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Optimization and Synthesis of a Robot Fish Motion

Janis Viba, Semjons Cifanskis and Vladimirs Jakushevich

Riga Technical University
Latvia

1. Introduction

Inverse method algorithm for investigation of mechatronic systems in vibration technology is used for robotic systems motion control synthesis. The main difference of this method in comparison with simple analysis method is that before synthesis of a real system the optimal control task for abstract initial subsystem is solved [1 - 4]. As a result of calculations the optimal control law is found that allows to synthesize series of structural schemes for real systems based on initial subsystem. Is shown that near the optimal control excitation new structural schemes may be found in the medium of three kinds of strongly non-linear systems:

- systems with excitation as a time function;
- systems with excitation as a function of phase coordinates only;
- systems with both excitations mixed [2 – 4].

Two types of vibration devices are considered. The first one is a vibration translation machine with constant liquid or air flow excitation. The main idea is to find the optimal control law for variation of additional surface area of machine working head interacting with water or air medium. The criterion of optimization is the time required to move working head of the machine from initial position to end position. The second object of the theoretical study is a fin type propulsive device of robotic fish moving inside water. In that case the aim is to find optimal control law for variation of additional area of vibrating tail like horizontal pendulum which ensures maximal positive impulse of motion forces acting on the tail. Both problems have been solved by using the Pontryagin’s maximum principle. It is shown that the optimal control action corresponds to the case of boundary values of area. One real prototype was investigated in linear water tank.

2. Translation motion system

First object with one degree of freedom x and constant water or air flow $V_0$ excitation is investigated (Fig. 1., 2.). The system consists of a mass m with spring c and damper b. The main idea is to find the optimal control law for variation of additional area $S(t)$ of vibrating mass m within limits (1.): 

$$S_1 \leq S(t) \leq S_2,$$  

(1)
where $S_1$ - lower level of additional area of mass $m$; $S_2$ - upper level of additional area of mass $m$, $t$ - time.

The criterion of optimization is the time $T$ required to move object from initial position to end position.

Then the differential equation for large water velocity $V_0 \geq |\dot{x}|$ is (2):

$$m \ddot{x} = -c x - b \dot{x} - u(t) \cdot (V_0 + \dot{x})^2,$$

where $u(t) = S(t) \cdot k$, $c$ - stiffness of a spring, $b$ - damping coefficient, $V_0$ - constant velocity of water, $S(t)$ - area variation, $u(t)$ - control action, $k$ - constant.

It is required to determine the control action $u = u(t)$ for displacement of a system (2) from initial position $x(t_0)$ to end position $x(t_1)$ in minimal time (criterion $K$) $K = T$, if area $S(t)$ has the limit (1).

Fig. 1. Working head construction

Fig. 2. Side-view of a mathematical model
2.1 Solution of optimal control problem

For solution of problem the Pontryagin’s maximum principle may be used [5 - 13]. We have the high-speed problem \( K = \int_{t_0}^{t_1} 1 \cdot dt \).

To assume \( t_0 = 0; \ t_1 = T \), we have \( K = T \).

From the system (2), we transform \( x_1 = x; \ \dot{x}_1 = x_2 \) or

\[
\dot{x}_1 = x_2; \quad \dot{x}_2 = \frac{1}{m} \left( -c x - b \dot{x} - u(t) \cdot (V_0 + \dot{x})^2 \right)
\]

and we have Hamiltonian (3):

\[
H = \psi_0 + \psi_1 x_2 + \psi_2 \left( \frac{1}{m} \left( -c x_1 - b x_2 - u(t) \cdot (V_0 + x_2)^2 \right) \right),
\]

here \( H = \psi \cdot X \), where (4)

\[
\psi = \begin{bmatrix} \psi_0 \\ \psi_1 \\ \psi_2 \end{bmatrix}; \quad X = \begin{bmatrix} 0 \\ \dot{x}_1 \\ \dot{x}_2 \end{bmatrix}.
\]

Scalar product of those vector functions \( \psi \) and \( X \) in any time (function \( H \)) must be maximal. To take this maximum, control action \( u(t) \) must be within limits \( u(t) = u_1; \ u(t) = u_2 \), depending only on function \( \psi_2 \) sign (5):

\[
H = \max H, \quad \text{if} \quad \psi_2 \cdot (-u(t) \cdot (V_0 + x_2)^2) = \max.
\]

If \( \psi_2 > 0 \), \( u(t) = u_1 \) and if \( \psi_2 < 0 \), \( u(t) = u_2 \), where \( u_1 = S_1 \cdot k \) and \( u_2 = S_2 \cdot k \), see (1).

