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Reconciling Orthodox and Heterodox Approaches to Economic Growth – A Modeling Proposal

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1. Introduction

Modern growth theories have paid little attention to the understanding of the demand side phenomenon, associated to the increasing relevance of consumption activities within the society (Silva and Teixeira, 2009). In fact, the orthodox theory on economic growth neglects the aggregate demand (Dutt, 2006), whether in its shape – initially neoclassic – (Solow, 1956), or in the new or endogenous theory of growth (see Barro & Sala-i-Martin, 1995 for a survey). In the orthodox theory of economic growth, the focus lies traditionally on the logic of capital accumulation and on the influence of the different ways of technical change. However, even though technology, production and supply are essential for an economy to grow, that does not constitute the whole story. Historically, there have been, and there still are, massive changes in the products and services offered to the final demand, as well as changes in the consumers’ behaviour and consumption patterns throughout the process of economic growth (Witt, 2001a).

Some authors within more heterodox tendencies, namely evolutionary (e.g., Metcalfe, 2001; Saviotti, 2001; Witt, 2001a,b), argue that what happens precisely on the demand side constitutes an essential part of the economic growth theory. According to this perspective, the consumer’s role as an “innovative” being has been deeply underestimated in the Schumpeterian approaches to innovation and growth, almost exclusively based on the supply side (Metcalfe, 2001).

Trying to requalify the important role played by the demand on the process of economic growth and thus reconcile, to a certain extent, the two theoretical approaches for growth (orthodox and evolutionary) mentioned above, this chapter aims at presenting an endogenous model of growth where growth is induced by improvements in the products’ quality. Using the basic models of Grossman & Helpman (1991a, 1991b), yet, on the contrary, a consumption index set up by differentiated goods and a homogeneous good, the proposed model highlights the crucial influence that the demand has, also granting consumers with the importance that they have in the real world.

An innovative result of the model developed for this article points out to the fact that if the weight of the differentiated goods on the consumption index is relatively lower than the
market balance rate, it will always be lower than what would be socially expected (optimum), regardless of the dimension of the technological advances. Additionally, when consumers present a relatively biased consumption as far as differentiated goods are concerned, there is a possibility of situations where the market balance ratio generates excessive incentives in terms of welfare, in the event of a higher profit caused by a higher demand for differentiated goods. Therefore, these incentives will be higher than what would be socially suitable.

This article is structured as follows: in the following section, the model is briefly described. In the third section, the analytical structure of the model is developed, with the aim of determining the market balance innovation rate and the growth rate of the consumption index. We also analyse matters of welfare (Section 4) since we are in the presence of a model that involves market structures of imperfect competition (for the goods differentiated by quality, the market price is higher than the production’s unit cost) and knowledge spillovers that involve scale increasing incomes. In concrete terms, the optimum growth rate is determined and, after it is compared with the market’s growth rate, we draw conclusions on the insufficient or excessive incentives to R&D. Finally, the article is concluded with a summary of the model’s main results. In order to facilitate reading fluency, we refer to the Annex section where we provide several technical steps for the model’s result derivation.

2. Brief description of the model

The composition of the economic system has deeply changed throughout time (Saviotti, 2001). The observation that the system has been subject to numerous qualitative changes throughout time is an unquestionable fact for any economist (Saviotti, 1988, 1991, 1994, 1996). Modern economies include a large number of entities (products, services, production methods, competences, individual and organisational actors, institutions) that are qualitatively new and different when compared to the ones in previous economic systems (Saviotti & Mani, 1995). In fact, the economic development and growth depend on the ability that the economic system has of creating new (or new versions of) goods and services. However, for them to contribute to economic growth, such goods and services will have to be acquired by the consumers. The dynamics of the development of demand thus constitutes a fundamental aspect for economic development (Saviotti, 2001; Sonobe, Hu & Otsuka, 2004).

Using this empirical intuition, in the theoretical model described in this section, the economy grows due to the continuous improvement of a set of differentiated products, thus originating an accrual of the consumers’ welfare. The utility/level of satisfaction of the consumers depends on an aggregate consumption index set up by a fixed number of differentiated products, and the increase in the level of satisfaction is exclusively determined by the quality improvements in each of those products.

The use of this economic growth approach via product innovation and based on the efforts of the companies specifically oriented to product improvement has to do with the fact that this framework is currently the one that theoretically constitutes a more complete formulation of the microeconomic basis of the aggregate phenomenon growth. Therefore, it is much more connected to the complexity of the behaviour of the economic agents that underlie the economic relations. At the same time, the use of this approach is the result (and perhaps mainly) of the empirical observation that most part of the companies’ research efforts are nowadays destined to improve the products already in the market, while radical
innovations (new products and new processes) are a less frequent phenomenon. Following that option, the human capital factor was introduced so that its importance in a county’s innovation process would be shown. As such, human capital is quite clear in the extraordinary performance of economies, such as the economies of South Korea and Taiwan, characterised by large investments in human capital (Collins, 1990).

The used structure and procedures are based essentially on the models presented by Aghion & Howitt (1992), Grossman & Helpman (1991a, 1991b) and Segerstrom (1991). Particularly, the type of explanation carried out by Grossman & Helpman (1991b) is followed closely. This is a double differentiation as far as products are concerned. There is a fixed set of differentiated products that are included in each individual’s consumption basket (horizontal differentiation), and each of them is available in an unlimited number of different qualities (vertical differentiation). It is the dynamics included in the products’ quality evolution that promotes economic growth. The economic growth rate depends on the composition, dimension and allocation of available resources at each moment in time and, particularly, of the human capital employed in the research that produces new qualities for each differentiated product.

As we have seen in Grossman & Helpman (1991a, 1991b) and in Aghion & Howitt (1992), Research and Development (R&D) is seen as an essential activity and its success encourages the improvement in the quality of the existing products, thus promoting economic growth as well. Considering that human capital is the fundamental input of the R&D activity, it constitutes the accumulative resource that is crucial for the growth process. According to Romer (1990), human capital is the “scale variable” for economic growth. An economy with less human capital has a meaningless research sector and, therefore, it is unfit to cause improvements in the quality of products. Thus, this economy is incapable of generating economic growth.

