Inertia Matching Manipulability and Load Matching Optimization for Humanoid Jumping Robot

Zhaohong Xu; Tiansheng Lü and Xuyang Wang
School of Mechanical Engineering, Shanghai Jiaotong University, China
zhaoheongxu@sjtu.edu.cn

Abstract: Human jumping motion includes stance phase, flight phase and landing impact phase. Jumping robot belongs to a variable constraints system because every phase has different constraint conditions. An unified dynamics equation during stance phase and flight phase is established based on floated-basis space. Inertia matching is used to analyze actuator/gear systems and select the optimum gear ratio based on the transmission performance between the torque produced at the actuator and the torque applied to the load. Load matching is an important index which affects jumping performance and reflects the capability of supporting a weight or mass. It also affects the distributing of the center of gravity (COG). Regarding jumping robot as a redundant manipulator with a load at end-effector, inertia matching can be applied to optimize load matching for jumping robot. Inertia matching manipulability and directional manipulability are easy to analyze and optimize the load matching parameters. A 5th order polynomial function is defined to plan COG trajectory of jumping motion, taking into account the constraint conditions of both velocity and acceleration. Finally, the numerical simulation of vertical jumping and experimental results show inertia matching is in direct proportion to jumping height, and inertia matching manipulability is a valid method to load matching optimization and conceptual design of robot.

Keywords: jumping robot, inertia matching, directional manipulability, load matching

1. Introduction

Legged robots have better mobility, versatility and autonomous capability on non-structural environment contrasting to wheeled and tracked vehicles among mobile robotics family. Legged robots can be classified into two groups which are static locomotion robots and dynamic locomotion robots (S. Kajita et al, 2004). Static locomotion (e.g. walking and swing motion) is defined that the COG of robot is always ensured within the support polygon constructed by the supporting legs. Dynamic locomotion (e.g. jumping and running motion) is defined that the COG is allowed to not be in the support polygon constructed by the supporting legs. Moreover, jumping motion can over an obstacle which size is similar or times to itself (V. Nunez et al, 2005), and jumping robots improves their movement space and capability.

Trajectory planning and motion optimization are two crucial steps in the design and control for jumping robots. Some researchers have engaged in those problems. Albro et al. solved the optimal control of platform dives using a hybrid recursive algorithm and computing exact analytic gradients of the objective function (J.V. Albro et al, 2000). Yokozawa et al. considered jumping robot as a variable constraint system, and formulated an optimal control problem to maximize the peak height based on the complementarity modeling (T. Yokozawa et al, 2000). Takahashi et al. have generated periodic trajectories by Particle Swarm Optimization method (T. Takahashi et al, 2006). Fujimoto has solved the minimum energy consumption trajectories for a biped running motion through the numerical study of a five link planar robot (Y. Fujimoto, 2004). Bobrow et al. have formulated the equations of motion of complex multibody systems in resulting optimization problems by computing exact analytic gradients of the objective function without resorting to numerical approximations (J.E. Bobrow et al, 2001). Although some optimal control and trajectory optimization methods have discussed, they don’t solved the relation between objective parameters and jumping performance. It is still a problem how to improve the jumping performance.

The concept of kinematic manipulability (A. Bowling et al, 2005) was introduced to perform task space analysis of robotic manipulators. Differential kinematics and kineto-static show that the axes of ellipsoids representing the range both applicable velocities and forces coincide with their magnitude being of inverse proportion. The manipulator manipulability (N. Naksuk et al, 2005) can evaluate the acceleration and force capability between the angular velocities at each joint and the linear or angular velocity at the end-effector of the manipulator. Manipulability measure (N. Naksuk et
al, 2005) is the acceleration radius defined as the minimum upper bound of the magnitude for end-effector accelerations over the entire manipulator workspace. Some indices have been proposed for the evaluation of manipulator manipulability. The manipulability ellipsoid (A. Bowling et al, 2005) was introduced to evaluate the static performance of a robot manipulator as an index of the relationship between the angular velocities at each joint and the linear or angular velocity at the end-effector of the manipulator. The manipulating force ellipsoid (S.H. Jeong et al, 2006) is an index that evaluates the static torque-force transmission from the joints to the end-effector. The dynamic manipulability ellipsoid [8] is a measure for the dynamic performance of a robot manipulator based on the acceleration of the end-effector. Inertia ellipsoid (D.Z. Chen et al, 1991, R. Kurazume et al, 2004) is a geometric representation for the inertial properties of a manipulator. These manipulability indices have been applied to conceptual design of mechanical arm (S.H. Lee, 2005), singularity avoidance (J. Kim et al, 2004), the impact measure for redundant manipulator (I.D. Walker, 1994), global task space optimization and coordination control for multi-arm system (C.Y. Kim et al, 1997), the fault tolerant property of redundant robot (C. Caldwell et al, 2002) and so on.

