

Application of Receding Horizon Kalman Filter to Underwater Navigation Systems

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1. Introduction

In this chapter, the receding horizon Kalman filter is applied to underwater navigation systems. The ocean covers about two-thirds of the earth and has a great effect on human beings. However, the ocean is overlooked while we focus our attention on land and atmospheric issues; we have not been able to explore the full depths of the ocean, its abundant livings and non-living resources. For example, only recently we have discovered, by using manned submersibles, that a large amount of methane and carbon dioxide comes from the seafloor and extraordinary groups of organisms live in hydrothermal vent areas. However, a number of complex issues due to the unstructured and hazardous undersea environment make it difficult to survey in the ocean even though today's technologies have allowed humans to land on the moon and robots to travel to Mars.

Unmanned underwater vehicles (UUVs) can help us better understand marine and other environmental issues, protect the ocean resources of the earth from pollution, and efficiently utilize them for human welfare. The UUV is a platform for a variety of sensors: acoustic, magnetic, gravimetric and chemical ones. Most commercial UUVs are tethered and remotely operated, referred to as remotely operated vehicles (ROVs). Extensive use of manned submersibles and ROVs are currently limited to a few applications because of very high operational costs, operator fatigue and safety issues. The demand for advanced underwater vehicle technologies is growing and will eventually lead to fully autonomous and reliable underwater vehicles. Autonomous underwater vehicles (AUVs) were initially developed to perform missions that were not easy for ROVs and manned underwater vehicles. Since the autonomy allows AUVs to be used for risky missions such as a mine countermeasure (MCM) or under-ice operations, AUVs are replacing ROVs towed vehicles as well as manned underwater vehicles (Whitcomb, 2000). For detailed ocean surveys, an AUV acts as a more stable platform for precision sensors than ROVs or towed vehicles because an AUV is not subject to physical disturbances transmitted along the cable to the surface vessel. This absence of physical attachment also allows AUVs to measure ocean characteristics at specific depths and perform bottom-following missions as owing to its autonomy. In short, An AUV provides marine researchers with a new form of access to deeper ocean.

For an AUV to successfully complete a typical survey mission, it must follow a path specified by the operator as closely as possible and arrive at a precise location for collecting data. When an AUV is not able to follow the path accurately during the mission, critical

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features may not be recorded and the position of any features during the mission will be uncertain. When the final position of an AUV is not accurate, the AUV may even be unrecoverable. Since AUVs normally carry out missions in wide, unstructured and highly dynamic environments, an accurate and 'reliable' navigation system is required. Thus the most important part of an AUV is the navigation system.

There are many navigation systems for underwater vehicles. Global positioning system (GPS) based navigation systems offer good navigation solutions due to the absolute positioning capability (Farrell & Barth, 1999). However, the drawback of the GPS based navigation systems is that the AUVs need to rise to shallow water depths in order to communicate with satellites, which is a time and energy consuming task (Marco & Healey, 2001). An alternative to the GPS based devices is an acoustic based positioning system (Larsen, 2000, Lee et al., 2004). An acoustic system uses external sound emitting beacons in order to triangulate its position. While precise, navigation based on this type of device incurs a high cost and limits the mission space to the area covered by the beacons. Dead reckoning systems, such as the inertial navigation system (INS) or the Doppler velocity log (DVL), estimate the position of an AUV with respect to the initial position by measuring linear and angular velocities and/or accelerations. Using this type of sensor offers a practical and inexpensive navigation method. Nevertheless, these sensors accumulate drifts over time because position errors tend to increase with any new measurement.

Unlike ground and aerial vehicles, an AUV presents a uniquely challenging navigational problem because it operates autonomously in a highly unstructured environment where satellite based navigations such as the GPS are not directly available. In this respect, many AUVs basically navigate with the help of underwater navigation systems which are comprised of optimal filters and sensors such as an inertial measurement unit (IMU), a DVL, a current meter and a magnetic compass.

The primary challenge of navigation systems for AUVs is maintaining the accuracy of the position of the AUVs over the course of a long mission. An initially accurate position may quickly become uncertain through variations in the motion of the AUVs. While this effect can be reduced by using accurate acceleration, heading and velocity sensors, but these sensors cannot be made as accurate as required. During long missions, these inaccuracies become significant. Temporal and strong currents or other underwater disturbances that affect the motion of the AUV cannot be precisely modelled in advance and these may deteriorate the accuracy further. If the position of an AUV is not externally referenced, the position accuracy will inevitably degrade over the course of the mission. The absence of an easily observable, external reference makes the navigation of AUVs very difficult. External references must be used for any AUV navigation systems that yield to an accurate navigation over long missions.

As stated above, the navigation system may employ multiple measuring devices to enhance the accuracy and reliability of the system. This practice becomes important because existing systems can be upgraded by supplementing more accurate and economic navigation devices. This implies the necessity of finding a new way of sharing and merging data obtained from the various devices. Since the data collected by each device represent only partial information of the phenomenon under survey, a process of 'sensor data fusion' is required in order to acquire complete information. With measurement fusion, some measured data for both steady and unsteady systems are passed directly to the fusion centre for centralized Kalman filter. Because of its relatively lower state estimation errors, the measurement fusion is a widely used method (Titterton & Weston, 1997).

However, it is well known that an AUV often experiences position errors caused by environmental disturbances as mentioned above. This is particularly the case for an inertial navigation system with velocity measurement and when the standard Kalman filter is influenced by the action of temporary currents (Jo et al., 2006). This is the reason why robust filters for navigation systems have been studied (Yu et al., 2004, Seo et al., 2006). This article focuses on the receding horizon Kalman filter (RHKF), which is a kind of robust filter.

The estimation of the state of a dynamic system by using the measurement obtained from the most recent time is defined in various ways, including fixed memory, receding horizon or sliding window estimation (Bierman, 1975). These estimation methods were originally reported in the work of Jazwinski (1968). Since then, the estimation method has been widely used in many application areas, where measurement uncertainties hinder the proper use of the Kalman filter.

Finite impulse response (FIR) filters utilize finite measurements over the most recent time interval. FIR structured filters have actually become a standard filter commonly used because they are more robust against numerical errors and temporary uncertainties than infinite impulse response (IIR) structured filters, which utilize all measurements on the infinite interval.

Kwon et al. (1989) and Kwon et al. (1999) suggested the use of optimal FIR filters and the receding horizon Kalman FIR filter for discrete linear time invariant systems for state estimation. When using these filters, error covariance is minimized in discrete-time stochastic state-space models. The optimal FIR filter has several advantages over existing optimal filters such as the celebrated Kalman filter, which is a kind of an optimal IIR filter. Since the optimal FIR filter utilizes finite measurement over the most recent time interval, it is robust against temporary modelling and measurement uncertainties that may cause a divergence of the estimated state. Divergence was a problem for the optimal IIR structured filter. An optimal FIR filter can be effectively derived by modifying the Kalman filter because the Kalman filter is easy to use and widely applied in many engineering problems. In this article, the standard Kalman filter is combined with the receding horizon strategy which has already been adopted in many optimal control and estimation methods. We denote it as the RHKF for time-varying systems.