Following Schultz (1961), Becker, Murphy & Tamura (1990), Romer (1990), Grossman & Helpman (1991b) and Barro (1991), human capital is the accumulation of effort destined to education and learning. This capital is considered to be constant at each moment, neglecting the cumulative effects that are inherent to this factor. This apparently extreme assumption is justified by the fact that the model is one of technological progress. Here, the interest is focused on the relation human capital \(\Rightarrow\) technological progress, and not the other way around.

1 According to a study from the Gabinete de Estudos e Planeamento do Ministério da Indústria e Energia (GEPIE, 1992), April 1992, based on a survey carried out on 3276 industrial undertakings (25% of the companies in mining and manufacturing industries with more than 10 employees) during the period of 1987-1989, the improvement of the existing products constituted the innovation of the most frequent product (69.1%), followed by the introduction of several new products and a new product, registering 26.8% and 15.2%, respectively. However, it is important to mention that such percentages tend to underestimate the importance of vertical differentiation since many of the new products end up replacing the ones that used to perform similar functions.

2 Grossman and Helpman (1992, ch.3) and Romer (1990) present models that are similar to the one we are going to develop. However, in this model, economic growth is based on the increase in the number of differentiated products.

3 Even though in a finite life horizon an individual’s human capital cannot grow without a limit, the qualifications that an individual acquires may be applied to a set production technologies, from where the value of that capital will continue to increase throughout time, as well as the growth rate. This cumulative effect is neglected by assumption.
Unlike Grossman & Helpman (1991b) and Romer (1987), workforce is not seen as an input base for research activity. Even though it is plausible that an economy with a higher amount of work carries out more R&D, thus generating a higher product innovation rate, its consideration would imply that larger economies (with a higher labour/population) would, *ceteris paribus*, tend to grow more rapidly and that is precisely the result that should be avoided. Generally, a faster growth will only take place if there is an increase in the amount of factors that the economic growth promoting activities (R&D in this model) use more frequently. Thus, in this model, human capital constitutes the correct measure of an economy’s scale and not the population’s (Romer, 1990).

The recent theoretical literature on industrial research as an endogenous growth engine focuses on two fundamental concepts: the first concept portrays the fact that the companies that maximise profit, seek to increase their market power through the production of goods (quality goods) that are better than their direct competitors’ goods. In this context, recent goods and services replace the older ones throughout time, taking advantage of a temporary profit and considering that afterwards they will also be replaced. Here, there is an implicit idea of what Aghion & Howitt (1992), following Schumpeter, designate as “the effect of creative destruction” which simultaneously promotes and limits the private value of industrial innovation; the second concept points to the fact that knowledge is a “public good” that promotes scale growing incomes for the economy as a whole. Here, the technological spillovers play a fundamental role when a company (innovator) puts a new product on the market, thus enabling researchers (potential innovators) to obtain information on production technology and on the new product’s features. Thus, competitors may then start joining efforts to carry out research in order to improve the “state-of-the-art” product, even if they haven’t succeeded previously while developing the product. According to Caballero & Jaffe (1993, p.16), “[i]n the process of creating new goods, inventors rely and build on the insights embodied in previous ideas; they achieve their success partly by ‘standing upon the shoulders of giants’. ”. This way, inventions contribute to the public knowledge and that same knowledge facilitates subsequent innovation.

In this context, companies (innovators) invest in resources, hoping to discover something with a commercial value, which means that they are hoping to be capable of getting a positive profit from their research efforts. This way, in order to restore their initial investments in research, these companies must be capable of selling their products at prices that exceed the respective unit costs. This means that “... some imperfect competition in product markets is necessary to support private investments in new technologies.” (Grossman & Helpman, 1994, p. 32).

The process of innovation “à la Schumpeter” represents the fact that the successful innovator (the most recent one) replaces the previous leader, taking part of the profits in the product’s industry.

The modelling of the growth process on a microeconomic level shows that the process is uneven and stochastic. Companies compete amongst themselves in order to launch the new generation of the product and in certain industries there may be long periods when they do not succeed, whereas other industries may experience continuous and quick successful research. Despite that fact, on a macroeconomic level, the aggregation partly dissolves this microeconomic turbulence – and, considering the existence of a high number of quality
improvements in the (fixed) set of products, the economy as a whole grows at a steady pace. Thus, research costs and benefits determine the rhythm of long-term growth.

3 Structure of the basic model

3.1 Initial remarks

Here, we will consider a continuum of differentiated products, indexed by $k$, where $k \in [0, N]$, and $N$ is constant. In order to simplify, we will normalise $N$ to 1.

Each $k$ differentiated product has its own quality and it can have an unlimited number of qualities.

Each innovation is built from the inside, which means that when a research company achieves a technological advance (product improvement) in the line of product $k$, the state-of-the-art product in that industry moves forward one generation.\(^4\)

Where, $q_j(k) \equiv$ quality of generation $j$ of product $k$ ($j = 0, 1, 2, ...$).

Each new generation of the product provides $\lambda$ times more services than the product of the previous generation $q_j(k) = \lambda q_{j-1}(k) \quad \forall j, k$, where $\lambda > 1$ is exogenous, constant and common to all products ($\lambda \equiv$ dimension of the innovation).

The units are chosen so that the lowest quality of each product (the one available in generation 0) provides a service unit, $q_0(k) = 1$. Here, we consider that $q_j(k) = \lambda^j \quad \forall j, k, \lambda > 1$.

We define the state-of-the-art product as the most recently invented product available (i.e. the one with the highest quality), for each moment in time.

At each moment in time, entrepreneurs put their efforts on a sole product and compete amongst themselves in order to produce the next generation of that same product. Thus, the entrepreneurs that have the ability to create the state-of-the-art product or any generation prior to that product will compete amongst themselves as oligopolists. The result of this competition is a profit flow that represents the reward that the companies receive for their successful research activities. In this context, the successful research efforts promote new improvements that cause researchers to continuously compete in order to place the next generation of a certain product on the market.

As a result, the product’s quality distribution goes through an upward evolution throughout time (as well as the welfare of the economic agents), and each product follows a stochastic progression of quality sequences. Despite this randomness at the level of industry, the process is constant in aggregate terms. As it will be confirmed, in the steady-state, the consumption index grows at a constant and determined rate.

