Chapter from the book *Recent Advances in Robust Control - Theory and Applications in Robotics and Electromechanics*

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1. Introduction

Switched reluctance motors (SRMs) can be applied in many industrial applications due to their cost advantages and ruggedness. The switched reluctance motor is simple to construct. It is not only features a salient pole stator with concentrated coils, which allows earlier winding and shorter end turns than other types of motors, but also features a salient pole rotor, which has no conductors or magnets and is thus the simplest of all electric machine rotors. Simplicity makes the SRM inexpensive and reliable, and together with its high speed capacity and high torque to inertia ratio, makes it a superior choice in different applications.

The dynamics of these systems are highly nonlinear and their models inevitable contain parametric uncertainties and unmodeled dynamics. The application of non linear robust control techniques is a necessity for successful operation electrical system. The industrial applications necessitate speed/position variators having high dynamic performances, a good precision in permanent regime, a high capacity of overload and robustness to the different perturbations. Thus, the recourse to robust control algorithms is desirable in stabilization and in tracking trajectories [1, 2].

Variable structure control with sliding mode, is one of the effective non linear robust control approaches. Sliding Mode Control (SMC) has attracted considerable attention because it provides a systematic approach to the problem of maintaining stability. It has been studied extensively to tackle problems of the nonlinear dynamic control systems. The sliding mode control can offer many good properties such as good performance against unmodelled dynamics systems, insensitivity to parameter variation, external disturbance rejection and fast dynamic [5, 9].

Sliding mode control has long proved its interests. Among them, relative simplicity of design, control of independent motion (as long as sliding conditions are maintained), invariance to process dynamics characteristics and external perturbations, wide variety of operational modes such as regulation, trajectory control [1], model following [2] and observation [3].

However, the motor is highly nonlinear and operates in saturation to maximize the output torque. Moreover, the motor torque is a nonlinear function of current and rotor position. This highly coupled nonlinear and complex structure of the SRM make the design of the controller difficult [4].
Section 2, investigates a case study of sliding mode control. In a more general study, the third section develops sliding mode controllers for switched reluctance motor drive; the proposed controller is described, and used to control the speed of the switched reluctance motor. Simulation results are given to show the effectiveness of this controller. Conclusions are summarized in the last section.

2. SRM model

2.1 Description of the system

In a switched reluctance machine, only the stator presents windings, while the rotor is made of steel laminations without conductors or permanent magnets. This very simple structure reduces greatly its cost. Motivated by this mechanical simplicity together with the recent advances in the power electronics components, much research has been developed in the last decade. The SRM, when compared with the AC and DC machines, shows two main advantages:

- It is a very reliable machine since each phase is largely independent physically, magnetically, and electrically from the other machine phases;
- It can achieve very high speeds (20000 - 50000 r.p.m.) because of the lack of conductors or magnets on the rotor;

The switched reluctance machine motion is produced because of the variable reluctance in the air gap between the rotor and the stator. When a stator winding is energized, producing a single magnetic field, reluctance torque is produced by the tendency of the rotor to move to its minimum reluctance position [5].

A cross-sectional view is presented in figure 1.

![Switched reluctance motor](image_url)

**Fig. 1. Switched reluctance motor.**

The schematic diagram of the speed control system under study is shown in figure 2. The power circuit consists with the \( H \)-bridge asymmetric type converter whose output is connected to the stator of the switched reluctance machine. Each phase has two IGBTs and two diodes. The parameters of the switched reluctance motor are given in the Appendix [5, 6].

