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1. Introduction

One of the main goals of geophysical studies is to apply suitable mathematical and statistical techniques to extract information about the subsurface properties. Well logs are largely used for characterizing reservoirs in sedimentary rocks. In fact it is one of the most important tools for hydrocarbon research for oil companies. Several parameters of the rocks can be analyzed and interpreted in term of lithology, porosity, density, resistivity, salinity and the quantity and the kind of fluids within the pores.

Geophysical well-logs often show a complex behavior which seems to suggest a fractal nature (Pilkington & Tudoeschuck, 1991; Wu et al., 1994; Turcotte, 1997; Ouadfeul, 2006; Ouadfeul and Aliouane 2011; Ouadfeul et al, 2012). They are geometrical objects exhibiting an irregular structure at any scale. In fact, classifying lithofacies boundary from borehole data is a complex and non-linear problem. This is due to the fact that several factors, such as pore fluid, effective pressure, fluid saturation, pore shape, etc. affect the well log signals and thereby limit the applicability of linear mathematical techniques. To classify lithofacies units, it is, therefore, necessary to search for a suitable non-linear method, which could evade these problems.

The scale invariance of properties has led to the well known concept of fractals (Mandelbrot, 1982). It is commonly observed that well log measurements exhibit scaling properties, and are usually described and modelled as fractional Brownian motions (Pilkington & Tudoeschuck , 1991; Wu et al. 1994; Kneib 1995; Bean, 1996; Holliger 1996; Turcotte 1997; Shiomi et al. 1997; Dolan et al 1998; Li 2003; Ouadfeul, 2006; Ouadfeul and Aliouane, 2011; Aliouane et al; 2011). In previous works (Ouadfeul, 2006; Ouadfeul and Aliouane, 2011; Aliouane et al, 2011), we have shown that well logs fluctuations in oil exploration display scaling behaviour that has been modelled as self affine fractal processes. They are therefore
considered as fractional Brownian motion (fBm), characterized by a fractal \( k-H \) power spectrum model where \( k \) is the wavenumber and \( H \) is related to the Hurst parameter (Hermman, 1997; Ouadfeul and Aliouane, 2011). These processes are monofractal whose complexity is defined by a single global coefficient, the Hurst parameter \( H \), which is closely related to the Hölder degree regularity. Thus, characterizing scaling behavior amounts to estimating some power law exponents.

Petrophysical properties and classification of lithofacies boundaries using the geophysical well log data is quite important for the oil exploration. Multivariate statistical methods such as principle component and cluster analyses and discriminate functions analysis have regularly been used for the study of borehole data. These techniques are, however, semi-automated and require a large amount of data, which are costly and not easily available every time.

The modern data modeling approach based on the Artificial Neural Network (ANN) techniques is inherently nonlinear and completely data-driven requiring no initial model and hence provide an effective alternative approach to deal with such a complex and nonlinear geophysical problem. Some researchers have been engaged in classifying lithofacies units from the recorded well logs data. They have recently employed statistical and ANN methods (Aliouane et al., 2011).

In this work, we show that the fractal analysis is not able to improve lithofacies classification from well-logs data using the Self-Organizing Map neural Network. We analyze several petrophysical properties recorded in two boreholes, Well01 and Well02 located in Berkine basin in the northeast of the Saharan platform (Algeria). This basin is considered as a vast Palaeozoic depression in which the crystalline basement is covered by an important sedimentary series. Lithologically, the explored geological unit at the drill site consists of four main facies units: clay, sandstone and alternations of clayey sandstone and Sandy Clay (Well Evaluation Conference., 2007).

A fractal model is assumed for the logs and they are analyzed by the Continuous Wavelet Transform (CWT) which maps the measured logs to profiles of Hölder exponents. We use the estimated wavelet Hölder exponents rather than the raw data measurements to check the classification process initiated by a self organizing map of Kohonen procedure.

In this chapter we first present a short mathematical description to show that the CWT is the suitable tool used to analyze concept of a self affine process. Second, we describe the neural network method, particularly, the Kohonen’s Self Organizing Map (SOM) and its derived processing algorithm. Finally, we show the fractal analysis effect on the Self-Organizing Map neural network for lithofacies classification.

2. Wavelet analysis of scaling processes

Here we review some of the important properties of wavelets, without any attempt at being complete. What makes this transform special is that the set of basis functions, known as
wavelets, are chosen to be well-localized (have compact support) both in space and frequency (Arneodo et al., 1988; Arneodo et al., 1995; Ouadfeul and Aliouane, 2010). Thus, one has some kind of “dual-localization” of the wavelets. This contrasts the situation met for the Fourier Transform where one only has “mono-localization”, meaning that localization in both position and frequency simultaneously is not possible.

