1. Introduction

Magnetorheological fluid (MRF) is a non-colloidal suspension of magnetizable particles that are on the order of tens of microns (20-50 microns) in diameter. Generally, MRF is composed of oil, usually mineral or silicone based, and varying percentages of ferrous particles that have been coated with an anti-coagulant material. When inactivated, MRF displays Newtonian-like behavior. When exposed to a magnetic field, the ferrous particles that are dispersed throughout the fluid form magnetic dipoles. These magnetic dipoles align themselves along lines of magnetic flux. The fluid was developed by Jacob Rabinow at the US National Bureau of Standards in the late 1940’s. For the first few years, there was a flurry of interest in MRF but this interest quickly waned. In the early 1990’s there was resurgence in MRF research that was primarily due to Lord Corporation’s research and development. Although similar in operation to electro-rheological fluids (ERF) and Ferro-fluids, MR devices are capable of much higher yield strengths when activated. For this advantage, many MRF-based mechanisms have been developed such as MR damper, MR brake, MR clutch, MR valve... and some of them are now commercial. As well-known that performance of MRF based systems significantly depends on the activating magnetic circuit, therefore, by optimal design of the activating magnetic circuit, the performance of MRF-based systems can be optimized. Recently, there have been various researches on optimal design of MRF-based devices such as MR damper, MR valve, MR brake. The results from these studies showed that performance of MRF-based systems can be significantly improved via optimal design of the magnetic circuit of the systems.

Consequently, this chapter focuses on the methodology of optimal design of MRF-based devices. The chapter is organized as follows; in section 2, fundamentals and the theory behind MRF are overviewed. Section 3 deals with the modes used in MRF based devices and several different devices featuring MRF are discussed. In the fourth section, optimal
design methodology of MRF-based mechanisms is considered. In this section, firstly the necessity of optimal design and the state of the art are discussed. Then the magnetic circuit analysis and the modeling of MRF devices are considered. In addition, the optimization problem of MRF devices is figured out and the methods to solve the problem are investigated. Section 5 is devoted to deal with a case study of MR valve optimal design. In this case study, several valve configurations such as single-coil, multiple-coil and annular-radial MR valves are considered. The chapter is then closed by the conclusion.

2. Fundamentals of MRF and applications

2.1. Composition of MRF

Generally, MRF consists of non-colloidal suspensions, magnetically soft ferromagnetic, ferrimagnetic or paramagnetic elements and compounds in a non-magnetic medium. In practice, MRF usually consists of suitable magnetizable particles such as iron, iron alloys, iron oxides, iron nitride, iron carbide, carbonyl iron, nickel and cobalt [1, 2]. Among these, a preferred magnetic responsive particle that is commonly used to prepare MRF is carbonyl iron. The possible maximum yield stress induced by MR effect is mainly determined by the lowest coercivity and the highest magnitude of saturation magnetization of the dispersed particles. Therefore, soft magnetic material with high purity such as carbonyl iron powder appears to be the main magnetic phase for most of the practical MRF composition [3]. Other than carbonyl iron, Fe-Co alloys and Fe-Ni alloys can also be used as MR materials, whereby, Fe contributes to the high saturation magnetization. In contrary, some of the ferrimagnetic materials such as Mn-Zn ferrite, Ni-Zn ferrite and ceramic ferrites have low saturation magnetizations and are therefore suitable to be applied in low yield stress applications [1]. MR particles are typically in the range of 0.1 to 10μm [4, 5], which are about 1000 times bigger than those particles in the ferrofluids [6]. In the MRF, magnetic particles within a certain size distribution can give a maximum volume fraction without causing unacceptable increasing in zero-field viscosity. For instance, fluid composition that consists of 50% volume of carbonyl iron powder was used in the application of electromechanically controllable torque-applying device.

The carrier liquid forms the continuous phase of the MRF. Examples of appropriate fluids include silicone oils, mineral oils, paraffin oils, silicone copolymers, white oils, hydraulic oils, transformer oils, halogenated organic liquids, diesters, polyoxyalkylenes, fluorinated silicones, glycols, water and synthetic hydrocarbon oils [7, 2]. A combination of these fluids may also be used as the carrier component of the MRF. In the earlier patents, inventors were using magnetizable particles dispersed in a light weight hydrocarbon oil [8], either a liquid, coolant, antioxidant gas or a semi-solid grease [9] and either a silicone oil or a chlorinated or fluorinated suspension fluid [10]. However, when the particles settled down, the field-induced particle chains formed incompletely at best in which MR response was critically degraded. Later, in order to prevent further sedimentation, new compositions of MRF with consideration on viscoplastic [11] and viscoelastic continuous phases [12] were formulated, so that the stability could be improved immensely. In addition, a composite MRF has been
prepared by Pan et al. [13] with a combination of iron particles powder, gelatine and carrier fluids. They showed that the MR effects were superior under low magnetic field strength, and had a better stability compared to pure iron carbonyl powder alone.

Surfactants, nanoparticles, nanomagnetizable or coating magnetizable particles can be added to reduce the sedimentation of the heavy particles in the liquid phase [14, 13]. The sedimentation phenomenon can cause a shear-thinning behaviour of the suspension [15]. With further sedimentation, with MRF under the influence of high stress and high shear rate over a long period of time, the fluid will thicken (in-use-thickening) [16, 17]. Sedimentation phenomenon will reduce the MR effect where the particles in the MRF are settled down and form a hard “cake” that consists of firmly bound primary particles due to incomplete chain formation [18]. Anti-settling agent such as organoclay can provide soft sedimentation. When the composition of MRF has relatively low viscosity, it does not settle hard and can easily redisperse [2]. Coating of the polymer layer also influences magnetic properties of the particles and cause them to easily re-disperse after the magnetic field is removed [19]. However, specific properties of MRF such as shear and yield stresses under the same conditions were enormously degraded inevitably by addition of the coating layer. This is due to the shielding of the polymer layer that affects the magnetic properties of the particles [19, 20]. In addition, some additives can improve the secondary properties like oxidation stability or abrasion resistance.

2.2. Magnetic properties of MRF

The static magnetic properties of MRF are important to design any MRF-based devices and generally can be characterized by B-H and M-H hysteresis. Through the magnetic properties, the dependence of the MRF response on the applied current in the device can be predicted. Under the influence of the magnetic field, a standard model for the structure is used to predict the behaviour of the particle of MRF [21]. The model is based on a cubic network of infinite chains of the particles arranged in a line with respect to the direction of the magnetic field as shown in Figure 1.

![Figure 1. Schematic presentation of the affine deformation of a chain of spherical particles](image)

The chains are considered to deform with the same distance between any pair of neighbours in the chains and increase at the same rate with the strain when the MRF is strained. This model is quite simple since the chains, in actual case, are formed into some more compact aggregates of spheres in which can be constituted in the form of cylinders. Under shear stress, these aggregates might deform and eventually break. Even though the particles...
develop into different complicated structures under different conditions [22], the standard model still can be used in order to give a valid prediction of the yield stress [21]. The equation of motion of each particle under a magnetic field is required in order to evaluate the bulk property of MRF. At a very low magnetic field, the magnetic force tensor $F_{ij}$ is obtained as point-dipole similar to the pair interaction, the magnetic dipole moment induced by other particles and surrounding walls for an unmagnetized and isolated sphere under a uniform magnetic field is given by [23]:

$$F_{ij} = \frac{3}{4\pi\mu_0\mu_p} \left[ m^2 \frac{r_{ij}^3}{r_{ij}^5} - 5(mr_{ij})^2 \frac{r_{ij}}{r_{ij}^7} + 2(mr_{ij})m \frac{1}{r_{ij}^3} \right]$$

where $F_{ij}$ is the magnetic force tensor acts on particle $i$ from $j$, $\mu_p$ is the specific permeability of particles, $\mu_0$ is the vacuum permeability, $r_{ij}$ is position from particle $j$ to $i$ and $m$ is magnetic dipole moment induced in particles within MRF given by [24],

$$m = 4\pi\mu_0\beta a^3 H$$

where $H$ is the uniform magnetic field, $a$ is the diameter of the particles and $\beta$ is given by,

$$\beta = \frac{\mu_f - \mu_p}{\mu_f - 2\mu_p}$$

where $\mu_f$ is the specific permeability of carrier liquid.