The velocity-aided inertial navigation system is relatively underdeveloped for underwater applications, although it has been proven to be a good method for land applications (Marco & Healey, 2001). The DVL is a kind of measuring device used for velocity information. However, the DVL can give reliable velocity information to AUVs only when the AUVs cruise close to the bottom or the surface. Current meters are used more often for an AUV than the DVL devices because current meters measure the relative speed between the AUV and the surrounding fluid. The devices have no limit of relative distances to surface or bottom. The disadvantage of current meters is that their accuracy may be deteriorated by ocean current. In general, the ocean current often occurs temporarily and its magnitude can exceed the speed of the vehicle. In this case, the velocity-aided navigation system with the Kalman filter is deteriorated by temporary disturbances. In the worst case, the position error of the system may even 'blow up'. However, as it will be shown, the underwater navigation system based on the RHKF is robust against temporary disturbances. Therefore the current meters can be used for an accurate the velocity-aided navigation system for AUVs with help of the RHKF.

The statistical process and measurement noises are often unknown a priori in most practical situations. In such a situation, the Kalman filter may fail to accurately estimate state

variables or may even cause system failure due to divergence (Fitzgerald, 1971, Sangsuklam & Bullock, 1990). With these uncertainties, it is difficult to develop an accurate navigation system with the Kalman filter. The difficulty can be overcome by using a number of estimation methods of noise covariance and some have been proposed. The problem is also important to make the navigation systems accurate. But it must be kept in mind that it remains herein. More information can be found in Mehra (1970), Belanger (1974), Um et al. (2000) and Jo et al. (submitted).

This chapter contains the following information: In section II, the error equations for a velocity-aided underwater navigation system are introduced. In section III, the RHKF for time-varying systems is derived in an iterative form. In section IV, simulations for a linear time-varying system and the navigation system are explained. Finally, a short summary is given.

2. Dynamic model for navigation errors

2.1 Coordinate systems

Coordinate systems used in this chapter are the inertial frame (i-frame), the Earth-fixed frame (e-frame), the navigational frame (n-frame) and the body frame (b-frame). The i-frame is fixed with the centre of the Earth as the origin. The e-frame coincides with the i-frame at the origin but rotates with the Earth rate. The n-frame is a local level frame in which the vertical axis is parallel with the gravity vector. Fig. 1 shows the reference frames.

In the strapdown inertial navigation system (SDINS), the determination of the transformation matrix from the b-frame to the n-frame is very important and error generated during the computation of the transformation matrix becomes one of the main error sources of the system.

Various mathematical representations are used to define the attitude of a body with respect to a coordinate reference frame. In three-dimensional space, spatial rotations can be parameterized using both Euler angles and unit quaternions. The parameters associated with each method may be stored in computer and updated as the vehicle rotates, using the measurements of turn rate by the SDINS. Theorems derived from the Euler angles state that any given sequence of rotations can be represented as a single rotation about a single fixed axis. In the Euler angle representation, the attitude is defined by roll (Φ), pitch (θ) and yaw (ψ) angles, which constitute a direction cosine matrix (DCM) (Siouris, 1993). The yaw angle is equivalent to the heading angle while the roll and pitch angles represent the levelling angles.

However, among the popular methods of attitude computations, quaternions are known to be the most effective one due to their simplicity and easy normalization procedure, in modern navigation systems (Siouris, 1993). The representation of rotation as a quaternion is more compact than the representation of the DCM or the Euler angles. Furthermore, for a given axis and angle, one can easily construct the corresponding quaternion and, conversely, for a given quaternion one can easily read off the axis and the angle. When composing several rotations on a computer, round off errors inevitably accumulate. A quaternion that is slightly off still represents a rotation after being normalised. The quaternion also avoids the phenomenon called gimbal lock which can result when, for example in the Euler angle representation, the pitch angle is rotated by 90° up or down, so that yaw and roll correspond to the same motion, and a degree of freedom of rotation is lost. In the SDINS, for instance, this could lead to a disastrous result if the vehicle is in a steep

dive or ascent. For more rigorous and mathematical analysis, refer to Altmann (1986). In this chapter, the dynamic equation for attitude is mainly described by quaternions.

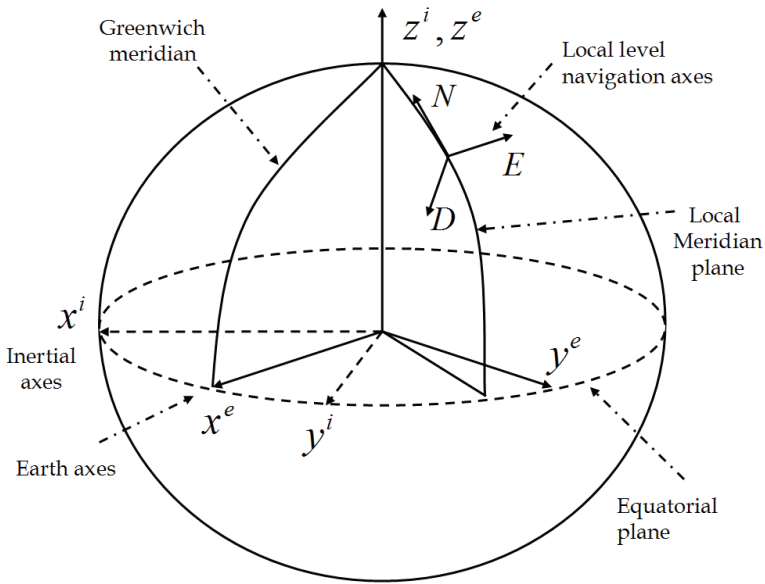


Fig. 1. Reference frames

2.2 Error model for SDINS

The SDINS error model plays an important role in implementing an optimal filter for alignment or for velocity-aided navigation algorithms. The quaternion error model with the attitude and the velocity error state for SDINS was first developed in the i-frame by Friedland (1978), and later in the n-frame by Shibata (1986). The axes of the n-frame point in the directions of north, east and downward in that order. In the n-frame, the rate vector is represented as follows:

$$\begin{aligned} \omega_m^n &= \omega_{ie}^n + \omega_{en}^n \\ \omega_{ie}^n &= [\Omega_N \quad 0 \quad \Omega_D]^T = [\Omega \cos L \quad 0 \quad -\Omega \sin L]^T \\ \omega_{en}^n &= [\rho_N \quad \rho_E \quad \rho_D]^T = [\dot{l} \cos L \quad -\dot{L} \quad -\dot{l} \sin L]^T \end{aligned}$$

where ω_m^n is the rate of the n-frame relative to the i-frame expressed in the n-frame, ω_{ie}^n is the Earth rate, ω_{en}^n is the rate of the n-frame relative to the e-frame, L denotes the latitude, l represents the longitude and Ω is the magnitude of the Earth rate. The superscripts n and b denote the n-frame and the b-frame, respectively.

The dynamic equations of the position, velocity and attitude of SDINS in the n-frame are given as follows (Titterton & Weston, 1997):

$$\dot{L} = \frac{V_N}{R_m + h}, \quad \dot{l} = \frac{V_E}{(R_t + h) \cos L}, \quad \dot{h} = -V_D \tag{1}$$

$$\dot{v}^n = C_b^n f^b - (2\omega_{ie}^n + \omega_{en}^n) \times v^n + g^n \tag{2}$$

$$\dot{C}_b^n = C_b^n \Omega_{nb}^b \tag{3}$$

with $R_m = R_0(1 - 2e + 3e \sin^2 L)$ and $R_t = R_0(1 + e \sin^2 L)$, where R_0 is the radius of the Earth at the equator, e denotes the major eccentricity of the Earth, h is the height, g^n is the gravitational force represented in the n-frame, $f^b (= [f_x \ f_y \ f_z]^T)$ denotes the specific force measured at the accelerometer, $v^n (= [V_N \ V_E \ V_D]^T)$ is the velocity of the vehicle represented in the n-frame, $\omega_{nb}^b (= [\omega_x \ \omega_y \ \omega_z]^T)$ is the rate of the b-frame relative to the n-frame, and Ω_{xy}^z is the skew-symmetric matrix for the rate ω_{xy}^z .