The examples of control action with one, two and three switch points are shown in Fig. 3. – Fig. 5.

Fig. 3. Example of optimal control with one switch point
2.2 Theoretical synthesis of control action

For realizing of optimal control actions (in general case) system of one degree of freedom needs a feedback system with two adapters: one to measure a displacement and another one to measure a velocity. There is a simple case of control existing with only one adapter when directions of motion are changed (Fig. 5) [3]. It means that control action is like negative dry friction and switch points are along zero velocity line. In this case equation of motion is (6):

\[
m \cdot \ddot{x} = -c \cdot x - b \cdot \dot{x} - \left[ k \cdot (V_0 + \dot{x})^2 \cdot A_1 \cdot \left( 0, 5 - 0,5 \cdot \frac{\dot{x}}{|\dot{x}|} \right) \right] -
\]

\[
+ \left[ k \cdot (V_0 + \dot{x})^2 \cdot A_2 \cdot \left( 0,5 + 0,5 \cdot \frac{\dot{x}}{|\dot{x}|} \right) \right],
\]

where \( m \) – mass; \( c, b, k, V_0 \) – constants. Examples of modeling are shown in Fig. 6. – Fig. 9.

Fig. 4. Example of optimal control with two switch points in the system with damping

Fig. 5. Example of optimal control with three switch points in the task of system excitation
Fig. 6. Full control action in time domain

Fig. 7. Displacement in time domain

Fig. 8. Motion in phase plane starting from inside of limit cycle

A search for the case with more than one limit cycle was investigated in a very complicated system with linear and cubic restoring force, linear and cubic resistance force and dry friction. It was found that for a system with non-periodic excitation (like constant velocity of water or air flow) more than one limit cycles exist (Fig. 10., 11.).
Fig. 9. Motion with initial conditions outside of limit cycle

\[
\begin{bmatrix}
  x_0 \\
  v_0
\end{bmatrix} = \begin{bmatrix}
  1.164209 \\
  1.500808
\end{bmatrix}
\]

Fig. 10. Motion in phase plane for small limit cycle

\[
\begin{bmatrix}
  x_0 \\
  v_0
\end{bmatrix} = \begin{bmatrix}
  0.09 \\
  0
\end{bmatrix}
\]

Fig. 11. Motion for large limit cycle

\[
\begin{bmatrix}
  x_0 \\
  v_0
\end{bmatrix} = \begin{bmatrix}
  9 \\
  -0
\end{bmatrix}
\]
2.3 Synthesis of a real system with rotating blades

Scheme of a system includes main mass of the object, spring and viscous damper (Fig. 12.). Part of mass with blades does not rotate. Second part of main mass like blades rotates around fixed axis inside body. Blades have symmetric areas A1 and A2.

Fig. 12. System with rotating blades

Results of investigation are shown in Fig. 13. – Fig. 14.

Fig. 13. Area change function in time domain when blades rotate with constant angular velocity $\omega$

Investigation shows that system is very stable because of air excitation and damping forces depending from velocity squared.
2.4 Synthesis of real system inside tube with variable hole area - A

Adaptive control where analyzed by next area exchange functions $f_1(A)$ (Fig. 15, 16.):

$$f_1(A) = A_2 \cdot (1 + a \cdot \frac{\mathbf{v} \cdot \mathbf{x}}{\|\mathbf{v}\| \cdot \|\mathbf{x}\|}),$$

where $A_2$, $a$ – constants, $\mathbf{v}$, $\mathbf{x}$ – velocity and coordinate of moving mass $m_2$.

Fig. 14. Motion in phase plane for parameters close to the first resonance

Fig. 15. Synthesis of system inside tube
It is shown that adaptive systems are also very stable because of water or air excitation and damping forces depending from velocity squared.

