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1. Introduction

The problem of designing an accurate and reliable control for an Autonomous Underwater Vehicle (AUV), which is being subjected to environmental disturbances as well as configuration related changes, is critical in order to accomplish a successful mission. Any real-world problem solving system must deal with the issue of uncertainty, since the system's knowledge of the world is always incomplete, imprecise, and uncertain. This situation is aggravated for an AUV, due to the complex oceanic environment, and the inevitable noise of the sensory system.

Some major facts that contribute to the difficulty of the underwater vehicle control are:

- the dynamic behavior of the vehicle is highly nonlinear,
- hydrodynamic coefficients cannot be easily obtained, hence making up uncertainties in the model knowledge,
- the vehicle main body can be disturbed due to the ocean currents and vehicle motion.

Therefore, it is difficult to obtain high performance by using the conventional control strategies. The control system should be able to learn and adapt itself to the changes in the dynamics of the vehicle and its environment.

Many control methods have been proposed by researchers during the last decade, and there still exists a trend towards finding a better control law to achieve exponential stability while accounting for environmental changes and vehicle uncertainties. Focusing on the low level motion control of AUVs, most of the proposed control schemes take into account the uncertainty in the model by resorting to an adaptive strategy ((Corradini & Orlando, 1997), (Fossen & Sagatun, 1991a) and (Narasimhan & Singh, 2006)), or a robust approach ((Marco & Healey, 2001) and (Healey & Lienard, 1993)). In (Healey & Lienard, 1993) an estimation of the dynamic parameters of the vehicle NPS AUV Phoenix is also provided. Other relevant works on the adaptive and robust control of underwater vehicles are (Cristi & Healey, 1989), and (Cristi et al., 1990). (Leonard & Krishnaprasad, 1994) considers the control of an AUV in the event of an actuator failure. Experimental results on underwater vehicle control have been addressed by many researchers (e.g. see (Antonelli et al., 1999), (Antonelli et al., 2001), and (Zhao & Yuh, 2005)). An overview of control techniques for AUVs is reported in (Fossen, 1994).

The aim of this chapter is to design a control system that would achieve perfect tracking for all configuration variables (e.g. sway and yaw motions) for any desired trajectory. To this end, we present the application of nonlinear control methods to an AUV that would lead to
a successful uncertainty management, while accounting for the effect of saturation: an unwanted implementation problem which is seldom addressed by researchers.

Three control methods are presented and applied to a two-dimensional model of an AUV, and their capabilities to cope with the issues of parameter uncertainties and environmental disturbances are studied and compared. The considered model is a nonlinear multi-input multi-output (MIMO) system, therefore we intend to shed a light on the complexities encountered when dealing with such systems. This model also serves as an example, and helps clarify the application of the given methods. All the methods presented, guarantee perfect tracking for all configuration variables of the system. The performance of the presented methods, are compared via simulation studies.

We begin by designing a control law using the computed torque control method. Although simple in design, the stability achieved by this method is sensitive to parameter variations and noise of the sensory system. Moreover, the maximum amount of disturbance waves that can be conquered by this method is somewhat lower relative to the other methods given here. Next we present the adaptive approach to computed torque control method. It will be shown that this method can withstand much higher values of disturbance waves and remain stable. Furthermore, parameter variations are compensated through an adaptation law. The third method presented, is the suction control method in which we employ the concepts of sliding surfaces, and boundary layers. This method, being robust in nature, achieves an optimal trade-off between control bandwidth and tracking precision. Compared to the computed torque control method, this method has improved performance with a more tractable controller design. Finally, the effect of saturation is studied through a novel approach, by considering the desired trajectory. A condition is derived under which saturation will not occur. The chapter will be closed by proposing topics for further research.

2. Nonlinear control methodologies

All physical systems are nonlinear to some extent. Several inherent properties of linear systems which greatly simplify the solution for this class of systems, are not valid for nonlinear systems (Shinner, 1998). The fact that nonlinear systems do not have these properties further complicates their analysis. Moreover, nonlinearities usually appear multiplied with physical constants, often poorly known or dependent on the slowly changing environment, thereby increasing the complexities. Therefore, it is important that one acquires a facility for analyzing control systems with varying degrees of nonlinearity.

This section introduces three nonlinear control methods for tracking purposes. To maintain generality, we consider a general dynamic model of the form

\[ T = H(q)\dot{q} + C(q,\dot{q})\dot{q} + G(q) \]  

(1)

that can represent the dynamic model of numerous mechanical systems such as robotic vehicles, robot manipulators, etc, where \( H(q) \) is an \( n \times n \) matrix, representing mass matrix or inertia matrix (including added mass for underwater vehicles), \( C(q,\dot{q}) \) represents the matrix of Coriolis and centripetal terms (including added mass for underwater vehicles), and \( G(q) \) is the vector of gravitational forces and moments. For the case of underwater vehicles, which is the main concern of this chapter, the term \( C(q,\dot{q}) \) will also represent the hydrodynamic damping and lift matrix. The methods given in this section, will be applied to an underwater vehicle model in section 3.
2.1 Computed torque control method

This section presents a nonlinear control method, apparently first proposed in (Paul, 1972) and named the computed torque method in (Markiewicz, 1973) and (Bejczy, 1974). This method is based on using the dynamic model of the system in the control law formulation. Such a control formulation yields a controller that suppresses disturbances and tracks desired trajectories uniformly in all configurations of the system (Craig, 1988).

