1. Introduction
Quantum Mechanics represents one of the greatest triumphs of the scientific enterprise of the twentieth century. The stunning success of quantum theory has led to many revolutionary inventions and its extraordinary concepts describe the heart of several important real world applications like transistors and lasers. The theory makes accurate predictions about a wide range of physical phenomena and has historically withstood the tests and scrutiny of every experimental investigation. However, inspite of the fact that quantum theory is widely regarded by the scientific establishment as the fundamental theory of nature and is immensly successful and useful, its conceptual framework makes many predictions which are difficult to comprehend “classically”. The theory is, paradoxically, powerful and confusing at the same time. Quantum theory’s unusual predictions originate from its basic formalism which involves concepts like probability amplitudes and the linear superposition principle (Dirac, 1947). The quantum view appears abstract and counterintuitive and at odds with classical perceptions. Many of the conceptual problems of quantum mechanics are encompassed in what is known as the quantum measurement problem (Peres, 1986; von Neumann, 1932; Wheeler & Zurek, 1983). Conventionally, the measurement paradox is supposedly ‘resolved’ by forcing a notion of a sudden collapse of the state vector of the system being measured. However, the nature of this mechanism is at odds with the basic tenets of quantum mechanics and hence may lie outside its realm thus questioning the validity of the theory it self. Closely related with the problem of measurement in quantum mechanics is the question of its connection with the emergence of classicality and the elusive boundary between quantum and classical worlds. What is the connection between the ‘classical’ and the ‘quantum’ worlds? Is there a definite relationship? Are classical mechanics and quantum mechanics two mutually exclusive incompatible theories or are they two aspects of the same underlying philosophy? Classical objects are eventually composed of elements of the microworld which can be described quantum mechanically. So, how and where can there be a boundary between the two worlds? In the following section we begin by introducing the quantum measurement problem. In the next section we will discuss attempts to understand and explain away the underlying paradoxes in quantum theory as highlighted in the quantum measurement problem and the question of the quantum-classical connection. From among the various explanations that seek a resolution to the conceptual problems of quantum mechanics, we focus on the ‘environment induced decoherence theory’ - an approach that employs the methods developed by several authors to analyse the quantum mechanics of
a system in interaction with its environment (Zeh, 1970). The central idea of this approach is that classicality is an emergent property triggered in open systems by their environments and it is the influence of environmental interactions that explains the perceived outcomes of quantum measurements (Zurek, 1981). We illustrate this approach through some specific system-apparatus models and highlight some key results of other researchers and ours. Following this, the next section will address a specific aspect of the decoherence theory, i.e., the notion of a ‘preferred basis’ or a ‘pointer basis’. Our experience of the classical world suggests that unlike quantum systems, which are allowed to exist in all possible states, classical systems only exist in a few select states. The decoherence approach demonstrates that such states are singled out by the environment from a larger quantum menu. These special states are the ‘preferred basis’, also referred to as the ‘pointer states’ in a quantum-measurement-like scenario. What is the ‘preferred’ or ‘pointer’ basis? This question is examined via specific system-apparatus models and answered through some key results of our work and that of other researchers. Some of these results show that the ‘pointer states’ could emerge independent of the initial state of the apparatus. In the light of several advances in technology and high precision experiments, many of the questions relating to the conceptual problems of quantum mechanics are no longer merely ‘academic’ in nature. These questions and theories like environment-induced decoherence now offer themselves to experimental tests. Many recent experiments have provided important insights into the role of the environment in bringing about classicality. The decoherence theory is strengthened by these spectacular observations and there is no doubt that this approach has provided many important insights into the actual mechanism of the loss of quantum coherence. However, many researchers believe that many of the conceptual problems of quantum mechanics are still unresolved and the decoherence explanation is not adequate. In the concluding section of this chapter we summarize the main ideas that are presented in it and also highlight some of the difficulties and unresolved issues and their implications.

2. Measurement in quantum mechanics

Though quantum theory is widely accepted as the fundamental theory of nature and is immensely successful and satisfying, its conceptual framework makes predictions which are difficult to comprehend “classically”. Many of these conceptual problems are encompassed in what is known as the quantum measurement problem. While the basic formalism of quantum mechanics was developed between 1925 and 1927, the standard interpretation of quantum measurement is attributed to von Neumann’s theory presented in his book in 1932 (von Neumann, 1932). The quantum mechanical description of a system is contained in its wave function or state vector $|\psi\rangle$ which lives in an abstract “Hilbert space”. The dynamics of the wavefunction is governed by the Schrödinger equation

$$i\hbar \frac{d}{dt} |\psi\rangle = H |\psi\rangle.$$  \hspace{1cm} (1)

Here $H$ is the Hamiltonian of the system and the equation is linear, deterministic and the time evolution governed by it is unitary. Dynamic variables or observables are represented in quantum mechanics by linear Hermitian operators which act on the state vector. An operator $\hat{A}$, corresponding to the dynamical quantity $A$ is associated with eigenvalues $a_i$, and corresponding eigenvectors $\{\alpha_i\}$ which form a complete orthonormal set. Any arbitrary state vector, $|\psi\rangle$ can, in general, be represented as a linear superposition of these eigenvectors:

$$|\psi\rangle = \Sigma c_i |\alpha_i\rangle.$$  \hspace{1cm} (2)
A basic postulate of quantum mechanics regarding measurement is that any measurement of the quantity \( A \) can only yield one of the eigenvalues, \( a_i \), but the result is not definite in the sense that different measurements for the quantum state \( |\psi\rangle \) can yield different eigenvalues. However, quantum theory predicts only that the probability of obtaining eigenvalue \( a_i \) is \( |c_i|^2 \). An additional postulate of quantum mechanics is that the measurement of an observable \( A \), which yields one of the eigenvalues \( a_i \) (with probability \( |c_i|^2 \)) culminates with the reduction or collapse of the state vector \( |\psi\rangle \) to the eigenstate \( |a_i\rangle \). This means that every term in the linear superposition vanishes, except one. This reduction is a non unitary process and hence in complete contrast to the unitary dynamics of quantum mechanics predicted by the Schrödinger equation and this is where the crux of the conceptual difficulties encountered in quantum theory lies. These two stages of quantum measurement are captured in the well-know von Neumann model through two distinct processes - first, where the system and apparatus interact through linear unitary Schrödinger evolution via an appropriate interaction Hamiltonian, and second - the nonlinear, indeterministic collapse (von Neumann, 1932). In this sense, the idea of measurement is very different from what we understand for classical systems. Classical systems are independent from measurements - the act of measurement does not disturb the state of the system or its ‘properties’. In the language of quantum mechanical wave functions, the von Neumann measurement scheme can be illustrated as follows:

Measurements are described by treating both the system and the measuring apparatus as quantum objects. Let the quantum system be in the superposition state \( |\psi_S\rangle = \sum_n c_n |\psi_{Sn}\rangle \), where \( |\psi_{Sn}\rangle \) are the eigenstates of the operator that needs to be measured. For a measurement to be affected, the measured system described by \( |\psi_S\rangle \) needs to interact with the measuring apparatus described by \( |\phi_A\rangle \), so that the total wave function before the interaction is \( |\psi_S\rangle|\phi_A\rangle \). During the interaction of the system and the apparatus, the unitary evolution realizes the following transition from the initial to the final total wave function:

\[
|\psi_S\rangle|\phi_A\rangle \rightarrow \sum_n c_n |\psi_{Sn}\rangle|\phi_{An}\rangle \quad \text{(measurement of the first kind). (3)}
\]

Here \( |\phi_{An}\rangle \) are orthonormal states of the measuring apparatus. This unitary evolution is referred to as premeasurement. The transition

\[
|\psi_S\rangle \rightarrow \sum_n |c_n|^2 |\psi_{Sn}\rangle \langle \psi_{Sn}| \quad \text{(4)}
\]

is often referred to as the wave function collapse. The final density operator corresponding to the system is calculated as \( \sum_n |c_n|^2 |\psi_{Sn}\rangle \langle \psi_{Sn}| \). This density operator describes an ensemble of system states, which, after the measurement will be found in the state \( |\psi_{Sn}\rangle \) with probability \( |c_n|^2 \). The transition

\[
|\psi_S\rangle \rightarrow \sum_n |c_n|^2 |\psi_{Sn}\rangle \langle \psi_{Sn}| \rightarrow |\psi_{Sn}\rangle, \quad \text{(5)}
\]

corresponds to an additional selection of a subensemble by means of observation. In measurements of the second kind, the unitary evolution during the interaction of the system and measuring apparatus is described as:

\[
|\psi_S\rangle|\phi_A\rangle \rightarrow \sum_n c_n |\chi_{Sn}\rangle|\phi_{An}\rangle, \quad \text{(6)}
\]

in which the states \( |\chi_{Sn}\rangle \) of the system are determined by the nature of the interaction between system and measuring apparatus. As in the case of measurements of the first kind, the
final state of the system will be \(|\chi_{Sn}\rangle\) with probability \(|c_n|^2\). The concept that quantum mechanics does not yield an objective description of microscopic reality but deals only with probabilities (as illustrated in the measurement process) is an essential part of the Copenhagen interpretation of quantum mechanics which is regarded as the "standard" interpretation of quantum mechanics. The von Neumann measurement scheme is in tune with the Copenhagen interpretation of quantum mechanics which was one of the first attempts to understand quantum mechanics, initiated by Niels Bohr, and supported by Werner Heisenberg, Max Born and others. The von Neumann scheme described above would typically involve a coupling between the microscopic system and a 'macroscopic' apparatus (meter), resulting in states like (3) and (6), called entangled states which are uniquely quantum mechanical states for the composite. The term entanglement was coined by Schrödinger and describes a correlated state that is "not separable" (Schrödinger, 1935). Today, entanglement is considered one of the most defining concepts in quantum mechanics - a uniquely quantum mechanical possibility with no classical analouge. In quantum information and quantum computation, entanglement is viewed as a resource for computing tasks that can be performed faster or in a more secure way than is classically possible and there are intensive experimental efforts to create entangled states in the laboratory. The entangled state describing the system-apparatus, as in (3) above should contain one-to-one correlations between the states of the system, \(|\psi_{Sn}\rangle\), and the states of the apparatus \(|\phi_{An}\rangle\), so that a read out of the apparatus or 'meter' states gives information about the states of the system. Consider a simple example of a two-level system for which the entangled system-apparatus state after the measurement interaction should look like

\[
|\psi_S\rangle|\phi_A\rangle \rightarrow |\psi_{S1}\rangle|\phi_{A1}\rangle + |\psi_{S2}\rangle|\phi_{A2}\rangle.
\]

(7)

Such an entangled state is like a two-particle superposition state. The problem with such an entangled state is that it seems to allow the 'meter' (apparatus) to exist in a coherent superposition of the two states \(|\phi_{A1}\rangle\) and \(|\phi_{A2}\rangle\) which could be macroscopically distinct - a situation hard to reconcile with classical intuition. Historically, it was the Einstein, Podolsky, Rosen (EPR) paper (Einstein et al., 1935) which first highlighted the problem of quantum entanglement. In response to the EPR work, Schrödinger posed a thought experiment which is now famously known as Schrödinger’s Cat paradox (Schrödinger, 1935). Schrödinger’s cat is the unfortunate victim of a nasty contraption where the decay of a radioactive atom triggers a device which kills the cat. The quantum mechanical description of this scenario demands that a superposition state of ‘decayed’ and ‘not decayed’ for the atom lead to an entangled state of the kind (7) for the atom-cat composite with the cat being in a superposition state of ‘dead’ and ‘alive’. This amounts to interpreting the quantum state of the cat as being in a coherent superposition of ‘dead’ and ‘alive’ states - a situation which is completely at odds with our familiar classical perceptions. Schrödinger’s Cat paradox is often presented as an illustration of the conceptual problems of quantum mechanics. It is worth mentioning at this point that the density matrix is a convenient formal tool to compare and contrast quantum and classical systems in terms of probabilities. Some of the conceptual problems of quantum measurement become more transparent when analyzed in this language. It can be easily seen that the density matrix corresponding to the entangled state (7) is