The SMC inputs are obtained by manipulating the speed reference and feedback, while the SMC output is integrated to produce the current reference.
2.2 Machine equation
The switched reluctance motor has a simple construction, but the solution of its mathematical models is relatively difficult due to its dominant non-linear behaviour. The flux linkage is a function of two variables, the current I and the rotor position (angle $\theta$). The instantaneous voltage across the terminals of a phase of an SR motor winding is related to the flux linked in the winding by Faraday’s law as:

$$ V_j = RI_j + \frac{\partial \Psi_j(i, \theta)}{\partial t} $$

(1)

With $j = 1, ..., 3$

Because of the double salience construction of the SR motor and the magnetic saturation effects, the flux linked in an SRM phase varies as a function of rotor position $\theta$ and the phase current. Equation (1) can be expanded as

$$ V_j = RI_j + \frac{\partial \Psi_j(i, \theta)}{\partial i} \frac{di}{dt} + \frac{\partial \Psi_j(i, \theta)}{\partial \theta} \omega $$

(2)

In which:

$$ \omega = \frac{\partial \theta}{\partial t} $$

where $\frac{\partial \Psi_j}{\partial i}$ is defined as $L(\theta, i)$, the instantaneous inductance, and term $\frac{\partial \Psi_j}{\partial \theta} \frac{\partial \theta}{\partial t}$ is the instantaneous back e.m.f.

While excluding saturation and mutual inductance effects, the flux in each phase is given by the linear equation

$$ \Psi_j(\theta, i_j) = L(\theta)i_j $$

(3)
It can be written as

\[ V_j = R I_j + L(\theta) \frac{\partial i}{\partial t} + i \frac{\partial L(\theta)}{\partial \theta} \omega \quad j = 1, \ldots, 3 \] (4)

The total energy associated with the three phases \( n = 3 \) is given by

\[ W_{\text{total}} = \frac{1}{2} \sum_{j=1}^{3} L(\theta + (n - j - 1)\theta_s) I_j^2 \]

with

\[ \theta_s = 2\pi \left( \frac{1}{N_r} - \frac{1}{N_s} \right) \] (5)

Each phase inductance displaced by an angle \( \theta_s \).

The average torque can be written as the superposition of the torque of the individual motor phases:

\[ T_e = \sum_{\text{phase}=1}^{n} T_{\text{phase}} \] (6)

and the motor total torque by

\[ T_e = \frac{\partial W_{\text{total}}}{d\theta} = \frac{1}{2} \sum_{j=1}^{3} \frac{\partial L(\theta + (n - j - 1)\theta_s)}{\partial \theta} I_j^2 \] (7)

The mechanical equations are

\[ J \frac{d\omega}{dt} = T_e - T_i - f\omega \] (8)

Where \( V \) - the terminal voltage, \( I \) - the phase current, \( R \) - the phase winding resistance, \( \Psi \) - the flux linked by the winding, \( J \) - the moment of inertia, \( f \) - the friction coefficient, \( L(\theta) \) - the instantaneous inductance, \( N_r \) - number of rotor poles, \( N_s \) - number of stator poles, \( T_i \) is the torque load and \( T_e \) is the total torque.

3. SRM sliding mode speed controller

3.1 Sliding mode principle

Sliding modes is phenomenon may appear in a dynamic system governed by ordinary differential equations with discontinuous right-hand sides. It may happen that the control as a function of the system state switches at high frequency, this motion is called sliding mode. It may be enforced in the simplest tracking relay system with the state variable \( x(t) \) \([7, 8]\):

\[ \frac{\partial x}{\partial t} = f(x) + u \] (9)
With the bounded function $f(x) \mid f(x) < f_0 \quad f_0 \text{ const} \tan t$ and the control as a relay function (figure(3)) of the tracking error $e = r(t) - \frac{\dot{e}}{\dot{t}} \quad r(t)$ is the reference input and $u$ is given by:

$$u = \begin{cases} u_0 & \text{if } e > 0 \\ -u_0 & \text{if } e < 0 \end{cases} \quad \text{or } \quad u = u_0 \text{sign}(e) \quad u_0 = \text{const} \tan t$$

![Fig. 3. Relay control.](image)

The values of $e$ and $\frac{\dot{e}}{\dot{t}} = \frac{\partial r}{\partial t} - f(x) - u_0 \text{sign}(e)$ have different signs if $u_0 > f_0 + \left| \frac{\partial r}{\partial t} \right|$.  