In order to get values of parameters in equations, experiments were made in the wind tunnel. It should be noted that the air flow is similar to the water flow, with one main difference in the density. It is therefore convenient to perform synthesis algorithm experiments done with air flow in the wind tunnel. Experimental results obtained in the wind tunnel to some extent may be used to analyze water flow excitation in robot fish prototypes. Using “Armfield” subsonic wind tunnel several additional results were obtained. For example, drag forces, acting on different shapes of bodies, were measured and drag coefficients were calculated by formula $F_d = C_d \cdot S \cdot \frac{\rho \cdot V^2}{2}$, where $\rho$ - density, $V$ - flow velocity, $S$ - specific area, $C_d$ - drag coefficient. These coefficients depend on bodie’s geometry, orientation relative to flow, and non-dimensional Reynolds number.
All experiments were done at constant air flow velocity in the range of 10 – 20 m/s. This corresponds to $Re$ value of about 40 000.

Drag coefficient for a plate perpendicular to the flow depends on plate’s length and chord ratio. For standard plates $C_d$ it is about 1.2. For infinitely long plates $C_d$ reaches the value of 2. For a cone $C_d = 0.5$; for a cylinder along the flow - about 0.81; for both cylinder and cone in one direction - 0.32. All these forms can be used for robot fish main body form designing, (together with the tail) in real robot fish models (see part 3.4, where cone and cylinder were used). Depth control of prototypes was provided by two front control wings (see part 3.4).

Parameters of subsonic wind tunnel were as follows: length 2.98m, width 0.8m., height 1.83m. Variable speed motor drive unit downstream of the working section permits stepless control of airspeed between 0 and 26 ms-1. Experiments prove that air flow excitation is very efficient.

3. Investigation of a rotating system

The second object of the theoretical study is a fin type propulsive device of robotic fish moving inside water. The aim of the study is to find out optimal control law for variation of additional area of vibrating tail like horizontal pendulum, which ensures maximal positive impulse of motion forces components acting on a tail parallel to x axis (Fig.18).

![Fig. 18. Side-view of a robot fish tail like horizontal pendulum](image)

Problem has been solved by using Pontryagin’s maximum principle. It is shown that optimal control action corresponds to the case of boundary values of area.

Mathematical model of a system consists of rigid straight flat tail moving around pivot (Fig. 18,19.). Area of the tail may be changed by control actions. For excitation of the motion a moment $M(t, \phi, \omega)$ around pivot must be added, where $\omega = dq/dt$ – angular velocity of a rigid tail. To improve performance of a system additional rotational spring with stiffness $c$ may be added (Fig. 19.).
Fig. 19. Top-view of a robot fish tail like horizontal pendulum. Simplified model with fixed pivot A

The principal task described in this report is to find optimal control law for variation of additional area \( B(t) \) of vibrating tail within limits (7):

\[
B_1 \leq B(t) \leq B_2,
\]

where \( B_1 \) - lower level of additional area of a tail; \( B_2 \) - upper level of additional area of a tail, \( t \) - time.

The criterion of optimization is full reaction \( Ax^* \) impulse in pivot, acting to a hull (indirect reaction to a tail) which is required to move an object from one stop - initial position \((\varphi_{\text{max}}, \varphi = 0)\) to another stop - end position \((\varphi_{\text{min}}, \varphi = 0)\) in time \( T(8) \):

\[
K = -\int_{0}^{T} Ax^* \cdot dt,
\]

Differential equation of motion of the system with one degree of freedom (including a change of angular momentum around fixed point A) is (9):

\[
I_{x}\ddot{\varphi} = M(t, \varphi, \varphi) - c \cdot \varphi - k \cdot B \cdot \text{sign}(\varphi \cdot \varphi) \cdot \int_{0}^{L} (\dot{\varphi} \cdot \xi)^2 \cdot \xi \cdot d\xi.
\]

Here \( I_{x} \) - moment of inertia of the tail around pivot point; \( \varphi \) - angular acceleration of a rigid straight tail; \( M(t, \varphi, \varphi) \) - excitation moment of the drive; \( c \) - angular stiffness; \( k \) - constant coefficient; \( B \) - area; \( \int_{0}^{L} (\dot{\varphi} \cdot \xi)^2 \cdot \xi \cdot d\xi \) - component of the moment of resistance force expressed as an integral along tail direction; \( \dot{\varphi} \) - angular velocity; \( L \) - length of the tail.