Suppose that the system’s dynamics is governed by Eq. (1). The control objective is to track a desired trajectory $q_d$. Such a trajectory may be preplanned by several well-known schemes (Craig, 1989). We define a tracking error $\dot{q}$

$$\dot{q} = q_d - q,$$  

(2)

and make the following proposition.

**Proposition 2.1** The control law

$$T = H(q)u + C(q,\dot{q})\dot{q} + G(q),$$  

(3)

can track any desired trajectory $q_d$, as long as the matrices $H$, $C$, and $G$ are known to the designer. The servo law, $u$, is given by

$$u = q_d + K_s q + K_p \dot{q},$$  

(4)

where $K_s$ and $K_p$ are called servo gain matrices.

**Proof.**

Substituting the proposed control law into the equation of motion, Eq. (1), we obtain

$$H(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = H(q)(q_d + K_s q + K_p \dot{q}) + C(q,\dot{q})\dot{q} + G(q),$$

which yields the following error dynamics

$$\ddot{q} + K_s \dot{q} + K_p \dot{q} = 0.$$

A proper choice of the servo gain matrices will lead to a stable error dynamics. One such example is given by the following matrices

$$K_p = \text{diag}[\lambda_1^2, \lambda_2^2, \ldots, \lambda_n^2]$$  

(5)

$$K_s = \text{diag}[2\lambda_1, 2\lambda_2, \ldots, 2\lambda_n],$$  

(6)

where $\lambda_i$ are adjustable design parameters.

It can be seen that this control formulation exhibits perfect tracking for any desired trajectory. But this desired performance is based on the underlying assumption that the values of parameters appearing in the dynamic model in the control law match the parameters of the actual system, which makes the implementations of the computed torque control less than ideal due to the inevitable uncertainties of the system, e.g. resulting from unknown hydrodynamic coefficients. In the existence of uncertainties, the control law (3) must be modified to

$$T = \hat{H}(q)u + \hat{C}(q,\dot{q})\dot{q} + \hat{G}(q),$$  

(7)
where \( \hat{[} \) denotes the estimation of matrix \([\cdot]\). One can show that substitution of the above control law into the equation of motion will lead to the following error dynamics

\[
\dot{q} + K_q \dot{q} + K_p q = \hat{\mathbf{T}}^T,
\]

(8)

where \( \hat{\mathbf{T}} = \hat{\mathbf{H}}q + \mathbf{C}q + \mathbf{G} \), and the tilde matrices are defined by \( \hat{[} = \[ - \hat{[} \). Since the right hand side of the error dynamics is not zero anymore, this method becomes inefficient in the presence of uncertainties. This problem is conquered by the adaptive counterpart of the computed torque control method.

2.2 Adaptive computed torque control method

In this section, we introduce the adaptive computed torque control method, and derive an adaptation law to estimate the unknown parameters. The control of nonlinear systems with unknown parameters is traditionally approached as an adaptive control problem. Adaptive control is one of the ideas conceived in the 1950’s which has firmly remained in the mainstream of research activity with hundreds of papers and several books published every year. One reason for the rapid growth and continuing popularity of adaptive control is its clearly defined goal: to control plants with unknown parameters. Adaptive control has been most successful for plant models in which the unknown parameters appear linearly. But in many mechanical systems, the unknown parameters appear in a nonlinear manner. For such systems we define parameter functions \( P \), such that the system have a linear relationship with respect to these parameter functions. Fortunately, such a linear parameterization can be achieved in most situations of practical interest (Kristic et al., 1995). We only consider such systems throughout this work.

In the linear parameterization process, we partition the system into a model-based portion and a servo portion. The result is that the system’s parameters appear only in the model-based portion, and the servo portion is independent of these parameters. This partitioning involves the determination of parameter functions \( P \), such that the error dynamics is linear in the parameter functions. When this is possible, one can write

\[
\hat{\mathbf{T}} = \hat{\mathbf{H}}(q)\dot{q} + \mathbf{C}(q, \dot{q})q + \mathbf{G}(q) = \mathbf{W}(q, \dot{q}, \ddot{q})\hat{P},
\]

(9)

where \( \mathbf{W} \) is a \( n \times k \) matrix, called the regression matrix, and \( \hat{P} \) is a \( k \times 1 \) vector, representing the parameter function estimation errors and is defined by \( \hat{P} = P - \hat{P} \).

Once the parameterization process is done successfully, one can employ the following adaptation law to estimate the parameter functions.

**Proposition 2.2** For a system with either constant or slowly varying unknown parameters, the adaptation law

\[
\dot{\hat{P}} = \Gamma \mathbf{W}^T \hat{\mathbf{H}}^{-T} \mathbf{Y},
\]

(10)

estimates the parameter functions, such that the error dynamics of Eq. (8) becomes stable. Definitions of \( \Gamma \) and \( \mathbf{Y} \) are given in the following proof.