At high magnetic fields, the magnitude of the moment can be considered as independent point dipoles as magnetization of particles reaches saturation. In this case, the magnetic moment is given by [25].

$$m = \frac{4}{3} \pi a^3 \mu_s M_s$$

where $\mu_s M_s$ is the saturation magnetization of the particle, which is about $1.7 \times 10^6 A/m$ for bulk iron and $0.48 \times 10^6 A/m$ for the magnetite.

2.3. Fundamentals of rheological properties

Rheology is the response of materials to an applied stress [26]. Rheology is an interdisciplinary field and is used to describe the properties of a wide variety of materials such as oil, food, ink, polymers, clay, concrete, asphalt and others. Rheology measurements and parameters can be used to determine the processing behaviour of non-Newtonian materials, viscoelastic behaviour as a function of time, the degree of stability of a formulation at rest condition or during transport, and zero shear viscosity or the maximum viscosity of the fluid phase to prevent sedimentation [27]. The viscosity equation on the basis of a hydrodynamic theory for dilute dispersions of spherical particles has been developed by Einstein about 100 years ago [28]. The equation has been derived as
where $\eta_r$ is the relative viscosity of the suspension and $\phi$ is the volume fraction of the suspended solutes or particles assumed to be spherical. The addition of the solid particles to a liquid will increase the amount of particles and consequently increases the volume fraction of the particles. Therefore, as the volume fraction of particles increases, there will be an increase in the fluid’s viscosity. Shook [29] has suggested that the maximum concentration of the particles $\phi_{\text{max}}$ should be incorporated in the relationship between viscosity and concentration as

$$\eta_r = \frac{\phi}{(1 - \phi)^{2.5} \phi_{\text{max}}}$$

(6)

However, these equations do not depend on the particle size but instead depend on the particle shape and solid concentration. Thus, Toda and Furuse [30] extended the equation in order to satisfy the viscosity behaviour of concentrated dispersion for small and large particles, respectively given by,

$$\eta_r = \frac{1 - 0.5 \phi}{(1 - \phi)^3}$$

(7)

$$\eta_r = \frac{1 + 0.5 \kappa \phi - \phi}{(1 - \kappa \phi)^3(1 - \phi)}$$

(8)

where $\kappa$ is the correction factor that may depend on the size and concentration of the particles. The viscosity of the fluid can be increased with additional amounts of the solid particles. However, at the same time, the fluid behaviour will change and diverge from a Newtonian fluid. Generally, shear stress $\tau$ increases with the shear rate $\frac{du}{dy}$ which often can be represented by the relationship

$$\tau = \tau_y + \eta \left(\frac{du}{dy}\right)^n$$

(9)

where $\tau_y$, $\eta$ and $n$ are constants, $\tau_y$ is the yield stress and $\eta$ is the dynamic viscosity. Newtonian fluids occur when the fluids show no yield stress or $\tau_y$ is equal to zero and $n$ is equal to one. The viscosity of a Newtonian fluid is independent of time and shear rate. Figure 2 shows the classification of fluids based on rheological properties. As shown in the figure, the behaviour of the fluids can be classified into Newtonian fluids and non-Newtonian fluids such as plastic, Bingham plastic, pseudo-plastic and dilatant fluids [31]. Fluids are said to be plastic when the shear stress must reach a certain minimum value before it begins to flow. If $n$, in Eq. (9), is equal to one, the material is known as a Bingham plastic. For the pseudo-plastic or shear-thinning fluid, the dynamic viscosity decreases as the shear rate increases. On the other hand, a shear-thickening or dilatant fluid exhibits the converse property of pseudo-plastic for which the dynamic viscosity increases as the shear rate increases. The shear thickening fluid is represented by $n > 1$ and shear thinning fluid by $n < 1$. 

$$\eta_r = 1 + 2.5\phi$$

(5)
2.4. Rheology of MRF

MRF responds to the external field, where the particles are held together to form chains parallel to the applied field. The interaction between the particles impedes to a certain level of the shear stress without breaking and simultaneously increases the viscosity of the fluids [32]. In many cases, the effect of MRF is described by Bingham Plastic model [33]. A modified or extended Bingham model, or a combination of Bingham model with other models such as viscous and coulomb friction have also been used to describe the behaviour of MRF [34]. In the absence of an external field, MRF behave like a normal fluid which is known as Newtonian fluid. There are many factors that influence the rheological properties of controllable MRF such as concentration and density of particles, particle size and shape distribution, properties of the carrier fluid [35], additional additives, applied field and temperature. The relationships of all these factors are very complex and are important in establishing methodology to improve efficiency of these fluids for suitable applications. Excellent MRF must have low viscosity and coercivity of particles without the influence of an external magnetic field and can achieve maximum yield stress in the presence of the external magnetic field. Gross [8] in his invention related to the valve for magnetic fluids, found that the advantage of large particle sizes or heavy suspensions can increase the size of the gap which also increases the flow of the fluid. Conversely, the large particles of the magnetically active phase of MRF lead to a strong tendency for particles to settle out of the liquid phase [19].

Some of the techniques are typically necessary in order to increase the yield stress; either by increasing the volume fraction of MR particles or by increasing the strength of the applied magnetic field. However, neither of these techniques is desirable since a higher volume fraction of the MR particles can add significant weight to the MR devices as well as increases the overall off-state viscosity of the material. In that connection, restricting the size and
geometry of the MR device capable of utilizing that material, and a higher magnetic field significantly increases the power requirement of the device. To overcome this difficulty, Carlson [36] in his patent introduced alloy-particles material that was used as a solid particle instead of the common carbonyl iron. This MRF independently increases the yield stress without requiring increment of either the volume fraction of particles or magnetic field strength.

2.5. MRF models

MRF models play an important role in the development of MRF based devices. Moreover, accurate models that can predict the performance of these MRF devices are an important part of implementation of such devices. MRF demonstrates nonlinear behavior when subjected to external magnetic fields. The rheological behavior of these materials can be separated into distinct preyield and post-yield regimes. A wide variety of nonlinear models have been used to characterize MRF, including the Bingham plastic model [37, 38], the biviscous model [39], the Herschel–Bulkley model [40, 41], and Eyring plastic model [42]. Although there have been several models have been developed and applied for MRF the two most popular models have been widely used with reasonable accuracy and computational cost are the Bingham plastic model and the Herschel–Bulkley plastic model. Therefore, in this chapter these two constitutive models are used.

i. Bingham plastic model

The so-called Bingham plastic model includes a variable rigid perfectly plastic element connected in parallel to a Newtonian viscosity element. This model assumes that the fluid exhibits shear stress proportional to shear rate in the post-yield region and can be expressed as [37, 38]

$$\tau = \tau_y(H) \text{sgn}(\dot{\gamma}) + \eta \dot{\gamma}$$

(10)

where \(\tau\) is the shear stress in the fluid, \(\tau_y\) is the yielding shear stress controlled by the applied field \(H\), \(\eta\) is the post-yield viscosity independent of the applied magnetic field, \(\dot{\gamma}\) is the shear strain rate and \(\text{sgn}(\cdot)\) is the signum function. That is, the fluid is in a state of rest and behaves viscoelastically until the shear stress is greater than the critical value \(\tau_y\), whereas it moves like a Newtonian fluid when such a critical value is exceeded. The Bingham plastic model is shown in Figure 3 to represent the field-dependent behaviour of the yield stress. The simplicity of this two-parameter model has led to its wide use for representation of field-controllable fluids, especially ER and MRF.

ii. Herschel-Bulkley plastic model

In cases where the fluid experiences post-yield shear thickening or shear thinning, especially when the MRF experiences high shear rate, this choice of constitutive equation can result in an overestimation. In this case, the Herschel-Bulkley plastic model is more suitable [43]. The Herschel-Bulkley model can be expressed by
\[ \tau = (\tau_y(H) \text{sgn}(\dot{\gamma}) + \ k |\dot{\gamma}|^{1/m} \text{sgn}(\dot{\gamma})) \]  

(11)

where \( K \) is the consistency parameter and \( m \) is fluid behavior index of the MRF. For \( m > 1 \), equation (4) represents a shear thinning fluid, while shear thickening fluids are described by \( m < 1 \). Note that for \( m = 1 \) the Herschel–Bulkley model reduces to the Bingham model.

