)^4 + 11.2(j\omega)^3 + 11.5(j\omega)^2 + 5.6(j\omega) + 1. \quad (41)$$

### 5.1 Low Frequency Analysis

Starting from the TS fuzzy dynamic model, Eq. (37), and applying the concepts presented in the subsection 4.1, the steady-state response for sinusoidal input at low frequencies for the SLPV dynamic system can be obtained as follows:

$$\begin{aligned} \lim_{\omega \rightarrow 0} \tilde{W}(j\omega, \tilde{v}) = & \left| 2\gamma_1 \frac{0.6(j\omega)^4 + 3.6(j\omega)^3 + 6.5(j\omega)^2 + 4.5(j\omega) + 1}{Den_{[\tilde{W}(j\omega, \tilde{v})]}} + \right. \\ & 1.5\gamma_2 \frac{0.1(j\omega)^4 + 1.6(j\omega)^3 + 4.2(j\omega)^2 + 3.7(j\omega) + 1}{Den_{[\tilde{W}(j\omega, \tilde{v})]}} + \\ & \left. \gamma_3 \frac{0.1(j\omega)^4 + 0.8(j\omega)^3 + 2.7(j\omega)^2 + 3(j\omega) + 1}{Den_{[\tilde{W}(j\omega, \tilde{v})]}} \right| \angle \arctan \\ & \left[ 2\gamma_1 \frac{0.6(j\omega)^4 + 3.6(j\omega)^3 + 6.5(j\omega)^2 + 4.5(j\omega) + 1}{Den_{[\tilde{W}(j\omega, \tilde{v})]}} + \right. \\ & 1.5\gamma_2 \frac{0.1(j\omega)^4 + 1.6(j\omega)^3 + 4.2(j\omega)^2 + 3.7(j\omega) + 1}{Den_{[\tilde{W}(j\omega, \tilde{v})]}} + \\ & \left. + \gamma_3 \frac{0.1(j\omega)^4 + 0.8(j\omega)^3 + 2.7(j\omega)^2 + 3(j\omega) + 1}{Den_{[\tilde{W}(j\omega, \tilde{v})]}} \right]. \quad (42) \end{aligned}$$

As  $\omega$  tends to zero, Eq. (42) can be approximated as follows:

$$\lim_{\omega \rightarrow 0} \tilde{W}(j\omega, \tilde{v}) = |2\gamma_1 + 1.5\gamma_2 + \gamma_3| \angle \arctan [2\gamma_1 + 1.5\gamma_2 + \gamma_3]. \quad (43)$$

Hence

$$\lim_{\omega \rightarrow 0} \tilde{W}(j\omega, \tilde{v}) = |2\gamma_1 + 1.5\gamma_2 + \gamma_3| \angle 0^\circ. \tag{44}$$

Applying the *Theorem 3.1*, proposed in section 3, the obtained boundary conditions at low frequencies are presented in Tab. 1. The fuzzy frequency response of the SLPV dynamic system, at low frequencies, presents a magnitude range in the interval [0; 6.0206] (dB) and the phase is 0°.

Table 1. Boundary conditions at low frequencies.

Activated Rule	Boundary Condition	Magnitude (dB)	Phase (Degree)
1	$\gamma_1 = 1; \gamma_2 = 0$ and $\gamma_3 = 0$	6.0206	0°
2	$\gamma_1 = 0; \gamma_2 = 1$ and $\gamma_3 = 0$	3.5218	0°
3	$\gamma_1 = 0; \gamma_2 = 0$ and $\gamma_3 = 1$	0	0°

