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Feature Extraction Based on Wavelet Moments and Moment Invariants in Machine Vision Systems

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1. Introduction

Recently, there has been an increasing interest on modern machine vision systems for industrial and commercial purposes. More and more products are introduced in the market, which are making use of visual information captured by a camera in order to perform a specific task. Such machine vision systems are used for detecting and/or recognizing a face in an unconstrained environment for security purposes, for analysing the emotional states of a human by processing his facial expressions or for providing a vision based interface in the context of the human computer interaction (HCI) etc.

In almost all the modern machine vision systems there is a common processing procedure called feature extraction, dealing with the appropriate representation of the visual information. This task has two main objectives simultaneously, the compact description of the useful information by a set of numbers (features), by keeping the dimension as low as possible.

Image moments constitute an important feature extraction method (FEM) which generates high discriminative features, able to capture the particular characteristics of the described pattern, which distinguish it among similar or totally different objects. Their ability to fully describe an image by encoding its contents in a compact way makes them suitable for many disciplines of the engineering life, such as image analysis (Sim et al., 2004), image watermarking (Papakostas et al., 2010a) and pattern recognition (Papakostas et al., 2007, 2009a, 2010b).

Among the several moment families introduced in the past, the orthogonal moments are the most popular moments widely used in many applications, owing to their orthogonality property that comes from the nature of the polynomials used as kernel functions, which they constitute an orthogonal base. As a result, the orthogonal moments have minimum information redundancy meaning that different moment orders describe different parts of the image.

In order to use the moments to classify visual objects, they have to ensure high recognition rates for all possible object’s orientations. This requirement constitutes a significant operational feature of each modern pattern recognition system and it can be satisfied during
the feature extraction stage, by making the moments invariant under the basic geometric transformations of rotation, scaling and translation.

The most well known orthogonal moment families are: Zernike, Pseudo-Zernike, Legendre, Fourier-Mellin, Tchebichef, Krawtchouk, with the last two ones belonging to the discrete type moments since they are defined directly to the image coordinate space, while the first ones are defined in the continuous space.

Another orthogonal moment family that deserves special attention is the wavelet moments that use an orthogonal wavelet function as kernel. These moments combine the advantages of the wavelet and moment analyses in order to construct moment descriptors with improved pattern representation capabilities (Feng et al., 2009).

This chapter discusses the main theoretical aspects of the wavelet moments and their corresponding invariants, while their performance in describing and distinguishing several patterns in different machine vision applications is studied experimentally.

2. Orthogonal image moments

A general formulation of the \((n+m)\)th order orthogonal image moment of a \(NxN\) image with intensity function \(f(x,y)\) is given as follows:

\[
M_{nm} = NF \times \sum_{i=1}^{N} \sum_{j=1}^{N} \text{Kernel}_{nm}(x_i, y_j) f(x_i, y_j)
\]  

(1)

where \(\text{Kernel}_{nm}(.)\) corresponds to the moment’s kernel consisting of specific polynomials of order \(n\) and repetition \(m\), which constitute the orthogonal basis and \(NF\) is a normalization factor. The type of Kernel’s polynomial gives the name to the moment family by resulting to a wide range of moment types. Based on the above equation (1) the image moments are the projection of the intensity function \(f(x,y)\) of the image on the coordinate system of the kernel’s polynomials.

The first introduction of orthogonal moments in image analysis, due to Teague (Teague, 1980), made use of Legendre and Zernike moments in image processing. Other families of orthogonal moments have been proposed over the years, such as Pseudo-Zernike, Fourier-Mellin etc. moments, which better describe the image in process and ensure robustness under arbitrarily intense noise levels.

However, these moments present some approximation errors due to the fact that the kernel polynomials are defined in a continuous space and an approximated version of them is used in order to compute the moments of an image. This fact is the source of an approximation error (Liao & Pawlak, 1998) which affects the overall properties of the derived moments and mainly their description abilities. Moreover, some of the above moments are defined inside the unit disc, where their polynomials satisfy the orthogonality condition. Therefore, a prior coordinates’ transformation is required so that the image coordinates lie inside the unit disc. This transformation is another source of approximation error (Liao & Pawlak, 1998) that further degrades the moments’ properties.