\[
\hat{\rho}_{S+A} = |\psi_{S1}\rangle \langle \psi_{S1}| \hat{\rho}_{S} + |\psi_{S2}\rangle \langle \psi_{S2}| \hat{\rho}_{S} + |\psi_{A1}\rangle \langle \psi_{A1}| \hat{\rho}_{A} + |\psi_{A2}\rangle \langle \psi_{A2}| \hat{\rho}_{A}.
\]

(8)

While (8) represents a perfectly legitimate solution of the Schrödinger equation, the physical interpretation in the usual language of probabilities leads to difficulties. While the diagonal
elements (the first and second terms) can be easily interpreted as probabilities corresponding to the system being in state $|\psi_S^1\rangle$ or $|\psi_S^2\rangle$ (with the corresponding correlations with the apparatus states $|\phi_A^1\rangle$ and $|\phi_A^2\rangle$, respectively), the off-diagonal elements represented by the third and fourth terms are difficult to interpret classically in terms of probabilities. In order to make ‘classical’ sense, the density matrix corresponding to the pure state ensemble described by the entangled state (8) must reduce to a statistical mixture which is diagonal in some basis with appropriate system-apparatus correlations. Such a mixed density matrix would look like

$$
\rho_{\text{mixed}} \sim |\psi_S^1\rangle\langle\psi_S^1| |\phi_A^1\rangle\langle\phi_A^1| + |\psi_S^2\rangle\langle\psi_S^2| |\phi_A^2\rangle\langle\phi_A^2|.
$$

(9)

Several interpretations of quantum mechanics seek to explain this $\rho_{\text{pure}} \rightarrow \rho_{\text{mixed}}$ transition (von Neumann’s irreversible ‘reduction’ process) and a resolution to the mechanism for the apparently nonunitary ‘collapse’ in a quantum measurement (Wheeler & Zurek, 1983; Zurek, 1991). In recent years, the decoherence approach (Joos et al., 2003) has been widely discussed and accepted as the mechanism responsible for this transition. The central idea of this approach has been that ‘classicality’ is an emergent property of systems interacting with an environment. The theory also predicts that in a quantum measurement, the apparatus will have correlations with the system in a set of ‘preferred states’ (Joos et al., 2003; Zurek, 1981; 1991) selected by the environment. In the next two sections we describe the progress made in adopting this approach to explain the mechanism for the perceived outcomes of a quantum measurement as well as the emergence of classicality from an underlying quantum world. The strength of this approach lies in the fact that it provides a reasonably satisfying explanation within the realm of quantum mechanics.

3. Decoherence

In the previous section, we have seen that there is a serious interpretational problem with the way quantum mechanics deals with the act of measurement. In particular, the problem lies in von Neumann’s postulate of an irreversible reduction process which takes the quantum superposition to a statistical mixture which is supposedly classically interpretable and meaningful. However, the non-unitary nature of this reduction is at odds with the inherent unitary nature of the Schrödinger equation, implying, somehow that the mechanism seems to lie outside the realm of quantum mechanics. From among the various explanations that seek a resolution to the conceptual problems of quantum mechanics, in this section we focus on the ‘environment induced decoherence theory’. As pointed out in the previous section, the problem lies with the off-diagonal elements of the density matrix describing the entangled state of the system-apparatus composite, (8). These off-diagonal elements are the signatures of quantum correlations. In quantum mechanics, wave functions evolve according to the Schrödinger equation which is linear and deterministic and this evolution is unitary in nature. Unitary evolutions ensure that eigenvalues are preserved. There is no way that some terms of the density matrix can vanish in the course of a unitary evolution. How, then, can ‘classical behaviour’ (as discussed above) ever emerge from this substrate of the quantum world where entanglements and coherences are ubiquitous and inevitable? How can a pure state density matrix become a “classically interpretable” statistical mixture? The decoherence approach seeks to answer this problem by providing a mechanism which leads to the loss of quantum coherence, thus bringing about the much desired $\rho_{\text{pure}} \rightarrow \rho_{\text{mixed}}$ transition, and hence, classicality. The answer lies in realizing the fact that the Schrödinger equation driving unitary evolutions is strictly applicable only to completely isolated systems. In reality, we know that macroscopic systems are almost never isolated from their surroundings but are known to
be constantly interacting with a complex environment. In a measurement like situation, the apparatus is almost always a macroscopic object from which one reads out the measured property of the system. In fact, the apparatus is not only considered macroscopic, it is also regarded as ‘classical’ in its dynamics. How does the apparatus (which starts off as a quantum object) end up appearing ‘classical’?

