Magnetic Properties and Size Effects of Spin-1/2 and Spin-1 Models of Core-Surface Nanoparticles in Different Type Lattices

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1. Introduction

Dimension in the range of 1 to 100 nm, is called the nano regime. In recent years, nanoparticles/quantum dots are in a class of magnetic nanostructures (Aktaş et al., 2003, 2006; Kartopu & Yalçın, 2010). Nanoparticles (NPs) have been steadily interesting in Physics, Chemistry, Biology, Biomedicine, Spintronics, etc. As the dimensions of magnetic NPs decrease down to the nanometer scale, these core-surface NPs start to exhibit new and interesting physical properties mainly due to quantum size effects. Even the intrinsic physical characteristics of NPs are observed to change drastically compared to their macroscopic counterparts. The potential applications of NPs are very attractive for magneto-sensor, bio-sensor, magneto-electronics, data storage media, computer hard disks, microwave electronic devices, nano-transistors, etc. Especially, the studies of core-surface NPs are extremely important for technology because of transmission of data at high density to optical computer, nanorobot to assemble, compose rigid disk. The nanoparticles have relevance to thin film devices in the new breed of magnetoelectronics, spin-valve, spin-transistors, spin-dependence tunneling devices and etc. (Babin, et al., 2003). The hysteresis in fine magnetic particles applied to new technologies such as Magnetic Random Access Memory (MRAM).

In generally, a nanoparticle is divided into the inner, outer and intermediate regions. These zones are called core (C), surface (S) and core-surface (CS), respectively. The size effects of core-surface NPs are very important for technological and biomedicine applications (Fraerman et al., 2001; Pankhurst et al., 2003). Especially, superparamagnetic (single-domain) NPs are important for non surgical interfere of human body. The ferromagnetic (FM) orders in magnetic systems were dominated as mono-domain (or single-domain) nanoparticles consisting of FM surface and antiferromagnetic (AFM) core regions which couple with each other (Rego & Figueiredo, 2001; Leite & Figueiredo, 2004). At the lower temperatures, the FM surface and AFM core are only ordered in the noninteracting (monodomain) NPs. Stoner-Wohlfarth (Stoner & Wohlfarth, 1948) and Heisenberg model (Heisenberg, 1928) to describe the fine structure were fistly used in detail. Magnetic
evolutions with temperatures (Babin, et al., 2003; Szlaferek, 2004; Usov & Gudoshnikov, 2005), thermodynamic properties (Vargas et al., 2002) and experimental techniques (Wernsdorfer et al., 1995; Wernsdorfer et al., 2000) were performed by different type works for the core-surface NPs. A simple (Bakuzis & Morais, 2004) and the first atomic-scale models of the ferrimagnetic and heterogeneous systems in which the exchange energy plays a central role in determining the magnetization of the NPs, were studied (Kodama et al., 1996, 1999; Kodama & Berkowitz, 1999).

Ising models and real magnets have provided a rich and productive field for the interaction between theory and experiment over the past 86 years (Ising, 1925; Peierls, 1936). Ising models (Erdem, 1995; Keskin, & Erdem 1997; Erdem & Keskin, 2001; Erdem, 2009; Erdem, 2008; Chen & Levy, 1973) and thier variants such as Blume-Capel (Blume, 1966; Capel, 1966; Bakchich, et al., 1994), Blume-Emery-Griffiths (Blume, et al., 1971; Achiam, 1985; Hoston, & Berker, 1991; Bakkali, 1996; Goveas & Mukhopadhyay, 1997; Keskin, et al., 1999; Temizer, 2008) and mixed spin (Benayad & Dakhma 1997; Kaneyoshi, 1998; Albayrak, & Yigit, 2005; Albayrak, & Yigit, 2006; Albayrak, 2007; Albayrak, 2007; Deviren, et al., 2009) models were regarded as theoretical simplifications, designed to model the essential aspects of cooperative system (Kikuchi,1951) without detailed correspondence to specific materials.