From the principle of the motion of mass center \( C \) it follows (10):
After integration from equations (9) and (10) we have (11, 12):

\[
\dot{\varphi} = \frac{1}{I_A} \left[ M(t, \varphi, \dot{\varphi}) - c \cdot \varphi - k_1 \cdot B \cdot \text{sign}(\varphi) \cdot \dot{\varphi}^2 \cdot \frac{L^4}{4} \right].
\]

\[
A_x = m \cdot (\varphi^2 \cdot \frac{L}{2} \cdot \cos(\varphi) + \frac{1}{I_A} \left[ M(t, \varphi, \dot{\varphi}) - c \cdot \varphi - k_1 \cdot B \cdot \text{sign}(\varphi) \cdot \dot{\varphi}^2 \cdot \frac{L^4}{4} \right] \cdot \frac{L}{2} \cdot \sin(\varphi)) +
+k_1 \cdot B \cdot \sin(\varphi) \cdot \text{sign}(\varphi \cdot \dot{\varphi}) \cdot \dot{\varphi}^2 \cdot \frac{L^3}{3}.
\]

Then the criterion of optimization (full impulse) is:

\[
K = -\int_0^T \left[ m \cdot (\varphi^2 \cdot \frac{L}{2} \cdot \cos(\varphi) + \frac{1}{I_A} \left[ M(t, \varphi, \dot{\varphi}) - c \cdot \varphi - k_1 \cdot B \cdot \text{sign}(\varphi) \cdot \dot{\varphi}^2 \cdot \frac{L^4}{4} \right] \cdot \frac{L}{2} \cdot \sin(\varphi)) +
+k_1 \cdot B \cdot \sin(\varphi) \cdot \text{sign}(\varphi \cdot \dot{\varphi}) \cdot \dot{\varphi}^2 \cdot \frac{L^3}{3} \right] \, dt,
\]

Equations (13) will be used to solve the optimization problem.

### 3.1 Task of optimization

Solution of optimal control problem for a system with one degree of freedom (11) by using the Pontryagin’s maximum principle includes following steps [8 - 13]:

1. Formulation of a criterion of optimization (13):

\[
K = -\int_0^T A_x \, dt.
\]

2. Transformation of the equation (9) and equation (13) in the three first order equations with new variables \(\varphi_0, \varphi_1\) and \(\varphi_2\) (phase coordinates):

\[
\varphi_0 = (-1) \cdot \left[ m \cdot (\varphi^2 \cdot \frac{L}{2} \cdot \cos(\varphi) + \frac{1}{I_A} \left[ M(t, \varphi_1, \varphi_2) - c \cdot \varphi_1 - k_1 \cdot B \cdot \text{sign}(\varphi_2) \cdot \varphi_2^2 \cdot \frac{L^4}{4} \right] \cdot \frac{L}{2} \cdot \sin(\varphi)) +
+k_1 \cdot B \cdot \sin(\varphi_1) \cdot \text{sign}(\varphi_1 \cdot \varphi_2) \cdot \varphi_2^2 \cdot \frac{L^3}{3} \right]
\]

\[
\varphi_1 = \varphi_2;
\]

\[
\varphi_2 = \frac{1}{I_A} \left[ M(t, \varphi_1, \varphi_2) - c \cdot \varphi_1 - k_1 \cdot B \cdot \text{sign}(\varphi_2) \cdot \varphi_2^2 \cdot \frac{L^4}{4} \right]
\]