The following Table 1, summarizes the main characteristics of the most used moment families.
<table>
<thead>
<tr>
<th>Moment Family</th>
<th>Kernel Form</th>
<th>Normalization Factor (NF)</th>
<th>Type</th>
<th>Coordinate System</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zernike</td>
<td>$K_{n,m}(r,\theta) = \left( \sum_{j=0}^{n-m} (-1)^j \frac{(n-s)!}{s!(r+</td>
<td>m</td>
<td>+1-s)!} \cdot r^{n+2s} \right) e^{-2\pi i \theta}$</td>
<td>$\frac{n+1}{\pi}$</td>
</tr>
<tr>
<td>(Mukundan, &amp; Ramakrishnan, 1998)</td>
<td></td>
<td></td>
<td></td>
<td>polar coordinates</td>
</tr>
<tr>
<td>Pseudo-Zernike</td>
<td>$K_{n,m}(r,\theta) = \left( \sum_{j=0}^{n-m} (-1)^j \frac{(2n+1-s)!}{s!(r+</td>
<td>m</td>
<td>+1-s)!} \cdot r^{n+2s} \right) e^{-2\pi i \theta}$</td>
<td>$\frac{n+1}{\pi}$</td>
</tr>
<tr>
<td>(Mukundan, &amp; Ramakrishnan, 1998)</td>
<td></td>
<td></td>
<td></td>
<td>polar coordinates</td>
</tr>
<tr>
<td>Fourier-Mellin</td>
<td>$K_{n,m}(r,\theta) = \left( \sum_{j=0}^{n-m} (-1)^j \frac{(n+k+1)!}{(n-k)!k!(k+1)!} \cdot r^{n+k} \right) e^{-2\pi i \theta}$</td>
<td>$\frac{n+1}{\pi}$</td>
<td>continuous</td>
<td>disc</td>
</tr>
<tr>
<td>(Mukundan, &amp; Ramakrishnan, 1998)</td>
<td></td>
<td></td>
<td></td>
<td>polar coordinates</td>
</tr>
<tr>
<td>Legendre</td>
<td>$K_{n,0}(x,y) = P_n(x) \cdot P_m(y)$ with $P_n(x) = \frac{1}{2^n n!} \frac{d^n(x^2-1)^n}{dx^n}$</td>
<td>$\frac{(2n+1)(2m+1)}{4}$</td>
<td>continuous</td>
<td>$[-1,1]$</td>
</tr>
<tr>
<td>(Mukundan, &amp; Ramakrishnan, 1998)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tchebichef</td>
<td>$K_{n,0}(x,y) = t_n(x) \cdot t_m(y)$ with $t_n(x) = \sum_{k=0}^{n} (-1)^k \frac{(N-1-k)!}{n-k!(k)!} \frac{x^n}{n}$</td>
<td>$\frac{F(n,N) \cdot F(m,N)}{2n(2n+1)}$</td>
<td>discrete</td>
<td>image dimensions</td>
</tr>
<tr>
<td>(Mukundan et al., 2001)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Krawtchouk</td>
<td>$K_{n,0}(x,y) = K_n(x;p,N) \cdot K_m(y;p,N)$ with $K_n(x;p,N) = \sum_{k=0}^{n} a_{n,p,k} x^k$</td>
<td>1</td>
<td>discrete</td>
<td>image dimensions</td>
</tr>
<tr>
<td>(Yap et al., 2003)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 1. Orthogonal moments’ characteristics.
Apart from some remarkable attempts to compute the theoretical image moment values (Wee & Paramesran, 2007), new moment families with discrete polynomial kernels (Tchebichef and Krawtchouk moments) have been proposed, which permit the direct computation of the image moments in the discrete domain.

It is worth pointing out that the main subject of this chapter is the introduction of a particular moment family, the wavelet moments and the investigation of their classification capabilities as compared to the traditional moment types.

However, before introducing the wavelet moments, it is useful to discuss two important properties of the moments that determine their utility in recognizing patterns.

### 2.1 Information description

As it has already been mentioned in the introduction, the moments have the ability to carry information of an image with minimum redundancy, while they are capable to enclose distinctive information that uniquely describes the image’s content. Due to these properties, once a finite number of moments up to a specific order $n_{\text{max}}$ is computed, the original image can be reconstructed by applying a simple formula, inverse to (1), of the following form:

$$\hat{f}(x,y) = \sum_{n=0}^{n_{\text{max}}} \sum_{m=0}^{n} \text{Kernel}_{nm}(x,y) M_{nm}$$

where $\text{Kernel}_{nm}(\cdot)$ is the same kernel of (1) used to compute moment $M_{nm}$.

Theoretically speaking, if one computes all image moments and uses them in (2), the reconstructed image will be identical to the original one with minimum reconstruction error.