In the scope of this chapter, we give a detailed analysis for both spin $S = 1/2$ and $S = 1$ Ising models of homegeneous and core-surface composite NPs to describe the magnetic properties of these particles. These models are based on the pair approximation in the Kikuchi version (Kikuchi, 1974; Keskin, 1986; Erdinç & Keskin, 2002; Yalçın, et al. 2008, Özüm, 2010; Çiftçi, 2011). Incorporating the pair correlations between the spins inside the NPs, we calculated the free energy and minimized with respect to pair variables to obtain the field-cooled magnetization. The field cooling magnetization ($M$) curves of homogeneous and composite NPs are given as a function of the reduced temperature with different radius and different type lattices. Hysteresis loops and coercive fields with their linear fit to the data were plotted as a function of radius and temperature of different NPs. We compared our result with other works (Kaneyoshi, 2005; Kodama, 1999; Usov & Gudoshnikov, 2005).

2. Theoretical model

2.1 Ising model

Ising model, which was introduced in the field of magnetism, is one of the most studied models in modern statistical physics. Although its greatest success during the last century has been in the theory of phase transitions, the model today is viewed as a mathematical structure which can represent a variety of different physical phenomena. In this section, we give a brief summary for the basics of the model before its application to the nanoparticle (NP) magnetism.

Ising model is considered on a regular lattice where each interior site has the same number of nearest-neighbour sites. This is called the coordination number of the lattice and will be denoted by $\gamma$. The system under consideration is composed of the magnetic atoms (also called the spins) located at the lattice sites. It is assumed that, in the thermodynamic limit, boundary sites can be disregarded and that, with $N$ sites, the number of nearest-neighbour site pairs is $N\gamma/2$. The standard Hamiltonian for the the simplest Ising model is given by
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\[ H \{ S_i \} = -J \sum_{\langle ij \rangle} S_i S_j - h \sum_{\langle ij \rangle} (S_i + S_j), \quad \text{with } S_i = \pm 1, \quad (1) \]

where \( h \) is the external magnetic field at the site \( i \) and the summation is performed for nearest-neighbour sites. \( J \) is the exchange interaction between neighbouring sites \( \langle ij \rangle \). Two distinctive cases corresponding to different signs of intersite interaction is considered, i.e., \( J < 0 \) (ferromagnetic (FM) coupling) and \( J > 0 \) (antiferromagnetic (AFM) coupling). The fractions of \( S_i = \pm 1 \) spins given by \( X_i \) are called the point (or state) variables. The \( X_i \) are normalized by \( \sum_{i=1}^{2} X_i = 1 \). The long-range order parameter in the model is called the magnetization \( M \) and it is defined by \( M = X_1 - X_2 \). From this definition and the normalization condition the point variables can be written as

\[ X_1 = \frac{1}{2}(1 + M), \quad X_2 = \frac{1}{2}(1 - M). \quad (2) \]

On the other hand, Eq. (1) may be extended by allowing values \( s = 0, \pm 1, \pm 2, ..., \pm S \) for the variables. It is then possible to consider higher order interactions such as \( K \sum_{\langle ij \rangle} S_i^2 S_j^2 \) or a chemical potential such as \( \Delta \sum_i S_i^2 \). These generalizations are regarded as extensions of the Blume-Emery-Griffiths (BEG) model (Blume et al., 1971). Recently, there have been many theoretical studies of mixed spin Ising systems. These are of interest because they have less translational symmetry than their single-spin counterparts since they consist of two interpenetrating inequivalent sublattices. The latter property is very important to study a certain type of ferrimagnetism, namely molecular-based magnetic materials which are of current interest (Kaneyoshi et al., 1998).

2.2 Pair approximation

In the pair approximation, we consider the pair correlations between the spins. Besides the point variables \( (X_i) \), we introduce new variables \( (Y_{ij}) \), indicating the average number of the states in which the first member of the nearest-neighbour pair is in state \( i \) and the second member in state \( j \). These will be called the pair or bond variables. The bond variables are normalized by \( \sum_{i,j=1}^{n} Y_{ij} = 1 \) and related to the state variables by the relations \( X_i = \sum_{j=1}^{n} Y_{ij} \). Here \( n \) is the number of spin states in the given spin \( S \) model. The interaction energy \( E \) and entropy \( S_E \) can be written in terms of \( Y_{ij} \) as