**Proof.**

The error dynamics is given by Eq. (8). Substituting for \( \hat{\mathbf{T}} \) from the linear parameterization law, Eq. (9), we have
The aim of the adaptation law is to estimate the parameter functions \( \mathbf{P} \), so as to make the right hand side of the above equation approach zero, i.e. by making \( \mathbf{P} \) approach zero. One can write Eq. (11) in state space form by defining the state vector \( \mathbf{X} \) as

\[
\mathbf{X} = [\mathbf{X}_1, \mathbf{X}_2, \ldots, \mathbf{X}_n]^T, \quad \mathbf{X}_i \triangleq [\dot{q}_i, \dot{q}_i]^T,
\]

and the output vector \( \mathbf{Y} \) as

\[
\mathbf{Y} \triangleq \dot{\mathbf{q}} + \mathbf{\Phi} \mathbf{q}, \quad \mathbf{\Phi} = \text{diag} [\phi_1, \phi_2, \ldots, \phi_n],
\]

where \( \mathbf{\Phi} \) is the filtering matrix, and \( \mathbf{Y} \) represents the vector of filtered errors. The values of \( \phi_j \) must be chosen such that the transfer function

\[
\frac{s + \phi_j}{s^2 + K\phi_j s + K_p^2}
\]

is \emph{strictly positive real} (SPR). Therefore

\[
\dot{\mathbf{X}} = \mathbf{A} \mathbf{X} + \mathbf{B} (\dot{\mathbf{H}}^T \mathbf{W} \dot{\mathbf{P}})
\]

\[
\mathbf{Y} = \mathbf{C} \mathbf{X}.
\]

Having written the error dynamics in state space form, we employ a Lyapunov-based approach to derive the adaptation law. Consider the following Lyapunov candidate,

\[
\dot{\mathbf{V}} = \mathbf{X}^T \mathbf{P} \mathbf{X} + \dot{\mathbf{P}} \Gamma^{-1} \dot{\mathbf{P}}, \tag{12}
\]

where \( \mathbf{P} \) is a positive definite matrix, and \( \Gamma = \text{diag}[\gamma_1, \gamma_2, \ldots, \gamma_r] \) with \( \gamma_i > 0 \). Taking the time derivative of (12) yields

\[
\dot{\mathbf{V}} = \dot{\mathbf{X}}^T \mathbf{P} \mathbf{X} + \mathbf{X}^T \mathbf{P} \dot{\mathbf{X}} + 2 \dot{\mathbf{P}} \Gamma^{-1} \dot{\mathbf{P}}.
\]

Substitution of the state space equations of error dynamics into (13) results

\[
\dot{\mathbf{V}} = 2 \dot{\mathbf{P}}^T \left[ \Gamma^{-1} \dot{\mathbf{P}} + \mathbf{W}^T \dot{\mathbf{H}}^T \Gamma P \mathbf{X} \right] + \mathbf{X}^T (\mathbf{A}^T \mathbf{P} + \mathbf{P} \mathbf{A}) \mathbf{X}.
\]

This equation can further be simplified, by adopting the following lemma.

\textbf{Lemma 2.1 (Kalman-Yakubovich-Popov)} Consider a controllable linear time-invariant system

\[
\dot{\mathbf{x}} = \mathbf{A} \mathbf{x} + \mathbf{b} \mathbf{u}
\]

\footnote{A transfer function \( h(p) \) is \emph{positive real} if

\[
\text{Re}[h(p)] \geq 0 \quad \text{for all} \quad \text{Re}[p] \geq 0
\]

It is \emph{strictly positive real} if \( h(p - \varepsilon) \) is positive real for some \( \varepsilon > 0 \).}
The transfer function \( h(p) = c[pI - A]^{-1}b \) is SPR if, and only if, there exist positive definite matrices \( P \) and \( Q \) such that

\[
A^TP + PA = -Q
\]

\[
Pb = c^T.
\]

According to the above lemma, one can write \( (A^TP + PA) = -Q \) in Eq. (14). The adaptation law is found by setting the first term on the right side of (14) equal to zero

\[
2\hat{p}^T \left[ \Gamma X^T \hat{\dot{p}} + W^T (\hat{H}^T) B^T P X \right] = 0.
\]

Rearranging the above equation and noting that \( \hat{p} = \hat{\dot{p}} \) and using Lemma 2.1, the adaptation law is found as

\[
\hat{\dot{p}} = \Gamma W^T \hat{H}^T Y,
\]

and Eq. (14) will become

\[
\dot{V} = -X^T Q X,
\]

which is a stable Lyapunov function. 

Even though \( H^{-1} \) always exists in a physical problem, a vigilant reader might question the existence of \( \hat{H}^{-1} \). It is shown in (Craig, 1988), that \( \hat{H} \) will remain positive definite and invertible, if we ensure that all parameters remain within a sufficiently small range near the actual parameter value. See (Craig, 1988) for the details of how this is done.

### 2.3 Suction control

One major approach to dealing with model uncertainty is the robust control. Broadly speaking, robustness is a property which guarantees that essential functions of the designed system are maintained under adverse conditions in which the model no longer accurately reflects reality. In modeling for robust control design, an exactly known nominal plant is accompanied by a description of plant uncertainty, that is, a characterization of how the true plant might differ from the nominal one. This uncertainty is then taken into account during the design process (Freeman & Kokotovic, 1996).

For simplicity, we explain the method for a single-input system. The extension to multi-input systems is straightforward, as will be illustrated in the AUV example. A more detailed discussion of this method is given by (Slotine, 1985), (Slotine & Sastry, 1983), and (Slotine & Li, 1991).