The relationships among the quaternion, Euler angles and DCM are given by

$$C_b^n = \begin{bmatrix} \cos\theta \cos\psi & \sin\phi \sin\theta \cos\psi - \cos\phi \sin\psi & \cos\phi \sin\theta \cos\psi + \sin\phi \sin\psi \\ \cos\theta \sin\psi & \sin\phi \sin\theta \sin\psi + \cos\phi \cos\psi & \cos\phi \sin\theta \sin\psi - \cos\psi \sin\phi \\ -\sin\theta & \sin\phi \cos\theta & \cos\phi \cos\theta \end{bmatrix} \tag{4}$$

$$= \begin{bmatrix} q_0^2 + q_1^2 - q_2^2 - q_3^2 & 2(q_1q_2 - q_0q_3) & 2(q_1q_3 + q_0q_2) \\ 2(q_1q_2 + q_0q_3) & q_0^2 - q_1^2 + q_2^2 - q_3^2 & 2(q_2q_3 - q_0q_1) \\ 2(q_1q_3 - q_0q_2) & 2(q_2q_3 + q_0q_1) & q_0^2 - q_1^2 - q_2^2 + q_3^2 \end{bmatrix}$$

The attitude equation by the quaternion is

$$\dot{q} = 0.5q * \omega_{nb}^b = 0.5q * (\omega_{ib}^b - C_n^b (\omega_{ie}^n + \omega_{en}^n)) \tag{5}$$

where $q = q_0 + \hat{i}q_1 + \hat{j}q_2 + \hat{k}q_3$ is a quaternion, ω_{ib}^b denotes the measurement of gyroscopes and * is the quaternion product which is defined as

$$\begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix} * \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} q_0 & -q_1 & -q_2 & -q_3 \\ q_1 & q_0 & -q_3 & q_2 \\ q_2 & q_3 & q_0 & -q_1 \\ q_3 & -q_2 & q_1 & q_0 \end{bmatrix} \begin{bmatrix} 0 \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} \tag{6}$$

The error model of SDINS can be obtained by the perturbation method under several assumptions (Titterton & Weston, 1997)

$$\delta \dot{L} = \frac{\rho_E R_{mm} \delta L + \rho_E \delta h + \delta v_N}{R_m + h} \tag{7}$$

$$\delta \dot{l} = \rho_N \sec L \left(\tan L - \frac{R_t}{R_t + h} \right) \delta L - \frac{\rho_N \sec L \delta h - \sec L \delta v_E}{R_t + h} \tag{8}$$

$$\delta \dot{h} = -\delta V_D \tag{9}$$

$$\delta \dot{v} = [C_b^n f^b] \times \varepsilon - (2\omega_{ie}^n + \omega_{en}^n) \times \delta v^n + C_b^n \delta f^b + v^n \times (\delta 2\omega_{ie}^n + \delta \omega_{en}^n) \tag{10}$$

$$\dot{\varphi} = -\omega_{in}^n \times \varphi - C_b^n \delta \omega_{ib}^b + \delta \omega_{in}^n \tag{11}$$

with $R_{mm} = \partial R_m / \partial L = 6R_0 e \sin L \cos L$ and $R_{tt} = \partial R_t / \partial L = 2R_0 e \sin L \cos L$, where δL , δl and δh are errors of latitude, longitude and altitude, respectively, and $\delta v^n = [\delta v_N \ \delta v_E \ \delta v_D]^T$ is a velocity error in the n-frame.

$\varphi = [\varphi_N \ \varphi_E \ \varphi_D]^T$ is a tilt angle that is approximately equal to the Euler angle error under a small angle assumption and $\delta \omega_{ie}^n$, $\delta \omega_{en}^n$ and $\delta \omega_{in}^n$ are defined as

$$\delta \omega_{ie}^n = [-\Omega \sin L \delta L \quad 0 \quad -\Omega \cos L \delta L]^T \tag{12}$$

$$\delta \omega_{en}^n \approx \begin{bmatrix} -\frac{\rho_N}{R_t + h} \delta h + \frac{1}{R_t + h} \delta v_E \\ -\frac{\rho_E}{R_m + h} \delta h - \frac{1}{R_m + h} \delta v_N \\ -\rho_N \sec^2 L \delta L - \frac{\rho_D}{R_t + h} \delta h + \frac{\rho_D}{v_E} \delta v_E \end{bmatrix} \tag{13}$$

$$\delta \omega_{in}^n = \delta \omega_{ie}^n + \delta \omega_{en}^n \tag{14}$$

In (10) and (11), δf^b is an accelerometer error vector and $\delta \omega_{ib}^b$ is a gyro error vector. These errors of inertial sensors may be simply modelled as a sum of random constant and white Gaussian noise.

$$\delta f^b = \nabla_a + w_a(t), \quad w_a(t) \sim N(0, Q_a) \\ \dot{\nabla}_a = 0, \quad \nabla_a = [\nabla_x \ \nabla_y \ \nabla_z]^T \tag{15}$$

$$\delta \omega_{ib}^b = \varepsilon_g + w_g(t), \quad w_g(t) \sim N(0, Q_g) \\ \dot{\varepsilon}_g = 0, \quad \varepsilon_g = [\varepsilon_x \ \varepsilon_y \ \varepsilon_z]^T \tag{16}$$

Herein, the biases of the accelerometers and gyros, ∇_a and ε_g , are assumed to be random constant even though they generally vary very slowly.

2.3 Error model of measurements

Some auxiliary sensors should be used to compensate for the navigation errors of the SDINS. A pressure sensor, current meters and the magnetic compass can be used as auxiliary sensor in the navigation system for AUVs. A surface navigation system has been successfully developed by integrating position fixing systems such as GPS. As stated above, introducing GPS to the navigation system for an AUV is limited to the case of shallow water vehicles repeatedly surfacing to update the position information. If available, long-base line

(LBL) and short-base line (SBL) systems can be generally used for positioning underwater vehicles. Furthermore, near surface navigation systems can be utilized when a vehicle operates near the water surface or intentionally approaches the surface. The navigation systems for long-range cruising-type AUVs are structured by dead-reckoning. However, the inevitable growth of errors within the navigation system motivates the need for on-line calibration methods such as GPS-aided or Loran-C-aided navigation. Larsen (2000) and Lee et al. (2004) proposed hybrid navigation systems based on the IMU combined with acoustic velocity sensors.

We simply assumed such a situation for a numerical example. The modelled vehicle is assumed to be equipped with an IMU, a 3-axis magnetic-type current meter, a 3-axis magnetic compass and a pressure sensor to obtain depth information. They are also modeled to be the sum of random constant and white Gaussian noise.