\[ \beta E = N \sum_{i,j=1}^{n} \eta_{ij} Y_{ij}, \quad (3) \]

\[ S_E = Nk \left( (\gamma - 1) \sum_{i,j=1}^{n} X_i \ln(X_i) - \sum_{i,j=1}^{n} Y_{ij} \ln(Y_{ij}) \right), \quad (4) \]

where \( \beta = 1/kT \) (\( k \) Boltzmann’s constant and \( T \) temperature). In Eq. (3), the parameters \( \eta_{ij} \) are called the bond energies for the spin pairs \( (i,j) \) and determined from Eq. (1). The free energy per site \( \Phi \) can be found from
\[ \Phi = \frac{\beta F}{N} = \frac{\beta}{N}(E - TS_E) \cdot \tag{5} \]

For the system at equilibrium, the minimization of Eq. (5) with respect to \( Y_{ij} \) (\( \partial \Phi / \partial Y_{ij} = 0 \)) leads to the following set of self-consistent equations:

\[ Y_{ij} = \frac{1}{Z} (X_i X_j) \bar{\gamma} e^{-\beta \eta_{ij}} = \frac{e_{ij}}{Z}, \tag{6} \]

where \( \bar{\gamma} = (\gamma - 1) / \gamma \) and \( Z \) is the partition function:

\[ Z = \exp(2\beta \lambda / \gamma) = \sum_{i,j=1}^{n} e_{ij}. \tag{7} \]

In Eq. (7), \( \lambda \) is introduced to maintain the normalization condition. Applications of the above formulation to \( S = 1/2 \) and \( S = 1 \) Ising systems can be found in many works in the literature (Meijer et al., 1986; Keskin & Meijer, 1986; Keskin & Erdinç, 1995; Erdinç & Keskin, 2002). These applications are summerized for comparison in Table 1.

<table>
<thead>
<tr>
<th>Spin state variables (( X_i ))</th>
<th>( S = 1/2 )</th>
<th>( S = 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( X_1, X_2 )</td>
<td>( X_1, X_2, X_3 )</td>
<td></td>
</tr>
</tbody>
</table>

| Spin values (\( S_i \))        | \( +1, -1 \) | \( +1, 0, -1 \) |

<table>
<thead>
<tr>
<th>Bond variables ( Y_{ij}(S_i, S_j) )</th>
<th>( Y_{11}(+1,+1), Y_{12}(+1,-1) )</th>
<th>( Y_{11}(+1,+1), Y_{12}(+1,0), Y_{13}(+1,-1) )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( Y_{21}(-1,+1), Y_{22}(-1,-1) )</td>
<td>( Y_{21}(0,+1), Y_{22}(0,0), Y_{23}(0,-1) )</td>
</tr>
<tr>
<td></td>
<td>( Y_{31}(-1,+1), Y_{32}(-1,0), Y_{33}(-1,-1) )</td>
<td></td>
</tr>
</tbody>
</table>

| Normalization                       | \( \sum_{i=1}^{2} X_j = 1, \sum_{i,j=1}^{2} Y_{ij} = 1 \) | \( \sum_{i=1}^{3} X_j = 1, \sum_{i,j=1}^{3} Y_{ij} = 1 \) |

| Relations between point variables and pair variables | \( X_i = \sum_{j=1}^{2} Y_{ij} \) | \( X_i = \sum_{j=1}^{3} Y_{ij} \) |

| | | |
| | \( X_1 = Y_{11} + Y_{12} \) | \( X_1 = Y_{11} + Y_{12} + Y_{13} \) |
| | \( X_2 = Y_{21} + Y_{22} \) | \( X_2 = Y_{21} + Y_{22} + Y_{23} \) |
| | \( X_3 = Y_{31} + Y_{32} + Y_{33} \) | |

<table>
<thead>
<tr>
<th>Avarage magnetization (( M \equiv \langle S_i \rangle ))</th>
<th>( M = X_1 - X_2 )</th>
<th>( M = X_1 - X_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M = Y_{11} + Y_{12} - (Y_{21} + Y_{22}) )</td>
<td>( M = Y_{11} + Y_{12} + Y_{13} )</td>
<td></td>
</tr>
<tr>
<td>( - (Y_{31} + Y_{32} + Y_{33}) )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| Quadrupole moment \( Q \equiv \langle Q \rangle \equiv \langle S_i^2 \rangle \) | \( Q = X_1 + X_3 \) | \( Q = Y_{11} + Y_{12} + Y_{13} + Y_{31} + Y_{32} + Y_{33} \) |