According to procedure of Pontryagin’s maximum principle, Hamiltonian \(H\) is [9 -12]:

\[
H = m \cdot \left( \varphi^2 \cdot \frac{L}{2} \cdot \cos(\varphi) + \frac{1}{I_A} \left[ M(t, \varphi, \dot{\varphi}) - c \cdot \varphi - k_1 \cdot B \cdot \text{sign}(\varphi) \cdot \dot{\varphi}^2 \cdot \frac{L^4}{4} \right] \cdot \frac{L}{2} \cdot \sin(\varphi)) +
+k_1 \cdot B \cdot \sin(\varphi) \cdot \text{sign}(\varphi \cdot \dot{\varphi}) \cdot \dot{\varphi}^2 \cdot \frac{L^3}{3} \right).
\]
Optimization and Synthesis of a Robot Fish Motion

\[ H = \psi_0 \cdot (-1) \cdot \left[ m \cdot q^2 \cdot \frac{L}{2} \cdot \cos(\varphi_1) + \frac{1}{J_A} \cdot [M(t, \varphi_1, \varphi_2) - c \cdot \varphi_1 - k_i \cdot B \cdot \text{sign}(\varphi_2) \cdot q^2 \cdot \frac{L^4}{4} \cdot \frac{L}{2} \cdot \sin(\varphi_1)] + k_i \cdot B \cdot \sin(\varphi_1) \cdot \text{sign}(\varphi_1 \cdot \varphi_2) \cdot q^2 \cdot \frac{L^3}{3} \right] + \psi_1 \cdot \varphi_2 + \psi_2 \cdot \frac{1}{J_A} \cdot [M(t, \varphi_1, \varphi_2) - c \cdot \varphi_1 - k_i \cdot B \cdot \text{sign}(\varphi_2) \cdot q^2 \cdot \frac{L^4}{4}] \]

(15)

here \( H = \psi \cdot \Phi \), where (10, 11)

\[ \psi = \begin{pmatrix} \psi_0 \\ \psi_1 \\ \psi_2 \end{pmatrix} \]

(16)

\[ \Phi = \begin{pmatrix} \Phi_0 \\ \Phi_1 \\ \Phi_2 \end{pmatrix} \]

(17)

\( \Phi_0, \Phi_1, \Phi_2 \) - right side of system equations (14).

For functions \( \Phi_0, \Phi_1, \Phi_2 \) calculations are following differential equations [9 - 12]:

\[ \Phi_0 = -\frac{\partial H}{\partial \varphi_0} ; \Phi_1 = -\frac{\partial H}{\partial \varphi_1} ; \Phi_2 = -\frac{\partial H}{\partial \varphi_2} , \]

(18)

where left side is the derivation in time \( t \): \( \dot{\Phi}_{0,1,2} = \frac{d\Phi_{0,1,2}}{dt} \).

From equations (15) and (18) we get nonlinear system of differential equations to find functions \( \psi_0, \psi_1, \psi_2 \). Solution of such system is out of the scope of this report because it depends from unknown moment \( M(t, \varphi, \omega) \). But some conclusions and recommendations may be given from Hamiltonian if excitation moment \( M(t, \varphi, \omega) \) does not depend from phase coordinates \( \varphi = \varphi_1, \omega = \varphi_2 \):

\[ M = M(t). \]

In this case scalar product of two last vector functions \( \psi \) and \( \Phi \) in any time (Hamiltonian \( H [11] \)) must be maximal (supremum - in this linear B case) [8 - 12]. To have such maximum (supremum), control action \( B(t) \) must be within limits \( B(t) = B_1; \quad B(t) = B_2 \), depending only from the sign of a function (19) or (20):
From inequalities (19) and (20) in real system synthesis following quasi-optimal control action may be recommended (21):

\[
B = B_2; \\
B = \left[ B_2 \cdot (0,5 - 0,5 \cdot \text{sign}(\varphi_1 \cdot \varphi_2)) + B_1 \cdot (0,5 + 0,5 \cdot \text{sign}(\varphi_1 \cdot \varphi_2)) \right],
\]

or

\[
B = \left[ B_2 \cdot (0,5 - 0,5 \cdot \text{sign}(\varphi \cdot \varphi)) + B_1 \cdot (0,5 + 0,5 \cdot \text{sign}(\varphi \cdot \varphi)) \right].
\]

### 3.2 Synthesis of mixed system with time-harmonic excitation and area adaptive control

In the case of time-harmonic excitation moment \( M \) in time domain is (see equations (9) and (11)):

\[
M(t) = M_0 \cdot \sin(k \cdot t).
\]

Results of modeling are shown in Fig. 20. – 25. Comments about graphics are given under all Fig. 20. – 25.

