Adaptive Control of Nonlinear Dynamics Systems Based on RBF Network

Ho Dac Loc, Nguyen Thi Phuong Ha & Luong Van Lang

1. Introduction

As it was mentioned in (Narendra et al., 1990; Ching-Teng Lin et al., 1996), many adaptive control techniques, such as self-tuning control, self-tuning PID, etc. have been developed and successfully implemented in many applications. More recently, many adaptive control systems which use artificial intelligent techniques are developed to deal with increasing complexity in control systems, such as non-linearity, unexpected load disturbances, variable time delay, etc. In many direct adaptive control approaches, the actual system error is usually used to tune the controller parameters. Several adaptive control strategies also make use of predictive models in formulating their adaptation laws. A large number of these intelligent adaptive control systems use neural network and fuzzy logic, and also combination of other new paradigms (Kosko B., 1994; Lewis F.L., 1995).

In this chapter, we will develop an adaptive controller based on RBF network for unknown nonlinear dynamic system. Design of adaptive controller is based on the theory optimal control. Some applications of the proposed methodology are introduced. The simulation results made by matlab showed that the synthesized adaptive control system have good performance.

2. Statement of the problem

Let us consider the nth-order nonlinear dynamic system in the form:

\[
\begin{align*}
\dot{x}_1 &= f_1(x) \\
\dot{x}_2 &= f_2(x) \\
&\vdots \\
\dot{x}_n &= f_n(x) + u \\
y &= x_1,
\end{align*}
\]

(1)

where \( x = (x_1, x_2, \ldots, x_n)^T \) is the state vector; \( f_i, i = 1, n \) are nonlinear continuous functions; \( y \) is the output signal; \( u \) is the control signal from controller.

The control objective is a determination of a feedback control \( u \) such that the following conditions are met:

the close-loop system must be globally stable.

b)

\[
\lim_{t \to \infty} |y_m - y| = \lim_{t \to \infty} |y_m - x_1| = \lim_{t \to \infty} |e|
\]

(2)
where \( y_m \) is the output reference signal.

If the functions \( f_i, i = 1, n \) are known, then the following control law can ensure the control objective (Kolesnikov A.A, 1994):

\[
\hat{u} = - \left[ \frac{\partial \Psi(x)}{\partial x_n} \right]^{-1} \left[ \frac{1}{T} \varphi(\Psi) + \sum_{i=1}^{n} \frac{\partial \Psi(x)}{\partial x_i} f_i(x) \right]
\]  

(3)

where \( T \) is a positive constant; \( \Psi(x) \) is a differentiable function of state variables and \( \Psi(0) = 0, \varphi(\Psi) \) is a continuous differentiable function responding the following conditions:

a) \( \varphi(0) = 0 \); b) \( \varphi(\Psi)^T \Psi > 0 \) for \( \forall \Psi \neq 0 \).

The control law (3) cannot be implemented since: a) \( f_i, i = 1, n \) are unknown; b) part of state vector is not measured variables. There are many practical situations where both a) and b) are true. The adaptive controller developed in this chapter is suitable for these situations.

3. Design of the adaptive controller

The basic architecture of the adaptive controller is a standard radial basis function neural network, plus an adaptive block which adjusts the parameters of the controller.

To begin, let \( \Psi(x) = e_1 + \alpha e_2 \), where \( \alpha \) is positive constant; \( \varphi(\Psi) = \Psi \). Using (1), we have:

\[
\frac{\partial \Psi}{\partial x_1} = -1 - \alpha \frac{\partial f_1(x)}{\partial x_1}
\]

\[
\frac{\partial \Psi}{\partial x_2} = -\alpha \frac{\partial f_1(x)}{\partial x_2}
\]

\[
\vdots
\]

\[
\frac{\partial \Psi}{\partial x_n} = -\alpha \frac{\partial f_1(x)}{\partial x_n}
\]

Substituting (4) into (3), we have:

\[
\hat{u} = \left( \alpha \frac{\partial f_1(x)}{\partial x_n} \right)^{-1} \left[ \Psi - \left( 1 + \alpha \frac{\partial f_1(x)}{\partial x_1} \right) f_1(x) - \alpha \frac{\partial f_1(x)}{\partial x_2} f_2(x) - \alpha \frac{\partial f_1(x)}{\partial x_n} f_n(x) \right]
\]

(5)

If the functions \( f_i, i = 1, n \) are known, then using control law of (5) applied to (1), we can write the following differentiable equation for \( x_1 \):

\[
x_1 = \sum_{i=1}^{n} f_i(x) x_i = \frac{1}{\alpha} \Psi - \frac{1}{\alpha} f_1(x) = \frac{1}{\alpha} \left( y_m - x_1 - \alpha x_i \right) - \frac{1}{\alpha} x_i
\]