It is noteworthy that, in the above, the post-yield parameters of the MRF such as the post-yield viscosity, the consistency parameter and the fluid behavior index are assumed to be independent on the applied magnetic field. In practice, these parameters are slightly affected by the applied magnetic field. Zubieta et al. [44] have proposed field-dependent plastic models for MRF based on the original Bingham plastic and Herschel-Bulkley plastic models. The models were then applied in several researches [45, 46] with experimental agreement. In the field dependent Bingham and Herschel-Bulkley model, the rheological properties of MRF depend applied magnetic field and can be estimated by the following equation

\[ Y = Y_\infty + (Y_0 - Y_\infty)(2e^{-B\alpha Y} - e^{-2B\alpha Y}) \]  

(12)

where \( Y \) stands for a rheological parameters of MRF such as yield stress, post yield viscosity, fluid consistency and flow behavior index. The value of parameter \( Y \) tends from the zero applied field value \( Y_0 \) to the saturation value \( Y_\infty \). \( \alpha \) is the saturation moment index of the \( Y \) parameter. \( B \) is the applied magnetic density. The values of \( Y_0, \ Y_\infty, \alpha \) are determined from experimental results using curve fitting method.

3. MRF mode of operation and its application

3.1. Valve mode

Figure 3 schematically show the valve mode which have been used in many MR devices where the flow of the MRF between motionless plates or a duct is created by a pressure drop. The magnetic field, which is applied perpendicular to the direction of the flow, is used
to change the rheological properties of the MRF in order to control the flow. Therefore, the increase in yield stress or viscosity alters the velocity profile of the fluid in the gap between two plates. A typical velocity profile for Bingham-plastic of the valve mode is illustrated in Figure 4b. The velocity profile contains a pre-yield region, where the velocity gradient is zero across the plug region. The velocity profile of MRF between two parallel plates can be represented by the following relation [47]

\[
\frac{u_1(y)}{n+1} = \frac{\Delta P}{KL} \left[ \left( \frac{d-\delta}{2} \right)^{1+1/n} + \left( \frac{2y+\delta}{2} \right)^{1+1/n} \right], \quad \frac{u_2(y)}{n+1} = \frac{\Delta P}{KL} \left( \frac{d-\delta}{2} \right)^{1+1/n}
\]

Here, \(n=1/m\), \(u_1\) and \(u_3\) are velocity profiles of the post-yield flow regions adjacent to the walls of the rectangular duct, and \(u_2\) is the velocity profile across the central pre-yield or plug region. \(\delta\) is the plug region thickness, which is a key parameter of the flow. As the field increases, so does the pre-yield thickness, thereby, constricting the flow through the duct, increasing the pressure drop. A high resistance produced by the valve mode can be used in many applications such as dampers, valves and actuators [48-51].

\[ \text{Figure 4. Valve mode in the MR application} \]

\[ \text{3.2. Shear mode} \]

The second working mode for controllable fluid devices is the direct shear mode. An MRF is situated between two surfaces, whereby one surface slides or rotates in relation to the other, with a magnetic field applied perpendicularly to the direction of motion of these shear surfaces. Figure 5 shows the concept of shear mode in MRF application.

\[ \text{Figure 5. The concept of shear mode} \]
The direct shear mode has been studied thoroughly especially in the MR damper technology. Masri et al. [52] proposed a curve fitting technique for representing the nonlinear restoring force of an ER device in order to characterize the ER material behaviour under static and dynamic loading over a wide range of electric fields. Spencer et al. [53] developed a phenomenological model which is based on the improved Bouc-Wen hysteresis model to represent MR dampers. Moreover, Wereley et al. have proposed a non-dimensional approach to model different types of shear damper (linear shear mode, rotary drum and rotary disc damper) [54]. In the research, the Bingham–plastic, biviscous, and Herschel–Bulkley models are considered. In terms of the behaviour of the damper under conditions of high-velocity and high field input, Lee et al. [55] recommended the Herschel-Bulkley shear model to analyze the performance of impact damper systems. Furthermore, Neelakantan et al. [56] incorporated a volume fraction profile of particles with an analytical technique for calculating the torque transmitted in clutches experiencing particle centrifuging. The effect of centrifuging at high rotational speeds and the subsequent sealing problems associated with it can be mitigated by the proposed model. Extraordinary features of the direct shear mode like simplicity, fast response, simple interface between electrical power input and mechanical power output using magnetic fields, and controllability are features that make MRF technology suitable for many applications such as dampers, brakes, clutches and polishing devices [56-59].

3.3. Squeeze mode

The third working mode of MRF is the squeeze mode shown in Figure 6. This mode has not been widely investigated. Squeeze mode operates when a force is applied to the plates in the same direction of a magnetic field to reduce or expand the distance between the parallel plates causing a squeeze flow. In squeeze mode, the MRF is subjected to dynamic (alternate between tension and compression) or static (individual tension or compression) loadings. As the magnetic field charges the particles, the particle chains formed between the walls become rigid with rapid changes in viscosity. The displacements engaged in squeeze mode are relatively very small (few millimeters) but require large forces. The squeeze mode was disclosed by Stanway et al. [60] in 1992. They studied the usage of ER fluids in squeeze mode and found that the yield stress produced under DC excitation could be several times greater than the shear mode. The same outcome was later confirmed by Monkman [61] for fluids under compressive stress. Consequently, systematic investigations have been carried out by many researchers to evaluate the mechanical and electrical properties of ER and MRF in squeeze flow. Despite the fact that the Bingham plastic model was used to describe the behaviour of ER fluid in shear mode, Nilsson and Ohlson [62] have not recommended to utilize that model in squeeze mode. Bingham parameters tested from shear mode are not well-founded for the calculation of the squeeze mode behaviour. Sproston et al. [63] characterized the performance of ER fluids in dynamic squeeze mode using a bi-viscous model under a constant potential difference or by a constant field. Later on, Sproston and El Wahed [64] utilized the model to assess the fluid’s response to a step-change in the applied field and the influence of the size of solid phase. Even though the model was useful to...
predict the peak values of the input and transmitted forces [65], a more refined model is needed, according to authors, to predict the detailed temporal variations. Therefore, a new approach of modified bi-viscous model was developed by El Wahed et al. in order to model the behaviour of an ER squeeze mode cell under dynamic conditions [66, 67]. Furthermore, Yang and Zhu [68] extended this latter model by incorporating Navier slip condition to obtain the radial velocity, pressure gradient, pressure and squeeze force.

![Figure 6. The concept of squeeze mode](image)

The stress produced by the squeeze mode is the highest stress among other modes and can be used in damping vibrations with low amplitudes and high dynamic forces [69, 70]. In vibration isolation of structural system, the unwanted vibration in a relatively high frequency range can be attenuated by activating the MR mount. Examples for vibration control are isolation engine mount [71], turbo-machinery [72] and squeeze film damper [73]. Another interesting application on the squeeze mode is related to haptic devices where the user can feel the resistance forces by touching and moving a tool [74].

### 3.4. Combination of modes

Some of the applications of field responsive fluids take advantage of the combination of two modes for a greater strength and functionality. For instance, dampers can be constructed in three different modes [75]. In a general manner, shear mode exhibits Couette flow through the annular bypass, while a valve flow is characterized by Poiseuille flow through the annular bypass. The combination of them often gives higher yield stress as compared to stress produced by individual operation modes. Kamath et al. [76] have shown in their analysis and testing of Bingham plastic behaviour that mixed (valve and shear) mode dashpot dampers have higher passive damping than flow mode dampers. The mixed mode damper has a secondary effect of viscous drag as a result of the motion of piston head, instead of relying on the pressure gradient developed by the piston head to push the fluid through the gap created by the fixed electrodes. Wereley and Pang [75] have developed nonlinear quasi-steady ER and MR damper models using idealized Bingham plastic shear flow mechanism to characterize the equivalent viscous damping constant of the dampers. Plug thickness is the strongest variable that contributed to the damper behaviour for both flow and mixed modes.
In another experimental study done by Kulkarni et al. [62], the performance ofhe combination of squeeze and shear modes of MRF in dynamic loading was investigated. Even though squeeze mode can produce the highest strength among all modes, the addition of squeeze mode to shear mode did not always give a better strength than the shear mode alone. However, Tang et al. [77] demonstrated that the yield shear stress can be significantly improved by compressing the MRF along the magnetic field direction before the shear process is performed.