In this paper, the unified dynamics for jumping motion is established, and the manipulability measure which combines inertia matching and directional manipulability is applied to the load matching optimization of humanoid jumping robot. In Section 2, the unified dynamics including stance and flight phase for jumping robot is derived. In Section 3, the concept of inertia matching is introducing to geared mechanism and humanoid jumping robot. The inertia matching manipulability and directional manipulability for jumping robot are obtained. In Section 4, load matching for jumping robot is optimized by inertia matching manipulability and directional manipulability. And moreover, a 5th order polynomial function is defined to plan COG trajectory of jumping motion taking into account the constraint conditions of both velocity and acceleration. In Section 5, numerical simulation and experimental results of load matching optimization with inertia manipulability and directional manipulability are dedicated. In addition, we show the relation between inertia matching and jumping performance. Section 6 summarizes the findings of this paper and gives some future work.

2. Dynamics model for jumping robot

Humanoid jumping process can be divided into three phases based on constraint conditions. They are stance phase, flight phase and landing impact phase. Fig.1 depicts a humanoid jumping motion process. Every phase has a dynamics equation because of their different constraint conditions. So the dynamics of jumping motion belongs to the dynamics of a various constraint system.

Some equivalent models have been used to analysis the jumping robot model, such as telescopic leg mode (Y. Fujimoto, 2004), mass-spring model (W.J. Schwind et al, 2000) and spring load inverted pendulum (J. Vermeulen et al, 2003). The motion space of humanoid robot can be decoupled into sagittal and lateral plane. Our model is a revolute-jointed robot driven by electrical DC motors in sagittal plane. The jointed configuration of multibody is shown in Fig. 2. XOY is an inertia reference frame, and $X_{cm}O_{cm}Y_{cm}$ is a floated-basis frame which is fixed to the mass center of jumping robot. The vector of body coordinates $q_i$ is the relative angles $\left(q_i, \ldots, q_{i+n-1}\right)^T$, which describes the shape of the robot. The direction of anticlockwise is positive. The absolute orientation of robot is given by $q_a$. There are three constraints in COG, viz. two holonomic constraints resulting from the fact that the COG tracks a parabolic trajectory, and one non-holonomic resulting from the conservation of angular momentum with respect to COG. The robot’s absolute position of COG $r_{cm}$ is specified by the Cartesian coordinates $(x_{cm}, y_{cm})$. The vector of generalized coordinates $q_i$ is denoted as $\left(q_i, q_{i+n}, r_{cm}\right)^T$. The mass of i-th rigid body is $m_i$, its length is $l_i$, the moment of inertia around its center of mass is $I_i$, and the position of its center of mass is given by $\frac{\Delta C_i}{2} = \alpha l_i$. During flight phase, the reference base is free-floating, and there is a non-holonomic dynamics being the conservation of angular momentum with respect to the center of gravity (COG) of robot system. We establish the unified dynamics equations including both stance phase and flight phase based on floated-basis space and Lagrange formula.
2.1. Dynamics equation for flight phase

The dynamics model can be determined from Lagrange's equation

\[
\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} = \Gamma
\]

Where Lagrangian is defined as \( L = K - P \), including the total kinetic energy \( K \) and the total potential energy \( P \). Defining the variable \( \xi_i \) as the mass percent of \( i \)-th rigid body to the whole body, viz. \( \xi_i = \frac{m_i}{\sum_{j=1}^{n} m_j} \cdot \) The COG of robot can be denoted into \( \sum_{i=1}^{n} \xi_i \cdot \mathbf{r}_i = 0 \) if \( \mathbf{r}_i \) is the vector between \( O_{cm} \) and the COG of \( i \)-th rigid body. When analyzing the COG vector of \( i \)-th rigid body in frame \( X_{cm}O_{cm}Y_{cm} \) along the COG vector of robot in frame \( XOY \), it can yield \( \sum_{i=1}^{n} m_i \cdot \mathbf{r}_i \cdot \mathbf{r}_i = 0 \). So we decouple rotational motion from translational motion.