Consider the dynamic system

\[
x^{(\alpha)}(t) = f(X; t) + b(X; t)u(t),
\]

where \( u(t) \) is the control input and \( X = [x, \dot{x}, \ldots, x^{(\alpha-1)}]^T \) is the state vector. It is assumed that the generally nonlinear function \( f(X; t) \) is not exactly known, but the extent of imprecision on \( f \) is upper-bounded by a known continuous function of \( X \) and \( t \). Similarly the control gain \( b(X; t) \) is not exactly known, but is of constant sign and is bounded by known continuous functions of \( X \) and \( t \). The control problem is to track the desired trajectory
in the presence of model imprecisions on \( f \) and \( b \). Defining the tracking error as usual, \( \hat{X} = X - X_d \), we assume that

\[
\hat{X} = 0. \tag{16}
\]

A time-varying sliding surface \( S(t) \) is defined in the state space \( \mathbb{R}^n \) as

\[
s(X; t) = x^n - \lambda x, \quad \lambda > 0,
\]

where \( \lambda \) is a positive constant. Given the initial condition (16), the problem of tracking \( X_d \) is equivalent to that of remaining on the surface \( S(t) \) for all \( t > 0 \). Now a sufficient condition for such positive invariance of \( S(t) \) is to choose the control law \( u \) of Eq. (15) such that outside of sliding condition \( S(t) \), the following holds:

\[
\frac{1}{2} \frac{d}{dt} s^2(X; t) \leq -k |s|, \tag{18}
\]

where \( k \) is a positive constant. Sliding condition (18) constraints state trajectories to point toward the sliding surface \( S(t) \). Geometrically, it looks like the trajectories are sliding down \( S(t) \) to reach the desired state. Satisfying Eq. (18) guarantees that if condition (16) is not exactly verified, the surface \( S(t) \) will nonetheless be reached in a finite time, while definition (17) then guarantees that \( \hat{X} \rightarrow 0 \) as \( t \rightarrow \infty \) (Slotine, 1985).

The controller design procedure in the suction control method, consists of two steps. First, a feedback control law \( u \) is selected so as to verify sliding condition (18). Such a control law is discontinuous across the surface, which leads to control chattering. Chattering is undesirable in practice because it involves high control activity and further may excite high-frequency dynamics neglected in the course of modeling. Thus in a second step, discontinuous control law \( u \) is suitably smoothed to achieve an optimal trade-off between control bandwidth and tracking precision. While the first step accounts for parametric uncertainty, the second step achieves robustness to high-frequency unmodeled dynamics. Construction of a control law to verify the sliding condition (18) is straightforward, and will be illustrated in section 3.4 through an example.

3. A two-dimensional model of a MIMO AUV

In this section the problem of tracking the configuration variables (position and attitude) of an AUV in the horizontal plane is considered. Two rudders in front and rear side of the vehicle are used as control inputs, and the methods of previous section are applied. A schematic diagram of the system under consideration is shown in Fig. 1.

3.1 Dynamic modeling

The dynamic behavior of an underwater vehicle is described through Newton’s laws of linear and angular momentum. The equations of motion of such vehicles are highly nonlinear, time-varying and coupled due to hydrodynamic added mass, lift, drag, Coriolis and centripetal forces, which are acting on the vehicle and generally include uncertainties (Fossen & Sagatun, 1991b). Detailed discussions on modeling and system identification techniques are given in (Fossen, 1994) and (Goheen & Jefferys, 1990).
It is convenient to write the equations of motion in accordance with the Society of National Architects and Marine Engineers (SNAME, 1950). Restricting our attention to the horizontal plane, the mathematical model consists of the nonlinear sway (translational motion with respect to the vehicle longitudinal axis) and yaw (rotational motion with respect to the vertical axis) equations of motion. According to (Haghi et al., 2007), these equations are described by

\[
\dot{v}[m - Y_v] + \dot{r}[mx_G - Y_r] = Y_{\delta_s} \delta_* u^2 + Y_{\delta_b} \delta_* u^2 - d_v(v, r) + Y_{uv} + (Y_r - m)ur \tag{19}
\]

\[
\dot{v}[mx_G - N_v] + \dot{r}[I_z - N_r] = N_{\delta_s} \delta_* u^2 + N_{\delta_b} \delta_* u^2 - d_r(v, r) + N_{uv} + (N_r - mx_G)ur, \tag{20}
\]

where \(d_1(v, r)\) and \(d_2(v, r)\) are defined as

\[
\begin{align*}
    d_1(v, r) & \triangleq \frac{D}{2} \int_{\xi_{\text{nose}}}^{\xi_{\text{tail}}} C_{D_p} h(\xi) \frac{(v + \xi r)^3}{|v + \xi r|} d\xi \\
    d_2(v, r) & \triangleq \frac{D}{2} \int_{\xi_{\text{nose}}}^{\xi_{\text{tail}}} C_{D_p} h(\xi) \frac{(v + \xi r)^3}{|v + \xi r|} \xi d\xi.
\end{align*}
\]

Equations (19) and (20), along with the expressions for the vehicle yaw rate and the inertial position rates, describe the complete model of the vehicle. For control purposes it is convenient to solve Eqs. (19) and (20) for \(\dot{v}\) and \(\dot{r}\). Therefore the complete set of equations of motion is

\[
\dot{v} = a_1 uv + a_2 ur + d_1(v, r) + b_1 \mu^2 \delta_* + b_2 \mu^2 \delta_b \tag{21}
\]

\[
\dot{r} = a_3 uv + a_4 ur + d_2(v, r) + b_3 \mu^2 \delta_* + b_4 \mu^2 \delta_b \tag{22}
\]
\begin{align*}
\dot{y} &= r 
\end{align*}
(23)

\begin{align*}
\dot{x} &= u \cos \psi - v \sin \psi 
\end{align*}
(24)

\begin{align*}
\dot{y} &= u \sin \psi + v \cos \psi, 
\end{align*}
(25)

where \( a_i, b_i \) and \( c_i \) are the related coefficients that appear when solving (19) and (20) for \( \dot{v} \) and \( \dot{r} \).