### 2.2 Invariant description

Apart from the ability of the moments to describe the content of an image in a statistical fashion and to reconstruct it perfectly (orthogonal moments) according to (2), they can also be used to distinguish a set of patterns belonging to different categories (classes). This property makes them suitable for many artificial intelligence applications such as biometrics, visual inspection or surveillance, quality control, robotic vision and guidance, biomedical diagnosis, mechanical fault diagnosis etc. However, in order to use the moments to classify visual objects, they have to ensure high recognition rates for all possible object’s orientations. This requirement constitutes a significant operational feature of each modern pattern recognition system and it can be satisfied during the feature extraction stage, where discriminative information of the objects is retrieved.

Mainly, two methodologies used to ensure invariance under common geometric transformations such as rotation, scaling and translation, either by image coordinates normalization and description through the geometric moment invariants (Mukundan & Ramakrishnan, 1998; Zhu et al., 2007) or by developing new computation formulas which incorporate these useful properties inherently (Zhu et al., 2007).

However, the former strategy is usually applied for deriving the moment invariants of each moment family, since it can be applied in each moment family in a similar way.
According to this method and by applying coordinates normalization (Rothe et al., 1996) the Geometric Moment Invariants (GMIs) of the following form, are constructed:

\[
GMI_{nm} = GM_{00}^{\gamma} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} \left[ (x-x) \cos \theta + (y-y) \sin \theta \right]^{m} \left[ (y-y) \cos \theta -(x-x) \sin \theta \right]^{n} f(x,y)
\]

with

\[
\gamma = \frac{n+m}{2} + 1, \quad \bar{x} = \frac{GM_{10}}{GM_{00}}, \quad \bar{y} = \frac{GM_{01}}{GM_{00}}
\]

\[
\theta = \frac{1}{2} \tan^{-1} \left( \frac{2\mu_{11}}{-\mu_{20} - \mu_{02}} \right)
\]

where \((\bar{x}, \bar{y})\) are the coordinates of the image’s centroid, \(GM_{nm}\) are the geometric moments and \(\mu_{nm}\) are the central moments defined as:

\[
GM_{nm} = \sum_{x=0}^{N-1} \sum_{y=0}^{M-1} x^{n} y^{m} f(x,y)
\]

\[
\mu_{nm} = \sum_{x=0}^{N-1} \sum_{y=0}^{M-1} (x-\bar{x})^{n} (y-\bar{y})^{m} f(x,y)
\]

which are translation invariant. The value of angle \(\theta\) is limited to \(-45^\circ \leq \theta \leq 45^\circ\) and additional modifications (Mukundan & Ramakrishnan, 1998) have to be performed in order to extend \(\theta\) into the range \(0^\circ \leq \theta \leq 360^\circ\).

By expressing each moment family in terms of geometric moment invariants the corresponding invariants can be derived. For example, Zernike moments are expressed (Wee & Paramesran, 2007) in terms of GMIs as follows:

\[
ZMI_{nm} = \frac{n+1}{\pi} \sum_{k=m}^{n} \sum_{i=0}^{k} B_{imk} \sum_{j=0}^{m} w \left( \left( \begin{array}{c} n \\ i \end{array} \right) \left( \begin{array}{c} m \\ j \end{array} \right) \right) GMI_{k-2i-j,2i+j}
\]

where \(n\) is a non-negative integer and \(m\) is a non zero integer subject to the constraints \(n-|m|\) even and \(|m| \leq n\) and

\[
w = \begin{cases} -i, & m > 0 \\ +i, & m \leq 0 \end{cases} \quad \text{with} \quad i = \sqrt{-1}, \quad s = \frac{1}{2} (k-m)
\]

\[
B_{imk} = ( -1 )^{(n-k) \over 2} \frac{(n+k)!}{(k+m)! (k-m)!}.
\]
3. Wavelet-based moment descriptors

In the same way as the continuous radial orthogonal moments such as Zernike, Pseudo-
Zernike and Fourier-Mellin ones are defined in a continuous form (9), one can define the
wavelet moments by replacing the function $g_n(r)$ with a wavelet basis functions.

$$M_{nm} = \int \int g_n(r)e^{-jnm\theta} f(r, \theta) r dr d\theta$$

(9)

Based on Table 1 it can be deduced that by choosing the appropriate function $g_n(r)$, the
Zernike, Pseudo-Zernike and Fourier moments are derived. If one chooses wavelet basis
functions of the following form

$$\psi_{a,b}(r) = \frac{1}{\sqrt{a}} \psi\left(\frac{r-b}{a}\right)$$

(10)

where $a \in \mathbb{R^+}$, $b \in \mathbb{R}$ are the dilation and translation parameters and $\psi(\cdot)$ the mother wavelet
that is used to generate the whole basis.