The kinetic energy of robot can be given by the following integral

\[
K_i = \frac{1}{2} \sum_{i=1}^{n} \left( m_i \mathbf{r}_i^T \mathbf{r}_i + \frac{1}{2} m_i \mathbf{r}_i^T \mathbf{r}_i \right) + \frac{1}{2} m \mathbf{r}_{cm}^T \mathbf{r}_{cm}
\]

\[
= \frac{1}{2} \mathbf{q}_i^T \mathbf{M}(\mathbf{q}_i) \mathbf{q}_i + \frac{1}{2} m \mathbf{r}_{cm}^T \mathbf{r}_{cm}
\]

Where

\[
\mathbf{M}(\mathbf{q}_i) = \begin{bmatrix} \mathbf{M}(\mathbf{q}_i) \\ \mathbf{0}_{(n-i) \times 2} \end{bmatrix}, \quad \mathbf{q}_i = \begin{bmatrix} \mathbf{q}_i \\ \mathbf{0}_{(n-i) \times 2} \end{bmatrix}, \quad i = 1, \ldots, n-1
\]

And \( \mathbf{M}(\mathbf{q}_i) \) is the inertia matrix.

The potential energy of the system is obtained as

\[
P_i = m g y_{cm}
\]

By the principle of virtual principle, the general torques matrix is

\[
\Gamma_t = \begin{bmatrix} \tau_1 \cdots \tau_{n-1} & 0 & 0 \end{bmatrix}^T
\]

Using Eq. (2), (3) and (4), Lagrange's equation can be written as

\[
\mathbf{D}(\mathbf{q}_i) \mathbf{q}_i + \mathbf{H}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i) \dot{\mathbf{q}}_i + \mathbf{G}_i(\mathbf{q}_i) = \Gamma_t
\]

Where \( \mathbf{H}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i) \) contains Coriolis and centrifugal terms

\[
\mathbf{H}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i) = \frac{\partial}{\partial \dot{\mathbf{q}}_i} \left[ \frac{1}{2} \dot{\mathbf{q}}_i^T \mathbf{D}_i(\mathbf{q}_i) \dot{\mathbf{q}}_i \right]
\]

And \( \mathbf{G}_i(\mathbf{q}_i) \) is the gravity vector

\[
\mathbf{G}_i(\mathbf{q}_i) = \begin{bmatrix} 0 & \cdots & 0 & m g \end{bmatrix}^T
\]

2.2. Dynamics equation for stance phase

Assuming without slide and rotation under the robot foot, the motion constraint during stance phase is holonomic. The generalized coordinates \( \mathbf{q}_i \) can be written as \( \begin{bmatrix} \mathbf{q}_i \end{bmatrix}^T \). In the past, most people established math model during stance phase using a fixed frame. Presently this paper contributes to describe dynamics from floated-basis space in flight phase.

According to the Cartesian coordinate space, the COG of robot during stance phase can be denoted as

\[
\mathbf{r}_{cm} = \begin{bmatrix} x_{cm} \\ y_{cm} \end{bmatrix} = \mathbf{R}_{ani} \begin{bmatrix} (1-\alpha_i)l_i \\ 0 \end{bmatrix}
\]

\[
- \sum_{i=1}^{n} \mathbf{R}_{ani} \left[ \begin{bmatrix} (1-\alpha_i)\xi_i + \frac{1}{\sum_{j=1}^{n} \xi_j} \xi_i \end{bmatrix} l_i \right] = f_{cm}(\mathbf{q}_i)
\]

Hence, the relationship of generalized coordinates between flight phase and stance phase is

\[
\mathbf{q}_i = \begin{bmatrix} \frac{1}{\sum_{j=1}^{n} \xi_j} \end{bmatrix} \mathbf{q}_i
\]
Substituting Eq. (7) into (2), the kinetic energy during stance phase yields

\[
K_s = \frac{1}{2} \dot{q}_i^T \left[ \frac{I_{\text{nea}}}{\partial q_i} \right]^T D_i(q_s) \left[ \frac{I_{\text{nea}}}{\partial q_i} \right] \dot{q}_s = \frac{1}{2} \dot{q}_i^T D_i(q_s) \dot{q}_s
\]  

(8)

Where

\[
D_i(q_s) = M(q_s) + \frac{\partial}{\partial q_i} \left[ \frac{I_{\text{nea}}}{\partial q_i} \right] \frac{\partial f_m}{\partial q_i}
\]