During regular cruising, the drag related terms \( d_s(v,r) \) and \( d_v(v,r) \) are small, and can be neglected (Yuh, 1995). Note that all the parameters \( a_i \) and \( b_i \), include at least two hydrodynamic coefficients, such as \( Y_s, Y_r, N_s, N_r, \ldots \); hence uncertainties. In the proceeding sections, we apply the nonlinear control methods of the previous section to this model. Our goal is to achieve perfect tracking for both sway and yaw motions of the vehicle.

### 3.2 Computed torque control method

Suppose that it is desired that the sway motion of the vehicle tracks the preplanned trajectory \( y_d \), and that the yaw motion of the vehicle tracks the preplanned trajectory \( \psi_d \).

Let the tracking errors be defined by

\begin{align*}
\tilde{y} &= y - y_d 
\end{align*}
(26)

\begin{align*}
\tilde{\psi} &= \psi - \psi_d. 
\end{align*}
(27)

The control law is given by Eqs. (3) and (4). One can observe that Eq. (3) is obtained by replacing the acceleration term of the equations of motion, \( \ddot{q} \), by the servo law \( u \). Since this process involves the acceleration terms, we take the time derivative of Eqs. (23) and (25), and substitute (21) and (22) into the results. Therefore

\begin{align*}
\ddot{y} &= u \psi \cos \psi + (a_i \mu v + a_{2i} \mu \psi + b_i u^2 \delta_s + b_{12} u^2 \delta_b) \cos \psi - v \psi \sin \psi 
\end{align*}
(28)

\begin{align*}
\ddot{\psi} &= a_i \mu v + a_{2i} \mu \psi + b_i u^2 \delta_s + b_{12} \mu u^2 \delta_b. 
\end{align*}
(29)

Next we replace \( \ddot{y} \) with the servo law \( \mu \), and \( \ddot{\psi} \) with the servo law \( \nu \), and solve these equations for the rudder deflections \( \delta_s \) and \( \delta_b \) to obtain the control law

\begin{align*}
\delta_s &= (b_{22}(\mu \sec \psi + \nu \tan \psi - u \psi - a_i \mu v - a_{2i} \mu \psi) - b_{12}(\nu - a_i \mu v) - a_{12} \mu \psi)}/(b_{11} b_{22} - b_{12} b_{12}) u^2 
\end{align*}
(30)

\begin{align*}
\delta_b &= -(b_{22}(\mu \sec \psi + \nu \tan \psi - u \psi - a_i \mu v - a_{2i} \mu \psi) - b_{11}(\nu - a_i \mu v) - a_{12} \mu \psi)}/(b_{11} b_{22} - b_{12} b_{12}) u^2, 
\end{align*}
(31)

where the servo laws \( \mu \) and \( \nu \) are given according to Eqs. (4), (5), and (6)

\begin{align*}
\mu &= \ddot{y}_d - 2 \lambda \ddot{y} - \lambda^2 \dot{y} 
\end{align*}
Note that the negative signs in the above equations, are due to the definition of tracking error in Eqs. (26) and (27) which differs in a minus sign from the definition of Eq. (2).

### 3.3 Adaptive computed torque control method

In this method, the control law estimates the unknown parameters. As stated before, all the parameters $a_y$ and $b_y$ comprise hydrodynamic uncertainties which must be estimated. On the other hand, the vehicle's forward velocity $u$ is assumed to be constant, but subjected to changes from environment, and ocean currents. Thus all terms including $u$ must also be estimated. But instead of estimating all the $a_y$ and $b_y$ terms, we define parameter functions, $p_i$, in a linear parameterization process. This process does not reveal a unique parameterization and the results depend on the way one defines $p_i$s. One can show that a possible parameterization of Eqs. (30) and (31) is given by

\[
\delta_s = p_1 \left( \frac{\mu + vr \sin \psi}{\cos \psi} \right) + p_2 y' + p_3 r - p_4 v
\]

\[
\delta_b = -p_5 \left( \frac{\mu + vr \sin \psi}{\cos \psi} \right) + p_7 y' + p_8 r + p_9 v,
\]

where

\[
p_1 = \frac{b_{22}}{(b_1 b_{22} - b_2 b_{12})u^2}
\]

\[
p_2 = \frac{b_{31}a_1 \mu - b_{22}a_3 \mu}{(b_1 b_{22} - b_2 b_{12})u^2}
\]

\[
p_3 = \frac{b_{13}a_2 \mu - b_{22}a_4 \mu}{(b_1 b_{22} - b_2 b_{12})u^2}
\]

\[
p_4 = \frac{b_{12}}{(b_1 b_{22} - b_2 b_{12})u^2}
\]

\[
p_5 = \frac{b_{21}a_2 \mu - b_{22}a_3 \mu}{(b_1 b_{22} - b_2 b_{12})u^2}
\]

\[
p_6 = \frac{b_{23}a_1 \mu - b_{12}a_3 \mu}{(b_1 b_{22} - b_2 b_{12})u^2}
\]

\[
p_7 = \frac{b_{22}a_2 \mu - b_{22}a_3 \mu + b_3 \mu}{(b_1 b_{22} - b_2 b_{12})u^2}
\]

\[
p_8 = \frac{b_{11}}{(b_1 b_{22} - b_2 b_{12})u^2}
\]

Since all the $p_i$s include uncertainties, the control law is modified as follows:

\[
\delta_s = \hat{p}_1 \left( \frac{\mu + vr \sin \psi}{\cos \psi} \right) + \hat{p}_2 y' + \hat{p}_3 r - \hat{p}_4 v
\]

\[
\delta_b = -\hat{p}_5 \left( \frac{\mu + vr \sin \psi}{\cos \psi} \right) + \hat{p}_7 y' + \hat{p}_8 r + \hat{p}_9 v,
\]

where $\hat{p}_i$ represents parameter estimations, and the servo signals $\mu$ and $v$ are defined as before. The next step is to derive the adaptation law.