Consumers, on the other hand, have at their disposal a set of differentiated goods (represented by vector $X=[x_1, x_2, ..., x_k]$) and a homogeneous good, represented by scalar $Y$ - cash\(^5\), and the price is by definition a unit price.

\(^4\) Hence the expression “... ‘standing upon the shoulders of giants.’” by Newton and quoted by Caballero and Jaffe (1993, p. 16).

\(^5\) The prices of $X$ have to do with the homogeneous product $Y$. 

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The market of the homogeneous good, \( Y \), is one of perfect competition, whereas the market of the differentiated goods, \( X \), is characterised by a Bertrand competition.

There are two production factors, unskilled labour (\( L \)) and human capital stock (\( H \)). As far as the intensity of factor use is concerned, it is assumed that the R&D activity and the production of differentiated goods, \( X \), exclusively employ human capital, while the production of the homogeneous good demands human capital and unskilled labour (specific factor of \( Y \)).

Since the intention here is to emphasize the importance of human capital on economic growth, included in an endogenous technological progress model, we do not pay attention to important aspects such as population growth and work supply, thus avoiding questions regarding fertility, participation in workforce and variation of the working hours. We also consider that the population’s human capital stock (i.e. given at any moment in time) and the human capital’s fraction that is supplied to the market are exogenous. Thus, the aggregate offers of human capital stock, \( H \), and labour, \( L \), are fixed. Therefore, this is a model of endogenous growth in relation to technological progress and not to human capital.

### 3.2 Preferences of the consumers

The preferences of the consumers are similar to the ones proposed in Grossman & Helpman (1991b), with the additional consideration of the homogeneous good. The economy consumers provide a fixed amount of unskilled labour and human capital in exchange for a salary. At the same time, they receive interests on assets, they buy goods for consumption and save money by accumulating additional assets. While carrying out their plans, consumers take the welfare of their descendants into consideration (Barro, 1974). Therefore, the present generation maximises intertemporal utility in an infinite time frame.

Thus, the representative consumer maximises total utility, \( U_t \), provided by the following intertemporal utility function:

\[
U_t = \int_0^\infty e^{-\rho t} C(t) dt \\
U_t = \int_0^\infty e^{-\rho t} \left[ \alpha \log u(t) + (1 - \alpha) \log Y(t) \right] dt
\]

where

\( \rho \) is the intertemporal discount rate, \( \rho > 0 \); \(^7\)

\(^6\) We specify a logarithmic utility function that constitutes a special case of the more general utility function, \( U_t = \int_0^\infty e^{-\rho t} \frac{C(t)^{1-\sigma} - 1}{1 - \sigma} dt, \sigma > 0 \), where the elasticity of the intertemporal substitution is unitary (\( \sigma = 1 \)). Even though this last specification allows a wider interval of substitution elasticity, the general part of what we can take from this does not contribute much to the analysis. Therefore, we decided to sacrifice generality in order to simplify the analysis.

\(^7\) A positive value for \( \rho \) means that utility is less valued when it is received later. Ramsey (1928) assumes that \( \rho = 0 \), taking the maximising agent into account. The maximising agent is more of a social planner...
\( \alpha \) is the utility fraction of the differentiated goods on the total utility of the individual; 
\( u(t) \): is the aggregate consumption of the differentiated products, in moment \( t \); 
\( y(t) \): is the consumption of the homogeneous good, in moment \( t \); 
\( u(t)^{\alpha}y(t)^{1-\alpha} \): is the consumption index, in moment \( t \).

The instant utility for each individual on the aggregate consumption of \( X \) is provided by:

\[
\alpha \log u(t) = \alpha \int_0^1 \log \left( \sum_j q_j(k)d_{jt}(k) \right) dk ,
\]

where

- \( q_j(k) \): is the quality of generation \( j \) of product \( k \); 
- \( d_{jt}(k) \): is the consumption of product \( k \), by generation \( j \), in moment \( t \).

The sum in (2) includes the set of generations of product \( k \) that are available in moment \( t \). It is important to highlight that quality, \( q_j \), positively contributes to utility.

Consumers are subject to an intertemporal budgetary constraint provided by:

\[
\int_0^\infty e^{-R(t)} \left[ E(t) + Y(t) \right] dt \leq A(0)
\]

where

- \( E(t) + Y(t) \): is the total expenditure, in moment \( t \); 
- \( R(t) \): is the factor of the accumulated interest until moment \( t \), \( R(t) = \int_0^t r(\tau) d\tau \); 
- \( A(0) \): is the current value of the factors’ income flow, plus the assets initially held by the individual.

On the other hand, \( E(t) \) is the total expense in differentiated products and \( Y(t) \) is the expense with the homogeneous good, respectively provided by

\[
E(t) = \int_0^1 \sum_j p_{jt}(k)d_{jt}(k) dk
\]

\[
Y(t) = y(t)p_Y(t) = y(t) \text{ considering that } p_Y(t) = 1
\]

where

- \( p_{jt}(k) \): is the price of a unit of product \( k \) from generation \( j \), in moment \( t \); 
- \( d_{jt}(k) \): is the consumption of product \( k \) by generation \( j \), in moment \( t \); 
- \( p_Y(t) \): is the price of the homogeneous good, \( Y \) (cash), in moment \( t \).

The goal of the representative consumer is to maximise (1), respecting the budgetary constraint (3). Considering that we are dealing with regular goods with unitary elasticity

\[
\frac{\partial u(t)}{\partial p} = \frac{y(t)}{p_Y(t)}
\]

than a consumer that chooses to consume and save, whether from the current generation or from future generations.
income, the consumer’s optimisation process may be (by analytical convenience and easiness of exposure) divided into three stages:

1. Given the expense aggregated in differentiated goods at a certain moment in time, $E(t)$, it is necessary to find the optimum allocation of this expense among those goods;
2. Given the total expense at a certain moment in time, $E(t)+Y(t)$, it is necessary to find the optimum distribution between $E(t)$ and $Y(t)$;
3. Taking into consideration the allocation of the expense that maximises the consumer’s utility at each moment (whether it is on the differentiated products’ shares in $E(t)$, or on the $E(t)$ and $Y(t)$ shares in the total expense), it is necessary to find the distribution of $E(t)$ throughout time in order to maximise the intertemporal preferences.