### 3.2 Sliding mode controller

The equivalent total phase power becomes [9, 10]

$$P_{eq}(t) = I_c^2(t) \omega \frac{\partial L(\theta)}{\partial \theta} = V_{dc}I_c(t)$$  \hfill (10)

The electromagnetic torque over the switching period is then

$$T_c = \left( \frac{V_{dc}}{\omega} \right) I_c(t)$$  \hfill (11)

If $I_c(t) = K_t \left( \frac{\omega}{V_{dc}} \right) I_t(t)$ then electromagnetic torque can be further simplified as

$$T_c = K_t I_t(t)$$  \hfill (12)

Where $K_t$ is a proportional torque constant and $I_t(t)$ is the equivalent dc-link current providing electromagnetic torque.

The electromagnetic dynamic model of a switched reluctance motor and loads can be expressed as follows [11,12, 13]:

$$\frac{\partial \omega}{\partial t} = \frac{(T_c - T_l - f \omega)}{J}$$  \hfill (13)
From (11) and (12), (13) can be obtained:

\[
\frac{\partial \omega}{\partial t} = \frac{(K_I I_r(t) - T_i - f \omega)}{J}
\]  

(14)

Speed control can be implemented by a sliding-mode variable structure controller, but a discontinuous torque control signal would cause chattering of the speed response. In order to enable smooth torque control and reduce the chattering problem, \( I_r(t) \) must be smoothed according to (11). The phase variable state representation of Fig. 4 can be used to develop the required control scheme. It can be simplified as:

\[
\begin{bmatrix}
\frac{\partial x_1}{\partial t} \\
\frac{\partial x_2}{\partial t}
\end{bmatrix} = \begin{bmatrix}
\frac{-f}{J} & \frac{K_t}{J} \\
0 & -\frac{R}{L(\theta)}
\end{bmatrix} \begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} + \begin{bmatrix}
0 \\
\frac{1}{L(\theta)}
\end{bmatrix} U + \begin{bmatrix}
1 \\
0
\end{bmatrix} \frac{\partial \omega_{ref}}{\partial t}
\]

(15)

Where \( x_1 = \omega - \omega_{ref} \), \( \omega_{ref} \) is the demand rotor speed, \( x_2 = I - I_{ref} \), and \( U \) is a control signal which is used to control the current error, irrespective of drive system parameter variations. The sliding line in the phase plane diagram [Fig. 4] can be described as follows:

\[
S = \omega - \omega_{ref}
\]

(16)

from the equation (13) and (15), we can be obtains

\[
\frac{\partial S}{\partial t} = \frac{K_t f}{J} - \frac{f \omega - T_i}{J} - \frac{\partial \omega_{ref}}{\partial t}
\]

(17)

the current of control is given by

\[
I_c = I_c^{eq} + I_c^n
\]

With

\[
I_c^{eq} = \frac{1}{K_t} \left( \frac{\partial \omega_{ref}}{\partial t} + f \omega + T_i \right)
\]

(18)

\[
I_c^n = K_p \text{sgn}(S(\omega))
\]

To satisfy the existence condition of the sliding-mode speed controller, the following must be satisfied:

\[
\lim_{s \to 0} S \frac{dS}{dt} \leq 0
\]

(19)

The controller can be designed as follows:

\[
U = a x_1 + b \frac{\partial x_1}{\partial t}
\]
Where:

\[
a = \begin{cases} 
\alpha_1 & \text{if } Sx_1 > 0 \\
\beta_1 & \text{if } Sx_1 < 0 
\end{cases}
\]

\[
b = \begin{cases} 
\alpha_2 & \text{if } S\frac{\partial x_1}{\partial t} > 0 \\
\beta_2 & \text{if } S\frac{\partial x_1}{\partial t} < 0 
\end{cases}
\]

(20)

\[d x_1/\partial t, S(x_1, dx_1/\partial t)=0, Sliding line\]

a and b are proportional and derivative gain constant respectively, and \(\alpha_1, \alpha_2, \beta_1\) and \(\beta_2\) are real constants.