Table 1. Comparison of the \( S = 1/2 \) and \( S = 1 \) Ising models under the pair approximation.
3. Magnetic properties of $S=1/2$ and $S=1$ Ising nanoparticles

The magnetic particles become single domain below a critical size in contrast with the usual multidomain structure of the bulk materials. Therefore, in the scope of this section, we study size effects and magnetic properties of monodomain NPs manifestations. We consider a noninteracting monodomain NP with Ising spins on both hexagonal and square lattices for any two-dimensional (2D) regular arrays which can also be extended to hexagonal closed packed (hcp) and simple cubic (sc) lattices for the three-dimensional (3D) case as in Fig.1. The shells and their numbers originate from the nearest-neighbor pair interactions for the hexagonal and square lattices in 2D. In this structure, number of shells for hexagonal and square lattices can be associated with radius ($R$) of the NPs. This behaviour can be seen explicitly in Fig. 2 for hexagonal lattice and in Fig. 3 for square lattice. The value of $R$ includes number of shells and the size of a NP increases as the number of shells increases. Therefore, we have considered Ising spins in three parts that are core ($C$), core-surface ($CS$) and surface ($S$) within the NP. Each of these parts contain core spin number ($N_C$), core-surface spin number ($N_{CS}$) and surface spin number ($N_S$), respectively. The total number of spins ($N$) in a single NP involves core and surface spin numbers, i.e. $N = N_C + N_S$. The $C$ and $S$ spins interact ferromagnetically ($J < 0$) or antiferromagnetically ($J > 0$). The $S=1/2$ and $S=1$ Ising model Hamiltonians with dipol-dipol interaction ($J$) for a NP is given by

$$H = H_C + H_{CS} + H_S,$$

with

$$H_C = -J_C \sum_{\langle ij \rangle} S_i S_j - h \sum_{\langle ij \rangle} (S_i + S_j),$$

$$H_{CS} = -J_{CS} \sum_{\langle ij \rangle} S_i \sigma_j,$$

$$H_S = -J_S \sum_{\langle ij \rangle} \sigma_i \sigma_j - h \sum_{\langle ij \rangle} (\sigma_i + \sigma_j),$$

where $J_C$, $J_{CS}$ and $J_S$ represent exchange interactions for $C$, $CS$ and $S$ atoms, respectively. If $J_C = J_{CS} = J_S$, the NP is known as a homogeneous NP. It is called a composite NP when $J_C \neq J_{CS} \neq J_S$, $J_C = J_{CS} \neq J_S$, $J_C \neq J_{CS} = J_S$ or $J_{CS} \neq J_C = J_S$. In Eqs. (9), $S_i$ is called the core spin values and $\sigma_i$ is the surface spin values. These variables take on the values $\pm 1$ for $S=1/2$ and $0, \pm 1$ for $S=1$ Ising systems.

The interaction energies for $S=1/2$ and $S=1$ models of an Ising NP in 2D can be written shortly in term of $Y_{ij}$ as

$$\beta E = \sum_{\langle ij \rangle} (N^C_P \eta^C_{ij} + N^{CS}_P \eta^{CS}_{ij} + N^S_P \eta^S_{ij}) Y_{ij},$$
where the numbers of spin pairs for $C$, $CS$ and $S$ regions are defined by $N_p^C = (N_C \gamma_C / 2) - N_{CS}$, $N_p^{CS} = 2N_{CS} \gamma_{CS} / 2$ and $N_p^S = N_S \gamma_S / 2$, respectively. Similarly $\gamma_C$, $\gamma_{CS}$, $\gamma_S$ denote the coordination numbers for these regions. Since we consider the arrays of Ising spins for a structure made up of bigger particles in 2D, we choose $\gamma_C = 6$, $\gamma_{CS} = \gamma_S = 2$ for hexagonal lattice and $\gamma_C = 4$, $\gamma_S = 0$, $\gamma_{CS} = 2$ for square lattice, as depicted in Figs. 2 and 3, respectively. The values of these numbers for both structures in 2D are given in Table 2. The expressions for the bond energies $\eta_{ij}^C$, $\eta_{ij}^{CS}$ and $\eta_{ij}^S$ of three regions are found using Eq. (9) for both models, as listed in Table 3.