As far as the static allocation of the aggregate expense for the differentiated goods is concerned, the consumer maximises instant utility, selecting a sole generation $j$ for each of those goods with the lowest price adjusted by quality. This means that $J_t(k)$ is selected, such that $p_{j_t(k)}/q_{j_t(k)} = \min\{p_{j_t(k)}/q_{j_t(k)}; j = 0, 1, \ldots, \infty\}$.

Such procedure will originate the static demand functions for differentiated products,

$$d_{j_t}(k) = \begin{cases} \frac{E(t)}{p_{j_t(k)}}, & \text{para } j = J_t(k) \\ 0, & \text{para } j \neq J_t(k) \end{cases}$$  \hspace{1cm} (6)

As a consequence, the instant utility for the differentiated products is obtained by

$$\alpha \log u(t) = \alpha \left[ \int_0^1 \log \sum_j q_j(k) \frac{E(t)}{p_{j_t(k)}} dk \right], \text{ or by}$$

$$\alpha \log u(t) = \alpha \left[ \int_0^1 \log \sum_j q_j(k) \frac{1}{p_{j_t(k)}} + \log E(t) \right] dk .$$

Since the consumer selects only the leader quality for each product (and supposing that the price is uniform), (i.e. the state-of-the-art product), then:

$$\alpha \log u(t) = \alpha \log E(t) + \alpha \left[ \int_0^1 \log q_j(k) \frac{1}{p_j(k)} dk \right]$$

$$\alpha \log u(t) = \alpha \log E(t) + \alpha \left[ \int_0^1 (\log q_t(k) - \log p_t(k)) dk \right].$$  \hspace{1cm} (7)

In order to maximise its global instant utility (including the differentiated products and the consumption of the homogeneous good), the consumer distributes the total expense ($E+Y$) proportionately between differentiated goods and the homogeneous good (see Annex):
This way, the expressions of utility and budgetary constraints of the representative consumer are obtained as follows:

\[
U_t = \int_0^\infty e^{-\rho t} \left[ \log E(t) + \alpha \int_0^1 \left( \log q_i(k) - \log p_i(k) \right) dk + \left( 1 - \alpha \right) \log \left( \frac{1 - \alpha}{\alpha} \right) \right] dt \quad (1')
\]

\[
\frac{1}{\alpha} \int_0^\infty e^{-R(t)} E(t) dt \leq A(0) \quad (3')
\]

The solution for the consumer’s dynamic problem consists of determining the maximum intertemporal utility function \((1')\), which is subject to the budgetary constraints \((3')\).

\[
\text{Max } U_t \quad \text{s.a. } \frac{1}{\alpha} \int_0^\infty e^{-R(t)} E(t) dt \leq A(0).
\]

The solution is the following optimum temporal trajectory of the expenses (see Annex):

\[
\frac{\dot{E}}{E} = r - \rho, \quad (9)
\]

where \(r\) is the interest rate, at a moment in time and \(\dot{E} = \frac{dE}{dt}\).  

### 3.3 Supply-side – The producers

As it was mentioned in the beginning of this section, there are two production factors in this economy: unskilled labour, \(L\), and human capital, \(H\). It is also admitted that, given the strong emphasis on human capital, there is a particular case related to the frequency of use of the factors for each of the considered sectors: the R&D and differentiated product sectors use human capital on an exclusive basis, while the sector of the homogeneous good uses human capital and unskilled labour (this input is specific for this sector).

For the activities of research and production of differentiated goods, we admit fixed coefficient technologies. Thus, in order to carry out research activities, an R&D unit requires \(a_{HI}\) human capital units, while a unit that produces differentiated good requires \(a_{HX}\) human capital units. As far as the production of the differentiated goods is concerned, \(Y\), we consider a neoclassical technology with scale constant incomes and substitutability between the inputs \((L, H)\) that are necessary for the production.

### 3.3.1 Industry of the homogeneous good

Since the market structure for the homogeneous good, \(Y\), is one of perfect competition and since all of the unskilled work is used in its production, the producers, profit maximisers,  

\[\text{8 An identical solution will henceforth be used in order to reference a derivative of a variable in time (t).}\]
produce an amount of $Y$, such that $p_Y = c_Y(w_H, L)$, where $c_Y(.)$ is the marginal cost of the production of good $Y$; $p_Y$ is the price of good $Y$ (provided); $w_H$ is the price of each human capital unit. Since good $Y$ is cash, $I = c_Y(w_H, L)$.

### 3.3.2 Industry of differentiated products

At this point, we followed the balance derivation carried out by Grossman & Helpman (1991a, 1991b) assuming that all companies compete as far as prices are concerned. There are laws on patents that indefinitely protect the intellectual property rights of the innovators (which provide the innovative companies the exclusive right to sell the goods that they invent), so patent licensing is not possible. This way, it is guaranteed that the whole production of differentiated goods is carried out by companies that have successfully developed new state-of-the-art products.

In order to describe the price-fixing process in this industry, we will imagine that there are two different companies: one (leader) has access to state-of-the-art technology, while the other one (follower) is capable of producing the good that is inferior to the leader product in quality terms.

**1st Situation**

At a given moment, the follower company fixes the price, $c_X$, which is the lowest price consistent with non-negative profits. The consumers are willing to pay a premium for a state-of-the-art product. However, they will choose the product of the previous generation if the leader fixes a price that exceeds $\lambda c_X$ ($\lambda$: accrual in services / quality provided by the leader good relatively to the closest rival product).