The equations of the sensors can be formulated by

$$\hat{h}_{INS} - h_m = h_{true} + \delta h - (h_{true} + h_{bias}) = \delta h - h_{bias} \tag{17}$$

$$\begin{aligned} \hat{v}_{INS}^n - v_m &= \hat{v}_{true}^n + \delta v^n - C_b^n (v_{true}^n + v_{bias}) \\ &\approx \delta v^n - v^n \times \phi - C_b^n v_{bias} \end{aligned} \tag{18}$$

$$\begin{bmatrix} \hat{\phi}_{INS} \\ \hat{\theta}_{INS} \\ \hat{\psi}_{INS} \end{bmatrix} - \begin{bmatrix} \phi_m \\ \theta_m \\ \psi_m \end{bmatrix} = - \begin{bmatrix} \phi_N \\ \phi_E \\ \phi_D \end{bmatrix} - \begin{bmatrix} \phi_{bias} \\ \theta_{bias} \\ \psi_{bias} \end{bmatrix} \tag{19}$$

where $\hat{\cdot}$ denotes the estimated value by SDINS and $[\phi \ \theta \ \psi]^T$ denotes the roll, pitch and yaw angle of the vehicle, respectively. The subscript m, INS and bias denotes the measured value of the sensors, the estimated value of optimal filters and sensor bias. Optimal filters for the navigation system can be updated with these equations.

Combining (1)-(19) with the differential equations for sensor errors yields the following error equation for the velocity-aided navigation system.

$$\dot{x}(t) = F(t)x(t) + w(t), \quad w \sim N(0, Q(t)) \tag{20a}$$

where

$$\begin{aligned} x &= \begin{bmatrix} x_{INS}^T & x_{bias}^T \end{bmatrix}^T \\ x_{INS} &= \begin{bmatrix} \delta P^T & (\delta v^n)^T & \phi^T & \nabla_a^T & \varepsilon_g^T \end{bmatrix}^T \\ x_{bias} &= \begin{bmatrix} h_{bias} & v_{bias}^T & \phi_{bias} & \theta_{bias} & \psi_{bias} \end{bmatrix}^T \\ w &= \begin{bmatrix} 0_{1 \times 9} & w_a^T & w_g^T & 0_{1 \times 7} \end{bmatrix}^T \end{aligned}$$

The time-varying system matrix $F(t)$, which is a differential equation of SDINS can be used to estimate the position, velocity, attitude and the biases of the sensors (See Appendix for

details). The state variable $x(t)$ has 22 error states: δP denotes the position error in the latitude, longitude and altitude. δv^n denotes the velocity errors in the n-frame, while φ denotes the vector of roll, pitch and yaw angles.

The innovation (also called measurement residual) of the optimal filter is the difference between estimated values by SDINS and measurements of pressure, velocity and attitude. The innovation for a velocity-aided underwater navigation system at t_k may be expressed in terms of the error state variables as follows:

$$\begin{aligned}
 y(t_k) &= \begin{bmatrix} \hat{h}_{INS} \\ \hat{v}_{INS}^n \\ \hat{\phi}_{INS} \\ \hat{\theta}_{INS} \\ \hat{\psi}_{INS} \end{bmatrix} - \begin{bmatrix} h_m \\ v_m^n \\ \phi_m \\ \theta_m \\ \psi_m \end{bmatrix} \\
 &= \begin{bmatrix} 0_{1 \times 2} & 1 & 0_{1 \times 12} & -1 & 0_{1 \times 3} & 0_{1 \times 3} \\ 0_{3 \times 3} & I_{3 \times 3} & -V^n \times & 0_{3 \times 7} & -C_b^n & 0_{3 \times 3} \\ 0_{3 \times 3} & 0_{3 \times 3} & -I_{3 \times 3} & 0_{3 \times 7} & 0_{3 \times 3} & -I_{3 \times 3} \end{bmatrix} x(t_k) \\
 &\quad + \begin{bmatrix} v_h(t_k) \\ v_{cur}(t_k) \\ v_{com}(t_k) \end{bmatrix}
 \end{aligned} \tag{20b}$$

where v_h , v_{cur} and v_{com} are the measurement noise of the pressure sensor, current meters and compass, respectively.

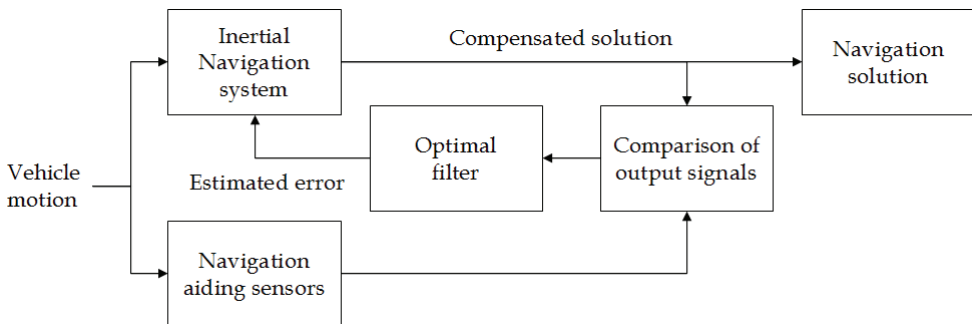


Fig. 2. Indirect feedback structure

In general, while the inertial navigation system is nonlinear, the error of the system can be assumed to be linear. This is the reason why the indirect feedback method is used for the navigation system. Fig.2 shows the block diagram of the indirect feedback method. In the figure, it can be seen that the navigation system predicts the position, velocity, attitude and biases of the sensors with the output of the SDINS and then compensates the error states for the system by using an optimal filter. The navigation solutions are updated indirectly each time when the depth, velocity and attitude are measured by the sensors. In the indirect scheme, the navigation solutions are already given as the predetermined nominal points,

while the Kalman filter estimates the navigation solution. The word 'indirect' means that the filter estimates not the navigation solutions but the error of the solutions. Incorporated with the SDINS equations, the indirect method becomes efficient. If the navigation system uses indirect feedback states, the error system consists of a linearized system with compensated states.

3. Receding horizon Kalman filter for underwater navigation systems

In this section, the receding horizon Kalman filter is introduced for the underwater navigation system. Consider a linear discrete time-varying state space model with control input

$$\begin{aligned}x_{k+1} &= F_k x_k + B_k u_k + G w_k \\ y_k &= H_k x_k + v_k\end{aligned}\quad (21)$$

where $x_k \in \mathfrak{R}^n$ is the state vector and $u_k \in \mathfrak{R}^l$ and $y_k \in \mathfrak{R}^q$ are the input vector and the measured output vector, respectively.

The initial state x_{k_0} is assumed to be random with a certain mean \bar{x}_{k_0} and a certain covariance Σ_{k_0} . The process and measurement noises are assumed to be zero-mean white Gaussian and are not correlated with each other. The covariances of w_k and v_k are denoted by Q and R , respectively. It is assumed that these noises are not correlated with the initial state x_{k_0} .

The following Kalman filter for a time-varying system provides a state estimate \hat{x}_k called, the one-step predicted estimate of the system state x_k , with control input

$$\hat{x}_{k+1} = F_k \hat{x}_k + K_k (y_k - H_k \hat{x}_k) + B_k u_k \quad (22)$$

$$K_k = F_k P_k H_k^T (R + H_k P_k H_k^T)^{-1} \quad (23)$$

$$P_{k+1} = F_k P_k F_k^T + G Q G^T - F_k P_k H_k^T (H_k P_k H_k^T + R)^{-1} H_k P_k F_k^T \quad (24)$$

with $\hat{x}_{k_0} = \bar{x}_{k_0}$, where P_k is the error covariance of the estimate \hat{x}_k with the initial value $P_{k_0} = \Sigma_{k_0}$.