Fig. 1. A spherical monodomain magnetic NP spaced coherently in a form of 3D arrays. The shape of a single NP consists of the hexagonal lattice. The dashed lines displayed shells of spins in a 2D finite arrays. The radius of NP ($R$) includes shell numbers. The insets exhibit coordination numbers ($\gamma$) of hexagonal closed packed (hcp) and simple cubic (sc) lattices in 3D as well as hexagonal and square lattices in 2D structure.
Fig. 2. Schematic representation of a NP on a hexagonal lattice in 2D exhibiting nine shells of spins. Small full coloured circles correspond to ten radius of the NP. Solid grey lines are number of the core-shell pairs. Solid coloured lines are number of shell pair (this line corresponds to core and shell number for $R = 2$).
Fig. 3. Same as Fig. 2 but for the NP on square lattice in 2D.

<table>
<thead>
<tr>
<th>Lattice Type</th>
<th>R</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
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<tbody>
<tr>
<td>Hexagonal Lattice in 2D</td>
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<td></td>
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<td></td>
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<td></td>
</tr>
<tr>
<td>( N_C )</td>
<td></td>
<td>7</td>
<td>19</td>
<td>37</td>
<td>61</td>
<td>91</td>
<td>127</td>
<td>169</td>
<td>217</td>
<td>271</td>
</tr>
<tr>
<td>( N_S )</td>
<td>12</td>
<td>18</td>
<td>24</td>
<td>30</td>
<td>36</td>
<td>42</td>
<td>48</td>
<td>54</td>
<td>60</td>
<td></td>
</tr>
<tr>
<td>( N_{CS} )</td>
<td>9</td>
<td>15</td>
<td>21</td>
<td>27</td>
<td>33</td>
<td>39</td>
<td>45</td>
<td>51</td>
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<tr>
<td>( N_P^C )</td>
<td>12</td>
<td>42</td>
<td>90</td>
<td>156</td>
<td>240</td>
<td>342</td>
<td>462</td>
<td>600</td>
<td>756</td>
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<tr>
<td>( N_P^S )</td>
<td>12</td>
<td>18</td>
<td>24</td>
<td>30</td>
<td>36</td>
<td>42</td>
<td>48</td>
<td>54</td>
<td>60</td>
<td></td>
</tr>
<tr>
<td>( N_P^{CS} )</td>
<td>18</td>
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<td>42</td>
<td>54</td>
<td>66</td>
<td>78</td>
<td>90</td>
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<td>Square Lattice in 2D</td>
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</tr>
<tr>
<td>( N_C )</td>
<td></td>
<td>5</td>
<td>13</td>
<td>25</td>
<td>41</td>
<td>61</td>
<td>85</td>
<td>113</td>
<td>145</td>
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<tr>
<td>( N_S )</td>
<td>8</td>
<td>12</td>
<td>16</td>
<td>20</td>
<td>24</td>
<td>28</td>
<td>32</td>
<td>36</td>
<td>40</td>
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</tr>
<tr>
<td>( N_{CS} )</td>
<td>6</td>
<td>10</td>
<td>14</td>
<td>18</td>
<td>22</td>
<td>26</td>
<td>30</td>
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<td>36</td>
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<td>100</td>
<td>144</td>
<td>196</td>
<td>256</td>
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<td>( N_P^{CS} )</td>
<td>12</td>
<td>20</td>
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<td>52</td>
<td>60</td>
<td>68</td>
<td>76</td>
<td></td>
</tr>
</tbody>
</table>

Table 2. Numbers of the spins and spin pairs within the \( C \), \( CS \) and \( S \) regions (Yalçın, et al., 2008).
### Table 3. Bond energies for $S = 1/2$ and $S = 1$ models.