Let the estimation error of parameters be $\tilde{p}_i = p_i - \hat{p}_i$. One can find the error dynamics by substituting (34) and (35) into the system dynamic equations. This results
or in matrix form

\[
\begin{bmatrix}
\dot{\hat{y}} - \mu \\
\cos\psi \\
\dot{\hat{\psi}} - \nu
\end{bmatrix}
= 
\begin{bmatrix}
\frac{\dot{y} + vr \sin\psi}{\cos\psi} & -v & -r & \psi & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{\dot{y} + vr \sin\psi}{\cos\psi} & v & r & \dot{\psi}
\end{bmatrix}
\hat{P}, \quad (36)
\]

One can write Eq. (37) in state space form by defining the state vector \( \mathbf{X} \) and the output vector \( \mathbf{Y} \) as defined in section 2.2

\[
\dot{\mathbf{X}} = \mathbf{A} \mathbf{X} + \mathbf{B} (\mathbf{H}^T \mathbf{W} \hat{p})
\]

\[
\mathbf{Y} = \dot{\mathbf{N}} + \Phi \dot{\mathbf{N}},
\]

where \( \Phi = \text{diag}[\phi_1, \phi_2] \), and \( \mathbf{N} = [\hat{y}, \hat{\psi}]^T \). Having defined the necessary matrices, we can utilize the adaptation law given by Eq. (10):

\[
\dot{\hat{p}} = \Gamma \mathbf{W}^T \mathbf{H}^T \mathbf{Y},
\]

where \( \mathbf{H} \) and \( \mathbf{W} \) are defined in Eq. (36).

### 3.4 Suction control

One can write the system’s governing dynamics, in matrix form as:

\[
\begin{bmatrix}
\dot{\hat{y}} \\
\dot{\hat{\psi}}
\end{bmatrix}
= 
\begin{bmatrix}
a_{11} \mu \cos\psi - r \sin\psi & u \cos\psi + a_{12} \mu \cos\psi \\
a_{12} \mu & a_{22} \mu
\end{bmatrix}
\begin{bmatrix}
\hat{y} \\
r
\end{bmatrix}
\]

or in vector form as

\[
\dot{\mathbf{N}} = \mathbf{A} \mathbf{X} + \mathbf{B} \mathbf{U}.
\]

With the objective of tracking desired trajectories, the sliding surfaces \( S_1 \) and \( S_2 \) are chosen as

\[
s_1(y, t) = \dot{y} + \lambda_1 \dot{y} = 0
\]

\[
s_2(\psi, t) = \dot{\psi} + \lambda_2 \dot{\psi} = 0.
\]

The terms \( s_1 \) and \( s_2 \) are also called combined tracking errors, and can be written as
Underwater Vehicles

\[ s_1 = \dot{y} - \dot{y}_r, \]
\[ s_2 = \dot{\psi} - \dot{\psi}_r, \]

where
\[ \dot{y}_r = \dot{y}_r - \lambda_1 \ddot{y} \]
\[ \dot{\psi}_r = \dot{\psi}_r - \lambda_2 \ddot{\psi} \]

are the reference signals. For notational simplicity, we define the vectors
\[ S = [s_1, s_2]^T, \] and \[ N_r = [\dot{y}_r, \dot{\psi}_r]^T. \] Considering the equality of the sliding condition (18), one can write
\[ \dot{s}_i = -k_i |s_i|, \]
or
\[ \dot{s}_i = -k_i \text{sgn}(s_i). \]

Defining a vector \( K \text{sgn}(S), \) with the elements \( k_i \text{sgn}(s_i), \) the sliding condition will be \( \dot{S} = -K \text{sgn}(S). \) Differentiation of \( S \) yields
\[ \dot{S} = \ddot{N} - \ddot{N}_r. \]

Substitution of the dynamic equation and solving the result for \( U, \) the control law is found to be
\[ U = B^{-1} \left( \ddot{N}_r - AX - K \text{sgn}(S) \right). \]

The above control law is discontinuous across the sliding surface. Since the implementation of the associated control law is necessarily imperfect (for instance, in practice switching is not instantaneous), this leads to chattering. Chattering is undesirable in practice, since it involves high control activity and further may excite high frequency dynamics neglected in the course of modeling (such as unmodeled structural modes, neglected time-delays, and so on). Thus, in a second step, the discontinuous control law is suitably smoothed. This can be achieved by smoothing out the control discontinuity in a thin boundary layer neighboring the switching surface (Slotine & Li, 1991):
\[ B(t) = \{ x, |s(x,t)| \leq \Phi \} \quad \Phi > 0, \]

where \( \Phi \) is the boundary layer thickness. In other words, outside of \( B(t), \) we choose control law \( u \) as before (i.e. satisfying the sliding condition); all other trajectories starting inside \( B(t = 0) \) remain inside \( B(t) \) for all \( t \geq 0 \). The mathematical operation for this to occur is to simply replace \( \text{sgn}(s) \) with \( \text{sat} \left( \frac{s}{\Phi} \right), \) with the saturation function defined as:
\[
\begin{align*}
\text{sat}(y) &= y \quad \text{if } |y| \leq 1 \\
\text{sat}(y) &= \text{sgn}(y) \quad \text{otherwise}
\end{align*}
\]