![Fig. 20](image-url)
Fig. 21. Angular acceleration of the tail as function of angle. At the end of transition process graph is symmetric.

Fig. 22. Angular acceleration $\varepsilon = \dot{\phi}$ of a tail in time domain (see equation (9)). Practically angular acceleration of the tail reaches steady-state cycle after one oscillation.

Fig. 23. Impulse $Ax(t)$ in time domain (see equation (12)). Impulse is non-symmetric against zero level (non-symmetry is negative).
3.3 Synthesis of a system with adaptive excitation and adaptive area control

In a case of adaptive excitation a moment $M$ may be used as the function of angular velocity in the form [3. 4]:

$$M(t) = M_0 \cdot \text{sign}(\omega).$$

Results of modeling are shown in Fig. 26. - 31. Comments about graphics are given under all Fig. 26. - 31.
Fig. 26. Tail angular motion in phase plane – angular velocity as the function of angle. Due to large water resistance the transition process is very short. Adaptive excitation is more efficient than harmonic excitation (see Fig. 20.)

Fig. 27. Angular acceleration of a tail as the function of angle. At the end of transition process graph is more symmetric than the graph shown in Fig. 22

Fig. 28. Angular acceleration $\varepsilon = \dot{\phi}$ of a tail in time domain (see equation (9)). Typically angular acceleration of the tail reaches steady-state cycle after half oscillation
Fig. 29. Impulse $Ax(t)$ in time domain (see equation (12)). Impulse is non-symmetric against zero level (non-symmetry is negative).

Fig. 30. Impulse $Ax(t)$ as a function of angle $\phi$

Impulse is non-symmetric against zero level (non-symmetry is negative), it means that criterion of optimization (8) is positive and force of pivot pushes fish hull to the right.

\[ IAx_n := \sum_{n} Ax_n \]

Fig. 31. Negative mean value of $Ax(t)$ in time domain. Force of pivot to push fish hull to the right (in order to have robotic fish motions to the right). This graph shows that adaptive excitation of a moment is about four times more efficient than harmonic excitation (see Fig. 25.)
3.4 Robot fish model
A prototype of robot fish for experimental investigations was made (Fig. 32, 33). This prototype was investigated in a linear water tank with different aims, for example: – find maximal motion velocity depending on the power pack capacity; – find minimal propulsion force, depending on the system parameters.
Additionally this prototype was investigated in a large storage lake with autonomy power pack and distance control system, moving in real under water conditions with waves (Fig. 34.). The results of the theoretical and experimental investigation may be used for inventions of new robotic systems. The new ideas of synthesising robotic systems in Latvia can be found in [15 - 23].

Fig. 32. Top-view of prototype

Fig. 33. Inside prototype there are: microcontroller 1; three actuators (for level 4, direction 5 and velocity 6 control); power supply 2 and radio signal detector 3)
4. Conclusion

Motion of robotic systems vibration by simplified interaction with water or air flow can be described by rather simple equations for motion analysis. That allows to solve mathematical problem of area control optimization and to give information for new systems synthesis. Control (or change) of object area under water or in air allows to create very efficient mechatronic systems. For realization of such systems adapters and controllers must be used. For this reason very simple control action have solutions with use of sign functions. Examples of synthesis of real mechatronic systems are given. As one example of synthesis is a system with time-harmonic moment excitation of the tail in the pivot. The second example of synthesis is a system with adaptive force moment excitation as the function of phase coordinates. In both systems area change (from maximal to minimal values) has control action as the function of phase coordinates. It is shown that real controlled systems vibration motion is very stable.

5. References

Optimization and Synthesis of a Robot Fish Motion


This book brings together some of the latest research in robot applications, control, modeling, sensors and algorithms. Consisting of three main sections, the first section of the book has a focus on robotic surgery, rehabilitation, self-assembly, while the second section offers an insight into the area of control with discussions on exoskeleton control and robot learning among others. The third section is on vision and ultrasonic sensors which is followed by a series of chapters which include a focus on the programming of intelligent service robots and systems adaptations.

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