4. Optimal design methodology of MRF-based mechanisms

4.1. Modeling of MRF based mechanisms

It is well-known that modeling of the MRF based systems is a coupled analysis problem: electromagnetic analysis and fluid system analysis. The purpose of the modeling of an MRF based device is to find the relation between the applied electric power (usually the current applied to the coils) and the output mechanical power such as pressure drops for MR valves, damping force for MR damper, braking torque for MR brakes and transmitted torque for MR clutches. In order to deal with modeling of MRF based devices, firstly the magnetic circuit of the MRF based devices should be solved. In general, the magnetic circuit can be analyzed using the magnetic Kirchoff’s law as follows:

\[ \sum H_k l_k = N_{\text{turns}} I \]  

(14)

where \( H_k \) is the magnetic field intensity in the \( k \)th link of the circuit and \( l_k \) is the effective length of that link. \( N_{\text{turns}} \) is the number of turns of the valve coil and \( I \) is the applied current. The magnetic flux conservation rule of the circuit is given by

\[ \Phi = B_k A_k \]  

(15)

where \( \Phi \) is the magnetic flux of the circuit, \( A_k \) and \( B_k \) are the cross-sectional area and magnetic flux density of the \( k \)th link, respectively. It is noteworthy that the more links are used the more exact solution can be obtained. However, this increases computation load. At low magnetic field, the magnetic flux density, \( B_k \), increases in proportion to the magnetic intensity \( H_k \) as follows:

\[ B_k = \mu_0 \mu_k H_k \]  

(16)

where \( \mu_0 \) is the magnetic permeability of free space (\( \mu_0 = 4\pi \times 10^{-7} \text{Tm/A} \)) and \( \mu_k \) is the relative permeability of the \( k \)th link material. As the magnetic field becomes large, its ability to polarize the magnetic material diminishes and the material is almost magnetically saturated. Generally, a nonlinear \( B-H \) curve is used to express the magnetic property of material. At low magnetic field, taking the linear relation (16) into consideration, the magnetic flux density and the field intensity of the \( k \)th link of magnetic circuit can be approximately calculated as follows:
Optimal Design Methodology of Magnetorheological Fluid Based Mechanisms

By assuming magnetic property of the structural materials of the MR devices is similar \((\mu_1=\mu_2=\ldots=\mu_n=\mu)\), the magnetic flux density and the field intensity across the active MRF volume can be approximately calculated as follows:

\[
B_k = \frac{\mu_0 N_{\text{turns}} I}{I_k + \sum_{i=1, i \neq k}^{n} \frac{\mu_i A_i}{\mu_k A_k}}
\]

(17)

\[
H_k = \frac{N_{\text{turns}} I}{I_k + \sum_{i=1, i \neq k}^{n} \frac{\mu_i A_i l_i}{\mu_k A_k}}
\]

(18)

where \(\mu_{\text{mr}}\) and \(\mu\) are the relative permeability of MRF and the structural materials of the MR devices, respectively. It is noted that the permeability of the MRF is much smaller than that of the valve core material, therefore from Eq. (20) the magnetic field intensity of the MRF link can be approximated by

\[
H_{\text{mr}} = \frac{N_{\text{turns}} I}{I_{\text{mr}} + \frac{\mu_{\text{mr}} A_{\text{mr}}}{\mu} \sum_{i}^{l_i A_i}}
\]

(19)

(20)

The inductive time constant \((T_{\text{in}})\) and the power consumption \((N)\) of the MRF based devices can be calculated as follows:

\[
T_{\text{in}} = \frac{L_{\text{in}}}{R_{\text{w}}}
\]

(22)

\[
N = I^2 R_{\text{w}}
\]

(23)

where \(L_{\text{in}}\) is the inductance of the coil given by \(L_{\text{in}}=N_{\text{turns}} \Phi / l\), \(R_{\text{w}}\) is the resistance of the coil wire which can be approximately calculated by

\[
R_{\text{w}} = l_{\text{w}} r_{\text{w}} = N_{\text{turns}} \pi \bar{d}_c \frac{r}{A_{\text{w}}}
\]

(24)

In the above, \(L_{\text{w}}\) is the length of the coil wire, \(r_{\text{w}}\) is the resistance per unit length of the coil wire, \(\bar{d}_c\) is the average diameter of the coil, \(A_w\) is the cross sectional area of the coil wire, \(r\) is the resistivity of the coil wire, \(r = 0.01726 \times 10^{-6} \Omega \cdot \text{m}\) for copper wire, \(N_{\text{turns}}\) is the number of coil turns which can be approximated by \(N_{\text{turns}}=A_d / A_{\text{w}}\), and \(A_{\text{v}}\) is the cross-sectional area of the coil.
In some applications, an electromagnet is used in combination with a permanent magnet to control the rheological properties of the MRF as shown in Figure 7. In this case, the permanent magnet is used to shift the off-state (no current in the coil) viscosity of the MRF to a selected value and the electromagnet is used to control the viscosity variations around this value. A frequent situation is that where the magnetic circuit is designed in such a way that the MRF viscosity is maximum when no current flows through the coil. This is particularly useful when the device based on such magnetic circuit has to be blocked the major part of its operation time (such as in release mechanisms, for instance). The magnetic intensity across the active volume of the MRF is determined by [78]

\[ H_{mr} = \frac{\left( \mu_r / I_m \right) N_{turn} I - B_r / \mu_0}{\mu_r l_m / I_m + \mu_{mr} A_{mr} / A_m} \]  

(25)

where \( l_m \) and \( A_m \) are the length and cross-sectional area of the permanent magnet, \( B_r \) and \( \mu_r \) are remanent flux density and relative permeability of the magnet (\( \mu_r \approx 1 \)). In order to cancel the flux inside the MRF, \( NI \) has to be equal to the magnetomotive force, \( F_m = B_r l_m / \mu_0 \), which is highly influenced by the magnet length. It should however be noticed that such a magnetic circuit will not be used in practice since it might lead to demagnetization of the permanent magnet. A solution to this problem is to include a secondary path inside the magnetic circuit as shown in Figure 8 [78]

Figure 7. Magnetic coil with MRF filled gap and permanent magnet: a) magnetic circuit b) electric equivalence

\[ H_{mr} = \frac{\left( \mu_r / l_m \right) N_{turn} I - B_r / \mu_0}{\mu_r l_m / I_m + \mu_{mr} A_{mr} / A_m} \]  

(25)

where \( l_m \) and \( A_m \) are the length and cross-sectional area of the permanent magnet, \( B_r \) and \( \mu_r \) are remanent flux density and relative permeability of the magnet (\( \mu_r \approx 1 \)). In order to cancel the flux inside the MRF, \( NI \) has to be equal to the magnetomotive force, \( F_m = B_r l_m / \mu_0 \), which is highly influenced by the magnet length. It should however be noticed that such a magnetic circuit will not be used in practice since it might lead to demagnetization of the permanent magnet. A solution to this problem is to include a secondary path inside the magnetic circuit as shown in Figure 8 [78]

Figure 8. Magnetic coil with MRF filled gap and permanent magnet with secondary path: a) magnetic circuit b) electric equivalence
The electromagnet will thus not be used to completely cancel the flux produced by the permanent magnet but will only redirect it to the secondary path. This secondary path comprises a higher reluctance air gap in order to concentrate the major part of the flux generated by the permanent magnet in the primary path (comprising the MRF gap) when no current is flowing through the coil. In this case, the magnetic intensity across the active volume of the MRF is determined by [78]

$$H_{mr} = \frac{(\mu_r/l_m + A_r\mu_a / A_{mr})N_{\text{turn}}l - B_r / \mu_0}{\mu_r/l_m + A_r\mu_a / A_{mr} + \mu_{mr}A_{mr} / A_m}$$

(26)

It is interesting to note that, if \(g_r \rightarrow \infty\), we come back to Eq. (25). In order to cancel the magnetic flux inside the MRF gap, we need:

$$N_{\text{turn}}l = \frac{B_rl_m}{\mu_r\mu_0\left(\frac{A_{mr}}{A_m}\left(\frac{l_m}{l_a}\frac{\mu_{mr}}{\mu_a} + 1\right)\right)}$$

(27)

This value may seem smaller than what was obtained in the previous case; however, to obtain the same magnetic field inside the MRF gap, the magnet has to be more powerful since it has to compensate for the loss of magnetic flux in the secondary circuit.