![Chart 1. Curve of the demand perceived by the industry leader](chart.png)

*Source: Grossman and Helpman (1993, p. 90)*

When the leader fixes a price lower than $\lambda c_X$, the whole industry demand is satisfied on its own since its product offers a price (per quality unit) that is lower than the follower’s price. If the prices fixed by the leader and by the follower are $\lambda c_X$ and $c_X$ respectively, then the leader can sell any amount throughout the BC segment. This way, the leader’s optimum response should be to establish a price that is infinitesimally lower than $\lambda c_X$ in view of the $c_X$ price fixed by its closest competitor. There are three reasons for that: 1) with that “limit price”, it is possible to keep the other company away from the market; 2) if they set a price...
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higher than $\lambda c_X$, they will lose clients to the follower; 3) if a price slightly lower than $\lambda c_X$ is set, that is not a rational decision because the functions of industry demand have a unitary elasticity, and it does not allow gains in the marginal revenue. With this behaviour, the leader absorbs the whole industry demand and thus it will be possible for the price to become closer to the monopoly price that the competition allows.\footnote{Barro & Sala-i-Martin (1995, ch. 7) present a detailed analysis of the differences between the limit price and the monopoly price, in the context of this type of model.}

2nd Situation

When the leader fixes the $\lambda c_X$ price, the optimum response of the follower will consist of fixing the $c_X$ price by two orders of reason: 1) if the price decreases (if it is set below the marginal cost), there will be losses; 2) if the price increases, the follower will be at a position of indifference, considering that the sales will be inexistent.

Therefore, as we can see in Grossman & Helpman (1993), the balance of Bertrand-Nash is reached when the leader practices the “limit price”, thus eliminating its most direct competitor (follower) from the market.

Since this balance may be extended to the most general case where the technological leaderships may overcome a generation ($\lambda$), we can conclude that the state-of-the-art product always has a lower (quality adjusted) price.

Considering what has been said regarding the technology for the development of the product and the nature of the property rights, it is guaranteed that in each industry and at each moment, there is only one leader. The leader is always one step ahead of its closer competitor (whichever the industry).

From what has been previously said, we can conclude that all state-of-the-art products (leaders in quality) for each industry have the same limit price $[p(k) = p]$:\footnote{This refers to differentiated products with a similar marginal cost.}

$$p = \lambda c_X,$$  \hspace{1cm} (10)

where

$p$: is the row vector ($1 \times k$) of similar prices for the differentiated products $k$;
$c_X$: is the row vector ($1 \times k$) of similar marginal costs for the differentiated products $k$.

Replacing (10) with (6), we obtain the following demand functions:

$$d_{ji} = \begin{cases} \frac{E(t)}{p} = \frac{E(t)}{\lambda c_X}, & \text{se} \; j = I_j(k) \\ 0, & \text{se} \; j \neq I_j(k) \end{cases}$$  \hspace{1cm} (11)

The monopoly profit flow is obtained by:
\[
\pi = pX - c_X X = pX - \frac{P}{\lambda} X = \left(1 - \frac{1}{\lambda}\right) pX = \left(1 - \frac{1}{\lambda}\right) E .
\] (12)

In balance, leaders do not invest any resources in order to improve their own state-of-the-art products. Therefore, the innovations are carried out by the followers. When the followers succeed, they will be one step ahead of the previous leader.\(^{11}\)

However, it is important to highlight that R&D is a risky activity. A company may put all its research efforts on any state-of-the-art product. If this company uses its resources to invest in R&D with an intensity of \(i\) during a time interval that lasts \(dt\), there is a \(idt\) probability of success for the development of the product’s next generation. Thus, the success in the R&D activity involves a Poisson probability distribution, with a rate of occurrence that depends on the level of the R&D activity. In order to reach a \(i\) R&D intensity, the company must invest \(a_{H|i}^i\) human capital units per time unit. This way, the probability that the company has of succeeding in its research is strictly proportional to the invested resources (human capital).

Analysing the state-of-the-art products available in the market, the potential producers may obtain precious technical information that allow them to fulfil their own research efforts.\(^{12}\) When the potential producer (follower) succeeds in its research, the company will take the leadership of the industry of the selected product, thus taking advantage of a profit flow provided by (12). This profit will cease as soon as the next research success case is achieved (by other company) for the same product line.

The profit flow in equation (12) is identical for all industries \(k\). Thus, as long as the expected duration of the leadership is identical as well, companies are indifferent as far as the industry in which they apply their research efforts is concerned. Like Grossman & Helpman (1991b, 1993), we consider that there is a symmetrical balance where all the products are subject to the same aggregate R&D intensity \((i)\), and that is why that indifference occurs.\(^{13}\)

**Free entry condition**

We will consider that \(v\) is the updated value of the profit (uncertain) that will flow to the industry’s leader, i.e. the market value of the leader company or, similarly, the amount of the premium guaranteed by the success in R&D.

A company may obtain \(v\) with a probability of \(idt\) if it invests an amount of \(a_{H|i}^i\) resources during a time interval of \(dt\), incurring in a cost of \(w_{H|i}^i dt\).

\(^{11}\) Leaders prefer to earmark the resources for the development of a leadership position in other markets instead of extending their advance (in terms of quality) in the market where they are already included. The reason for this is that the accrual in profit that would stem from this situation -
\[
\Delta \pi = \left(1 - \frac{1}{\lambda^2}\right) E - \left(1 - \frac{1}{\lambda}\right) \frac{E}{\lambda} = \left(1 - \frac{1}{\lambda}\right) E - \left(1 - \frac{1}{\lambda}\right) \frac{E}{\lambda} 
\] - is strictly lower than the accrual in profit that non-leaders would obtain if they were to succeed in their research efforts, that is, \(\left(1 - \frac{1}{\lambda}\right) E\).

\(^{12}\) This reflects the abovementioned spillover benefits of innovation, which are related to the nature of public good for technology.

\(^{13}\) Since researchers hope that future innovations have an equal probability in the different industries (i.e. that the flow of profits has an equal duration in each industry), they will be indifferent to the choice of industry.
The company’s financing is carried out through the issuing of securities, which grant the holders with the income flow associated to the industry’s leader if the research effort is successful. If, on the contrary, the research effort fails, the security holders incur in a loss that is equivalent to the total of the invested capital, \( v \). However, the security holders may neutralise the risk by holding diversified bases. For that, each of the companies must maximise the net profit expected for the respective research efforts, \( v_idt - w_i a_{HI} dt \).

Therefore, the new company should choose its research intensity in order to maximise the net benefit expected from that same research. This way, whenever the current value of the profit, \( v \), is lower than the amount invested in research, \( a_{HI} \omega_H \), there is no investment \( (i = 0) \). On the other hand, this investment tends to infinite when \( v > a_{HI} \omega_H \).