The CWT of a function \( s(z) \) is given by Grossmann and Morlet, (1985) as:

\[
C_s(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} s(z) \psi^*(z) dz,
\]

Each family test function is derived from a single function \( \psi(z) \) defined to as the analyzing wavelet according to (Torresiani, 1995):

\[
\psi_{a,b}(z) = \psi\left(\frac{z-b}{a}\right),
\]

Where \( a \in R^+ \) is a scale parameter, \( b \in R \) is the translation and \( \psi^* \) is the complex conjugate of \( \psi \). The analyzing function \( \psi(z) \) is generally chosen to be well localized in space (or time) and wavenumber. Usually, \( \psi(z) \) is only required to be of zero mean, but for the particular purpose of multiscale analysis \( \psi(z) \) is also required to be orthogonal to some low order polynomials, up to the degree \( n-1 \), i.e., to have \( n \) vanishing moments:

\[
\int_{-\infty}^{+\infty} z^n \psi(z) dz = 0 \quad \text{for} \quad 0 \leq n \leq p-1
\]

According to equation (3), \( p \) order moment of the wavelet coefficients at scale \( a \) reproduce the scaling properties of the processes. Thus, while filtering out the trends, the wavelet transform reveals the local characteristics of a signal, and more precisely its singularities.

It can be shown that the wavelet transform can reveal the local characteristics of \( s \) at a point \( z_0 \). More precisely, we have the following power-law relation (Hermann, 1997; Audit et al., 2002):

\[
|C_s(a,z_0)| \approx a^{h(z_0)} \quad \text{whence} \quad a \to 0^+
\]

where \( h \) is the Hölder exponent (or singularity strength). The Hölder exponent can be understood as a global indicator of the local differentiability of a function \( s \).

The scaling parameter (the so-called Hurst exponent) estimated when analysing process by using Fourier Transform (Ouadfeul and Aliouane, 2011) is a global measure of self-affine process, while the singularity strength \( h \) can be considered as a local version (i.e. it describes ‘local similarities’) of the Hurst exponent. In the case of monofractal signals, which are characterized by the same singularity strength everywhere (\( h(z) = \text{constant} \)), the Hurst exponent equals \( h \). Depending on the value of \( h \), the input signal could be long-range correlated (\( h > 0.5 \)), uncorrelated (\( h = 0.5 \)) or anticorrelated (\( h < 0.5 \)).
3. Neural network method

The Artificial Neural Network (ANN) based approaches have proved to be one of the robust and cost-effective alternative means to successfully resolve the lithofacies boundaries from well log data (Gottlob-Zeh et al., 1999;Aliouane et al., 2011). The method has its inherent learning ability to map some relation between input and output space, even if there is no explicit a priori operator linking the measured lithofacies properties to the well log response.

3.1. Self Organizing map of kohonen

A Self Organizing neural network, or SOM, is a collection of \( n \) reference vectors organised in a neighbourhood network, and they have the same dimension as the input vectors (Kohonen, 1998). Neighbourhood function is usually given in terms of a two-dimensional neighbourhood matrix \( \{W(i,j)\} \). In a two-dimensional map, each node has the same neighbourhood radius, which decreases linearly to zero during the self-organizing process. The conventional Euclidean distance is used to determine the best-matching unit (so called ‘winner’) \( \{W(iw, jw)\} \) on a map for the input vector \( \{X\} \). Kohonen’s SOMs are a type of unsupervised learning. The goal is to discover some underlying structure of the data. Kohonen’s SOM is called a topology-preserving map because there is a topological structure imposed on the nodes in the network. A topological map is simply a mapping that preserves neighbourhood relations. In the nets we have studied so far, we have ignored the geometrical arrangements of output nodes. Each node in a given layer has been identical in that each is connected with all of the nodes in the upper and/or lower layer. In the brain, neurons tend to cluster in groups. The connections within the group are much greater than the connections with the neurons outside of the group. Kohonen’s network tries to mimic this in a simple way. The algorithm for SOM can be summarized as follows (See Fig.1):

- Assume output nodes are connected in an array (usually 1 or 2 dimensional)
- Assume that the network is fully connected (i.e. all nodes in the input layer are connected to all nodes in the output layer). Use the competitive learning algorithm as follows:
- Randomly choose an input vector \( x \)
- Determine the "winning" output node \( i \), where \( W_i \) is the weight vector connecting the inputs to output node \( i \). Note the above equation is equivalent to \( W_i x \geq W_k x \) only if the weights are normalized.