In order to derive the RHKF for the stochastic systems, input and output information on the horizon $[k-N, k]$ are utilized together with information about the state at the starting point $k-N$. We write $k_N \equiv k-N$ for convenience. We refer to this state at t_k as the horizon initial state. It is logical to assume that the horizon initial state cannot be measured and thus is unknown. An information form of the Kalman filter is adopted as follows:

At first, let us define the new variables

$$\Omega_k \equiv P_k^{-1}, \quad \bar{\Omega}_k \equiv P_k^{-1} + H_k^T R^{-1} H_k$$

The estimation error covariance (24) can then be rewritten as

$$\Omega_{k+1} = [I + F_k^{-T} \bar{\Omega}_k F_k^{-1} G Q G^T]^{-1} F_k^{-T} \bar{\Omega}_k F_k^{-1} \quad (25)$$

with $\Omega_{k_0} = \Sigma_{k_0}^{-1}$

The information form of filter (22) becomes

$$\hat{x}_{k+1} = F_k \bar{\Omega}_k^{-1} (\Omega_k \hat{x}_k + H_k^T R^{-1} y_k) + B_k u_k. \tag{26}$$

The information form (26) of the Kalman filter uses all measurements starting from the initial time k_0 to provide the one-step predicted state estimate at the present time. By introducing the receding horizon strategy to filter (26), the RHKF at the present time k uses only the finite measurements on the horizon $[k_N, k]$ and discards the past measurements outside the horizon. We re-derived filter (26) on the horizon covering from the horizon initial time k_N to the present time k . The filter at time $k_N + i$ on the horizon $[k_N, k]$ is denoted as $\hat{x}_{k_N+i|k}$ with $0 \leq i < N$. In (24), the error covariance of the estimate \hat{x}_k , system F_{k_N+i} and measurement equations H_{k_N+i} are not correlated with the measurements for linear time-varying systems. The horizon initial condition is denoted as Ω_{k_N} . Since the measurements and control inputs are not estimated variables, these can be written as $y_{k_N+i|k} = y_{k_N+i}$ and $u_{k_N+i|k} = u_{k_N+i}$.

The filter can now be re-written by

$$\hat{x}_{k_N+i+1|k} = F_{k_N+i} \bar{\Omega}_{k_N+i}^{-1} (\Omega_{k_N+i} \hat{x}_{k_N+i|k} + H_{k_N+i}^T R^{-1} y_{k_N+i}) + B_{k_N+i} u_{k_N+i} \tag{27}$$

In (25), $\Omega_{k_N+i} > 0$ for all $i \geq n$, if $\{F_k, H_k\}$ is uniformly observable (Kwon & Pearson, 1978). In the above equation, n denotes the dimension of the system. This guarantees that a converged result is globally optimal to Kalman filter based FIR filters. Since it is difficult to know the horizon initial state $x_{k_N|k}$, it is assumed to be unknown. The horizon initial state must have an arbitrary mean and infinite covariance as $\Omega_{k_N} = 0$.

Theorem 1 RHKF for time-varying systems (Jo & Choi, 2006)

Assume that $\{F_k, H_k\}$ is uniformly observable and $N \geq n$. If the horizon initial state $x_{k_N|k}$ is assumed to be unknown on the horizon $[k_N, k]$, the RHKF for the state x_k is given by

$$\hat{x}_{k_N+i+1|k} = F_{k_N+i} \bar{\Omega}_{k_N+i}^{-1} (\Omega_{k_N+i} \hat{x}_{k_N+i|k} + H_{k_N+i}^T R^{-1} y_{k_N+i}) + B_{k_N+i} u_{k_N+i} \tag{28}$$

where

$$\Omega_{k_N+i+1} = [I + F_{k_N+i}^{-T} \bar{\Omega}_{k_N+i}^{-1} F_{k_N+i}^{-1} G Q G^T]^{-1} F_{k_N+i}^{-T} \bar{\Omega}_{k_N+i}^{-1} F_{k_N+i}^{-1} \tag{29}$$

Jo & Choi (2006) proved that the derived filter (28) has two important properties - it is unbiased and deadbeat and that the RHKF becomes a deadbeat observer when the filter is applied to the following noise-free system:

$$\begin{aligned} x_{k+1} &= F_k x_k + B_k u_k \\ y_k &= H_k x_k \end{aligned} \tag{30}$$

If (30) is contaminated by noises similar to (21), the RHKF optimally compensates for the noises. When the system has no noise, the RHKF can yield an exact estimate of the state. This deadbeat property indicates the finite convergence time and the fast tracking ability of the RHKF. Thus, we can expect that the suggested RHKF is appropriate for quick estimation and detection of AUV tracking even when occurrence of noises is not known.

4. Simulation

In this section, the rate at which the RHKF converges against temporary unknown disturbances is determined and its robustness is illustrated. At first, the RHKF is applied to a linear time-varying system. The RHKF is then applied to the velocity-aided underwater navigation system.

Consider the following linear system:

$$\begin{aligned}\dot{x}(t) &= A(t)x(t) + Gw(t) \\ y(t) &= H(t)x(t) + v(t)\end{aligned}\quad (31)$$

with

$$A(t) = \begin{bmatrix} -a + (a-b)\sin^2 \omega t + \delta(t) & \omega + (a-b)\sin \omega t \cos \omega t \\ -\omega + (a-b)\sin \omega t \cos \omega t & -a + (a-b)\cos^2 \omega t + \delta(t) \end{bmatrix}$$

$$G = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad H = [1 \quad 0], \quad \delta(t) = \begin{cases} 0.05 & 100 \leq t < 200 \\ 0 & \text{otherwise} \end{cases}$$

where $x(=[x_1 \quad x_2]^T)$ is the state vector, $w(=[w_1 \quad w_2]^T)$ denotes the process noise vector, y is the output and v is the measurement noise. When $a=0.003$, $b=0.007$ and $\omega=0.01$ are chosen, and the variance of the noises is assumed to be 0.05, the estimated error of x_1 is as shown in Fig. 3. The RHKF estimates the present state x_k with information on the prescribed horizon interval. The RHKF has some advantages (robustness against temporary disturbances and fast tracking ability), if the horizon interval is properly chosen. However, it takes a considerable amount of time to estimate the present state because the filter recedes from the horizon initial state each time. This is the reason why the RHKF takes more computing time (of about N times) than the Kalman filter, which has an IIR structure. A proper horizon interval must be chosen carefully for a real-time system.

In Fig. 3, as the horizon interval increases, the estimates obtained by the RHKF more closely approach those provided by the Kalman filter and it becomes less robust against the temporary disturbance. It can be also seen that, if the horizon interval is small, the estimates obtained by the RHKF fluctuate around the real value instead of converging to the real one. However, if the interval is large, it approaches closer to the estimate obtained from the Kalman filter. The horizon interval is important in the optimal estimation. Unfortunately, it is impossible to find an optimal horizon interval for general non-linear systems. Thus, a reasonable interval N must be chosen based on experience. The rule of thumb is to take it slightly larger than the limit of the index to converge to the covariance of the standard Kalman filter.