<table>
<thead>
<tr>
<th>Spin Model</th>
<th>Bond energy for Core ($\eta_{ij}^C$)</th>
<th>Bond energy for Core-Surface ($\eta_{ij}^{CS}$)</th>
<th>Bond energy for Surface ($\eta_{ij}^S$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S = 1/2$ ($n = 2$)</td>
<td>$\eta_{11}$: $-J_C - 2h$</td>
<td>$-J_{CS}$</td>
<td>$-J_S - 2h$</td>
</tr>
<tr>
<td></td>
<td>$\eta_{12}$: $+J_C$</td>
<td>$+J_{CS}$</td>
<td>$+J_S$</td>
</tr>
<tr>
<td></td>
<td>$\eta_{21}$: $+J_C$</td>
<td>$+J_{CS}$</td>
<td>$+J_S$</td>
</tr>
<tr>
<td></td>
<td>$\eta_{22}$: $-J_C + 2h$</td>
<td>$-J_{CS}$</td>
<td>$-J_S + 2h$</td>
</tr>
<tr>
<td>$S = 1$ ($n = 3$)</td>
<td>$\eta_{11}$: $-J_C - 2h$</td>
<td>$-J_{CS}$</td>
<td>$-J_S - 2h$</td>
</tr>
<tr>
<td></td>
<td>$\eta_{12}$: $-h$</td>
<td>$0$</td>
<td>$-h$</td>
</tr>
<tr>
<td></td>
<td>$\eta_{13}$: $+J_C$</td>
<td>$+J_{CS}$</td>
<td>$+J_S$</td>
</tr>
<tr>
<td></td>
<td>$\eta_{21}$: $-h$</td>
<td>$0$</td>
<td>$-h$</td>
</tr>
<tr>
<td></td>
<td>$\eta_{22}$: $0$</td>
<td>$0$</td>
<td>$0$</td>
</tr>
<tr>
<td></td>
<td>$\eta_{23}$: $+h$</td>
<td>$0$</td>
<td>$+h$</td>
</tr>
<tr>
<td></td>
<td>$\eta_{31}$: $+J_C$</td>
<td>$+J_{CS}$</td>
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</tr>
<tr>
<td></td>
<td>$\eta_{32}$: $+h$</td>
<td>$0$</td>
<td>$+h$</td>
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<tr>
<td></td>
<td>$\eta_{33}$: $-J_C + 2h$</td>
<td>$-J_{CS}$</td>
<td>$-J_S + 2h$</td>
</tr>
</tbody>
</table>

Using Eq. (6) we obtain four self-consistent equations of $Y_{ij}$ for $S = 1/2$ model of core-surface NPs:

\[
Y_{11} = \frac{1}{Z} (X_1 X_1)^T \exp \left[ -\beta \left( N_P^C \eta_{11}^C + N_P^{CS} \eta_{11}^{CS} + N_P^S \eta_{11}^S \right) \right] = \frac{e_{11}}{Z},
\]

\[
Y_{12} = \frac{1}{Z} (X_1 X_2)^T \exp \left[ -\beta \left( N_P^C \eta_{12}^C + N_P^{CS} \eta_{12}^{CS} + N_P^S \eta_{12}^S \right) \right] = \frac{e_{12}}{Z},
\]

\[
Y_{21} = \frac{1}{Z} (X_2 X_1)^T \exp \left[ -\beta \left( N_P^C \eta_{21}^C + N_P^{CS} \eta_{21}^{CS} + N_P^S \eta_{21}^S \right) \right] = \frac{e_{21}}{Z},
\]

\[
Y_{22} = \frac{1}{Z} (X_2 X_2)^T \exp \left[ -\beta \left( N_P^C \eta_{22}^C + N_P^{CS} \eta_{22}^{CS} + N_P^S \eta_{22}^S \right) \right] = \frac{e_{22}}{Z}.
\]

Similarly, nine self-consistent equations of $Y_{ij}$ for $S = 1$ model of these particles are
Eqs. (11) and (12) are solved numerically using Newton-Raphson method and normalized magnetization \( M \) is easily calculated for both \( S = 1/2 \) and \( S = 1 \) models of homogeneous and core-surface composite NPs. Results are shown as the magnetization curves and hysteresis loops in Figs. 4–9.