\[
|W_i - X| \leq |W_k - X| \quad \forall k
\]

- Given the winning node \( i \), the weight update is

\[
W_k^{(new)} = W_k^{(old)} + X(i,k) \times (X - W_k)
\]

Where \( X(i,k) \) is called the neighborhood function that has value 1 when \( i=k \) and falls off with the distance \( |\mathbf{n} - \mathbf{n'}| \) between units \( i \) and \( k \) in the output array. Thus, units close to the
winner as well as the winner itself, have their weights updated appreciably. Weights associated with far away output nodes do not change significantly. It is here that the topological information is supplied. Nearby units receive similar updates and thus end up responding to nearby input patterns. The above rule drags the weight vector \( W_i \) and the weights of nearby units towards the input \( x \).

Example of the neighbourhood function is given by the following relation

\[
X(i,k) = e^{-\frac{||h_k - i||^2}{\sigma^2}}
\]

Where \( \sigma^2 \) is the width parameter that can gradually be decreased as a function of time.

Figure 1. Schematic illustration of the Kohonen’s Self-Organizing Map principle
4. The processing algorithm

In this section we train five self-organizing map neural network machines, the inputs of these maps are:

- **Data Set1**: The five raw well-logs data which are: The Gamma ray, Density, Neutron porosity, Photoelectric absorption coefficient and sonic well-log.
- **Data Set2**: The estimated Hölder exponents using the continuous wavelet transform of the data set1.
- **Data Set3**: Data set1 and the three radioactive elements concentrations.
- **Data Set4**: The estimated Hölder exponents of the data set1 and the Hölder exponents of the radioactive elements concentrations.
- **Data Set5**: The estimated Hölder exponents of the data set1 and the three radioactive elements concentrations logs.

The goal is to choose the best map that will give more details about lithology of two boreholes named Well01 and Well02 located in the Algerian Sahara.

5. Application on real data

5.1. Geological setting

The Hassi Messaoud field is located in the central part of Algerian Sahara (Figure 2). It is known by its oil-producing wells, mainly from the Cambrian reservoirs. The Hassi Messaoud super-huge field is a structure covering an area of most 1600 km2 and it was discovered in 1956 by well Md1 drilled across the reservoirs in Cambro-Ordovician sandstone at a depth 3337m. The Cambrian deposits which are presented by sandstones and quartzites, are the best known and form the major reservoirs (Cambrian Ri and Ra).

We distinguish in the Cambrian four stratigraphic subdivisions (Algeria Well Evaluation Conference, 2007), which are (Figure 3):

**R3**: Consisting of 300 m of poorly consolidated microconglomeratic clay sandstones intercalated with clayey siltstone levels that cannot be exploited because of its poor matrix properties and its deep position, below the water table.

**R2**: Exploitable when in high position, consists of relatively clayey coarse sandstones with intercalated levels of clayey siltstones; the top part of this reservoir, whose thickness is on the order of 40 m, has the best matrix properties.

**Ra**: the main reservoir, whose thickness varies from 100 m in the east to 130 m in the west, it consists of two major superimposed units which are:

- **The lower Ra**: with 70 to 95 m as thickness, consisting of medium to coarse sandstones with inter-bedded siltstone levels.
- **The upper Ra**: which consists of 40 to 60 m of relatively fine clayey sandstones containing skolithos, with many siltstone levels.
4) Ri: Which has 45 to 50 m as thickness and consists of 3 units, produces from 5 to 10m of fine basal sandstones with abundant skolithos; siltstones predominate in the upper units.

Figure 2. Geographic situation of Hassi Messaoud field (Algeria Well Evaluation Conference, 2007)

5.2. Data description

Well-log is a continuous record of measurement made in borehole respond to variation in some physical properties of rocks through which the bore hole is drilled (Asquith and Krygowski, 2004). In this paper eight well-logs have been processed by the proposed technique of two wells named Well01 and Well02. The exploited well-logging are:

a. The gamma ray (Gr)

Gamma Ray is a high-energy electromagnetic waves which are emitted by atomic nuclei as a form of radiation. Gamma ray log is measurement of natural radioactivity in formation versus depth. It measures the radiation emitting from naturally occurring Uranium (U), Thorium (Th) and Potassium (K).

b. The Natural Gamma ray spectroscopy measurements

It measures the total number of Gamma Rays SGR as well as their energy from which is computed the percentage of Potassium (K), Thorium (Th), Uranium (U) and the corrected Gamma Ray from Uranium (CGR)
Total and spectrometry of natural Gamma Ray are also known as shale log. They reflect shale or clay content and used for:
- Correlation between wells.
- Determination of bed boundaries.
- Evaluation of shale content within a formation.
- Mineral analysis.

c. Neutron porosity (Nphi)

The Neutron porosity log is primarily used to evaluate formation porosity, but the fact that it is really just a hydrogen detector should always be kept in mind.