### 5.2 High Frequency Analysis

Likewise, starting from the TS fuzzy dynamic model, Eq. (37), and now applying the concepts seen in the subsection 4.2, the steady-state response for sinusoidal input at high frequencies for the SLPV dynamic system can be obtained as follows:

$$\begin{aligned} \lim_{\omega \rightarrow \infty} \tilde{W}(j\omega, \tilde{v}) = & \left| 2\gamma_1 \frac{0.6(j\omega)^4 + 3.6(j\omega)^3 + 6.5(j\omega)^2 + 4.5(j\omega) + 1}{Den_{[\tilde{W}(j\omega, \tilde{v})]}} + \right. \\ & 1.5\gamma_2 \frac{0.1(j\omega)^4 + 1.6(j\omega)^3 + 4.2(j\omega)^2 + 3.7(j\omega) + 1}{Den_{[\tilde{W}(j\omega, \tilde{v})]}} + \\ & \left. \gamma_3 \frac{0.1(j\omega)^4 + 0.8(j\omega)^3 + 2.7(j\omega)^2 + 3(j\omega) + 1}{Den_{[\tilde{W}(j\omega, \tilde{v})]}} \right| \angle \arctan \\ & \left[ 2\gamma_1 \frac{0.6(j\omega)^4 + 3.6(j\omega)^3 + 6.5(j\omega)^2 + 4.5(j\omega) + 1}{Den_{[\tilde{W}(j\omega, \tilde{v})]}} + \right. \\ & 1.5\gamma_2 \frac{0.1(j\omega)^4 + 1.6(j\omega)^3 + 4.2(j\omega)^2 + 3.7(j\omega) + 1}{Den_{[\tilde{W}(j\omega, \tilde{v})]}} + \\ & \left. \gamma_3 \frac{0.1(j\omega)^4 + 0.8(j\omega)^3 + 2.7(j\omega)^2 + 3(j\omega) + 1}{Den_{[\tilde{W}(j\omega, \tilde{v})]}} \right]. \tag{45} \end{aligned}$$

In this analysis, the higher degree terms of the transfer functions in the TS fuzzy dynamic model increase more rapidly than the other ones. Thus,

$$\lim_{\omega \rightarrow \infty} \tilde{W}(j\omega, \tilde{\nu}) = \left| 2\gamma_1 \frac{0.6(j\omega)^4}{0.1(j\omega)^6} + 1.5\gamma_2 \frac{0.1(j\omega)^4}{0.1(j\omega)^6} + \gamma_3 \frac{0.1(j\omega)^4}{0.1(j\omega)^6} \right| \angle \arctan \left[ 2\gamma_1 \frac{0.6(j\omega)^4}{0.1(j\omega)^6} + 1.5\gamma_2 \frac{0.1(j\omega)^4}{0.1(j\omega)^6} + \gamma_3 \frac{0.1(j\omega)^4}{0.1(j\omega)^6} \right]. \tag{46}$$

Hence

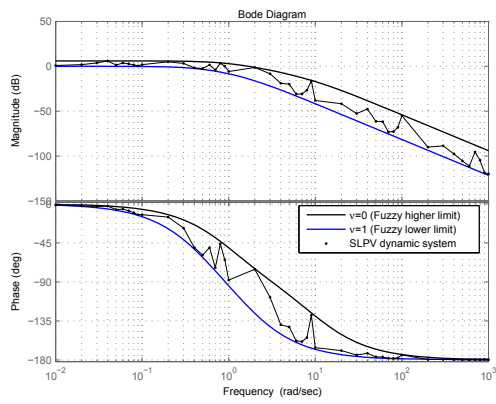
$$\lim_{\omega \rightarrow \infty} \tilde{W}(j\omega, \tilde{\nu}) = \left| 2\gamma_1 \frac{0.6}{0.1(j\omega)^2} + 1.5\gamma_2 \frac{0.1}{0.1(j\omega)^2} + \gamma_3 \frac{0.1}{0.1(j\omega)^2} \right| \angle -180^\circ. \tag{47}$$

Once again, applying the *Theorem 3.1*, proposed in section 3, the obtained boundary conditions at high frequencies are presented in Tab. 2. The fuzzy frequency response of the SLPV dynamic system, at high frequencies, presents a magnitude range in the interval  $\left[ \left| \frac{1}{(j\omega)^2} \right|, \left| \frac{12}{(j\omega)^2} \right| \right]$  (dB) and the phase is  $-180^\circ$ .