Fig. 4. A prescribed sliding line in phase plane.

4. Simulation result

The speed regulation of the SRM, despite its mechanical simplicity, is not simple to achieve. In the previous sections, we saw the importance that the values of the commutation angles have to torque oscillations. A linear controller as the PI regulator presents good results for the SRM speed control [8]. However, the controller will be only valid for a given operating point. Therefore, some authors have investigated recently non-linear controllers based on the sliding mode [11,12,16] applied to SRM speed control. In this section, we discuss and illustrate the advantages and drawbacks of the SRM speed control by using a PI regulator and a sliding mode controller.

To show the sliding mode controller performances we have simulated the system described in figure 2. The simulation of the starting mode without load is done. The simulation is realized using the SIMULINK software in MATLAB environment. Figure 5 shows the performances of the sliding mode controller.

The saturating function in the PI block diagram is necessary since during the transient, if the current demanded is high and if the speed reference is also high, then the f.e.m. produced will prevent the current to grow. Hence, the maximum current value of the block saturation has been fixed in 25A.
Fig. 5.b shows the speed regulation for a reference of 120 rad/s, with $\theta_{on} = 0^\circ$ and $\theta_{off} = 38^\circ$. Values of PI parameters $K_p$ and $K_i$ have been optimized in order to have the best compromise between response time and overshoot. Fig. 5.b shows good results for the speed regulation with weak speed oscillations in the permanent regime. Using a PI controller, a good compromise for $K_p$ and $K_i$ parameters has been found in order to have weak speed oscillations. The values found were: $K_p = 0.18$ and $K_i = 2.85$. However, if a load is applied, the speed oscillations will increase, as illustrated in Fig. 5.b. These results have been obtained with a load of $T_i = 1.5$ Nm applied at $t = 0.6$ s.

Fig. 5 shows the very good performance reached by the sliding mode controller. Indeed, one notes that the overshoot is less important in the case of the sliding mode regulator, with a best response time without increasing the overshoot. Follow, we show the robustness of the PI and the sliding mode controller for the same operating condition.

![Fig. 5. PI and sliding speed regulation with $T_i = 1.5$ Nm applied at $t = 0.6$ s.](image)

Follow, the speed regulation operates in a supplementary quadrant. That means we are going to do a speed regulation with a negative torque load but maintaining the speed reference. Fig. 6 shows the results obtained for a negative load of $T_i = -2$ Nm also applied at $t = 0.6$ s.

Fig. 6 shows that the speed regulation is not assured anymore after $t = 0.6$ s. From (8), the motor speed is given by

$$\omega = \frac{(T_e - T_i)}{f}$$

(21)

Therefore, the minimum speed for a possible regulation without producing a braking torque, meaning that the regulator will have its reference current to zero, will be

$$\omega_{min} = \frac{-T_i}{f}$$

(22)
In our case, the minimum speed stays \( \omega_{\text{min}} = 96.3 \text{ rd/s} \), as shown in Fig. 6. Now, if one wants to continue the speed regulation at 90 rd/s, it will be necessary to produce a braking torque when the speed error is negative. The controller by sliding mode gives good result compared to that of regulator PI. The increase speed is reduced during the application of a negative torque. However, when the negative torque load is applied at \( t = 0.5 \text{s} \), the speed oscillations become significant. Previous results in Fig. 6 showed significant oscillations in the speed signal due to an initial bad choice of the \( \theta_{\text{on}} \) value.