4. Result and discussions

4.1 Magnetization

The evolution of normalized magnetization \( M \) as a function of the reduced temperature \( \frac{k_B T}{J} \) and particle size dependence of the transition temperature \( T_C \) from FM to paramagnetic (PM) phases for homogeneous and composite Ising NPs are shown in Figs. 4 and 5, respectively. The magnetization curves in Fig. 4 are plotted for \( S = 1/2 \) and \( S = 1 \) models of homogeneous NPs using the FM core \( (J_0 = 1, J_C = 1) \), FM surface \( (J_S = J_0) \) and FM core-surface \( (J_{CS} = J_0) \) interactions and the curves in Fig. 5 are obtained for both models of the composite NPs based on FM core \( (J_C = J_0) \), FM surface \( (J_S = J_0) \) and AFM core-surface \( (J_{CS} = -J_0) \) interactions. In the plots, different values for the applied magnetic field are considered \( (h = 0.0-0.1) \). The solid curves in the figures correspond to hexagonal lattice while dotted ones denote the square lattice. As seen from the figures, the changes in the magnetization with the reduced temperature point out an interesting aspect for NPs on the hexagonal and square lattices in 2D. The magnetization curves are decreasing from one (1) to zero (0) value while the reduced temperature is increasing (Figs. 4(a), 4(b), 5(a), 5(b)). These decreases terminate at the phase transition temperature (or Curie temperature, \( T_C \)) from FM phase to PM phase for \( h = 0.0 \), seen in Figs. 4(c) and 5(c). To show the size dependence of the critical temperature we plot \( \frac{1}{T_C} \) vs \( R \) in Figs. 4(d) and 5(d). All critical temperature values follow a linear increase with the particle radius. With increase in the
particle radius it approaches to the Curie temperatures of the bulk materials. This is consistent with the mean-field approximation for the magnetic structure of Heisenberg NP (Usov & Gudoshnikov, 2005). On the other hand, it is interesting that composite $S = 1/2$ and $S = 1$ Ising NPs show smaller transition temperatures than their corresponding homogeneous NPs. This can easily be seen by comparing the same coloured fits in Figs. 4(d) and 5(d).

Fig. 4. Normalized magnetization ($M$) vs. reduced temperature ($k_B T / J_0$) and particle size dependence of the transition temperature $T_C$ from FM to PM phases for homogeneous $S = 1/2$ and $S = 1$ Ising NPs on the hexagonal and square lattices. $J_0 = J_C = J_{CS} = J_S = 1$ and $h = 0.0-0.1$. 
4.2 Hysteresis loops

The magnetic field evolution of normalized magnetization (or hysteresis loops) for the homogeneous $S = 1/2$ and $S = 1$ Ising NPs which has different particle sizes and their corresponding coercive field vs. $R^{-2}$ variation are given Figs. 6 and 7, respectively. We consider a FM coupling in core $(J_C = J_0)$, surface $(J_S = J_0)$ and core-surface $(J_{CS} = J_0)$ regions with $J_0 = 1$ on the hexagonal and square lattice structures. The hysteresis curves of small diameters, namely with radius $R = 2, 4, 5$ in Figs. 6(a)-6(d), are approximately the same. These behaviours are called superparamagnetic (SP) regime. However, the loops strongly depend on the size of NP. The hysteresis curves of high diameter values change sharply, as also shown in Figs. 6(a)-6(d). Moreover, the hysteresis curves for this type of NPs are broadening while the diameter of NPs is increasing so that it approaches to bulk.
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The size dependence of the coercive fields $h_C$ is determined from the hysteresis loops in Fig. 7. In Fig. 7, the full red and blue circles correspond to the curves obtained for $R = 2 - 9$ in the Figs. 6(a) and 6(c), respectively. Similarly, the open red and blue circles correspond to the curves obtained for $R = 3 - 11$ and $R = 4 - 10$ in Figs. 6(b) and 6(d), respectively. The straight solid and dotted lines are the results from a linear fit to the calculated data. From this fit, it is obvious that the coercive field ($h_C$) depends linearly on $1/R^2$.