The Neutron Log can be summarized as the continuous measurement of the induced radiation produced by the bombardment of that formation with a neutron source contained in the logging tool, which sources emit fast neutrons that are eventually slowed by collisions with hydrogen atoms until they are captured (think of a billiard ball metaphor where the similar size of the particles is a factor). The capture results in the emission of a secondary gamma ray; some tools, especially older ones, detect the capture gamma ray (neutron-gamma log). Other tools detect intermediate (epithermal) neutrons or slow (thermal) neutrons (both referred to as neutron-neutron logs). Modern neutron tools most commonly count thermal neutrons with an He-3 type detector.

The neutron porosity log is used for:
- Gas detection in certain situations, exploiting the lower hydrogen density, hydrogen index.
- Lithology and mineralogy identification in combination with density and sonic log

d. Density log (Rhob):

The formation density log (RHOB) is a porosity log that measures electron density of a formation. Dense formations absorb many gamma rays, while low-density formations absorb fewer. Thus, high-count rates at the detectors indicate low-density formations, whereas low count rates at the detectors indicate high-density formations. Therefore, scattered gamma rays reaching the detector are an indication of formation Density. The density log is used for:
- Lithology identification combined with neutron and sonic log
- Porosity evaluation
- Gaz beds detection

e. Sonic log (DT):

Acoustic tools measure the speed of sound waves in subsurface formations. While the acoustic log can be used to determine porosity in consolidated formations, it is also valuable in other applications, such as:
- Indicating lithology (using the ratio of compression velocity over shear velocity).
- Determining integrated travel time (an important tool for seismic/wellbore correlation).
- Correlation with other wells.
- Detecting fractures and evaluating secondary porosity.
- Evaluating cement bonds between casing, and formation.
- Determining mechanical properties (in combination with the density log).
- Determining acoustic impedance (in combination with the density log).

**Figure 3.** Cambrian stratigraphy of Hassi Messaoud field (Algeria Well Evaluation Conference, 2007)
The photoelectrical absorption coefficient (Pe):

The Photoelectric effect occurs when the incident gamma ray is completely absorbed by the electron. It is a low energy effect hence the photoelectric absorption index, Pe, is measured using the lowest energy window of the density tool.

Pe is related directly to the number of electrons per atom (Z) \( (\text{Asquith and Krygowski, 2004}) \)

\[
Pe = (\frac{Z}{A})^{3.6}
\]

Its unit is barns/electron. It is used also for lithology identification

5.3. Preliminary interpretation of natural gamma ray well-log

Natural gamma radiation occurs in rock formations in varying amounts. Uranium, Thorium, Potassium, and other radioactive minerals are associated with different depositional environments. Clay formations exhibit greater amounts of gamma radiation. A log of gamma radiation will give a positive indication of the type of lithology. Interpretation of gamma log data is done based on the relative low and high count rates associated with respective “clean” and “dirty” environments. Formations having high gamma count rates even though they may exhibit low water saturation are generally unfavorable for production in oil and water well environments.

In the description of the Cambrian stratigraphy, this interval is constituted only by sandstones and clays. Thus, our geological interval containing four lithofacies which are: The clay, sandy clays, clayey sandstones and clean sandstones.

This lithofacies classification is based on the gamma ray log value; three thresholds are used to distinguish between these lithologies. We distinguish four lithological units, differed by their gamma ray measurement value, which are:

\( 0<Gr<30\text{Api} \) is a clean sandstone.
\( 30\text{Api}<Gr<70\text{Api} \) is a clayey sandstone.
\( 70\text{Api}<Gr<90\text{Api} \) is a sandy clay.
\( Gr>90\text{Api} \) is a clay.

Figures 6a and 7a represent the obtained lithofacies classification based on this approach for the Well01 and Well02 boreholes respectively.