In the above, magnetic circuit of the MRF based devices is solved based on the approximation of the analytical analysis. This approach can only used in case of simple geometry. In case of complex geometry or several coils are used, the approach becomes very complicated. Therefore, practically, the magnetic circuit of the MRF based devices is solved by finite element method (FEM). Once the magnetic solution is obtained, the magnetic intensity and magnetic flux density across the active MRF volume can be calculated. The rheological properties of MRF in the active volume are then determined based on the behavior characteristics of the employed MRF. The behavior characteristics of MRF are usually obtained from experimental results with a curve-fitting algorithm. The most important parameter of MRF is the field-dependent yield stress. There have been several approximate functions have been used to express the dependence of the induced yield stress of MRF on the applied magnetic field. The two most widely used functions are the exponential function and the polynomial function. The former can well expressed the saturation of MRF yield stress as a function of the applied magnetic intensity. However, it exhibits large error at the small value of the applied magnetic intensity. In general, the approximate exponential function of induced yield stress is expressed as following

$$\tau_y(H) = \tau_0 + \alpha H^\beta$$

(28)

where \(\tau_y(H)\) is the induced yield stress of MRF as a function of the applied magnetic intensity \((H)\), \(\alpha\) and \(\beta\) are the curve parameters determined from experimental results using a curve-fitting algorithm, and \(\tau_0\) is the zero-field yield stress of the MRF.
The latter, the approximate polynomial function, can well predict the MRF yield stress at small value of the applied magnetic intensity. The higher order of the polynomial is the more accurate value of the yield stress can be predicted. In practice, the third order polynomial is often used. However, the polynomial function can not express the saturation of the induced yield stress. Therefore, a saturation condition should be added. The 3rd order approximate polynomial function of MRF yield stress can be expressed by

\[ \tau_y(H) = \tau_0 + c_1 H + c_2 H^2 + c_3 H^3 \]  

(29)

where \( c_1, c_2, \) and \( c_3 \) are the curve parameters determined from experimental results using a curve-fitting algorithm, and \( \tau_0 \) is the zero-field yield stress of the MRF.

In many researches, other characteristics of MRF such as the post yield viscosity (\( \eta \)) in Bingham model, the consistency parameter (\( K \)) and the fluid behavior index (\( m \)) are determined from experimental results on rheological properties of MRF and assumed to be independent of the applied magnetic. However, in practice, these parameters are slightly affected by the applied magnetic field. In order to take this into account, Zubieta et al. [44] have proposed a field-dependent plastic model for MRF based on original Bingham plastic and Herschel-Bulkley plastic models as mention in Section 2.5. Once obtaining the yield stress and other rheological parameters of the MRF, the output mechanical power such as pressure drops, damping force for MR damper, braking torque and transmitted torque can be determined on governing equations of the MRF based devices.

4.2. Optimization problems in design of MRF based devices

As aforementioned modeling of MRF based systems is a coupled analysis problem. Therefore, output mechanical power of these systems depends not only on their mechanics behaviors but also on their magnetic circuits. It is obvious that in order to improve performance of the MRF based systems, the optimal design should be taken into account. Generally, the objective of the optimal design is to find significant geometric dimensions of the MRF based devices that maximize an objective function considering typical characteristics such as pressure drop, damping force, dynamic range, braking torque, transmitted force, mass, time response constant and power consumption. Some constraints such as available space, allowable operating temperature, uncontrollable torque etc. may be also considered in the optimal design. There have been several researches focusing on optimal design of MRF devices. Rosenfield and Wereley [48] proposed analytical optimization design method for MR valves and dampers based on the assumption of constant magnetic flux density throughout the magnetic circuit to ensure that one region of the magnetic circuit does not saturate prematurely and cause a bottleneck problem. Nguyen et al. [79] proposed a FEM based optimal design of MR valves (single-coil, two-coil, three-coil and radial-annular types) constrained in a specified volume. This work considered the effects of all geometric variables of MR valves by minimizing the valve ratio calculated from the FE analysis. Later on Nguyen et al. [80] have developed an optimization procedure based on the finite element method in order to find the optimal geometry of MR valves...
constrained to a specific volume, satisfying a required pressure drop with minimal power consumption. The time response of the valves was also taken into account by considering the inductive time constant as a state variable. The optimization results showed the significance of the optimal design of the MR valves in order to minimize the power consumption. It was also shown that the wire diameter does not significantly affect the optimization solution and can be neglected. The optimal design of MR damper was also performed by Nguyen et al. [81], in which the objective function was proposed by a linear combination of the ratios of the damping force, dynamic range and the inductive time constant and their reference values using corresponding weighting factors. Recently, there have been several researches on the optimal design of MR brakes and clutches. Park et al. [82] have performed multidisciplinary design optimization of an automotive MR brake, in which a multi-objective function considering both braking torque and mass of the brake was considered. Nguyen et al. [45] have performed a thorough research on optimal design of MR brake for middle-sized vehicles considering the available space, mass, braking torque and steady heat generated by a zero-field friction torque of the MR brake on cruising at a speed of 100 km/h. Furthermore, different configurations of MR brake and different types of MRF are taken into account in that research. More recently, Nguyen et al. [83] have performed the optimal design of common types of MR brakes such as disc-type, drum-type, inverted drum-type, single-coil hybrid-types, inverted single-coil hybrid-types, two-coil hybrid-types, inverted two-coil hybrid-types and T-type. The objective of the optimization was to maximize the braking torque while torque ratio (the ratio of maximum braking torque and the zero field friction torque) is constrained not to exceed a certain value. Based on the optimal solutions, the advices on optimal selection of MR brakes type were addressed. It was showed that the guide on optimal selection of MR brake types can be applied for different types of MRF and different constrains of torque ratio.

4.3. Optimal design of MRF devices based on finite element analysis

As abovementioned, the magnetic circuit of the MRF based devices can be solved by an approximation of analytical solution or by FEM. Therefore, the optimal design of these devices can be performed based on either the analytical analysis or finite element analysis (FEA). The former is used only for simple devices such as single coil MR damper [84]. In this section, the optimal design of MRF devices based on FEA is introduced. First of all, an objective function should be proposed depending on the purpose of the optimal design and the application of the devices. It is noted that in the optimization problem the objective function is always minimized. Therefore, if the purpose of the optimization is to maximize a performance function of the devices, that function should be transform to an equivalent objective function. The equivalent objective function is the function that when it is minimized, the corresponding performance function is maximized. After the objective function is constructed, the design parameters of the optimization problem should be identified. In addition, the constraints of the optimization problem should be determined if there any. In the next step, an algorithm to obtain the optimal solution should be chosen. It is well-known that there have been numerous methods to find the optimal solution of an
optimization problem. They may be non-derivative, first-derivative or second-derivative methods. The non-derivative methods that do not require any derivative of the function are not usually applied to MRF based systems. Although they are generally easy to implement, their convergence properties are rather poor. They may work well in special cases when the function is quite random in character or the variables are essentially uncorrelated. Some typical non-derivative algorithms are the Simplex, Genetic Algorithms and Neural Networks. The second-derivative optimization methods are characterized by fast convergence and affine invariance. However, they require second derivatives and the solution of linear equation can be too expensive for large scale applications. The most popular optimization method, which is widely used in optimal design of MRF based devices, is the first order (derivative) method. Although the convergence rate of the first derivative method is somewhat slower than that of the second-derivative one, the first derivative method is still preferred in many applications because of its inexpensive cost for computation and programming. A typical first derivative optimization algorithm is the conjugate gradient method. The flow chart in Figure 9 shows how to find the optimal solution of MRF based devices using the first order method. The procedures from the flow chart can be easily extended to other finite element software.

First of all, initial value of the design variables (DV) should be decided. Computation time of the optimization process significantly depends on the initial value of the DVs. Therefore, the initial value of design variables should be calculated based on a draft calculation or based on practical experience. Then, an analysis file for solving the magnetic circuit and calculating performance characteristics of the devices such as control energy, the inductive time constant, pressure drops, damping force, braking torque and transmitted torque is built. In ANSYS, the analysis file is built using parametric design language (APDL). It is noted that this analysis file can be created from a graphic user interface (GUI) model of ANSYS by using the list>log file submenu from the file menu of the ANSYS software. In the analysis file, the DVs must be used as symbolic variables and initial value is assigned to them. Generally, in order to calculate performance characteristics of the devices, the magnetic flux density across the active volume of MRF should be calculated. The magnetic flux density ($B$) and magnetic intensity ($H$) are not constant along the MRF duct, so an average should be used. The average magnetic flux density and intensity across the MR ducts was calculated by integrating flux density along a predefined path, then divided by the path length [79, 80]. In order to calculate the inductive time constant, firstly the magnetic flux is determined as follows:

$$\Phi = 2\pi R_d \int_{L_p} B(s) ds$$  \hspace{1cm} (30)$$

where $B(s)$ is the magnetic flux density at each nodal point on the path, $s$ is a dummy variable for the integration. The integration was performed along the path length, $L_p$. It is noteworthy that geometric dimensions of the MRF devices change during the optimization process, so that the meshing size of FE model should be specified by the number of elements per line rather than element size.
After the analysis file is prepared, the procedures to achieve optimal design parameters of the MRF devices using the first order method of ANSYS optimization tool are performed as shown in Figure 9. Starting with initial value of DVs, by executing the analysis file, the initial value of the performance characteristics of the devices such as control energy, the inductive time constant, pressure drops, damping force, braking torque and transmitted torque are obtained. The ANSYS optimization tool then transforms the constrained optimization problem to an unconstrained one via penalty functions. The dimensionless, unconstrained objective function is formulated as follows:

\[
Q(x,q) = \frac{OBJ}{OBJ_0} + \sum_{i=1}^{n} P_i(x_i) + q \sum_{i=1}^{m} P_j(g_i)
\]  