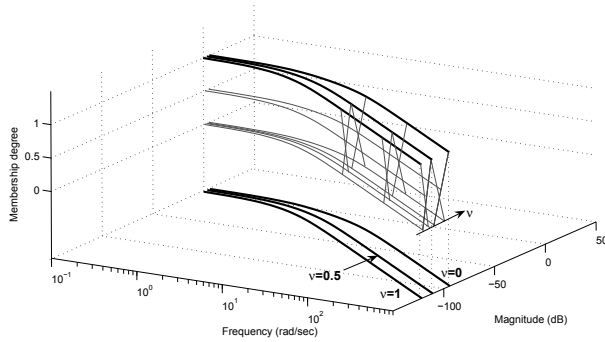
Table 2. Boundary conditions at high frequencies.

Activated Rule	Boundary Condition	Magnitude (dB)	Phase (Degree)
1	$\gamma_1 = 1; \gamma_2 = 0$ and $\gamma_3 = 0$	$\left  \frac{12}{(j\omega)^2} \right $	$-180^\circ$
2	$\gamma_1 = 0; \gamma_2 = 1$ and $\gamma_3 = 0$	$\left  \frac{1.5}{(j\omega)^2} \right $	$-180^\circ$
3	$\gamma_1 = 0; \gamma_2 = 0$ and $\gamma_3 = 1$	$\left  \frac{0.1}{(j\omega)^2} \right $	$-180^\circ$

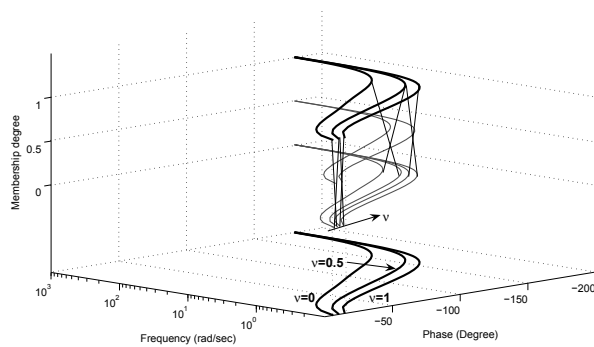
For comparative analysis, the fuzzy frequency response (boundary conditions at low and high frequencies from Tab. 1-2) and frequency response of the SLPV dynamic system are shown in Fig. 7. For this experiment, the frequency response of the SLPV dynamic system was obtained considering the mean of the stochastic parameter  $\nu$  in the frequency domain as shown in Fig. 8. The proposed structure for determining the frequency response of the SLPV dynamic system is shown in the block diagram (Fig. 9). It can be seen that the fuzzy frequency response is a region in the frequency domain, defined by the consequent linear sub-models  $W^i(s)$ , starting from the operating region of the antecedent space, as demonstrated by the proposed *Theorem 3.1*. This method highlights the efficiency of the fuzzy frequency response in order to estimate the frequency response of SLPV dynamic systems.



(a) Boundary region.

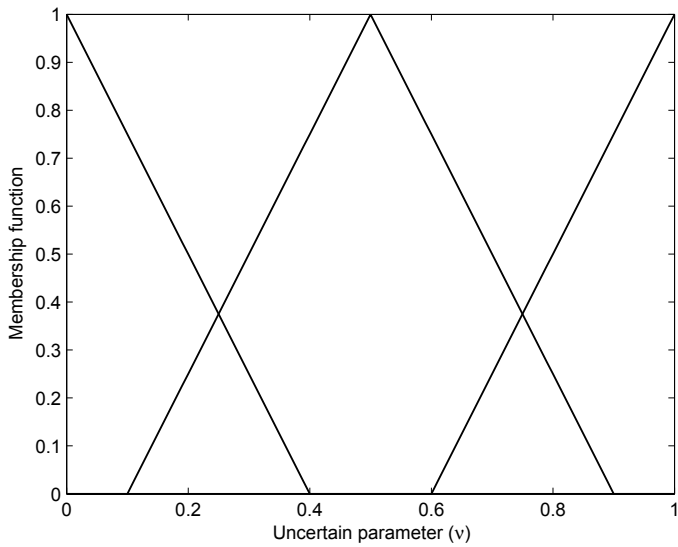


(b) Magnitude.