The ‘decoherence’ explanation is that it is the influence of the environment that makes a quantum system appear ‘classical’. The environment ‘washes away’ quantum coherence (‘decoherence’), leaving behind a system which looks and behaves like a classical object of our cherished commonsense world. The system no longer constitutes a closed system but an ‘open system’ which is coupled to a large number degrees of freedom which constitute the environment. However, one is always monitoring only a few degrees of freedom, which are of relevance. Technically, this amounts to ‘tracing’ over all other degrees of freedom. This tracing over has the effect of causing the transition:

\[ \rho_{\text{pure}} \rightarrow \rho_{\text{mixed}}. \]  

(10)

An illuminating and popular paradigm for understanding decoherence is the phenomenon of Brownian motion which describes the motion of a particle suspended in a liquid. Such a suspended particle, when examined closely, is seen to bounce around in a random, irregular, ‘zig-zag’ fashion. Einstein showed that this behaviour is exactly what should be expected if the suspended particle is being repeatedly ‘kicked’ by other unseen smaller particles. The random motion of the suspended particle can be statistically explained by taking into account its interaction with a large number of particles which constitute the reservoir of liquid molecules or the ‘environment’. When we see Brownian motion, we are only focussing on the dynamics of the suspended particle and do not monitor each and every particle of the environment. Mathematically, we trace over all the degrees of freedom of the environment and look only at the reduced system -the suspended particle. As a consequence, the tagged particle is found to show a dynamics that contains dissipation (a steady loss of energy or relaxation) and diffusion (the random zig-zag motion). Quantum Brownian motion describes a similar situation at the quantum mechanical level (Agarwal, 1971; Caldeira & Leggett, 1983). A simple example by Zurek (Zurek, 1991) illustrates this point (see Figure 1). Here an initial pure state constructed as a coherent superposition of two spatially separated Gaussian wavepackets decoheres into a statistical mixture (diagonal density matrix) when its dynamics incorporates the coupling to a large number of environmental degrees of freedom. While the pure state density matrix of the system (Fig. 1(a)) has both diagonal and off-diagonal elements, it can be seen that after a certain time, impacted by the environmental influence, the off-diagonal elements of the reduced system are diminished to give us a statistical mixture (Fig1(b)).

Let us now look at the quantum measurement situation through this approach. The microscopic system couples to a macroscopic apparatus, which in turn is interacting with a large number of degrees of freedom which constitutes the environment. Schrödinger’s equation is applied to the entire closed universe of system-apparatus-environment. Hamiltonian evolution drives this closed system from an initial uncoupled state into a gigantic entangled state containing all the degrees of freedom. A tracing over all the environmental degrees of freedom salvages the reduced system-apparatus combine from this mess. After a characteristic time, the apparatus, impacted by the environment, appears classical in its dynamics. Thus, the environment causes a general quantum state to decay into a statistical mixture of "pointer states" which can be understood and interpreted as classical probability distributions. This, in essence, is the approach of the decoherence theory to explain the
emergence of classicality and the perceived outcomes of quantum measurements (Zurek, 1991). In the following subsections we highlight studies done on two measurement models where the outcome of the system-apparatus interaction is explained by the decoherence approach.

![Fig. 1. (a) Density matrix of a superposition of two Gaussian wave packets (b) Density matrix after the off-diagonal elements have been partially washed away by decoherence](image)

3.1 Decoherence and the Stern-Gerlach measurement

Venugopalan et al. (Venugopalan et al., 1995a;b) first analysed a Stern-Gerlach-like measurement model through the decoherence approach. Their analysis shows that decoherence would bring out the desired ‘classical-like’ outcome in such a measurement scenario. Consider a simple model of measurement consisting of a free particle with spin (for simplicity, consider a two-state system or spin-1/2 which could represent a qubit). Here the spin degrees of freedom represent the system and the position and momentum degrees represent the apparatus. Let us first look at the bare system without the inclusion of additional environment degrees of freedom. The system and apparatus are coupled by a Stern-Gerlach measurement-like interaction such that the trajectory of the particle (position and momentum degrees) correlates with the spin states. The Hamiltonian describing this model is:

$$H = \lambda \sigma_z + \frac{p^2}{2m} + \epsilon z \sigma_z.$$  \hspace{1cm} (11)

While the first two terms represent the self Hamiltonians of the system and apparatus, respectively, the last term is the interaction Hamiltonian. $z$ and $p$ denote the position and momentum of the particle of mass $m$, $\lambda \sigma_z$ the Hamiltonian of the system and $\epsilon$ the product of the field gradient and the magnetic moment of the particle. The most general initial state for the system-apparatus combine can be written as

$$\psi = \{ a | \uparrow \rangle + b | \downarrow \rangle \} \otimes \phi(z).$$  \hspace{1cm} (12)
This is a product state of the most general spin state for the system and an arbitrary state, \( \phi(z) \) for the apparatus (free particle). \( |\uparrow\rangle \) and \( |\downarrow\rangle \) are the eigenstates of \( \sigma_z \). Following the measurement interaction governed by Hamiltonian evolution, this initial state becomes an entangled state between the system and apparatus. The density matrix of the entangled state, when the initial state, (12), undergoes a Hamiltonian evolution via (11) will be of the form

\[
\rho_{S+A} = |a|^2 |\uparrow\rangle \langle \uparrow| + |b|^2 |\downarrow\rangle \langle \downarrow| + \frac{1}{\sqrt{\sigma_\uparrow \sigma_\downarrow}} \exp\{-\frac{z^2}{2\sigma^2}\}.
\]

(13)

Here \( \rho_{\uparrow\uparrow} \) and \( \rho_{\downarrow\downarrow} \) correspond to the diagonal elements (in spin) of the density matrix \( \rho_d \) for the apparatus which could correlate with up and down spin states of the system and \( \rho_{\uparrow\downarrow} \) and \( \rho_{\downarrow\uparrow} \) correspond to the off-diagonal elements \( \rho_{od} \), and \( \rho(z,z',t) = \langle z|\rho|z'\rangle \). The specific form of \( \rho_d \) and \( \rho_{od} \) and the system correlations they would (or would not) contain depends on the initial state of the apparatus. When they contain one-to-one system-apparatus correlations, the states corresponding to \( \rho_d \) would be candidate pointer states. For the purpose of illustration we look at the situation when the apparatus starts off in an initial state which is a Gaussian wave packet as was first analyzed by Venugopalan et al. The initial system-apparatus state is thus given as

\[
\{a|\uparrow\rangle + b|\downarrow\rangle\} \otimes \phi(z) = \{a|\uparrow\rangle + b|\downarrow\rangle\} \otimes \frac{1}{\sqrt{\sigma_\uparrow \sigma_\downarrow}} \exp\{-\frac{z^2}{2\sigma^2}\}.
\]