With a positive, yet finite balance of research investment, we verified that \( v = a_{HI} \omega_H \), a case where research companies are indifferent to the scale of their research efforts. This way, the condition for free entry in the R&D sector is

\[
a_{HI} \omega_H \geq v \quad \text{with uniformity every time that } i > 0
\]

(13)

The industry leaders generate a flow of dividends of \( \pi dt \) during a time interval that lasts \( dt \). If none of the competitor research efforts succeed during that period of time, the shareholders of the leader company will benefit from capital earnings to the amount of \( \dot{v} dt \). This means that the shareholders will obtain \( \dot{v} dt \) capital earnings with a probability \((1 - idt)\), admitting that the companies’ research efforts are statistically independent. However, the leader product may be improved during the \( dt \) interval, with a probability of \( idt \). In this last case, the shareholders will suffer losses in the amount of the total invested capital, \( v \). This way, the expected rate of return for the company’s shares per time unit is

\[
\frac{\pi + \dot{v}}{v} - i
\]

Since the research results in the different industries are not contemporarily correlated, the risks that the leader faces are idiosyncratic. Therefore, shareholders may obtain a certain income if they have a diversified share base for each company in the different industries.\(^{14}\)

Thus, the market tends to value the companies where the rate of return of the shares is exactly equivalent to the interest rate on the obligations without risk, \( r \).

\[
\frac{\pi + \dot{v}}{v} - i = r
\]

(14)

The previous equation reflects the non-arbitrage condition.

### 3.4 Balance of the goods and labour markets

As it has been mentioned before, a research unit requires \( a_{H} \) human capital units. Thus, the total employment of this factor in the research activity equals \( a_{HI} \).

In the production of differentiated goods, each output unit requires \( a_{HX} \) human capital units. In order to simplify, yet without sacrificing generality, we assume that \( a_{HX} = 1 \). The

\(^{14}\) Here, there is no statistical correlation between the possibilities of success of the several contemporary research efforts, which causes no harm to the existence of spillovers.
aggregate demand of human capital carried out by the producers of these goods is represented by $\frac{E(t)}{\lambda c_X}$, since each industry of differentiated products produces $\frac{E(t)}{p} = \frac{E(t)}{\lambda c_X}$ [see (11)], and since the sector’s dimension is standardised by construction ($k \in [0,1]$). Because the production of these goods uses exclusively human capital as an input, $c_X = w_H$.

In order to produce a product unit, the industry of the homogeneous good, $Y$, requires human capital, $H$, and unskilled labour, $L$. Shephard’s lemma states that this industry’s demand for human capital and unskilled labour should be $a_{HY}(w_H, w_L)$ and $a_{LY}(w_H, w_L)$, respectively, per product unit, where $a_j(.)$ is the requirement of the factor $j$ ($j = H, L$) in the production of $Y$. This is equal to the partial derivative of $c_Y(.)$ in order of $w_j$.

Generically, the balance conditions for the factor market are $a_{HI}i + a_{HX}X + a_{HY}(w_H, w_L)Y = H$ and $a_{LY}(w_H, w_L)Y = L$. In the actual case,

$$a_{HI}i + \frac{E(t)}{\lambda w_H} + H_Y(w_H, L) = H$$

$$a_{LY}(w_H, w_L)Y(w_H, L) = L$$

(15)

where

$Y(w_H, L)$: is the supply of the homogeneous good, $Y$;

$H_Y(w_H, L)$: is the demand of human capital for the production of the good, $Y$, when the total unskilled labour is employed.

On the other hand, from Expense = Income, we can extract the following balance condition for the market of goods (differentiated and homogeneous):

$$E(t) + Y(t) = w_L L + w_H H + \pi(t) - a_{HI}w_Hi(t).$$

(16)

This condition, combined with condition (8) for the maximisation of instant utility, $\frac{\alpha}{1 - \alpha} = \frac{E(t)}{Y(t)}$, and the expression (12) of the monopoly profit flow $\pi(t) = \left(1 - \frac{1}{\lambda}\right)E(t)$, originates

$$E(t) = \frac{w_L L + w_H H - a_{HI}w_Hi(t)}{\left(\frac{1}{\alpha} + \frac{1}{\lambda} - 1\right)}.$$

(17)

i.e., the (simplified) balance condition in the market of goods (differentiated and homogeneous).

### 3.5 Market balance innovation rate

Taking into consideration the equations that describe the condition for consumption optimisation (9), the expression of profit (12) and the non-arbitrage condition (14), we have solved the system in a way that would make it possible to find the intensity of the balance
research effort (i.e. the innovation rate). Thus, replacing (9), which represents the optimum
time line of the expenses \( \hat{E} = r - \rho \), and (12), which represents the monopoly profit
flow \( \pi(t) = \left(1 - \frac{1}{\lambda}\right)E(t) \), in the condition of non-arbitrage (14), \( \frac{\pi + \hat{\nu}}{\nu} - i = r \), the result is as follows:

\[
\frac{\hat{\nu}}{\nu} = \rho + i + \frac{\left(1 - \frac{1}{\lambda}\right)E}{\nu}.
\]  
(18)

In the steady-state balance, \( \frac{\hat{\nu}}{\nu} = \frac{\hat{E}}{E} = 0 \), what was replaced in (18) and in (9) implies, respectively:

\[
\left(1 - \frac{1}{\lambda}\right)E = (\rho + i)v
\]

\[r = \rho.\]

(19)

(20)

From equation (19), we conclude that \( E = \frac{(\rho + i)v}{\left(1 - \frac{1}{\lambda}\right)} \), combined with the free entry condition (13), \( a_{HI}w_H = v \), originates \( E = \frac{(\rho + i)a_{HI}w_H}{\left(1 - \frac{1}{\lambda}\right)} \). By combining this last equation with the balance condition of the market of goods, (17), \( E(t) = \frac{w_L L + w_HIH - a_{HI}w_H i(t)}{\left(1 + \frac{1}{\alpha} - 1\right)} \), after some algebraic manipulation, we obtain the market’s balance innovation rate:

\[
i^c = \frac{\alpha \left(1 - \frac{1}{\lambda}\right)\left(\frac{w_L L + H}{w_H}\right)}{a_{HI}} - \frac{P}{\lambda} \left[\lambda - \alpha(\lambda - 1)\right].
\]

(21)

Give the specific nature of the use of the unskilled labour factor, this balance innovation rate is the result of the entrepreneurs’ decentralised choice that consists of using (in the most lucrative way) the human capital that is not used in the production of the homogeneous goods for the production of differentiated products versus production of innovations (research activities).