Substituting Eq. (8) into Lagrange’s equation, yielding

\[
\frac{\partial}{\partial q_i} \left[ \frac{1}{2} \dot{q}_i^T D_i(q_s) \dot{q}_s \right] + H_i(q_s, \dot{q}_s) \dot{q}_i + G_s(q_s) = \Gamma_s
\]  

(9)

Where \( H_i(q_s, \dot{q}_s) \) contains Coriolis and centrifugal terms

\[
H_i(q_s, \dot{q}_s) = \frac{\partial}{\partial q_i} \left[ \frac{1}{2} \dot{q}_i^T D_i(q_s) \dot{q}_s \right]
\]

\( G_s(q_s) \) is the gravity vector

\[
G_s(q_s) = \frac{\partial (m g v_{\text{nea}})}{\partial q_i}
\]

The general torques matrix is

\[
\Gamma_s = \begin{bmatrix} \tau_i \cdots \tau_s \end{bmatrix}^T
\]

3. Inertia Matching

The concept of inertia matching is widely used in the analysis of actuator and gear systems, primarily for selection of the optimum gear ratio based on the transmission performance between the torque produced at the actuator and the torque applied to the load (D.Z. Chen et al, 1991). In this process, the performance of torque transmission is maximized by setting the optimal balance of inertial properties between the actuator system (including inertia of the rotor and shaft) and the load. The concept of inertia matching for jumping robot is proposed in this paper as a new index of the dynamic performance. The proposed inertia matching ellipsoid characterizes the dynamic torque-force transmission efficiency between joint actuators and a load held by the end-effector of a manipulator, encompassing a wide range of previous concepts.

3.1. Inertia Matching for geared mechanism

Figure 3 shows a one DOF geared mechanism, the equation of motion is

\[
(I_{\text{op}} \dot{\xi}^2 + I_{\text{wp}}) \dot{\xi} = \xi \tau_{\text{wp}}
\]  

(10)

Where \( I_{\text{wp}} \) denotes the inertia of the input link, \( I_{\text{op}} \) denotes the inertia of the output link, \( \dot{\xi} = N_2/N_1 \) denotes the gear ratio, \( \dot{\xi} \) denotes the angular displacement of the output shaft, \( \tau_{\text{wp}} \) denotes the input torque, and \( I_{\text{op}} \dot{\xi}^2 \) denotes the inertia of the input link reflected at the output shaft.

Fig. 3. Geared Mechanism

Eq. (10) can be written as

\[
\dot{\xi} = \frac{\tau_{\text{wp}}}{\left( I_{\text{wp}} \dot{\xi}^2 - \sqrt{I_{\text{wp}} \dot{\xi}^4} \right)^2 + 2 \sqrt{I_{\text{wp}} I_{\text{op}}}}
\]  

(11)

Assuming that \( I_{\text{wp}} \) and \( I_{\text{op}} \) remain constant regardless of the change in gear ratio and there is no power loss in the gear mesh. Fig. 4 shows the relation between the output shaft acceleration \( \dot{\xi} \) and the gear ratio \( \xi \).

Fig. 4. The relation between output shaft acceleration and gear ratio

It is clear that there exists an optimum gear ratio which yields a maximum output acceleration. At the optimum design, the output acceleration and the gear ratio are given by

\[
\dot{\xi}_{\text{max}} = \frac{\tau_{\text{wp}}}{2 \sqrt{I_{\text{op}} I_{\text{wp}}}}
\]

\[
\xi = \frac{I_{\text{wp}}}{I_{\text{op}}}
\]  

(12)

By choosing the optimal ratio by Eq. (12), a large acceleration of the load is produced with small output torque at the actuator. It can be said that the gear ratio is chosen such that the reflected input inertia is matched with the output inertia. That is inertia matching.

3.2. Inertia Matching for jumping robot

The concept of inertia matching can be extended to jumping robot as follows. Humanoid jumping robot can be considered as a redundant manipulator with a load held at the end-effector (R. Kurazume et al, 2004). Fig. 5 shows the jumping robot and inertia matching ellipsoid (IME).
The motion and force equation analyzing to the load held at the end-effector can be written as

\[ \mathbf{F}_e = m_l (\dot{\mathbf{X}} + \mathbf{g}) \]  

(13)

Where \( \mathbf{X} \) is the acceleration of the end-effector, \( m_l \) is the mass inertia of load, \( \mathbf{g} = (0, -g) \) is the gravity vector.