(14)

Here \( \sigma \) is the width of the wavepacket. The wavepacket in quantum mechanics is often viewed as the most "nearly classical" state and is known to exhibit many striking classical properties and hence is a reasonably good choice for the initial state of the apparatus. Following Hamiltonian evolution via (11), the system-apparatus composite ends up in an entangled state whose density matrix is of the form (13). One examining the detailed form of the complete density matrix representing this entangled state, one can identify the parts of the "apparatus", i.e., \( \rho_{\uparrow\uparrow} \) and \( \rho_{\downarrow\downarrow} \) that correlate with the up and down spins, and it can be shown that the diagonal elements of \( \rho_{\uparrow\uparrow} \) and \( \rho_{\downarrow\downarrow} \)

\[
\rho_{\uparrow\uparrow}, \rho_{\downarrow\downarrow} \rightarrow \rho_d(z,t) = \frac{2}{\sigma} \sqrt{\frac{\pi}{N(t)}} \exp\{- \frac{4}{\sigma \sqrt{N(t)}} \left( \frac{z + \frac{Et}{2m}}{2m} \right)^2 \},
\]

(15)

in the position representation and

\[
\rho_{\uparrow\uparrow}, \rho_{\downarrow\downarrow} \rightarrow \rho_d(p,t) = 2\sigma \sqrt{\pi} \exp\{- 4\sigma^2 \left( \frac{p + \frac{Et}{\hbar}}{2m} \right)^2 \},
\]

(16)

in the momentum representation. It must be kept in mind, though, that the density matrix represents a pure state which has ‘nonlocal’ quantum correlations both in the spin space and the position and momentum space. In (15) & (16) above, we are looking at the diagonal elements of the position and momentum space density matrix and these show system-apparatus correlations. The up and down spin states of the system correlate with a Gaussian wavepackets centered round \( -\frac{Et}{2m} \) and \( +\frac{Et}{2m} \), in the position space, respectively, and around \( -\frac{Et}{\hbar} \) and \( +\frac{Et}{\hbar} \) in the momentum space. This pure entangled state of the system and the apparatus is akin to a ‘Schröödinger cat state’ which contains one-to-one correlations between the system and ‘macroscopic’ apparatus states with all quantum coherences intact. Since the dynamics, as governed by the Hamiltonian (11), is purely unitary, there is no
dissipation/decoherence involved, and the state remains a pure entangled state with all its non-diagonal elements ($p_{±z}$) as well as the spatial nonlocality. The establishment of system-apparatus correlations, therefore, is not enough to affect a measurement as the off diagonal elements of the density matrix have not vanished and we do not have the desired mixed state density matrix.

Now let us look at the situation where we include the interaction with the environment. A commonly used model to describe the environment is to consider it as a reservoir of quantum oscillators, each of which interacts with the apparatus in our case and is described by the Hamiltonian

$$H = \lambda z^2 + \frac{p^2}{2m} + \epsilon z \sigma_z + \sum_j \left[ \frac{p_j^2}{2M_j} + \frac{M_j^2 \Omega_j^2}{2} \left( Z_j - \frac{C_j z}{M_j \Omega_j^2} \right)^2 \right].$$

(17)

Here $z$ and $p$ denote the position and momentum of the particle (apparatus) of mass $m$. $\lambda z^2$ is the Hamiltonian of the system and $\epsilon$ is the strength of the system-apparatus coupling as mentioned earlier. The last term represents the Hamiltonian for the bath of oscillators (environment) and the apparatus-environment interaction. $Z_j$ and $P_j$ are the position and momentum coordinates of the jth harmonic oscillator of the bath, $C_j$s are the coupling strengths and $\Omega_j$s are the frequencies of the oscillators comprising the bath. Note that the coupling of the apparatus with the many environmental degrees of freedom is a coordinate-coordinate coupling. The dynamics for the closed universe of the system, apparatus and environment is governed by the Hamiltonian evolution via (17) and the Schrödinger equation. In the decoherence approach, a tracing over all the degrees of freedom of the environment results in an equation describing the dynamics of the reduced density matrix of the system-apparatus combine. The reduced density matrix evolves according to a master equation which is obtained by solving the Schrödinger equation for the entire universe of the system, apparatus and the environment and then tracing over the environment degrees of freedom. Several authors like Zeh and Zurek, among others, have worked extensively on the decoherence approach using the master equation for the reduced density matrix. The master equation for this kind of model of the environment was first derived separately by Caldeira and Leggett, Agarwal, and Dekker (Dekker, 1977) and others in the context of quantum Brownian motion and is a popular equation for the study of open quantum systems. For our purpose, we deal with the master equation for the system-apparatus-environment composite described by (17). The master equation for the density matrix, corresponding to the four elements of spin space ($\uparrow\uparrow, \uparrow\downarrow, \downarrow\uparrow, \downarrow\downarrow$) (see (13)) is

$$\frac{\partial \rho_{ss'}(z,z',t)}{\partial t} = \left[ -\frac{\hbar}{2im} \left( \frac{\partial^2}{\partial z^2} - \frac{\partial^2}{\partial z'^2} \right) - \gamma(z-z') \left( \frac{\partial}{\partial z} - \frac{\partial}{\partial z'} \right) - \frac{D}{4\hbar^2} (z-z')^2 + \frac{i\epsilon(zs-z's')}{\hbar} + \frac{i\lambda(s-s')}{\hbar} \right] \rho_{ss'}(z,z',t),$$