6. Fractal analysis of well-logs data

The first step consists to estimate the Hölder exponents of the eights raw well-logs data of the two boreholes OMJ 842 and WELL02 located in Hassi Messaoud field. The raw well-logs data are: the gamma ray (GR), the Uranium concentration (U), The Thorium concentration Th, the Potassium concentration (K), the slowness (DT), Photoelectric absorption coefficient (Pe), formations density (Rhob) and neutron porosity (Nphi). These data are presented in figures 4 and 5.
The Hölder exponents are estimated using the continuous wavelet transform for 929 samples at depths interval [3411.6m-3504.2m] (See figures 4 and 5).

The analyzing wavelet is the Complex Morlet (Morlet et al, 1982) defined by:

\[
\psi(Z) = \exp\left(-Z^2 / 2\right) \ast \exp(i \ast \Omega \ast Z) \ast (1 - \exp(-\Omega^2 / 4) \ast \exp(-Z^2 / 2))
\]

(6)

Where :

\( \Omega \): is the central frequency of the wavelet.

Source codes in C language are developed to calculate the continuous wavelet transform and to estimate the Hölder exponents at each depth.

Ouadfeul and Aliouane (2011) have showed that the optimal value of \( \Omega \) for a better estimation of the Hölder exponent is equal to 4.8.

Theoretically the Hölder exponent measures the singularity strength. Low exponent indicates a high singularity and a high exponent indicates a low singularity (Audi et al, 2002). Obtained results (figures 4 and 5) show that the main singularities in the raw well-logs data are manifested by spikes in the Hölder exponents graphs.

7. Holder exponents as an input of the Self-Organizing Map

Firstly we have applied the proposed idea at the Well01 borehole, the main depth interval is [3411.6m-3504.2m]. It contains only the four lithological units which are: The Clay, the Sandstone, the clayey sandstone and the sandy clay. The output of the neural machine should be one of these previous lithologies.

For the same reason it is sufficient on this case to use the information provided by the classical interpretation based in the gamma ray log (See figure 6a) for the SOM indexation (Sitao et al, 2003, Gottlib-Zeh et al, 1999).

The Numap7.1 software developed by the Neural Networks and Image Processing Lab of Univ. of Texas at Arlington is used for the training and running of the different self-organizing maps neural networks.

For each Kohonen’s map, the Input is used to train the SOM neural network; in this step weights of connection between neurons are calculated. After that outputs of each map are calculated. Figures 6b, 6c, 6d, 6e and 6f present the output of each Map.

The weights of connection calculated for the Well01 borehole are used to predict lithofacies for Well02, the different type of inputs used for the first well are used for the second one.

In this step we don’t need to the Self- Organizing Maps indexations, since the same maps are used. It means that the weights of connections calculated in the training of the first map using the Well01 borehole data and their Hölder exponents are used to calculate the outputs for second well (Well02).

Obtained lithofacies for the Well02 borehole and its corresponding classical interpretation based on the gamma ray log are presented in figure 7.
Figure 4. Measured well-logs data for Well01 borehole: GR, Vp, RHOB, PEF, NPHI and their corresponding Hölder exponents.

Figure 5. Measured well-logs data for Well02 borehole: GR, Vp, RHOB, PEF, NPHI and their corresponding Hölder exponents.
8. Results discussion and conclusion

By analyzing figures 6 and 7, one can remark that the Self Organizing neural network machines based on the raw well logs data as an input give more details than the classification based on the classical gamma ray interpretation.

Classifications based on the Hölder exponents of the five well-logs data as an input give less details, it means that they can’t provide details and thin geological details. However
lithofacies prediction based on the five raw well logs data combined with the spectrometric concentration gives more information about shaly character. This is due to the sensitivity of the concentration of radioactive elements to shale.

Finally the Self Organizing map based on the eight raw logs data can give a lot of details and thin facies intercalations. Reservoir model based on the self organizing map neural network machine with the raw data as an input is able to give a detailed information. The self-organizing map neural network model with the Hölder exponents estimated by the continuous wavelet transform as an input is not able to improve the lithofacies classification by the SOM. We suggest by this paper to use always the raw well-logs in a Self-Organizing Map artificial neural network model rather than the fractal analysis using by the CWT, this last processing decrease the details and hide geophysical information that contains the raw data.

**Figure 7.** Different lithofacies classification of Well02 borehole.
(a): Lithofacies classification based on the Gr.
(b): Lithofacies by SOM with data Set1 as an input.
(c): Lithofacies by SOM with data set2 as an input.
(d): Lithofacies by SOM with data set3 as an input.
(e): Lithofacies by SOM with data Set4 as an input.
(f): Lithofacies by SOM with Data Set 5 an input.
9. References