Two widely used mother wavelet functions are the cubic B-spline and the Mexican hat
functions defined as follows:

**Cubic B-spline Mother Wavelet**

$$\psi(r) = \frac{4a^{n+1}}{\sqrt{2\pi(n+1)}} \sigma_\psi \cos(2\pi f_0 (2r - 1)) \times e^{-\left(\frac{2r-1}{2\sigma_\psi(n+1)}\right)^2}$$

(11)

where

$$\begin{cases}
  n = 3 \\
  a = 0.697066 \\
  f_0 = 0.409177 \\
  \sigma_\psi^2 = 0.561145
\end{cases}$$

(12)

**Mexican Hat Mother Wavelet**

$$\psi(r) = \frac{2}{\sqrt{3\sigma}} \pi^{-1/4} \left(1 - \frac{r^2}{\sigma^2}\right) \times e^{-\left(\frac{r^2}{2\sigma^2}\right)}$$

(13)

with $\sigma = 1$.

The main characteristic of the above wavelet functions is that by adjusting the $a,b$
parameters a basis functions consisting of dilated (scaled) and translated versions of the
mother wavelets can be derived.

The graphical presentation of the above two mother wavelet functions for the above set of
their parameters, is illustrated in the following Fig.1

Since the $a,b$ parameters are usually discrete (this is mandatory for the case of the resulted
moments), a discretization procedure needs to be applied. Such a common method (Shen &
Ip, 1999) that also takes into consideration the restriction of $r \leq 1$, applies the following
relations.
\[ a = a_0^m = 0.5^m, \quad m = 0, 1, 2, 3... \]
\[ b = b_0 \times n \times a_0^m = 0.5 \times n \times 0.5^m, \quad n = 0, 1, ..., 2^{m+1} \]

With the above the wavelet basis is constructed by a modified formula of (10) having the form:

\[ \psi_{mn}(r) = 2^{m/2} \psi \left( 2^m r - 0.5n \right) \]

It has to be noted that the selection of \( b_0 \) to 0.5 causes oversampling, something which adds significant information redundancy when the wavelet moments are used to reconstruct the initial image, but it doesn’t seriously affect their recognition capabilities. Moreover, in order to reduce this affection a feature selection procedure can be applied to keep only the useful features, by discarding the redundant ones.

![Cubic B-spline mother wavelet](image)

(a)

![Mexican hat mother wavelet](image)

(b)

Fig. 1. Plots of (a) cubic B-spline and (b) Mexican hat mother wavelet functions.
Based on the previous analysis and by following the nomenclature of Table 1, the wavelet moments of a \( N \times N \) image \( f(x,y) \) are defined in the unit disc \( (r \leq 1) \) as follows:

\[
W_{m,n,q} = \sum_{x=1}^{N} \sum_{y=1}^{N} \psi_{m,n}(r) e^{-i\theta} f(r, \theta)
\]

with \( r = \sqrt{x^2 + y^2} \), \( \theta = \arctan(y / x) \).

The corresponding wavelet moment invariants can be derived by applying the methodologies presented in section 2.2. An important property of the radial moments defined by (9) is that their amplitudes are rotation invariant. The translation invariants are achieved by moving the origin to the mass center \((x_0, y_0)\) of the image, while scaling invariance is obtained by resizing the image to a fixed size having a predefined area (by a factor \( a = \sqrt{M_{00} / \text{area}} \)) by using the zero\(^{th}\) geometric moment order \((M_{00})\).

The inherent properties of the wavelet moments coming from the wavelet analysis (Strang & Nguyen, 1997), according to which a signal is breaking up into shifted and scaled versions of the base wavelet (mother wavelet), make them appropriate in describing the coarse and fine information of an image. This description has the advantage to study a signal on a time-scale domain by providing time and frequency (there is a relation between scale and frequency), useful information of the signal simultaneously.

4. Experimental study

In order to investigate the discrimination power of the wavelet moments, four machine vision experiments have been arranged by using some well-known benchmark datasets. The following Table 2, summarizes the main characteristics of the datasets being used, from different application fields (object, face, facial expression and hand posture recognition). Moreover, Fig. 2, illustrates six pattern samples from each dataset.