(31)
where \( OBJ_0 \) is the reference objective function value that is selected from the current group of design sets, \( q \) is the response surface parameter which controls constraint satisfaction. \( P_x \) is the exterior penalty function applied to the design variable \( x \). \( P_g \) is extended-interior penalty function applied to state variable (the constraint) \( g \). For the initial iteration \((j = 0)\), the search direction of DVs is assumed to be the negative of the gradient of the unconstrained objective function. Thus, the direction vector is calculated by

\[
d^{(0)} = -\nabla Q(x^{(0)}, 1)
\]  

(32)

The values of DVs in next iteration \((j+1)\) is obtained from the following equation,

\[
x^{(j+1)} = x^{(j)} + s_j d^{(j)}
\]  

(33)

where the line search parameter \( s_j \) is calculated by using a combination of a golden-section algorithm and a local quadratic fitting technique. The analysis file is then executed with the new values of DVs and the convergence of the objective function, \( OBJ \), is checked. If the convergence occurs, the values of DVs at the \( j^{th} \) iteration are the optimum. If not, the subsequent iterations will be performed. In the subsequent iterations, the procedures are similar to those of the initial iteration with the exception of the direction vectors which are calculated according to Polak-Ribiere recursion formula as follows:

\[
d^{(j)} = -\nabla Q(x^{(j)}, q_k) + r_{j-1} d^{(j-1)}
\]  

(34)

\[
r_{j-1} = \frac{[\nabla Q(x^{(j)}, q) - \nabla Q(x^{(j-1)}, q)]^T \nabla Q(x^{(j)}, q)}{\| \nabla Q(x^{(j-1)}, q) \|^2}
\]  

(35)

Thus, each iteration is composed of a number of sub-iterations that include search direction and gradient computations.

It is noted that ANSYS software supports optimal design problems by integrating an optimization tool. Therefore, in most cases the optimal solution of the MRF based devices can be solved directly by the ANSYS software without interfacing with any programming software. In order to use the ANSYS optimization tool, it is necessary to set up optimization parameters. To do this, firstly the analysis file should be manually executed once to load all parameters in the analysis file into software buffer memory. After that, from the Design Opt menu, we specify the analysis file which will be used during optimization process, the DVs with their limits and tolerances, the state variables (if there are any) with limits and tolerances, the objective function with a convergence criteria, the method for solving the optimal solution, and the optimal output control option if necessary. In some cases, it is expected to employ some advanced optimization algorithms such as Genetic Algorithms, Neural Network, or user defined algorithms, the interfacing between the ANSYS and other software to perform the optimization such as Matlab, FORTRAN, C languages is required.
5. Case studies on optimal design of MR valves

5.1. Configuration and modeling of MR valve

Figure 10 shows the structural configurations of the two typical types of MR valves: the annular MR relief valve (Figure 10a) and MR valve with both annular and radial flow paths (Figure 10b). The valve in Figure 10a consists of valve coil, cores and covers. The MRF flows through annular ducts between core A and core B. When the power of the coil is turned on, a magnetic field is exerted on the MRF, which causes the MRF flowing through the ducts to change its state into semi-liquid or solid and stop the flow. Only when the supply pressure gets high enough to offset the yield stress, the fluid can flow through the valve again. The valve in Figure 10b consists of the valve core, magnetic disk and valve housing form a magnetic circuit of the valve. A non-magnetic washer is used to warrant the required thickness of the radial duct. When the magnetic disk is placed coaxially with the valve housing using the cone-shape cap, the annular and radial ducts are formed between the disk and the valve housing, and the disk and the valve core, respectively. MRF flows from the inlet through the first annular and radial duct, then flow along the hole at the center of the core and after that follows the second radial and annular duct to the outlet.

![Schematic diagrams of MR valves.](image_url)

Figure 11a shows a simplified structure and significant dimensions of a single-coil annular MR valve. The valve geometry is featured by the overall effective length \( L \), the outside radius \( R \), the valve housing thickness \( t_h \), the MR duct gap \( t_g \), the core flange (pole) thickness \( t_f \), and the coil width \( w_c \).
By using Bingham plastic model, the pressure drop of the valve is calculated by \[80, 81\]

\[
\Delta P_A = \Delta P_{A,\eta} + \Delta P_{A,\tau} = \frac{6\eta L}{\pi t^2 R_d} Q + 2c \frac{t_f}{t_s} \tau_y
\]

(36)

where \(\Delta P_{A,\tau}\) and \(\Delta P_{A,\eta}\) are the field-dependent and viscous pressure drop of the single annular MR valve, respectively, \(Q\) is the flow rate through the MR valve, \(R_d\) is the average radius of annular duct given by \(R_d = R_t - 0.5t_s\), \(c\) is the coefficient which depends on flow velocity profile and has a value range from a minimum value of 2.0 (for \(\frac{\Delta P_{A,\tau}}{\Delta P_{A,\eta}} > 100\)) to a maximum value of 3.0 (for \(\frac{\Delta P_{A,\tau}}{\Delta P_{A,\eta}} < 1\)).

The multi-coil MR valve, which was first employed by Spencer et al. to make a high damping force MR damper used in seismic protection system \([85]\), is now widely used in many applications. For MR valve with two coils, the pressure drop is calculated by

\[
\Delta P_{2A} = \Delta P_{2A,\eta} + \Delta P_{2A,\tau} = \frac{6\eta L}{\pi t^2 R_d} Q + 2c_1 \frac{t_{f1}}{t_s} \tau_{y1} + c_2 \frac{t_{f2}}{t_s} \tau_{y2}
\]

(37)

where \(\Delta P_{2A,\tau}\) and \(\Delta P_{2A,\eta}\) are the field-dependent and viscous pressure drop of the two-coil annular MR valve respectively, \(\tau_{y1}\) and \(\tau_{y2}\) are the yield stresses of the MRF in the end ducts and the middle duct, respectively. \(c_1\) and \(c_2\) are coefficient which depends on flow velocity profile of MR flow in the end ducts and the middle duct, respectively.

Similarly, for the three-coil annular MR valve, the pressure drop is calculated by
\[
\Delta P_{3A} = \Delta P_{3A,\eta} + \Delta P_{3A,\tau} = \frac{6\eta L}{\pi t^3 R_d} Q + 2\left(c_1 \frac{t_f}{t_g} \tau_{y1} + c_2 \frac{t_f}{t_g} \tau_{y2}\right)
\]  

(38)

For the MR valve with both annular and radial flow paths shown in Figure 11b, the pressure drop can be calculated by

\[
\Delta P_{AR} = \Delta P_{AR,\eta} + \Delta P_{AR,\tau}
\]

(39)

where \(\Delta P_{AR,\tau}\) and \(\Delta P_{AR,\eta}\) are determined by

\[
\Delta P_{AR,\tau} = 2\left(c_\tau \frac{t_f}{t_g} \tau_{y\eta} + c_r \frac{R_2 - R_0}{t_g} \tau_{y\tau}\right)
\]

(40)

\[
\Delta P_{AR,\eta} = 2\left[\frac{6\eta t_f}{\pi t^3 R_d} Q + \frac{6\eta Q}{\pi t^3 S} \ln\left(\frac{R_d}{R_0}\right)\right] + \frac{8\eta (L - 2t_f)Q}{\pi R_0^4}
\]

(41)

In the above, \(\tau_{y\eta}\) and \(\tau_{y\tau}\) are the induced yield stresses of the MRF in the annular duct and the radial duct, respectively. \(R_0\) is the radius of the hole at the center of the valve core and \(R_2\) is the outer radius of the radial duct. Here, \(c_\tau\) and \(c_r\) are coefficients that depend on the velocity profile of MRF flowing through the annular and radial ducts, respectively.