where $s,s' = +1$ (for $\uparrow$) or $-1$ (for $\downarrow$). Here $\gamma$ is the Langevin friction coefficient and $D$ has the usual interpretation of the diffusion coefficient. $\gamma$ and $D$ are related to the parameters of the Hamiltonian of the total system. Without going into further details of the master equation, it suffices to point out that the equation naturally separates into three distinct terms, namely (i) a term describing the von Neumann equation which can be derived from the Schrödinger equation and thus represents the pure quantum evolution, (ii) a term that causes dissipation, which can be understood as a steady loss in energy or a relaxation process, and (iii) a term causing diffusion, which can be understood as the fluctuations or zig-zag
movement seen in Brownian motion. The initial work of Zeh, Zurek and subsequent work where the decoherence approach has been applied to typical quantum-measurement-like situations like the Stern Gerlach experiment have shown that in the dynamics governed by the master equation, coherent quantum superpositions persist for a very short time. They are rapidly destroyed by the action of the environment. In fact, results show that the larger the quantum superposition, the faster the decoherence. For truly macroscopic superpositions such as Schrödinger’s cat, decoherence occurs on such a short time scale that it is impossible to observe these quantum coherences. As mentioned above, the simple example by Zurek (Fig.1) illustrates this point. Several other authors have studied decoherence in other systems and their calculations are seen to contain two main features which can be seen as signatures of decoherence: (a) the decoherence time, over which the superpositions decay is much much shorter than any characteristic time scale of the system (the thermal relaxation time, $\gamma^{-1}$, and (b) the decoherence time varies versely as the square of a quantity that indicates the ‘size’ of the quantum superposition (e.g. $c^2/m$ in (15) is the "separation" in position space of the two Gaussians correlating with up and down spin states).

Venugopalan et al(Venugopalan et al., 1995a;b) have shown that if the evolution of the initial state (14) is via the mater equation, (18), there is a one-to-one correlation established between the system states (spin) and the apparatus states (position and momentum degrees of freedom). Further, the coupling with the environmental degrees of freedom causes decoherence of the pure density matrix of the entangled state into a statistical mixture. For more details of the exact solutions, the interested reader may see(Kumar, 1998; Venugopalan et al., 1995a;b; Venugopalan, 1997). The density matrix for an initial system-apparatus state described by (14), evolving via the master equation (18) has the form

$$\rho_{S+A} = |a|^2 |\uparrow\rangle \langle \uparrow| + |b|^2 |\downarrow\rangle \langle \downarrow| + |ab^*| |\uparrow\rangle \langle \downarrow| \rho_{\uparrow\downarrow}(z, z', t) + a^*b |\downarrow\rangle \langle \uparrow| \rho_{\downarrow\uparrow}(z, z', t) e^{-a t^3}. \quad (19)$$

Here the off-diagonal elements of the density matrix (last two terms) contain a multiplicative factor of the form $e^{-a t^3}$ which causes the decay of these terms to zero over a characteristic time making the density matrix diagonal in spin space. Thus, we are left with a mixed state density matrix which can be interpreted in terms of classical probabilities. It also turns out that the spatial nonlocality of the diagonal components $\rho_{\uparrow\uparrow}$ and $\rho_{\downarrow\downarrow}$ disappear over time and these are rendered completely diagonal in momentum space. The density matrix, thus ends up being completely diagonal with perfect correlations between the spin component value and the average momentum of the particle(Kumar, 1998; Venugopalan et al., 1995a). Thus, the inclusion of environmental interaction has destroyed the quantum corelations (signified by the off-diagonal elements of the density matrix) and rendered the reduced system-apparatus combine into a statistical mixture. This, in effect, explains the mechanism leading to the "collapse" in a quantum measurement, as well as the description of the way classicality emerges from an underlying quantum substrate.

### 3.2 Decoherence and the harmonic oscillator apparatus

In the previous section we have seen a measurement-like scenario where the apparatus was represented by the continuous position and momentum degrees of freedom of the particle who’s spin was measured. Venugopalan(Venugopalan, 2000) has looked at the same problem where the apparatus is a harmonic oscillator. The interaction of a quantum system (spin-1/2)
with a macroscopic quantum apparatus (harmonic oscillator) coupled to a bath of harmonic oscillators was analysed. The Hamiltonian for this system would be

\[ H = \lambda \sigma_z + \frac{1}{2} m \omega^2 z^2 + e z \sigma_z + \sum_j \frac{p_j^2}{2M_j} + \frac{M_j^2 \Omega_j^2}{2} \left( Z_j - \frac{C_j z}{M_j \Omega_j^2} \right)^2. \] (20)

Here \( \omega \) is the frequency of the oscillator and all the other symbols have the same meaning as explained in the previous section. The master equation for this system can be written exactly in the same way as for the free particle in the previous section. The initial system-apparatus state is considered to be a product state of any arbitrary state of the apparatus and a general superposition state of the spin-1/2 state of the system, of the form

\[ \psi = \{ a | \uparrow \} + b | \downarrow \} \otimes \phi(z). \] (21)