Defined the load matching \( \mathbf{M}_{load} \) as

\[ \mathbf{M}_{load} = \sum_{i=1}^{n} m_i + m_l \]  

(14)

Jumping performance is affected by not any motion control, but also the jumping posture and load capability. Jumping posture affects the attitude during flight phase, and the attitude determines the angular moment for COG of robot. Load matching is an important index which affects jumping performance and reflects the capability of supporting a weight or mass. It also affects the distributing of COG. We hope the robot can bear more loads and jump more height.

If the load mass is unknown, when a external moment and force is applied to the jumping robot, the dynamics equation for stance phase can be written as

\[
\mathbf{D}_s (\mathbf{M}_{load}, \mathbf{q}) \ddot{\mathbf{q}} + \mathbf{H}_s (\mathbf{M}_{load}, \mathbf{q}, \dot{\mathbf{q}}) \dot{\mathbf{q}} + \mathbf{G}_s (\mathbf{M}_{load}, \mathbf{q}) + \mathbf{J}(\mathbf{q})^T \mathbf{F}_e = \mathbf{\Gamma}_s
\]

(15)

Where \( \mathbf{J}(\mathbf{q}) \) is the Jacobian matrix, \( \mathbf{F}_e \) is the external moment and force.

The end-effector posture of manipulator \( \mathbf{X} \) is related to the joint space vector \( \mathbf{q} \) as \( \mathbf{X} = \mathbf{F}(\mathbf{q}) \), so

\[
\begin{align*}
\dot{\mathbf{X}} &= \mathbf{J}(\mathbf{q}) \dot{\mathbf{q}} \\
\mathbf{X} &= \mathbf{J}(\mathbf{q}) \mathbf{q} + \dot{\mathbf{J}}(\mathbf{q}) \dot{\mathbf{q}}
\end{align*}
\]

(16)

Substituting Eq. (15) and (16) into (14), the torque matrix can be obtained by

\[
\begin{align*}
\mathbf{\Gamma}_s &= \mathbf{D}_s (\mathbf{M}_{load}, \mathbf{q}) \mathbf{J}(\mathbf{q})^T \frac{(\mathbf{F}_e - m_l \mathbf{g} - m_l \dot{\mathbf{J}}(\mathbf{q}) \dot{\mathbf{q})/m_l)}{+ \mathbf{H}_s (\mathbf{M}_{load}, \mathbf{q}, \dot{\mathbf{q}}) + \mathbf{G}_s (\mathbf{M}_{load}, \mathbf{q}) + \mathbf{J}(\mathbf{q})^T \mathbf{F}_e = \mathbf{Q}(\mathbf{M}_{load}, \mathbf{q}) (\mathbf{F}_e - \mathbf{F}_{bias})}
\end{align*}
\]

(17)

Where

\[
\begin{align*}
\mathbf{Q}(\mathbf{M}_{load}, \mathbf{q}) &= \mathbf{J}(\mathbf{q})^T + \mathbf{D}_s (\mathbf{M}_{load}, \mathbf{q}) \mathbf{J}(\mathbf{q})^T / m_l \\
\mathbf{F}_{bias} &= \mathbf{W} \left[ \mathbf{J}(\mathbf{q})^T + \mathbf{D}_s (\mathbf{M}_{load}, \mathbf{q}) \mathbf{J}(\mathbf{q})^T / m_l \right] \times \\
&\quad \left[ \mathbf{D}_s (\mathbf{M}_{load}, \mathbf{q}) \mathbf{J}(\mathbf{q})^T (\mathbf{g} + \dot{\mathbf{J}}(\mathbf{q}) \dot{\mathbf{q}}) - \mathbf{H}_s (\mathbf{M}_{load}, \mathbf{q}, \dot{\mathbf{q}}) - \mathbf{G}_s (\mathbf{M}_{load}, \mathbf{q}) \right]
\end{align*}
\]

(18)

Here, \( \mathbf{F}_{bias} \) is the bias force matrix of angular velocity and acceleration. \( \mathbf{F}_e - \mathbf{F}_{bias} \) is the inertia matching for jumping robot. \( \mathbf{J}(\mathbf{q})^T \) is a pseudoinverse of the Jacobian matrix \( \mathbf{J}(\mathbf{q}) \). When the Jacobian matrix is a regular matrix, then \( \mathbf{J}(\mathbf{q})^T = \mathbf{J}(\mathbf{q})^{-1} \). In the case that Jacobian matrix is a regular matrix, then \( \mathbf{J}(\mathbf{q})^T = \mathbf{W}^{-1} \mathbf{J}^T (\mathbf{JW}^{-1} \mathbf{J}^T)^{-1} \).