Let us consider that an AUV is equipped with a low-degree IMU, a pressure sensor, a 3-axis current meter, and a 3-axis magnetic compass. The sampling rate of the IMU is 100 Hz and that of the pressure sensor, 3-axis current meter, and the 3-axis magnetic compass is all 1 Hz. The characteristics of the sensors are listed in Table 1.

In this simulation, the following assumptions are made: The vehicle moves directly north at a rate of 2m/s and downward at a rate of 0.05m/s without any change in attitude for 1200 seconds. It is known that SDINS has poor observability during straight forward running without any change in attitude because the attitude error model depends on the angular rate of the vehicle (Lee et al., 1993).

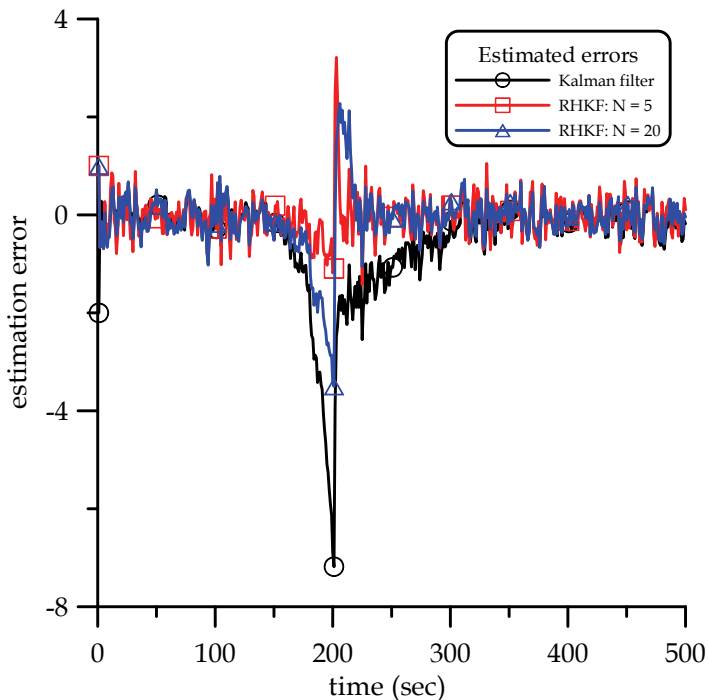


Fig. 3. Comparison of estimated error for the linear system

		Bias (1 σ)	Noise (1 σ)
IMU	Accelerometers	9.8 mm/s ²	0.49 mm/s ²
	Gyros	1deg/hr	0.35 deg/hr
Pressure sensor		0.1 m	0.1 m
Current meter		0.01 m/s	0.01 m/s
Compass		1 deg	2 deg

Table 1. Characteristics of Sample Sensors

Upwelling current of 0.5 m/s is introduced as a temporal disturbance. Fig. 4 shows the assumed velocity profile of the upwelling current in the NED coordinate. As shown in the figure, the upwelling current occurred temporarily during a time span between 385 sec and 655 sec. Fig. 5 and Fig. 6 show the reference trajectory of the vehicle under the action of the disturbance and Fig. 7 shows the reference velocity profile of the vehicle. It was assumed that the vehicle reacted against the current in 5 sec when it encountered with the current as shown in Fig. 7.

The trajectory and velocity of the vehicle under the upwelling current were estimated using the navigation filters, the standard Kalman filter and the RHKF. Figs. 8 shows the estimated velocity obtained by the Kalman filter. By comparing Fig. 7 with Fig. 8, it can be seen that the estimation error of the Kalman filter increases after the vehicle encounters with the current.

The choice of the horizon interval N affects the convergence speed of the RHKF. It is very difficult, as stated above, to find the optimal horizon interval N for general nonlinear systems. However, if a system is uniformly observable, the horizon interval N can be chosen as larger than the dimension of the system, because $\Omega_{k_N+i|k} > 0$ for all $i \geq n$, if $\{F_{k_N+i|k}, H_{k_N+i|k}\}$ is uniformly observable. Hereby n denotes the dimension of the system. It may be recommended with care that one may choose the interval to be larger than the dimension of the system, but less than 4 times. In this simulation, we chose $N = 30$.

As stated above, the RHKF is robust against temporary modelling and measuring uncertainties because it utilizes only the finite measurements on the most recent horizon. The estimated velocity obtained by the RHKF is shown in Fig. 9. In Fig. 10, it is seen that the navigation system based on the RHKF does not lost its position but the vehicle that was navigated by the Kalman filter has lost its position after the current occurs. While the pressure sensor helps the navigation system to compensate altitude errors, the deviation of estimated altitude obtained by the Kalman filter is larger than that estimated by the RHKF as shown in Fig. 11.

This simulation shows that, while the estimated navigation solution based on the RHKF may be somewhat noisy, the estimated errors do not blow up, which indicates the fast convergent rate and robustness of the RHKF under the temporary disturbances, as given in Fig. 4. Also, the estimation error of the RHKF is considerably smaller than that of the Kalman filter in the interval of measurement uncertainties. Therefore, it is concluded that the suggested RHKF for time-varying navigation systems is more robust than the standard Kalman filter when there are measuring uncertainties.