![Fig. 6. PI and sliding speed regulation with \( T_i = -2 \text{Nm} \) applied at \( t = 0.6 \text{s} \).](image)

Figure 7 shows the very good performances reached by the sliding mode controller. Indeed, one notes that the overshoot is less important in the case of the sliding regulator, with a best response time without increasing the overshoot. For this test, the sliding controller proves to be well more robust because the speed curve is hardly of its reference. On the other hand, the speed signal evolution obtained with the PI controller deviates about 10% from its reference value (figure 7). The speed tracking is satisfactory, and the torque ripple is low. These results demonstrate the robustness of the drive under unpredictable load conditions. The decreasing speed oscillations with the PI controller are owed to a slower reaction of the current, as shown in figure 7.

### 4.1 Robustness

In order to test the robustness of the proposed control, we have studied the speed performances. Two cases are considered:

1. Inertia variation,
2. Stator resistance variation.

The figure 8 shows the tests of the robustness: a) The robustness tests concerning the variation of the resistances, b) the robustness tests in relation to inertia variations.
PI controller
Fig. 7. Simulation results of speed control.
a) Different values of resistance of stator

b) Different values of moment of inertia

Fig. 8. Test of robustness.

Figure 8-b shows the parameter variation does not allocate performances of proposed control. The speed response is insensitive to parameter variations of the machine, without overshoot and without static error. The other performances are maintained.

For the robustness of control, a decrease or increase of the moment of inertia, the resistances doesn’t have any effects on the performances of the technique used (figure 8.a and 8.b). An increase of the moment of inertia gives best performances, but it presents a slow dynamic response (figure 8.b). The controller suggested gives good performances although the parameters are unknown.

5. Conclusion

This chapter presents a new approach to robust speed control for switched reluctance motor. It develops a simple robust controller to deal with parameters uncertain and external disturbances and takes full account of system noise, digital implementation and integral control. The control strategy is based on SMC approaches.
The simulation results show that the proposed controller is superior to conventional controller in robustness and in tracking precision. The simulation study clearly indicates the superior performance of sliding control, because it is inherently adaptive in nature. It appears from the response properties that it has a high performance in presence of the plant parameters uncertain and load disturbances. It is used to control system with unknown model. The control of speed by SMC gives fast dynamic response without overshoot and zero steady-state error. The controller contains only two structures and the only way of changing them is by switching. A major drawback of this system is chattering, which is caused by a fast switching of the controller structure

6. Appendix

Throughout this section the motor parameters used to verify the design principles are:
Number of phase 3, Number of stator poles 6, Pole arc 30°, Number of rotor poles 4, Pole arc 30°, Maximum inductance 60mH, Minimum inductance 8mH, resistance 1,3Ω, Moment of inertia 0,0013Kgm², friction 0,0183Nm/s, Inverter voltage 150v.

7. References

W. Perruquetti, J.P.Barbot “Sliding Mode Control In Engineering” Ed Marcel Dekker, 2002
Ahmed TAHOUR, Hamza ABID2, Abdel Ghani AISSAOUI ” Speed Control of Switched Reluctance Motor Using Fuzzy Sliding Mode” Advances in Electrical and Computer Engineering Volume 8 (15), Number 1 (29), 2008.


Robust control has been a topic of active research in the last three decades culminating in \( H_2/H_\infty \) and \( \mu \) design methods followed by research on parametric robustness, initially motivated by Kharitonov's theorem, the extension to non-linear time delay systems, and other more recent methods. The two volumes of Recent Advances in Robust Control give a selective overview of recent theoretical developments and present selected application examples. The volumes comprise 39 contributions covering various theoretical aspects as well as different application areas. The first volume covers selected problems in the theory of robust control and its application to robotic and electromechanical systems. The second volume is dedicated to special topics in robust control and problem specific solutions. Recent Advances in Robust Control will be a valuable reference for those interested in the recent theoretical advances and for researchers working in the broad field of robotics and mechatronics.

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