### 5.2. Optimization of MR valves considering pressure drop and dynamic range

The optimal objective is to minimize the valve ratio defined by the ratio of the viscous pressure drop to the field-dependent pressure drop of the MR valve. This ratio has great effect on the characteristics of the MR valve. It is desirable that the valve ratio takes a small value. The valves are constrained in a cylinder of the radius \(R=30\,\text{mm}\) and the height \(H=50\,\text{mm}\). Magnetic properties of valve components are given in Table 1. The post-yield viscosity of the MRF is assumed to be constant, \(\eta=0.092\,\text{Pa.s}\) and the flow rate of the MR valves is \(Q=3\times10^{-4}\,\text{m}^3/\text{s}\). The commercial MR fluid (MRF132-DG) from Lord Corporation is used. The induced yield stress of the MR fluid as a function of the applied magnetic field intensity \((H_{mr})\) can be approximately expressed by

\[
\tau_y = p(H_{mr}) = C_0 + C_1 H_{mr} + C_2 H_{mr}^2 + C_3 H_{mr}^3
\]

(42)

In Eq. (42), the unit of the yield stress is \(\text{kPa}\) while that of the magnetic field intensity is \(\text{kA/m}\). The coefficients \(C_0, C_1, C_2\), and \(C_3\), determined from experimental results by applying the least square curve fitting method, are respectively identified as 0.3, 0.42, -0.00116 and 1.05E-6.

It is noted that, a small change in the valve gap \(t_g\) would drastically alter the performance of the MR valve. Therefore, in MR valve design, a fixed gap is chosen according to each application. In this study, the valve gap is chosen as \(1\,\text{mm}\). From Eqs. (36)-(41), the valve ratios of the single-coil, two-coil and radial-annular MR valve are respectively calculated by
\[ \lambda_A = \frac{\Delta P_{A,\eta}}{\Delta P_{A,\tau}} = \frac{3\eta HQ}{\pi t^2 R^2 c t \tau_y} \]  

(43)

\[ \lambda_{2A} = \frac{\Delta P_{2A,\eta}}{\Delta P_{2A,\tau}} = \frac{3\eta HQ}{\pi t^2 R^2 c (t \tau_y + 0.5 \pi \tau_y a)} \]  

(44)

\[ \lambda_{AR} = \frac{2\eta Q}{\pi} \left[ \frac{3t}{t^2 R_d} \ln\left( \frac{R_y}{R_0} \right) + \frac{2t^2 (H - 2t)}{R_0^3} \right] c (t \tau_y + (R_2 - R_0) \tau_y) \]  

(45)

The ANSYS APDL program is the analysis ANSYS APDL code used in optimal design of the annular single MR valve. The analysis ANSYS APDL code for other types of MR valve can be prepared in the same manner.

<table>
<thead>
<tr>
<th>Valve Components</th>
<th>Material</th>
<th>Relative Permeability</th>
<th>Saturation Flux Density</th>
</tr>
</thead>
<tbody>
<tr>
<td>Valve Core</td>
<td>Silicon Steel</td>
<td>B-H curve (Fig. 12a)</td>
<td>1.5 Tesla</td>
</tr>
<tr>
<td>Valve Housing</td>
<td>Silicon Steel</td>
<td>B-H curve (Fig. 12a)</td>
<td>1.5 Tesla</td>
</tr>
<tr>
<td>Coil</td>
<td>Copper</td>
<td>1</td>
<td>x</td>
</tr>
<tr>
<td>MR Fluid</td>
<td>MRF132-DG</td>
<td>B-H curve (Fig. 12b)</td>
<td>1.6 Tesla</td>
</tr>
<tr>
<td>Nonmagnetic Cap/Bobbin</td>
<td>Nonmagnetic Steel</td>
<td>1</td>
<td>x</td>
</tr>
</tbody>
</table>

Table 1. Magnetic properties of the valve components

![Figure 12](image1.png)

(a) B-H curve of silicon steel

![Figure 12](image2.png)

(b) B-H curve of MR fluid

Figure 12. Magnetic properties of silicon steel and MR fluid
ANSYS APDL program 1

/COM, Magnetic-Nodal
/COM, Magnetic-Edge
/PREP7

Element definition

ET,1,PLANE13
KEYOPT,1,1,0
KEYOPT,1,2,0
KEYOPT,1,3,1
KEYOPT,1,4,0
KEYOPT,1,5,0

Material definition

MAT,1, ! Silicon Steel (B-H curve)
TBDEL,ALL,_MATL
MPDEL,ALL,_MATL
TB,BH,_MATL , 1, 9
TBPT, 130.000000 , 0.380000000
TBPT, 260.000000 , 0.750000000
TBPT, 400.000000 , 1.05000000
TBPT, 600.000000 , 1.30000000
TBPT, 800.000000 , 1.40000000
TBPT, 1000.00000 , 1.46500000
TBPT, 1100.00000 , 1.51000000
!
MAT,3, ! MR Fluid (B-H curve)
TBDEL,ALL,_MATL
MPDEL,ALL,_MATL
TB,BH,_MATL , 1, 8
TBPT, 5000.000000 , 0.300000000
TBPT, 10000.000000 , 0.600000000
TBPT, 15000.000000 , 0.900000000
TBPT, 20000.000000 , 1.30000000
TBPT, 25000.000000 , 1.58000000
TBPT, 30000.000000 , 1.64000000
!
MAT,1, ! Valve coil
TBDEL,ALL,_MATL
MPDEL,ALL,_MATL
TB,BH,_MATL , 1, 9
TBPT, 5000.000000 , 0.300000000
TBPT, 10000.000000 , 0.600000000
TBPT, 15000.000000 , 0.900000000
TBPT, 20000.000000 , 1.30000000
TBPT, 25000.000000 , 1.58000000
TBPT, 30000.000000 , 1.64000000
!
MAT,3, ! MR Fluid (B-H curve)
TBDEL,ALL,_MATL
MPDEL,ALL,_MATL
TB,BH,_MATL , 1, 8
TBPT, 5000.000000 , 0.300000000
TBPT, 10000.000000 , 0.600000000
TBPT, 15000.000000 , 0.900000000
TBPT, 20000.000000 , 1.30000000
TBPT, 25000.000000 , 1.58000000
TBPT, 30000.000000 , 1.64000000
!
MAT,1, ! Valve coil
pi=3.1416
msize=12 !Basic No. of elements/line

! Geometric definition
H=0.05 ! Height
R=0.03 ! Outer Radius
w=0.010 ! Coil width
t=0.017 ! Pole length
th=0.0065 ! Housing thickness
d=0.001 ! MRF duct gap
Rw=R-th-d
dc=0.00052 ! Wire Radius
res=0.0172e-6 ! Wire Resistivity
Ac=pi*dc**2/4
rrc=res/Ac
Nturn=w*(H-2*t)/Ac ! No of turns
Rc=rrc*Nturn*pi*Rw**2*(Rw-0.5*w) ! Wire Resistance
I=2.5 ! Applied current
J=I*4/dc/dc/pi ! Current density
PP=I**2*Rc ! Consumption Power

! geometric model
RECTNG,0,R,0,H,
RECTNG,Rw,R-Th,0,H,
RECTNG,Rw-W,Rw,t,H-t,
RECTNG,0,Rw,0.0,t,
RECTNG,0,Rw,H-t,H,
RECTNG,Rw,R,t,H-t,
!
FLST,2,6,5,ORDE,2
FITEM,2,1
FITEM,2,-6
AOVLAP,P51X
NUMCMP,LINE
NUMCMP,AREA
! Material assignment
! Housing
FLST,5,3,5,ORDE,3
FITEM,5,5
FITEM,5,9
FITEM,5,-10
Table 2. ANSYS APDL program