As we analyse in Section 3.7, the market balance innovation rate is all the higher as the economy’s investment in human capital increases \( H \); on the other hand, it will be all the lower as the economy’s investment in unskilled labour decreases \( L \), admitting that the substitution elasticity is lower in the production of the homogeneous goods; all the higher as
the research productivity \(\frac{1}{a_{HI}}\) is higher; as the families are more patient (and as the intertemporal preference rate is lower); as the dimension of technological advances is higher (quality “ladder”); and as the consumers’ degree of “sophistication” is higher \((\alpha)\).

### 3.6 Growth rate of the consumption index

Taking into consideration the equations referring to the instant utility of the aggregate consumption of \(X\) (2), the static demand functions for the differentiated products, (6), the balance of the market of goods (8) and the limit price fixing (10), it is possible to obtain (note that the expression on the left is the logarithm of the consumption index):

\[
\alpha \log u(t) + (1 - \alpha) \log y(t) = \alpha \int_0^1 \log \left( \sum_j q_j(k) dk \right) + \log E(t) - \alpha \log \lambda - \alpha \log w_H +
\]

\[
+ (1 - \alpha) \log \left( \frac{1 - \alpha}{\alpha} \right)
\]

(22)

Let us now consider that \(f(j, t)\): is the probability that any product \(k\) has of registering \(j\) quality improvements, during the time interval \(t\).

Since, in balance, every product has the same research intensity, \(f(j, t)\) represents the fraction of products that are improved \(j\) times before interval \(t\). This means that the counting of all products and possible generations will generate (Grossman & Helpman, 1991b)

\[
\int_0^1 \log q_i(k) dk = \sum_{j=0}^\infty f(j, t) \log \lambda^j.
\]

(23)

As \(\sum_{j=0}^\infty f(j, t) \log \lambda^j = \sum_{j=0}^\infty f(j, t) j \log \lambda = E(j)\), by the properties of Poisson’s distribution, it generates

\[
\int_0^1 \log q_i(k) dk = it \log \lambda.
\]

(24)

Replacing (24) with (22) we obtain:

\[
\alpha \log u(t) + (1 - \alpha) \log y(t) = \alpha it \log \lambda + \log E - \alpha \log w_H + (1 - \alpha) \log \left( \frac{1 - \alpha}{\alpha} \right) - \alpha \log \lambda
\]

(25)

\[15\text{ By Poisson’s distribution properties, } f(j, t) = p(j) = \frac{(it)^j e^{-it}}{j!} \text{ and } E(j) = it, \text{ where } E \text{ represents here the expected value (see Santos, 1988, p. 245).} \]
Thus, the growth rate of the consumption index is (note that, in balance, \( E \) and \( i \) are constant, while \( w_H \) is globally determined by the provision of factors that we consider to be fixed):

\[
g = \frac{d \log [u(t)^{\alpha} y(t)^{1-\alpha}]}{dt} = \alpha \lambda \log \lambda. \quad (26)
\]

### 3.7 Compared static analysis of the market balance

It is important to remember that, in (21), \( i \) is provided by

\[
i^e = \frac{\alpha \left(1 - \frac{1}{\lambda} \right) \left( \frac{w_L}{w_H} L + H \right)}{a_h} - \frac{\rho}{\lambda} \left[ \lambda - \alpha(\lambda - 1) \right].
\]

Thus,

\[
g = \alpha \lambda \log \lambda \left\{ \frac{\alpha \left(1 - \frac{1}{\lambda} \right) \left( \frac{w_L}{w_H} L + H \right)}{a_h} - \frac{\rho}{\lambda} \left[ \lambda - \alpha(\lambda - 1) \right] \right\}. \quad (27)
\]

From the analysis of (26), we conclude that the economy’s growth rate (\( g \equiv \) growth rate of the consumption index) is all the higher

a. as the innovation rate \( \frac{\partial g}{\partial i} = \alpha \lambda \log \lambda > 0 \) is faster.

b. as \( \alpha \), which is the weight of the differentiated products in the consumption index, is higher, \( \frac{\partial g}{\partial \alpha} = \lambda \log \lambda + \frac{\partial g}{\partial i} \cdot \frac{\partial i}{\partial \alpha} = \lambda \log \lambda + \alpha \lambda \log \lambda \left[ \frac{1 - \frac{1}{\lambda}}{w_H} - \frac{\rho}{\lambda} \left( \lambda - \alpha(\lambda - 1) \right) \right] > 0 \).

This means that the more sophisticated the consumers are, in a sense that they have a larger preference for quality differentiated products (i.e. they are more sensitive to quality), the higher is the economy’s growth rate.\(^\text{16}\)

From the analysis of (27), we additionally conclude that the innovation rate (therefore, the growth rate) is all the higher

c. as the economy’s investment in human capital is higher

\(^{16}\) It would be interesting to consider the chance that the highest sophistication of the consumers is positively correlated to the level of human capital. In that case (we do not analyse that case in particular here), the human capital would also have an influence on demand since \( \alpha \) would be a positive function of \( H \). Therefore, we would be in the presence of an additional influence channel for human capital on economic growth.
\[ \frac{\partial i}{\partial H} = \alpha \left( 1 - \frac{1}{\lambda} \right) \left[ \frac{w_L}{w_H} \left( \frac{\partial w_L}{\partial H} w_H - \frac{\partial w_H}{\partial L} w_L \right) \right] > 0 \text{ since } \frac{\partial w_L}{\partial H} > 0 \text{ e } \frac{\partial w_H}{\partial L} < 0. \]

The remunerations of the factors are fixed for each level of factor provision. Thus, an increase in the amount of available human capital will cause a relative shortage of \( L \), thus leading to an increase in this factor’s remuneration. On the other hand, given the relative abundance of human capital, this factor’s remuneration tends to decrease, thus reducing the costs of research activities and increasing the growth rate.

d. as the economy’s investment in unskilled labour is lower, admitting that the substitution elasticity in the production of the homogeneous good is not too high.