Exact solutions of the master equation show that the reduced density matrix of the system-apparatus combine decoheres to a statistical mixture where up and down spins eventually correlate with pointer states of the apparatus. The strength of this analysis is that unlike in the previous section where the initial state of the apparatus was considered to be a Gaussian wavepacket, no particular initial state of the harmonic oscillator was chosen. Venugopalan shows that for the zero temperature bath the system-apparatus combine ends up with spin-apparatus correlations in the coherent states of the harmonic oscillator for arbitrary initial states of the apparatus. For a high temperature bath, pointer states are Gaussian distributions or generalized coherent states (Venugopalan, 2000). For both cases, the off-diagonal elements in spin-space decohere over a time scale which goes inversely as the square of the "separation" between the "pointers". The "statistical mixture" into which the density matrix decoheres looks like

\[ \rho_{S+A} \sim |a|^2 |\uparrow\rangle \langle \uparrow| + |b|^2 |\downarrow\rangle \langle \downarrow| + \rho_{\downarrow\downarrow}(z, z', t) \] (22)

where the apparatus states are

\[ \rho_{\uparrow\uparrow} = \sqrt{\frac{m \omega}{\pi \hbar}} \exp \left\{ - \frac{m \omega}{\hbar} \left( R + \frac{e}{m \omega^2} \right)^2 - \frac{m \omega^2}{4\hbar} \right\}, \] (23)

\[ \rho_{\downarrow\downarrow} = \sqrt{\frac{m \omega}{\pi \hbar}} \exp \left\{ - \frac{m \omega}{\hbar} \left( R - \frac{e}{m \omega^2} \right)^2 - \frac{m \omega^2}{4\hbar} \right\}. \] (24)

with \( R = (z + z')/2 \), and \( r = (z - z') \). This is nothing but the density matrix corresponding to a coherent state, \( |a\rangle \), of a harmonic oscillator with zero mean momentum and mean positions \( = \pm e/m \omega^2 \), and

\[ |a|^2 = \frac{m \omega}{2 \hbar} \left( \frac{e}{m \omega^2} \right). \] (25)

The above result is for a zero temperature bath. For a high temperature bath, generalized coherent states constitute the pointer states. Exact results also demonstrate in an unambiguous way that the pointer states in this measurement model emerge independent of the initial state of the apparatus. For details of the analysis and exact solutions, the reader is referred to (Venugopalan, 2000). For both the zero temperature and high temperature cases analysed, the exact solutions for this model demonstrate the two main signatures of the decoherence mechanism in a quantum measurement, namely, (a) the decoherence time is much smaller than the thermal relaxation time, and (b) the decoherence time is inversely proportional to the
square of the "separation" between the two "pointers" that correlate with the system states. Thus, one again, one can see that the inclusion of the environmental degrees of freedom takes a pure entangled system-apparatus state to a statistical mixture which lets us interpret the measurement in terms of classical probabilities as well as lets us explain the emergence of classical correlations from an underlying quantum substrate.

4. Decoherence and the pointer basis

Finally, we focus on a specific aspect of the decoherence theory, i.e., the notion of a 'preferred basis' or a 'pointer basis'. Our experience of the classical world suggests that unlike quantum systems, which are allowed to exist in all possible states, classical systems only exist in a few select states which are singled out by the environment from a larger quantum menu(Joos et al., 2003; Zurek, 1981). These special states are the preferred basis, also referred to as the "pointer states" in a quantum-measurement-like scenario. In a measurement-like scenario the pointer basis should also be understood as those states of the apparatus in which correlations with the system states are eventually established (irrespective of the initial states of the apparatus). Inspite of many theoretical studies on the decoherence approach, not many systems have been analysed with the aim of predicting what the pointer basis should be in a given situation. Let us look at the issue of the pointer basis a little more closely by once again referring to the two quantum measurement examples discussed in the previous sections: (i) the spin-1/2 system with the free particle as an apparatus in the Stern-Gerlach-like interaction described in section 3.1 (ii) the spin-1/2 system with the harmonic oscillator as an apparatus described in section 3.2.

What is the preferred basis in a given scenario, and what decides in which pointer basis the system-apparatus will be finally established? For simplified models where the self Hamiltonian of the system has either been ignored or considered co-diagonal with the interaction Hamiltonian, the pointer variable has been shown to be the one which commutes with the interaction Hamiltonian(Zurek, 1981). However, in more general situations where all terms are included, as in the two examples discussed here, the various parts of the Hamiltonian may not commute. In such situations it is not obvious what decides the preferred basis. For example, in the case of the spin system and free particle apparatus with Stern-Gerlach like interaction discussed above, the coordinate-coordinate coupling indicates that the position basis is intuitively expected to emerge as the preferred basis. However, as we have seen, this is contrary to the conclusion of Venugopalan et al(Kumar, 1998; Venugopalan, 1994; Venugopalan et al., 1995a) in their analysis of the Stern-Gerlach measurement model where the spin components eventually correlate with distributions which are completely diagonal in the momentum basis and only approximately diagonal in the position basis(Kumar, 1998).

In the literature, the preferred basis has been variously described as the one in which the final state density matrix becomes diagonal or that set of basis states which are characterized by maximum stability or a minimum increase in linear or statistical entropy, decided by a predictability sieve(Zurek et al., 1993). Using the Markovian Master equation for a harmonic oscillator coupled to a heat bath and the criterion of the predictability sieve Zurek argues that coherent states emerge as the preferred basis. This is in tune with the results for the harmonic oscillator apparatus model where we have seen that pointer states end up being coherent states or generalized coherent states, irrespective of the initial state in which the apparatus starts off. This result also agrees with a study by Tegmark and Shapiro(Tegmark & Shapiro, 1999) where they show that generalized coherent states tend to be produced naturally when
one looks at the reduced Wigner distributions of infinite systems of coupled oscillators as $t \to \infty$. Paz and Zurek (Paz & Zurek, 1999) have investigated decoherence in the limit of weak interaction with the environment and show that the eigenstates of energy emerge as pointer states. Roy and Venugopalan have also obtained the exact solutions of the Master equation for a harmonic oscillator and a free particle in a compact factorizable form and have shown that the density matrix diagonalizes in the energy basis which is number states for the oscillator and momentum states for the free particle for arbitrary initial conditions (Roy & Venugopalan, 1999). It is intuitive that the pointer states should naturally be a consequence of the interplay between the various components of the total Hamiltonian and one should also expect them to be independent of the initial state of the system/apparatus.