![Diagram](https://www.intechopen.com)

Fig. 6. (a) Hysteresis loops of a homegeneous $S = 1/2$ Ising NP on the hexagonal lattice for various sizes. (b) Same as Fig. 6(a) but for NP on the square lattice. (c) Hysteresis loops of a homegeneous $S = 1$ Ising NP on the hexagonal lattice for different sizes. (d) Same as Fig. 6(c) but for NP on the square lattice. $J_0 = J_C = J_{CS} = J_S = 1$ and $T = 300J_0/k_B$.

Magnetic hysteresis loops of composite $S = 1/2$ and $S = 1$ Ising NPs on the hexagonal and square lattice (in 2D) structures for various values of particle sizes are shown in Fig. 8. The exchange interactions in the $C$ and $S$ regions are FM, i.e. $J_0 = J_C = J_S$, while the coupling
between $C$ and $S$ is an AFM exchange constant $I_{CS} = -I_0$ for each type of NP. From the figure, it is clear that the hysteresis loops strongly depend on the particle size. The loops for the $S = 1/2$ and $S = 1$ Ising NPs on the hexagonal lattice change suddenly in low radius values while those for the $S = 1/2$ and $S = 1$ Ising NPs on the square lattice in high radius values.

Fig. 7. The coercive field ($h_c$) plotted as a function of $R^{-2}$ for the hysteresis loops of the homogeneous NP in Fig. 6.
Finally, the evolutions of hysteresis loops and their coercive field according to the temperature of composite Ising NPs are seen to change monotonically as the temperature increases, illustrated in Fig. 9(a) and 9(b), respectively. Since the loops for both models of NPs on the hexagonal and square lattices display the same behaviour we have drawn only the loops of $S = 1/2$ Ising NP on the hexagonal lattice. In this case, hysteresis for the NP is in superparamagnetic (SP) regime at $700J_0/k_B$. But, the loops for the temperature regime between $150J_0/k_B - 600J_0/k_B$ belong to the FM phase (Fig. 9(a)). The temperature dependence of the coercivity ($h_C$) are determined from the hysteresis loops of Fig. 9(a), as given in Fig. 9(b).
5. Conclusion

In the scope of this chapter, we have focused on the magnetic properties with size effects for homogeneous and core-surface composite NPs which have Ising spins \( (1, 1/2) \) on 2D lattice structures (hexagonal, square). The transition for all NPs corresponds to a second-order phase transition in the absence of magnetic field \( (h \approx 0) \). The spin disorder can be caused by lower coordination of the surface atoms in core-surface NPs broken exchange interactions that produce spin-glass (SG) like state of spatially disordered spin in the surface captions with inhomogeneous surface effects (Kodama, 1999; Kaneyoshi, 2005). Our theoretical observations are scrutinized below briefly.

i. All critical temperature \( (\sqrt{I_C}) \) values of both types of Ising NPs on 2D lattice structures follow a linear increase with the particle size. With increase in the NP size it approaches to the Curie temperature of the bulk materials. These results agree with the mean-field magnetic structure of Heisenberg NPs (Usov & Gudoshnikov, 2005).

ii. From the hysteresis loops for the homogeneous \( S = 1/2 \) and \( S = 1 \) Ising NPs which have different sizes and corresponding coercive field \( (h_C) \) vs. \( R^2 \) variations, it is clearly seen that the coercivity strongly depends on the particle size. Due to the superparamagnetic regime the hysteresis curves of small diameters are almost independent of each other while the curves of big diameters sharply change. This shows that the NP approaches to bulk materials.

iii. The hysteresis loops at different temperatures show a monotonic change in the coercive field of composite Ising NPs on 2D lattice structures. This property probably is an important aspect in the future high-density magnetic data storage.
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7. References


In the last few years, Nanoparticles and their applications dramatically diverted science in the direction of brand new philosophy. The properties of many conventional materials changed when formed from nanoparticles. Nanoparticles have a greater surface area per weight than larger particles which causes them to be more reactive and effective than other molecules. In this book, we (InTech publisher, editor and authors) have invested a lot of effort to include 25 most advanced technology chapters. The book is organised into three well-heeled parts. We would like to invite all Nanotechnology scientists to read and share the knowledge and contents of this book.

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