Where \( \mathbf{W} \) is a weight matrix.

3.3. Inertia Matching manipulability

In Eq. (17), the coefficient matrix \( \mathbf{Q}(\mathbf{M}_{load}, \mathbf{q}) \) indicates the moment or force transmission efficiency between the torque produced at the actuators and the force or moment applied to the load by the end-effector.

Based on the theory of singular value decomposition, \( \mathbf{Q}(\mathbf{M}_{load}, \mathbf{q}) \) can be given by

\[
\mathbf{Q}(\mathbf{M}_{load}, \mathbf{q}) = \mathbf{U} \sum \mathbf{V}^T
\]

(19)

Where \( \mathbf{U} \in \mathbb{R}^{mn \times mn} \) and \( \mathbf{V} \in \mathbb{R}^{mn \times mn} \) are orthogonal matrices,

\[
\sum = \text{diag}(\sigma_1, \sigma_2, \ldots, \sigma_n) \in \mathbb{R}^{mn \times mn}, \quad \sigma_i \text{ is a nonnegative singular value.}
\]

The manipulability measure \( \omega \) of inertia matching \( \mathbf{F}_e - \mathbf{F}_{bias} \) can be expressed as the product of \( \sigma_i \), we obtain

\[
\omega = \sigma_1 \sigma_2 \cdots \sigma_n
\]

(20)

The principal axes are the product between the row vector \( (\mathbf{u}_1, \ldots, \mathbf{u}_n) \) of \( \mathbf{U} \) and the singular value vector \( (\sigma_1, \ldots, \sigma_n) \). And moreover, the singular value vector \( (\sigma_1, \ldots, \sigma_n) \) shows the motion capability of the corresponding principal axis. The manipulability measure of inertia matching synthetically evaluates the isotropic flexibility of robot, and it whole measures the manipulability of manipulator.

Generally, the torque limits at each actuator in jumping robot are assumed to be symmetrical and constrained, viz. \(-\tau_{max} \leq \tau_i \leq \tau_{max}\). The normalized joint torque \( \tilde{\mathbf{\Gamma}} \) can be obtained using a conversion matrix \( \mathbf{L} = \text{diag}(\tau_{max}, \ldots, \tau_{max}) \) as

\[
\tilde{\mathbf{\Gamma}} = \mathbf{L}^{-1} \mathbf{\Gamma}
\]

(21)
Therefore, when a normalized torque with magnitude of 1 is produced, the inertia matching ellipsoid can be obtained as

\[
(F_s - F_{bias})^T Q^T (M_{load}.q) L^2 Q(M_{load}.q)(F_s - F_{bias}) \leq 1 \quad (22)
\]

3.4. Directional manipulability for Inertia Matching
Inertia matching is a vector, it includes a value and direction. However, the inertia matching manipulability \( \Omega \) only describes the value, not expresses the direction. And moreover, jumping motion includes various jumping forms, such as vertical jumping, long jumping. In this paper, we introduce directional manipulability measure of inertia matching to analyze the jumping task and direction.
Assuming the moment and force vector applied to the center of load at end-effector is given by

\[ \Gamma = A_s P \]

(23)

Where \( A_s \) is the scalar quantity form of the moment or force \( \Gamma_s \), \( P = (\cos \alpha_s, \cos \alpha_s, \ldots, \cos \alpha_s) \in \mathbb{R}^n \) is the direction of the force in load at end-effector in Cartesian frame, and \( \alpha_s \) is the angle between moment/force at load and the positive direction of coordinate axis.
Substituting Eq. (23) into (17), the following equation can be obtained

\[
\Gamma_s = D(M_{load}.q) J(q)^T (X_{\text{cm}} - J(q)q) + H(M_{load}.q, q) \dot{q} + G(M_{load}.q) = \hat{Q}(M_{load}.q)(X_{\text{cm}} - X_{\text{cm}}) p
\]

(24)
The directional manipulability of inertia matching can be given by

\[
DM(M_{load}.q) = A_s^\dagger = \left[p^T Q^T (M_{load}.q) L^2 Q(M_{load}.q) p\right]^{-1}
\]

(25)
The directional manipulability measure of inertia matching \( DM(M_{load}.q) \) reflects the manipulability in specified direction of robot.