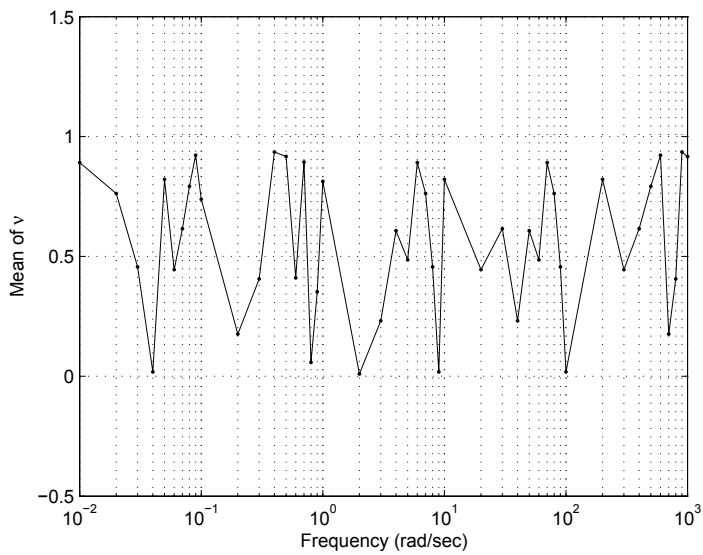


(c) Phase.

Fig. 7. Fuzzy frequency response of the SLPV dynamic system.



(a) Fuzzy sets of stochastic parameter ( $\nu$ ).



(b) Mean variation of stochastic parameter  $\nu$  in frequency domain.

Fig. 8. Fuzzy and statistic characteristics of the stochastic parameter ( $\nu$ ).



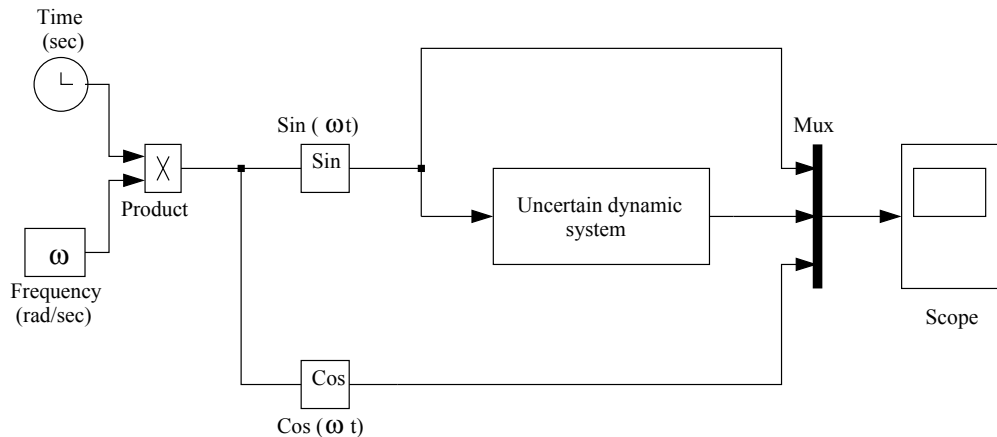


Fig. 9. Simulation diagram in Simulink to obtain Bode diagram of SLPV dynamic system.

## 6. Final Remarks

The Fuzzy Frequency Response: Definition and Analysis for Stochastic Linear Parameter Varying Dynamic Systems is proposed in this chapter. It was shown that the fuzzy frequency response is a region in the frequency domain, defined by the consequent linear sub-models  $W^i(s)$ , starting from operating regions of the SLPV dynamic system, according to the proposed *Theorem 3.1*. This formula is very efficient and can be used for robust stability analysis and control design for SLPV dynamic systems.

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