The control law derived by this method is robust in nature; therefore, insensitive to uncertainties and disturbances. One can adjust the robustness of the system by selecting
proper control gains. When the upper bounds and lower bounds of uncertainties and/or disturbances are known, one can include these bounds in the control law design, to assure the robustness of the system. See (Slotine & Sastry, 1983) for more information.

4. Simulations

For the purpose of simulations, the following numerical values have been used as in (Haghi et al., 2007). All values have been normalized. Time has also been non-dimensionalized, so that 1 second represents the time that it takes to travel one vehicle length.

\[
m = 0.0358 \quad I_z = 0.0022
\]
\[
Y_f = -0.00178 \quad Y_v = -0.03430
\]
\[
Y_v = -0.10700 \quad Y_{\delta_b} = 0.01241
\]
\[
N_{\phi} = -0.00047 \quad N_{\psi} = -0.00178
\]
\[
N_{\phi} = -0.00769 \quad N_{\psi} = -0.0047
\]
\[
x_G = 0.0014 \quad Y_r = 0.01187
\]
\[
Y_{\delta_b} = 0.01241 \quad N_r = -0.00390
\]
\[
N_{\delta_b} = 0.0035.
\]

Note that the hydrodynamic coefficients given, are not known by the control law, and are assumed to be the actual values that the estimations must converge to.

Simulation results are presented for two cases: one to examine the effectiveness of the proposed control law in the presence of disturbance waves (which result from ocean currents), and another to study the variation of parameters. The initial condition is assumed to be \([y_0, \psi_0]=[0, 30^\circ]\) in all simulations. It is assumed that the disturbance acts as a step wave that is actuated at some time \(t_1\) and is ended at time \(t_2\). Two types of disturbances are examined: one 10% of maximum input value, and the other 20% of maximum input value. In order to examine parameter variations, it is assumed that the variations are sinusoidal with a relatively low frequency (which corresponds to gradual variations). We have assumed that the parameter \(p\) varies according to

\[
p(t) = p + a \sin \omega t.
\]

Two cases are considered. For the first case, it is assumed that \(\omega = 0.5\) and \(a/p = 10\%\), whereas for the second case we consider \(\omega = 0.5\) and \(a/p = 50\%\). In other words, a 10% variation pertains to

\[
p(t) = p(1 + 0.1 \sin 0.5t),
\]

and a 50% variation pertains to

\[
p(t) = p(1 + 0.5 \sin 0.5t).
\]

Note that the control law is not aware of the parameter changes, i.e. the control law is designed for parameters of constant value \(p\), and that the variations are due to unknown environmental effects.
4.1 Results for computed torque control method

The control objective is to track the desired trajectories \( \{y_d, \psi_d\} = [\sin t, \cos 2t] \). Simulation results are rendered in Table 1.

<table>
<thead>
<tr>
<th>( \lambda_1 = \lambda_2 )</th>
<th>10% Disturbance</th>
<th>20% Disturbance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Unstable</td>
<td>Unstable</td>
</tr>
<tr>
<td>2</td>
<td>Unstable</td>
<td>Unstable</td>
</tr>
<tr>
<td>3</td>
<td>Unstable</td>
<td>Unstable</td>
</tr>
<tr>
<td>4</td>
<td>Stable, (-60 &lt; \delta_b &lt; 140)</td>
<td>Unstable</td>
</tr>
<tr>
<td>5</td>
<td>Stable, (-50 &lt; \delta_b &lt; 60)</td>
<td>Unstable</td>
</tr>
<tr>
<td>6</td>
<td>Stable, (-60 &lt; \delta_b &lt; 80)</td>
<td>Unstable</td>
</tr>
</tbody>
</table>

Table 1. The Required Range of Rudder Deflection For Stability in the Presence of Disturbance, for Different Design Parameters.

Fig. 2. System's behavior for the computed torque method: (a) \( \delta_b \) in the presence of disturbance (b) \( \dot{y} \) in the presence of disturbance (c) \( \delta_b \) in the presence of parameter variations (d) \( \ddot{y} \) in the presence of parameter variations.
It can be seen from Table 1, that a 20% disturbance will always lead to instability. Therefore, we only present the simulation results for a 10% disturbance. We also choose $\lambda = 5$, since it requires the least range of rudder deflection, according to Table 1. Fig. 2 shows the rudder deflection $\delta_b$ and the tracking error $\tilde{y}$ in the presence of disturbance and parameter variations. Although the tracking error does not converge to zero in the presence of parameter variations, it is still small when $a/p = 10\%$. Tracking error increases with increasing the ratio $a/p$, and as you can see, a 50% ratio does not yield satisfactory results. Comparing the simulation results of this controller, with the controllers given in the proceeding sections, one can conclude that the controller in this method is sensitive to parameter variations.