4. The load matching optimization for jumping motion
Jumping robot can be considered as a redundant manipulator with a load held at the end-effector. The link is rotary joint between end load and manipulator. This model or equivalent is a valid analysis method to mathematic modeling of jumping robot. In this paper, we consider the jumping height as the main jumping performance. Analyzing the jumping robot as a whole, we can find that the velocity at take-off determines the jumping height, and the COG trajectory after take-off is a parabola. The take-off motion can be regarded as a process from the initial posture \( q_0 = (q_{10}, q_{20}, \ldots, q_{n0}) \) to the ultimate posture \( q_f = (q_{1f}, q_{2f}, \ldots, q_{nf}) \) through the harmonious movement of joints. The inertia matching manipulability is a function of posture \( q \) and load matching \( M_{load} \), and it reflects the moment or force transmission efficiency between the torque produced at the actuators and the force or moment applied to the load by the end-effector. When the force transmission efficiency of interior joints is maximal, the time integral of ground reaction force will reach the maximization. So the jumping height is maximal. Although the posture \( q \) and load matching \( M_{load} \) all affect the jumping performance, we only discuss the relation between load matching and jumping performance in this paper. The following gives the optimization method of load matching which make the robot reach the maximization of jumping performance applying the inertia matching and directional manipulability.

4.1. The optimization of load matching
From the constraint condition of stance phase, viz. Eq. (6), the acceleration of COG can be written as

\[
\begin{bmatrix}
\dot{x} \\
\dot{y}
\end{bmatrix} = f_{\text{sta}}(q_s) q_s + f_{\text{sta}}(q_s) q_s
\]

(26)
The time integral of ground reaction force \( I_s \) can be obtained as

\[
I_s = \int_{t_0}^{t_f} N dt = \int_{t_0}^{t_f} \sum m_i (g + \dot{y}_i) dt
\]

(27)
Where \( t_0 \) is the initial time of take-off, \( t_f \) is the ultimate time of take-off, and \( N \) is the ground reaction force. The manipulability measure of inertia matching is a function of posture \( q \) and load matching \( M_{load} \). When the jumping form is given, the initial posture \( q_0 = (q_{10}, q_{20}, \ldots, q_{n0}) \) and the ultimate posture \( q_f = (q_{1f}, q_{2f}, \ldots, q_{nf}) \) are chosen. The optimization of load matching can be denoted by

\[
\begin{align}
\text{Maximize} & \quad \left[p^T Q^T (M_{load}.q) L^2 Q(M_{load}.q) p\right]^{-1} \\
\text{Subject to} & \quad f_i(q_s) = 0 \\
& \quad q_0 = f_i(q_{10}, q_{20}, \ldots, q_{n0}) \\
& \quad q_f = f_i(q_{1f}, q_{2f}, \ldots, q_{nf})
\end{align}
\]

(28)
Where the constraint condition \( f_i(q_s) = 0 \) is the jumping form (e.g. vertical jumping, long jumping and so on).

4.2. The motion plan for jumping motion
In this paper, we plan the jumping motion trajectory during stance phase and flight phase respectively. The
jumping motion in planar can be decoupled into the movement in vertical and horizontal direction. During the jumping movement, the COG has a time sequence from $t_0$ to $t_f$, a vertical displacement sequence from $0$ to $c_y(t_f)$, where $c_y(t_f)$ is the optimal take-off posture, a velocity sequence from $0$ to $v_y(t_f)$, where $v_y(t_f) > 0$, and a acceleration sequence from $0$ to $-g$.

Considering the constraints of velocity and acceleration, a 5th order polynomial function is defined to plan the COG trajectory during stance phase by as follows

$$
\begin{align*}
  y_c &= c_6 + c_5 t + c_4 t^2 + c_3 t^3 + c_2 t^4 + c_1 t^5 \\
  c_6 &= y_c(t_0), \quad c_5 = -10 y_c(t_f)/t_f^2 + 10 y_c(t_f)/t_f^2 - 4 v_y(t_f)/t_f^2 - g/2t_f \\
  c_4 &= 0, \quad c_3 = 15 y_c(t_f)/t_f^2 - 15 y_c(t_f)/t_f^2 + 7 v_y(t_f)/t_f^2 + g/2t_f \\
  c_2 &= 0, \quad c_1 = -6 y_c(t_f)/t_f^2 + 6 y_c(t_f)/t_f^2 - 3 v_y(t_f)/t_f^2 - g/2t_f 
\end{align*}
$$

(29)

During flight phase, the COG trajectory is a parabola, and it can be obtained by

$$
\begin{align*}
  y_c &= v_y t - g t^2 / 2 \\
  t_f &= v_y / g
\end{align*}
$$

(30)