<table>
<thead>
<tr>
<th>Dataset ID</th>
<th>Dataset Name</th>
<th>Type</th>
<th>Num. Classes</th>
<th>Instances / Class</th>
<th>Total Instances</th>
</tr>
</thead>
<tbody>
<tr>
<td>D1</td>
<td>COIL (Nene et al., 1996)</td>
<td>computer vision</td>
<td>10</td>
<td>12</td>
<td>120</td>
</tr>
<tr>
<td>D2</td>
<td>ORL (Samaria &amp; Harter, 1994)</td>
<td>face recognition</td>
<td>40</td>
<td>10</td>
<td>400</td>
</tr>
<tr>
<td>D3</td>
<td>JAFFE (Lyons et al., 1998)</td>
<td>facial expression recognition</td>
<td>7</td>
<td>30,29,32,31,30,31,30</td>
<td>213</td>
</tr>
<tr>
<td>D4</td>
<td>TRIESCH I (Triesch &amp; von der Malsburg, 1996)</td>
<td>hand posture recognition</td>
<td>10</td>
<td>24 (only the dark background)</td>
<td>240</td>
</tr>
</tbody>
</table>

Table 2. Characteristics of the benchmark datasets.

Since the wavelet moment invariants are constructed by applying the same methods as in the case of the other moment families and therefore their performance in recognizing geometrical degraded images is highly dependent on the representation capabilities of the wavelet moments, it is decided to investigate only the discrimination power of the wavelet moments under invariant conditions.
Fig. 2. Six pattern samples of the D1 (1st row), D2 (2nd row), D3 (3rd row) and D4 (4th row) datasets.

The performance of the wavelet moments (WMs) is compared to this of the well-known Zernike (ZMs), Pseudo-Zernike (PZMs), Fourier-Mellin (FMs), Legendre (LMs), Tchebichef (TMs) and Krawtchouk (KMs) ones. In this way, for each dataset, a set of moments up to a specific order per moment family is computed, by resulting to feature vectors of the same length. It is decided to construct feature vectors of 16 moments length which correspond to different order per moment family (ZMs(6th), PZMs(5th), FMs(3rd), LMs(3rd), TMs(3rd), KMs(3rd), WMs(1st)). Moreover, the wavelet moments are studied under two different configurations in relation to the used mother wavelet (WMs-1 uses the cubic B-spline and WMs-2 the Mexican hat mother wavelets respectively).

Furthermore, the Minimum Distance classifier (Kuncheva, 2004) is used to compute the classification performance of each moment feature vector. This classifier operates by measuring the distance of each sample from the patterns that represent the classes’ centre. The sample is decided to belong to the specific class having less distance from its pattern. For the purpose of the experiments the Euclidean distance is used to measure the distance of the samples from the centre classes, having the following form:

$$Euclidean\ Distance\ d(p,s) = \sqrt{\sum_{i=1}^{N} (p_i - s_i)^2}$$

The above formula measures the distance between two vectors the pattern \(p=[p_1, p_2, p_3, \ldots, p_n]\) and the sample \(s=[s_1, s_2, s_3, \ldots, s_n]\), which are defined in the \(R^n\) space. The following Table 3 and Table 4, summarize the classification rates (18) of the studied moment families for different set of training data used to determine the classes’ centres (percent of the entire data – 25%, 50%, 75%, 100%).

$$CRate = \frac{number\ of\ correct\ classified\ samples}{total\ number\ of\ samples}$$

(18)
Table 3. Classification performance of the moment descriptors.

<table>
<thead>
<tr>
<th>Moment Family</th>
<th>Datasets</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>D1</td>
</tr>
<tr>
<td>ZMs</td>
<td>25%</td>
</tr>
<tr>
<td></td>
<td>50%</td>
</tr>
<tr>
<td>PZMs</td>
<td>25%</td>
</tr>
<tr>
<td></td>
<td>50%</td>
</tr>
<tr>
<td>FMs</td>
<td>25%</td>
</tr>
<tr>
<td></td>
<td>50%</td>
</tr>
<tr>
<td>LMs</td>
<td>25%</td>
</tr>
<tr>
<td></td>
<td>50%</td>
</tr>
<tr>
<td>TMs</td>
<td>25%</td>
</tr>
<tr>
<td></td>
<td>50%</td>
</tr>
<tr>
<td>KMs</td>
<td>25%</td>
</tr>
<tr>
<td></td>
<td>50%</td>
</tr>
<tr>
<td>WMs – 1</td>
<td>25%</td>
</tr>
<tr>
<td></td>
<td>50%</td>
</tr>
<tr>
<td>WMs – 2</td>
<td>25%</td>
</tr>
<tr>
<td></td>
<td>50%</td>
</tr>
</tbody>
</table>

Table 4. Classification performance of the moment descriptors.