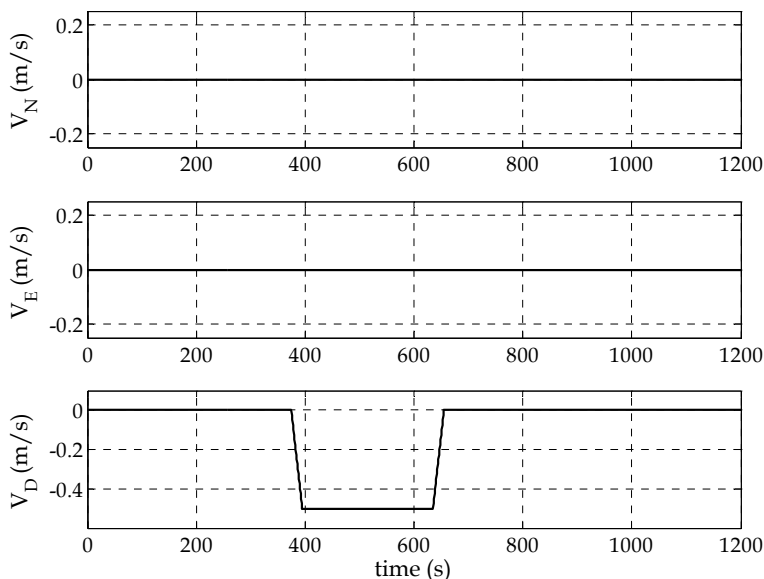


Fig. 4. Velocity profile of the upwelling current in the n-frame

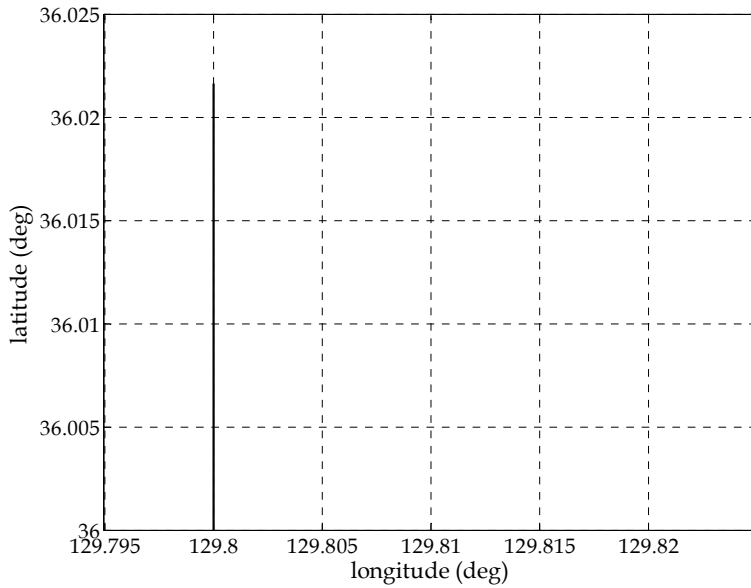


Fig. 5. Reference trajectory of the vehicle in the n-frame

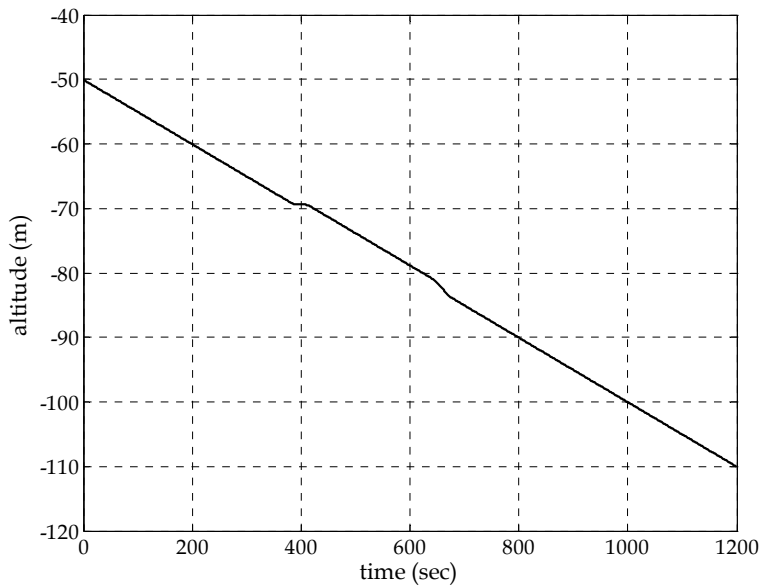


Fig. 6. Reference altitude of the vehicle in the n-frame

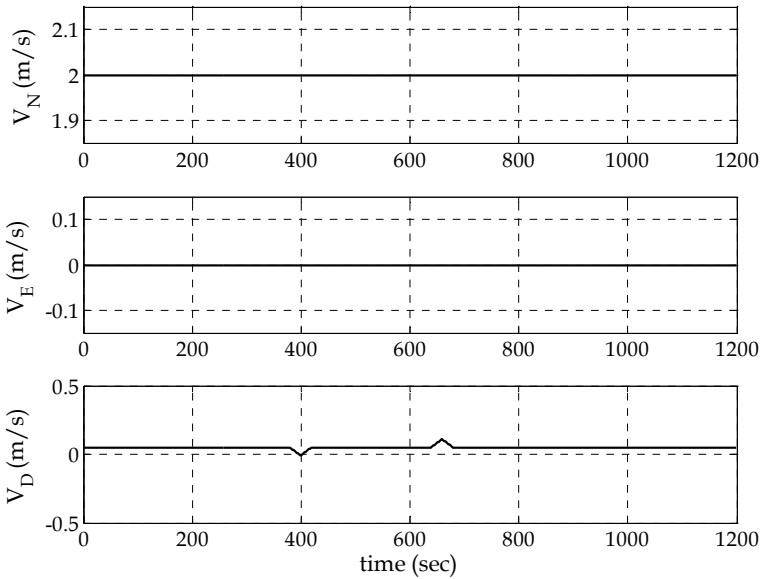


Fig. 7. Reference velocity of the vehicle in the n-frame

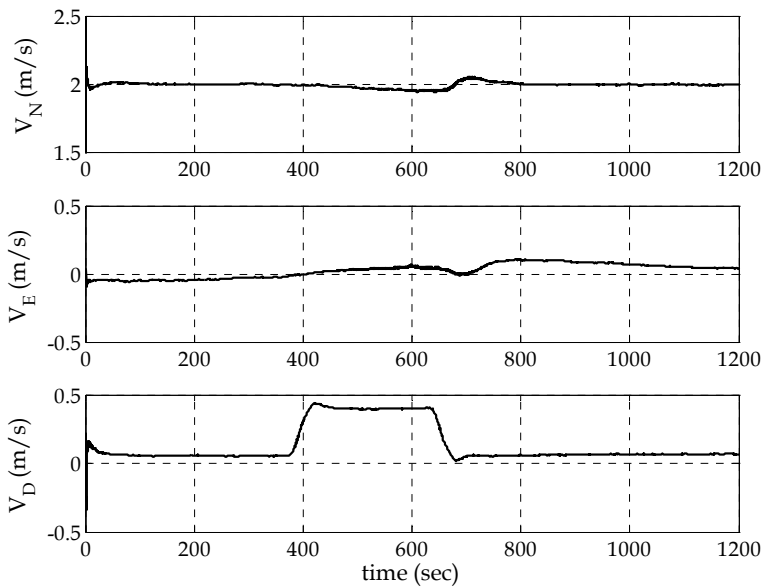


Fig. 8. Estimated velocity of the vehicle by the Kalman filter in the n-frame

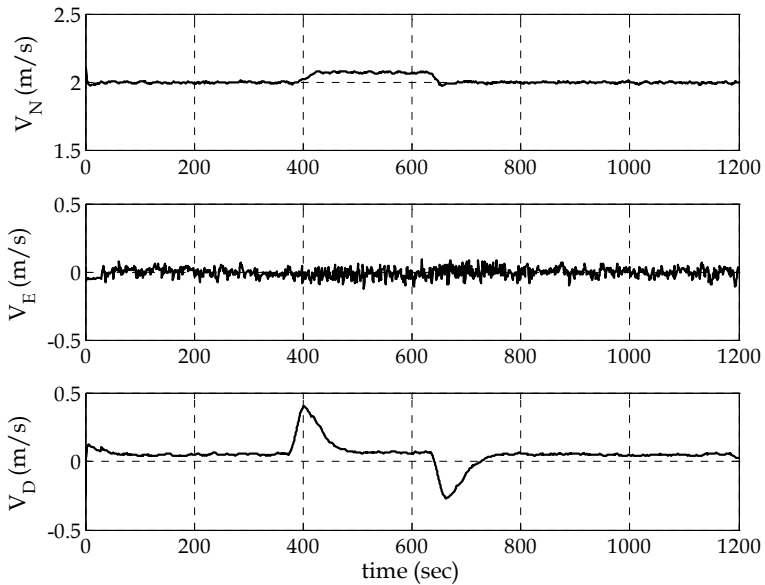


Fig. 9. Estimated velocity of the vehicle by the RHKF in the n-frame

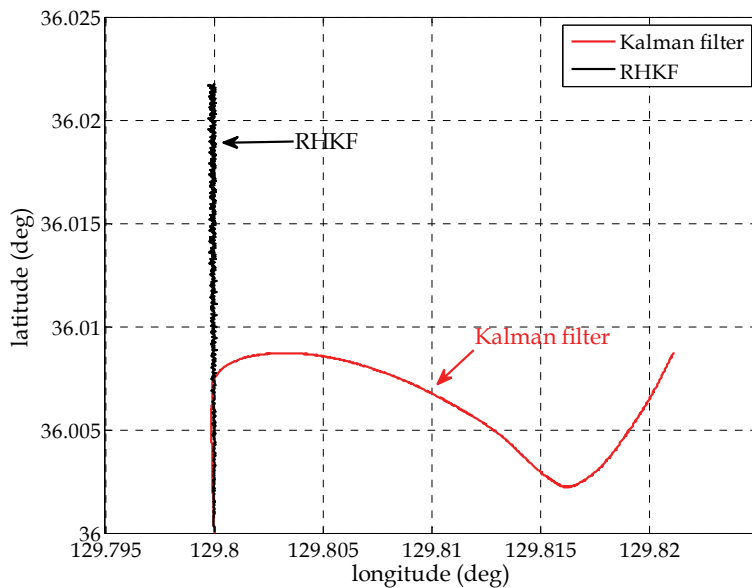


Fig. 10. Estimated trajectory of the vehicle by the Kalman filter in the n-frame

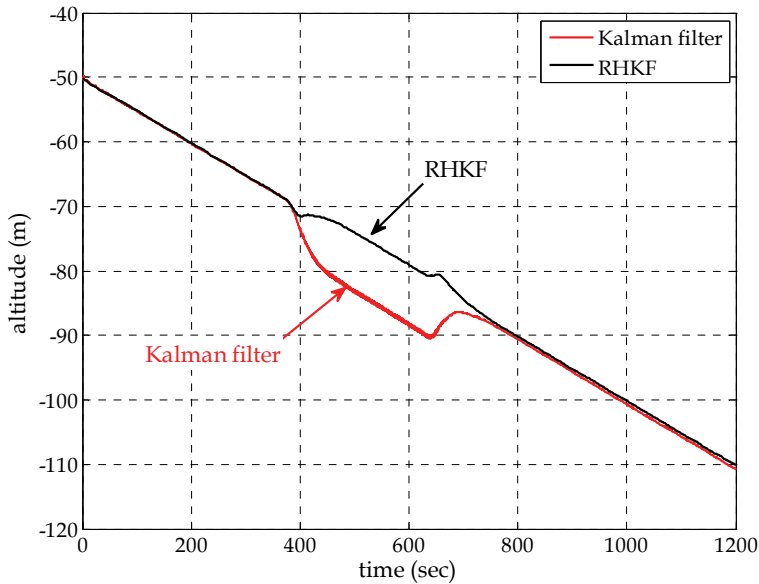


Fig. 11. Comparison of estimated altitudes in the n-frame

6. Summary

In this chapter, the RHKF and its application for a velocity-aided underwater navigation system are discussed. Firstly, the limitations of the standard Kalman filter in underwater navigation systems are described. And then it is shown how the RHKF can be replaced in order to improve the position accuracy of the navigation system when the vehicle is to be operated under uncertain environments. It is shown that the RHKF is robust against environmental uncertainties by tracking only the most recent finite measurement. When a navigation system is completely observable, the RHKF is exact for noise-free systems. This deadbeat property indicates the finite convergence time and fast tracking ability of the filter. Thus the filter is appropriate for fast estimation under temporary disturbances. Based on simulations for the velocity-aided navigation system, it is then demonstrated that the RHKF guides a better and faster transient performance of the underwater navigation system compared to the standard Kalman filter.

7. Appendix

The system matrix $F(t)$ of the SDINS system error model equation is as follows:

$$F = \begin{bmatrix} F_{11} & F_{12} & 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} \\ F_{21} & F_{22} & F_{23} & C_b^n & 0_{3 \times 3} & 0_{3 \times 3} \\ F_{31} & F_{32} & F_{33} & 0_{3 \times 3} & -C_b^n & 0_{3 \times 3} \\ & & & 0_{13 \times 22} & & \end{bmatrix}$$

where