5. Simulation and experiment

To verify the feasibility of the dynamics model and suggested optimal scheme we designed a 4-DOF rotary joints and five rigid bodies jumping robot (foot, crus, thigh, trunk and arm). Fig.6 shows our experimental device and jumping robot. The jumping robot can jump in a circle. The horizontal bar of device is attached at the robot’s body. The bar provides only lateral stability, and it does not prevent the robot falling forward, backward or down. Every joint of jumping robot is independently driven by servo motor. The type of servo motors is Tower Pro MG995, which made in Taiwan of China. Its rated velocity is $0.13/60 (6.0°) s^{-1}$, viz. $8.05 rad / s$, and its rated torque is $11Kg-cm$. In our jumping robot, there are not any assistant elastic components, such as spring, damp, hydraulic or pneumatic actuators. The controller is AVR system, and our main control chip is Atmega128 from ATMEL company. The ATMega128 is a low-power CMOS 8-bit microcontroller based on the AVR enhanced RISC architecture. It provides four flexible Timer/Counters with compare modes and PWM, and the PWM can directly drive servo motor.

In this paper, the simulation parameters are only related to physical parameters of foot, crus, thigh and load. So, we give those physical parameters as follows

$$
\begin{align*}
  I_1 &= 0.15m, \quad I_2 = I_3 = 0.2m, \quad \alpha = \alpha_2 = \alpha_3 = 1/2, \quad m_1 = m_2 = 0.2kg, \quad m_3 = 0.02kg
\end{align*}
$$

Jumping robot can reach the maximal height by experiments when $\bar{m} = 0.32kg$, viz. $M_{max} = 0.44$.

The following is the simulation figures. Fig.7 shows inertia matching ellipsoid, inertia matching, jumping height and ground reaction force from different load matching. Fig.8 shows the trajectory for angular angle, angular velocity and angular torque under the optimal jumping performance and load matching.
The numerical simulation shows inertia matching is in direct proportion to jumping height. The inertia matching reach a maximization when $M_{\text{load}} = 0.48$. From force transmission efficiency, the relation between load matching and inertia matching is as follows. Load matching is in direct proportion to load. When load matching increases to 0.48, inertia matching linearly increases to the maximal 0.89N by degrees, and the jumping height also linearly increases to the maximal 0.0168m by degrees. When load matching increases to 0.51, inertia matching rapidly reduces to 0.16N because of both the rapidly increase of angular torque and the decrease of jumping velocity, and the jumping height rapidly reduces to 0.002m. When load matching is on the increase, inertia matching gradually reduces to zero because of limit of maximal angular torque, and the jumping height also reduces to zero. Moreover, from the time integral of ground reaction force, when load matching is in the optimization, the time integral of ground reaction force and jumping height are maximal. Furthermore, the angular velocity and torque are under the rated values of motor performance.

The error percent of load matching between simulation value $\hat{M}_{\text{load}}$ and experiment value $\tilde{M}_{\text{load}}$ is 8.3%. The numerical simulation results agree with experiments value to a certain extent. The errors reflect the asymmetrical robot structure and the simplified math model. Numerical simulation and experiments show inertia matching is a valid optimization method to load matching.
Fig. 8. The angular trajectory under the optimal jumping performance and load matching. (a) The trajectory of angular angle under the optimal jumping performance. (b) The trajectory of angular velocity under the optimal jumping performance. (c) The trajectory of angular torque under the optimal jumping performance.

6. Conclusions

In this work, the unified dynamics of stance and flight phase for humanoid jumping robot is established based on various constraint conditions and floated-basis space. An effective strategy for optimizing humanoid jumping motion is introduced. The concept of inertia matching is introducing to optimization of humanoid jumping robot. Load matching is an important index which affects jumping performance and reflects the capability of supporting a weight or mass. It also affects the distributing of COG. The load matching for jumping robot is optimized by inertia matching and directional manipulability. Moreover, a 5th order polynomial function is defined to plan the COG trajectory of jumping motion taking into account the constraint conditions of velocity and acceleration, and moreover the trajectory during flight phase is obtained. Numerical simulation and experimental results show the time integral of ground reaction force and jumping height are at the maximization when inertia matching reaches to the optimization. Furthermore, inertia matching is in direct proportion to jumping performance, and inertia matching is a valid optimization method to load matching of jumping motion. The jumping performance includes jumping height and landing stability. In the future, we would like to apply multi-objective optimization to jumping motion plan, such as the functions of objective jumping parameters, jumping height and stability strategy.

7. References