4.2 Results for adaptive computed torque control method

In this case, it is desired to track the trajectories $[y_d, \psi_d] = [2\sin 0.3t, \cos 0.2t]$. Numerous simulations were performed and it was concluded that a good compromise between control effort and a good response, can be achieved using the following design parameters

$$\phi_1 = \phi_2 = 100$$
\[ \gamma_1 = \ldots = \gamma_{\bar{n}} = 0.01 \]
\[ \lambda_1 = 10, \quad \lambda_2 = 15 \]

Simulation results are shown in Fig. 3. It can be seen that while the computed torque method could not stabilize the system in a 20% disturbance, its adaptive counterpart has led to a successful response. Still more interesting is the system's response to parametric variations: the deviation of tracking error from zero, in the presence of a 50% variation is still small and acceptable.

4.3 Results for suction control

The control objective is to track the desired trajectories \([y_d, \psi_d] = [2\sin t, \sin t]\). The thickness of the boundary layer is taken to be 0.1, with the design parameters \(\lambda_1 = \lambda_2 = 5\), and \(k_1\) and \(k_2\) are chosen equal to 10 in the presence of disturbances, and 12 in the presence of parameter variations. The results are shown in Fig. 4. Though simple in design, this method has yield extraordinary results in conquering disturbances and parameter variations.

Fig. 4. System's behavior for the suction control method: (a) \(\dot{\psi}\) in the presence of disturbance (b) \(\dot{y}\) in the presence of disturbance (c) \(\delta_b\) in the presence of parameter variations (d) \(\tilde{y}\) in the presence of parameter variations.
4.4 Saturation of rudders

If the control signal generated by the feedback law is larger than possible or permissible for reasons of safety, the actuator will “saturate” at a lower input level. The effect of occasional control saturation is usually not serious: in fact a system which never saturates is very likely overdesigned, having a larger and less efficient actuator than is needed to accomplish even the most demanding tasks. On the other hand, if the control signals produced by the linear control law are so large that the actuator is always saturated, it is not likely that the system behavior will be satisfactory, unless the actuator saturation is explicitly accounted for in an intentionally nonlinear control law design. If such a design is not intended, the gain matrix should be selected to avoid excessively large control signals for the range of states that the control system can encounter during operation (Friedland, 1987).

A conventional value for the saturation of rudders in underwater vehicles is about 30°. (Haghi et al, 2007) showed that if saturation occurs, the tracking error will not converge to zero, leading to instability of the vehicle. Obviously saturation must be avoided. In attempt to answer “Why saturation occurs?”, we overlook the problem definition again. Previously, it was assumed that the vehicle had a constant forward velocity \( u \). The desired trajectory is a sine wave of the form \( y_d = a \sin \omega t \), with the amplitude \( a \) and the frequency \( \omega \). Imagine driving in a road full of sharp turns. Intrinsically, the driver will slow down, to avoid turning over the vehicle. Now if the vehicle's forward speed is constant, then there will be a limit to the frequency of the road turns, that the driver can conquer without smashing his car. Same line of reasoning is made for our underwater vehicle. If the frequency of the desired trajectory \( \omega \) is too much, the control signal that would be needed to keep the vehicle on the track will increase. If the control signal increases so that saturation occurs, the underwater vehicle will turn over and smash out of the road! Therefore, we conclude that there should be a margin to the maximum value of frequency \( \omega \) that we can conquer without decreasing the speed, under which saturation will not occur. This value was found for some design parameters \( \lambda_1 \) and \( \lambda_2 \), by making numerous simulations, utilizing the method known as Bisection method by numerical analyzers. Simulation results are summarized in Table 2. The value of \( \omega \) has been assumed to be the same for both \( y_d \) and \( \varphi_d \).

<table>
<thead>
<tr>
<th>( \lambda_1 = \lambda_2 )</th>
<th>( \omega_{saturation} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.267578</td>
</tr>
<tr>
<td>2</td>
<td>1.181562</td>
</tr>
<tr>
<td>3</td>
<td>0.84472656</td>
</tr>
</tbody>
</table>

Table 2. Saturation frequency as a function of design parameters

5. Conclusions and further research

Three nonlinear control methods were proposed for controlling underwater vehicles, and their capabilities to cope with the issues of environmental disturbances and parametric
uncertainties were examined through simulation results. It was shown that the stability achieved by the computed torque control method, is sensitive to parametric uncertainties. Moreover, the maximum amount of disturbance waves that can be conquered by this method was shown to be lower than its adaptive counterpart. The adaptive computed torque control method compensated parameter variations through an adaptation law. As a result, it could manage larger amounts of uncertainties. Finally the suction control method lead to a robust controller, insensitive to uncertainties or disturbances.

The theoretical analysis proposed in this chapter, verified by numerical simulations, has shown that the application of the proposed control laws can lead to a successful design, conquering drastic constraints such as uncertainties and environmental disturbances. The next step in evaluating the efficiency and reliability of these approaches passes necessarily through the practical implementation of such algorithms, and verification of the results by experimental studies.

6. References


For the latest twenty to thirty years, a significant number of AUVs has been created for the solving of wide spectrum of scientific and applied tasks of ocean development and research. For the short time period the AUVs have shown the efficiency at performance of complex search and inspection works and opened a number of new important applications. Initially the information about AUVs had mainly review-advertising character but now more attention is paid to practical achievements, problems and systems technologies. AUVs are losing their prototype status and have become a fully operational, reliable and effective tool and modern multi-purpose AUVs represent the new class of underwater robotic objects with inherent tasks and practical applications, particular features of technology, systems structure and functional properties.

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