$$F_{11} = \begin{bmatrix} \frac{\rho_E R_{mm}}{R_m + h} & \frac{\rho_E R_{mm}}{R_m + h} & \frac{\rho_E}{R_m + h} \\ \rho_N \sec L \left(\tan L - \frac{R_H}{R_t + h} \right) & 0 & -\frac{\rho_N \sec L}{R_t + h} \\ 0 & 0 & 0 \end{bmatrix}$$

$$F_{12} = \begin{bmatrix} \frac{1}{R_m + h} & 0 & 0 \\ 0 & \frac{\sec L}{R_t + h} & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$F_{21} = \begin{bmatrix} -(2\Omega_N + \rho_N \sec^2 L + \frac{\rho_D R_H}{R_t + h})V_E + \frac{\rho_E R_{mm}}{R_m + h} V_D & 0 & -\frac{\rho_D V_E}{R_t + h} + \frac{\rho_E V_D}{R_m + h} \\ (2\Omega_N + \rho_N \sec^2 L + \frac{\rho_D R_H}{R_t + h})V_N + (2\Omega_D + \frac{\rho_N R_H}{R_t + h})V_D & 0 & \frac{\rho_D V_N}{R_t + h} - \frac{\rho_N V_D}{R_t + h} \\ -\frac{\rho_E R_{mm}}{R_m + h} V_N - (2\Omega_D + \frac{\rho_N R_H}{R_t + h})V_E & 0 & -\frac{\rho_E V_N}{R_m + h} + \frac{\rho_N V_E}{R_t + h} \end{bmatrix}$$

$$F_{22} = \begin{bmatrix} \frac{V_D}{R_m + h} & 2\Omega_D + 2\rho_D & -\rho_E \\ -2\Omega_D - \rho_D & \frac{V_N \tan L + V_D}{R_t + h} & 2\Omega_N + \rho_N \\ 2\rho_E & -2\Omega_N - 2\rho_N & 0 \end{bmatrix}$$

$$F_{23} = \begin{bmatrix} 0 & -f_D & f_E \\ f_D & 0 & -f_N \\ -f_E & f_N & 0 \end{bmatrix}$$

$$F_{31} = \begin{bmatrix} \Omega_D - \frac{\rho_N R_H}{R_t + h} & 0 & -\frac{\rho_N}{R_t + h} \\ -\frac{\rho_E R_{mm}}{R_m + h} & 0 & -\frac{\rho_E}{R_m + h} \\ \Omega_N - \rho_N \sec^2 L - \frac{\rho_D R_H}{R_t + h} & 0 & -\frac{\rho_D}{R_t + h} \end{bmatrix}$$

$$F_{32} = \begin{bmatrix} 0 & \frac{1}{R_t + h} & 0 \\ -\frac{1}{R_m + h} & 0 & 0 \\ 0 & -\frac{\tan L}{R_t + h} & 0 \end{bmatrix}$$

$$F_{33} = - \begin{bmatrix} \Omega_N + \rho_N \\ \rho_E \\ \Omega_D + \rho_D \end{bmatrix} \times$$

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The Kalman filter has been successfully employed in diverse areas of study over the last 50 years and the chapters in this book review its recent applications. The editors hope the selected works will be useful to readers, contributing to future developments and improvements of this filtering technique. The aim of this book is to provide an overview of recent developments in Kalman filter theory and their applications in engineering and science. The book is divided into 20 chapters corresponding to recent advances in the field.

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