<table>
<thead>
<tr>
<th>Moment Family</th>
<th>Datasets</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>D1</td>
</tr>
<tr>
<td>ZMs</td>
<td>75%</td>
</tr>
<tr>
<td></td>
<td>100%</td>
</tr>
<tr>
<td>PZMs</td>
<td>75%</td>
</tr>
<tr>
<td></td>
<td>100%</td>
</tr>
<tr>
<td>FMs</td>
<td>75%</td>
</tr>
<tr>
<td></td>
<td>100%</td>
</tr>
<tr>
<td>LMs</td>
<td>75%</td>
</tr>
<tr>
<td></td>
<td>100%</td>
</tr>
<tr>
<td>TMs</td>
<td>75%</td>
</tr>
<tr>
<td></td>
<td>100%</td>
</tr>
<tr>
<td>KMs</td>
<td>75%</td>
</tr>
<tr>
<td></td>
<td>100%</td>
</tr>
<tr>
<td>WMs – 1</td>
<td>75%</td>
</tr>
<tr>
<td></td>
<td>100%</td>
</tr>
<tr>
<td>WMs – 2</td>
<td>75%</td>
</tr>
<tr>
<td></td>
<td>100%</td>
</tr>
</tbody>
</table>

From the above results it is deduced that the percent of the dataset used to determine the classes’ centres is crucial to the recognition performance of all the moment families. The performance of the wavelet moments is very low when compared to the other families. This behaviour is justified by the chosen order (1\textsuperscript{st}) that produces less discriminant features. It seems that the existence of the third parameter (n=0,1,...,2\textsuperscript{m+1}) does not add significant discriminative information to the feature vector, compared to that enclosed by the m and q parameters. As far as the performance of the other moment families is concerned, the experiments show that each moment family behaves differently in each dataset (highest rates: D1(ZMs), D2(LMs), D3(PZMs), D4(PZMs)) with the Pseudo-Zernike moments being the most efficient.

It is worth mentioning that the above rates are not optimized and they can be increased by using a more sophisticated classification scheme (e.g. neural classifier) or by constructing larger or appropriate selected feature vectors.

Besides the classification performance of the compared moment families discussed previously, it is also interesting to analyse their computational load. In almost all the non wavelet moment families (PZMs, FMs, LMs, TMs and KMs) the number of independent
moments that are computed up to the $p^{th}$ order is equal to $(p+1)^2$, while in the case of ZMs is $(p+1)(p+2)/2$ due to some constraints. On the other hand the number of wavelet moments that is computed for the $p^{th}$ order is $(p+1)^2 * (2^p + 1)$ due to the third parameter ($n$) of (16) defined in (14). From this analysis it is obvious that if a common computation algorithm is applied to all the moment families, the time needed to compute the wavelet moments up to a specific order ($p$) is considerable higher.

5. Discussion – Open issues

The previous analysis constitutes the first study of the wavelet moments’ classification performance in well-known machine vision benchmarks. The experimental results highlight an important weakness of the wavelet moments; the computation of many features for a given order (due to the third parameter), which do not carry enough discriminative information of the patterns. On the other hand, this additional parameter adds an extra degree of freedom to the overall computation which needs to be manipulated appropriately. The usage of a feature selection mechanism can significantly improve the classification capabilities of the wavelet moments by keeping only the useful features from a large pool. In this way, the multiresolution nature of the wavelet analysis can be exploited in order to capture the discriminative information in different discrimination levels.

Moreover, it has to be noted that a fast and accurate algorithm for the computation of the wavelet moments need to be developed, since their computation overhead is very high, compared to the other moment families, due to the presence of the third configuration parameter.

6. References


Recently, the algorithms for the processing of the visual information have greatly evolved, providing efficient and effective solutions to cope with the variability and the complexity of real-world environments. These achievements yield to the development of Machine Vision systems that overcome the typical industrial applications, where the environments are controlled and the tasks are very specific, towards the use of innovative solutions to face with everyday needs of people. The Human-Centric Machine Vision can help to solve the problems raised by the needs of our society, e.g. security and safety, health care, medical imaging, and human machine interface. In such applications it is necessary to handle changing, unpredictable and complex situations, and to take care of the presence of humans.

**How to reference**

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