<table>
<thead>
<tr>
<th>Command</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>CM,_Y,AREA</td>
<td>FINISH</td>
</tr>
<tr>
<td>ASEL,,,P51X</td>
<td>/POST1 ! Post processing</td>
</tr>
<tr>
<td>CM,_Y1,AREA</td>
<td>SET,LAST ! Choose the last set of solution</td>
</tr>
<tr>
<td>CMSEL,S_Y</td>
<td>! Define path</td>
</tr>
<tr>
<td>*</td>
<td>PATH,p1,2,30,20,</td>
</tr>
<tr>
<td>CMSEL,S_Y1</td>
<td>PPATH,1,0,R-th-d/2,0,0,0,</td>
</tr>
<tr>
<td>AATT, 1, 1, 0,</td>
<td>PPATH,2,0,R-th-d/2,t,0,0,</td>
</tr>
<tr>
<td>CMSEL,S_Y</td>
<td>! Calculate the magnetic intensity along the path</td>
</tr>
<tr>
<td>CMDELE,_Y</td>
<td>PDEF,H1,H,sum,AVG</td>
</tr>
<tr>
<td>CMDELE,_Y1</td>
<td>/PBC,PATH,,0</td>
</tr>
<tr>
<td>! Core</td>
<td>PCALC,INTG,H1in,H1,S,1,</td>
</tr>
<tr>
<td>FLST,5,3,5,ORDE,2</td>
<td>*GET,H1in,PATH, ,last,H1IN</td>
</tr>
<tr>
<td>FITEM,5,6</td>
<td>Hmr=1e-3*abs(H1IN)/t</td>
</tr>
<tr>
<td>FITEM,5,-8</td>
<td>!PADEL,P1 ! Delete path</td>
</tr>
<tr>
<td>CM,_Y,AREA</td>
<td>! Calculation of parameter</td>
</tr>
<tr>
<td>ASEL,,,P51X</td>
<td>neta=0.092</td>
</tr>
<tr>
<td>CM,_Y1,AREA</td>
<td>Q=0.0003</td>
</tr>
<tr>
<td>CMSEL,S_Y</td>
<td>c=2.5</td>
</tr>
<tr>
<td>CMSEL,S_Y1</td>
<td>C0=0.3</td>
</tr>
<tr>
<td>AATT, 1, 1, 0,</td>
<td>C1=0.42</td>
</tr>
<tr>
<td>CMSEL,S_Y</td>
<td>C2=0.00116</td>
</tr>
<tr>
<td>CMDELE,_Y</td>
<td>C3=1.0513e-6</td>
</tr>
<tr>
<td>CMDELE,_Y1</td>
<td>ty=(C0+C1<em>Hmr+C2</em>Hmr<strong>2+C3*Hmr</strong>3) ! yield st.</td>
</tr>
<tr>
<td>FLST,5,3,5,ORDE,2</td>
<td>R1=(R-Th-d/2)</td>
</tr>
<tr>
<td>FITEM,5,2</td>
<td>del_P1=(6<em>neta</em>H<em>Q/(3.14</em>d**3*R1))*0.00001</td>
</tr>
<tr>
<td>FITEM,5,-4</td>
<td>del_P2=delta_P+2<em>C</em>t<em>ty</em>0.01/d</td>
</tr>
<tr>
<td>CM,_Y,AREA</td>
<td>del_P=del_P1+del_P2 ! Pressure drop</td>
</tr>
<tr>
<td>ASEL,,,P51X</td>
<td>OBJ=del_P1/del_P2 ! Objective</td>
</tr>
</tbody>
</table>

Figure 13 shows the optimal solution of a single-coil annular MR valve constrained in the specific volume when a current of 2.5A is applied to the valve coil. Initial values of \(t\), \(w\) and \(t_h\) are 17mm, 10mm and 6.5mm, respectively. The valve ratio, pressure drop and power consumption of the valve at these initial values are \(\lambda_0=0.08274\), \(\Delta P_0=15\text{ bar}\) and \(N_0=38.83\text{ W}\), respectively. From the figure, it is observed that the solution is convergent after 13 iterations and the minimum value of valve ratio (objective function) is \(\lambda_{opt}=0.033\). The corresponding pressure drop is \(\Delta P_{opt}=37.32\text{ bar}\), which is also the maximum. At the optimum, the power consumption is \(N_{opt}=7.92\text{ W}\) which is much smaller than at the initial. The DVs at the optimum are \(t_{opt}=7.23\text{ mm}, \ w_{opt}=1.78\text{ mm}, \ t_{h, opt}=7.43\text{ mm}\).
Figure 13. Optimal solution of single-coil MR valves considering the valve ratio and pressure drop

Figure 14 shows the optimal solution of the two-coil annular MR valve. Initial values of \( a, t, w_c \) and \( t_h \) are 10 mm, 5 mm, 10 mm and 4 mm, respectively. The valve ratio, pressure drop and power consumption at these initial values are \( \lambda_0=0.0381, \Delta P_0=28.2 \text{bar} \) and \( N_0=83.2 \text{W} \), respectively. The solution is convergent after 11 iterations and the minimum value of valve ratio is \( \lambda_{opt}=0.023 \). The corresponding pressure drop is \( \Delta P_{opt}=48.6 \text{bar} \), which is also the maximum. The optimal DVs are \( a_{opt}=19.7 \text{ mm}, t_{opt}=10.6 \text{ mm}, w_{c,opt}=6.38 \text{ mm}, t_{h,opt}=5.33 \text{ mm} \).

Figure 15 shows the optimization solution of the annular-radial MR valve. Initial values of \( R_0, t, w_c \) and \( t_s \) are 6 mm, 10 mm, 6 mm and 8 mm, respectively. The valve ratio, pressure drop and power consumption at these initial values are \( \lambda_0=0.041, \Delta P_0=47 \text{bar} \) and \( N_0=44.3 \text{W} \), respectively. The convergence occurs at 10 iteration, at which the minimum value of valve ratio is \( \lambda_{opt}=0.023 \) and the optimal design parameters are \( R_{0,opt}=14.41 \text{ mm}, t_{opt}=6.47 \text{ mm}, \)
The corresponding pressure drop is $\Delta P_{opt} = 37.2\, \text{bar}$, which is not the maximum pressure drop. The reason for this is that the uncontrolled pressure drop (viscous pressure drop) of the valve significantly depends on the valve core radius. An increase of the valve core radius results in a decrease of the viscous pressure drop by which reduces the valve ratio. However, the increase of the valve core radius causes a decrease of the magnetic flux density, and by which reduces the pressure drop of the valve. In order to improve the valve performance, the valve core radius should be fixed at an appropriate. In case the valve core radius is fixed at $6\, \text{mm}$, it was found that the optimal value of valve ratio is $\lambda_{opt} = 0.0293$ and the corresponding pressure drop is $\Delta P_{opt} = 64.4\, \text{bar}$, which is also the maximum. The optimal DVs are $t_{c,opt} = 8.6\, \text{mm}$, $w_{c,opt} = 3.1\, \text{mm}$ and $t_{h,opt} = 6.36\, \text{mm}$. At these optimal DVs the power consumption is $N_0 = 29.1\, \text{W}$.

Table 3 summarizes the optimization results for MR valve design abovementioned. The results show that the geometry of MR valve has a great effect on the valve performance. By choosing an optimal geometry, the valve performance such as pressure drop can be much improved and the power consumption can be significantly reduced. Among the MR valves constrained in the same volume, the two-coil annular MR valve provides the best value of valve ratio while the annular-radial can provide the best pressure drop at the optimal design parameters. For MR valves with three coils or more, it was shown that the performance of these valves is not better than that of the two-coil MR valve at optimal design parameters.

It was also shown by Nguyen et al. [79] that the optimal solution is affected by the applied current. The higher value of the applied current is the better performance of the valve is. However, when the applied current increases to a certain value the optimal solutions tends to be saturation. Therefore, it is advised that the applied current should be set by it maximum allowable value in the optimization problem of the MR valve.
Table 3. Optimization results for MR valve design

6. Conclusion

In this chapter, the methodology of optimal design of MRF-based devices was introduced. The chapter was started by a review of fundamentals and the theory behind MRF in section 2. In section 3, the operating modes used in MRF based devices were considered and several different devices featuring MRF are discussed. In the fourth section, optimal design methodology of MRF-based mechanisms was considered. In this section, firstly the necessity of optimal design and the state of the art were discussed. The magnetic circuit analysis and the modeling of MRF devices were then considered. In addition, the optimization problem of MRF devices was figured out and the methods to solve the problem were investigated. Section 5 dealt with a case study of MR valve optimal design. In this case study, several valve configurations such as single-coil MR valve, multiple-coil MR valve and annular-radial MR valve were considered. The optimization problem is to minimize the valve ratio by which maximized the control range and pressure drop of the MR valves. The results have shown the significance and the effectiveness of the proposed optimization methodology. Base on this study case, the optimal design of other MRF-based devices such as MR dampers, MR brakes can be performed.
Author details
Quoc-Hung Nguyen and Seung-Bok Choi
Mechanical Engineering, Inha University, Korea

7. References