The studies mentioned above do shed light on the nature of the preferred basis but are inadequate and there is a need to analyze more systems. In particular, it is important to look at systems like the harmonic oscillator apparatus model which is fairly generic and exact solutions make it an interesting candidate to explore experimentally in the context of decoherence and quantum measurements. Also, this example indicates that it seems pertinent to look at a system-apparatus-environment like scenario for measurement to analyse the issue of the pointer basis and the states singled out by the environment.

Finally, we would like to refer to a recent work of Venugopalan (Venugopalan, 2011) which indicates that there is a need to look at the bare system-apparatus interaction in a measurement like scenario more carefully to get insights into what states would eventually end up as pointer states and be selected in the event of environmental influence. Venugopalan has looked at a simple one dimensional model for the system-apparatus interaction where the system is a spin-1/2 particle, and its position and momentum degrees constitutes the apparatus, like the Stern-Gerlach model discussed above. An analysis involving only unitary Schrödinger dynamics illustrates the nature of the correlations established in the system-apparatus entangled state. It is shown that even in the absence of any environment-induced decoherence, or any other measurement model, certain initial states of the apparatus -like localized Gaussian wavepackets - are preferred over others, in terms of the establishment of measurementlike one-to-one correlations in the pure system-apparatus entangled state. This result indicates that perhaps there already exist special states of the apparatus within the quantum menu, and it is these that end up being ultimately selected by the environment as the preferred states.

5. Conclusions

The central theme of the decoherence approach has been to explain, within the realm of quantum theory, the appearance of classicality in the macroscopic, familiar physical world. The strength of the theory has been often claimed to be the fact that it is within the realm of quantum mechanics, and uses the rules of quantum theory to explain the emergence of classicality. In the case of quantum measurement, decoherence is believed to “mimic” wave function collapse. This is achieved via the transition from a pure state density matrix to a statistical mixture with ‘classically’ meaningful terms. In a sense, decoherence explains the washing away of quantum coherences and the emergence of a state which makes classical sense. In this chapter we have seen this happen in the two measurement models considered above - one with free particle as an apparatus and one with a harmonic oscillator as the apparatus. We have seen that the environmental influence is crucial in not only destroying the quantum coherences, but also is selecting a special state or a preferred basis. A criticism levelled against the decoherence approach is often that it does not explain the fact that only
one of the mixed outcomes is actually observed, nor does it allow us to predict exactly which one will be observed. Inspite of these criticism, it is generally accepted that the decoherence explanation has certainly provided valuable insights into the actual mechanism of the loss of quantum coherences. In recent years, the predictions of the decoherence theory have been tested in several spectacular experiments which put the theory on a firm footing. Of these, two experiments are particularly noteworthy as they have succeeded in monitoring the decoherence mechanism, i.e., the actual transition from a pure entangled state to a statistical mixture. Moreover, the experiments also give a quantitative estimate of the decoherence time. These experiments have done no less than create Schrödinger-cat-like entangled states in the laboratory and seen them transform into classically recognizable objects under the influence of environmental coupling. Among the first successful attempts is an experiment by Brune et al (Brune et al., 1996) at the Ecole Normale Superieure in Paris. Using Rubidium atoms and high technology superconducting microwave cavities, Brune et al created a superposition of quantum states involving radiation fields. The superposition was the equivalent of a "system+ measuring apparatus" situation in which the "meter" was pointing simultaneously towards two different directions. This is a Schrönger-Cat-like entangled state. Through a series of ingenious "atom-interferometry" experiments, Brune et al. managed to not only "read" this pure state but also monitored the decoherence phenomenon as it unfolded, transforming this superposition state to a statistical mixture. Besides providing a direct insight into the role of the environment in a quantum measurement process, their experiment also confirmed the basic tenets of the decoherence theory. The two main signatures of the decoherence theory were clearly observed in this classic experiment. The environment in this experiment are the "modes" of the electromagnetic field in the cavity. At the National Institute of Standards and Technology, Boulder, Colorado, the group headed by Wineland (Monroe et al., 1996) created a Schrödinger-Cat-like state using a series of laser pulses to entangle the internal (electronic) and the external (motional) states of a Beryllium ion in a "Paul trap". The motion of this trapped ion couples to an electric field which changes randomly, thus simulating an environment. Monroe et al call this environment an engineered reservoir whose state and coupling can be controlled. Through their measurements, Wineland et al. have successfully demonstrated the two important signatures of the decoherence mechanism. Thus one can see that the qualitative and quantitative predictions of the decoherence theory are experimentally tested. These tests would be highly relevant to all experimental implementations of the novel ideas of quantum information and computation as decoherence would ruin the functioning of devices which use uniquely quantum mechanical effects for information processing. The above two experiments, along with others have provided important insights into the role of the environment in bringing about classicality and thus decoherence theory is strengthened by these spectacular observations. What, then, is the final verdict of the decoherence theory? Has it resolved the conceptual problems of quantum mechanics? There are many who believe that the conceptual problems of quantum mechanics are still unresolved and decoherence does not answer many issues. At the end of the day we can say that the decoherence explanation takes away some of the mystery from the idea of 'wave function collapse' and provides a conventional mechanism to explain the appearance of a classical world. Many physicists find this a satisfactory explanation and there is no doubt that the experiments discussed clearly show how decoherence washes away quantum coherences providing a fairly convincing evidence for explaining the absence of Schödinger’s Cats in the real world. For all practical purposes, the decoherence explanation finds favour as a satisfactory settlement of the quantum measurement problem.
6. References

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Perhaps quantum mechanics is viewed as the most remarkable development in 20th century physics. Each successful theory is exclusively concerned about "results of measurement". Quantum mechanics point of view is completely different from classical physics in measurement, because in microscopic world of quantum mechanics, a direct measurement as classical form is impossible. Therefore, over the years of developments of quantum mechanics, always challenging part of quantum mechanics lies in measurements. This book has been written by an international invited group of authors and it is created to clarify different interpretation about measurement in quantum mechanics.

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