\[ \frac{\partial i}{\partial L} = \alpha \left( 1 - \frac{1}{\lambda} \right) \left[ \frac{w_L}{w_H} + \frac{\partial w_L}{\partial L} w_H - \frac{\partial w_H}{\partial L} w_L \right] < 0 \text{ since } \frac{\partial w_L}{\partial L} < 0 \text{ e } \frac{\partial w_H}{\partial L} > 0 \]

and admitting that \[ \frac{\partial w_L}{\partial L} - \frac{\partial w_H}{\partial L} \frac{w_L}{w_H} \geq \frac{w_L}{L} \]

This result comes from the fact that the homogeneous good, \( Y \), requires human capital (\( H \)) and unskilled labour (\( L \)) in its production. Therefore, an increase in this sector’s specific factor – the \( L \) factor – decreases the respective remuneration and increases the remuneration of the human capital since it becomes relatively scarce. Considering that the effect of the increase in the relative remuneration of \( H \) is higher than the effect of the replacement of \( H \) by \( L \) in the production of \( Y \), the innovation rate will fall, as well as the economy’s growth rate.

e. as the research productivity \((1/a_{HI})\) is higher,

\[ \frac{\partial i}{\partial a_{HI}} = -\frac{\alpha \left( 1 - \frac{1}{\lambda} \right) \left( \frac{w_L}{w_H} L + H \right)}{(a_{HI})^2} < 0. \]

An increase in productivity increments the expected return of the research activity, thus encouraging efforts/investments in R&D and, consequently, by increasing the innovation rate, it increases growth.

f. as the families are more patient (when the intertemporal preference rate is lower, \( \rho \))

\[ \frac{\partial i}{\partial \rho} = -\frac{1}{\lambda} [\lambda - \alpha (\lambda - 1)] < 0. \]

---

17 Globally, the increase of \( L \) makes the human capital relatively scarce. However, at the same time, it releases part of the human capital for the production of differentiated goods and for research. What we admit here is that the substitution elasticity for the production of \( Y \) is not high enough to make the second effect dominant.
When families adopt a behaviour that is characterised by savings, it contributes to an increase in the innovation rate and, consequently, the growth rate.

Finally, from (25) and (26), it was possible to extract the effect of an increase in the dimension of the quality technological advances (λ). Growth is all the higher:

\[
\frac{\partial g}{\partial \lambda} = \alpha i + \alpha \log \lambda \frac{\partial i}{\partial \lambda} = \alpha i + \frac{\alpha^2 \log \lambda}{\lambda^2} \left[ \frac{w_H}{w_H a_{HI}} L + H \right] + \rho > 0.
\]

It is important to remember that an increase in the dimension of the quality advances (λ) promotes growth in two ways: directly, since the quality ladder is higher; or indirectly, which leads to a faster occurrence of technological advances, therefore increasing the expected return of the research. This will attract resources (human capital) to R&D activities.

As the consumers’ degree of “sophistication” is higher (α)

\[
\frac{\partial i}{\partial \alpha} = \left[ \frac{(1 - \frac{1}{\lambda})}{\frac{w_H}{a_{HI}} L + H} + \frac{\rho}{\lambda} (\lambda - 1) \right] > 0.
\]

This analysis has an important consequence, which is the prevision (ceteris paribus) of a growth rate that is lower for the larger economies (those with a larger amount of unskilled labour). On the other hand, the dimension of the human capital is crucial. The higher this dimension is, the faster will growth be.

This is in agreement with Romer’s conclusions (1990) “… that the correct measure of scale is not population but human capital…” (p. S78) and that an economy “… with a larger total stock of human capital will experience faster growth.” (p. S89).

In that context, a large economy with a large amount of skilled labour (H), carries out more industrial research because the R&D sector uses this factor more frequently. Such economy will grow faster than the other economy that has similar features, yet a smaller amount of human capital. However, a large economy, largely set up by unskilled individuals (high L), may grow more slowly than the other, which is identical, yet with a smaller population.

In other words, Grossman & Helpman (1994, p. 36) mention the same conclusion:18 “[t]he larger labor-abundant country, which specializes relatively in labor-intensive

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18 These conclusions implicitly suggest that, as it was mentioned above in the revision of the literature, the international free trade could contribute to accelerate growth. For a more complete and detailed analysis on the accumulation of human capital and the interconnections of human capital with the matters of international trade, see chapter 5 of Grossman & Helpman (1993).
production, might will conduct absolutely less industrial research than a smaller country with comparative advantage in R&D.”.

4. Conclusion

Trying to requalify the key role played by the demand on the process of economic growth, this article is the algebraic result of an endogenous growth model where growth is induced by improvements in the products’ quality.

Similarly to what happened in the models of Grossman & Helpman (1991a, 1991b), each innovation is built from the previous one and the same thing happens to each generation. Each new generation of the product proportionately supplies more services than the previous generation (i.e., we admit that, in a reasonable way, the dimension of innovation, identified in the model by parameter \( \lambda \), is always higher than 1).

However, as opposed to those authors, we have considered a consumption index set up by differentiated goods and by a homogeneous good. Thus, it is possible to highlight the influence of the demand on the economic growth and give the consumers the importance that they actually hold in the economy of the nations.

We can summarise the model’s predictions as follows - the economy’s growth is all the faster as, ceteris paribus:

- The economy’s investment in human capital is higher;
- The economy’s investment in unskilled labour is lower (assuming that there is a low substitution elasticity of the inputs in the production of the homogeneous good);
- The R&D productivity is higher;
- The economic agents are more patient;
- The dimension of innovation is higher (the products’ improvement “ladder”);
- The weight of the differentiated goods on the consumption index is higher.

5. References


Technological change is today central to the theory of economic growth. It is recognised as an important driver of productivity growth and the emergence of new products from which consumers derive welfare. It depends not only on the work of scientists and engineers, but also on a wider range of economic and societal factors, including institutions such as intellectual property rights and corporate governance, the operation of markets, a range of governmental policies (science and technology policy, innovation policy, macroeconomic policy, competition policy, etc.), historical specificities, etc. Given that technology is explicitly taken up in the strategies and policies of governments and firms, and new actors both in the national and international arenas become involved, understanding the nature and dynamics of technology is on demand. I anticipate that this book will decisively contribute in this regard.

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