(6)

or

\[
x_1 + \left( 1 + \frac{1}{\alpha} \right) x_1 + \frac{1}{\alpha} (x_1 - y_m) = 0
\]

(7)

Using \( z = x_1 - y_m \), we have:
\[
\ddot{z} + a_1 \dot{z} + a_0 z = 0 \tag{8}
\]

where \( a_1 = 1 + \frac{1}{\alpha} \); \( a_0 = \frac{1}{\alpha} \). Solving the differential equation (8), we have:

\[
z(t) = C_1 \exp\left[-\frac{1}{2} \left( a_1 - a_1^2 - 4a_0 \right) t \right] + C_2 \exp\left[-\frac{1}{2} \left( a_1 + a_1^2 - 4a_0 \right) t \right] \tag{9}
\]

where \( C_1 \) and \( C_2 \) are constants, which implies that \( \lim_{t \to \infty} |e| = 0 \) - main control objective.

Our purpose is to design a RBF neural network to approximate the optimal control of (5). The RBF network with two inputs, Gaussian basic functions and normalized output can be described as:

\[
u = \frac{\sum_{j=1}^{L} \left( \prod_{i=1}^{2} \mu_{A_i}(e_i) \right) \lambda}{\sum_{i=1}^{L} \left( \prod_{i=1}^{2} \mu_{A_i}(e_i) \right)} = \lambda^T \zeta(e) \tag{10}
\]

where \( \mu_{A_i}(e_i) \) is the activation function of neuron \( A_i \); \( L \) is the number of neurons in hidden layer; \( \lambda \) are unknown parameters of controller, which have been adjusted.

The equation (1) can be written as:

\[
\begin{align*}
\dot{x}_1 &= f_1(x) \\
\dot{x}_2 &= f_2(x) \\
& \vdots \\
\dot{x}_n &= f_n(x) + u + \dot{u} - \ddot{u}
\end{align*} \tag{11}
\]

Substituting (5) and approximated control signal by RBF-network output \( u = \lambda^T \zeta(e) \) into (11), we can write last equation of (11) in the form:

\[
\dot{x}_n = \left( \alpha \frac{\partial f_1(x)}{\partial x_n} \right)^{-1} \left[ \Psi - \left( 1 + \alpha \frac{\partial f_1(x)}{\partial x_1} \right) f_1(x) - \cdots - \alpha \frac{\partial f_1(x)}{\partial x_{n-1}} f_{n-1}(x) \right] + \lambda^T \zeta(e) \tag{12}
\]

Next, we develop an adaptive law to adjust the parameters of vector \( \hat{\lambda} \), which provides that the close-loop system is generally stable. Define the Lyapunov function candidate:

\[
V = \frac{1}{2} \Psi(e)^2 + \frac{1}{2} \alpha \frac{\partial f_1(x)}{\partial x_n} \zeta(e) \tag{13}
\]

where \( \theta = \hat{\lambda} - \lambda^* \) ; \( \gamma \) is positive constant. Using (11) and (12) we have a derivative of Lyapunov function as

\[
\dot{V} = \Psi \dot{\Psi} + \frac{1}{\gamma} \dot{\theta}^T \theta = -\frac{1}{T} \Psi^2 + \theta^T \left[ \frac{1}{\gamma} \lambda - \alpha \Psi \frac{\partial f_1}{\partial x_n} \zeta(e) \right] \tag{14}
\]

If we choose the adaptive law :
\[
\dot{\lambda} = \rho \psi \frac{\partial f_i}{\partial x_n} \zeta(e)
\]
then (14) becomes:
\[
\dot{V} = -\frac{1}{T} \psi^2 \leq 0
\] (15)
which guarantees that the close-loop system is generally stable. The overall scheme of the developed adaptive control system is shown in Fig.1 where C is an unit which inverts the error signal \( e \) into the error vector \( \xi \). Now, we make some few remarks.

Remark 1. The developed, in this paper, adaptive fuzzy control system using the error vector as inputs signal for the controller is suitable in situations, when the state vector is not measured variables.

Remark 2. The simple adaptive law makes easy to implement the adaptive NN-controller. The quality of the control system depends on form and parameters of function \( \psi \).

Remark 3. In using the adaptive law (18), the choice of the constant \( \gamma \) is very important. At present, there has been no theoretical guidance about the choice of \( \gamma \); trial-and-error seems to be the only practical option.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{The overall scheme of adaptive control system}
\end{figure}

4. Applications

4.1 Adaptive Control of Nonlinear System Sat Function

In this section, we apply the adaptive RBF-network controller developed by proposed method to control the following nonlinear dynamic system:
\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -x_2 + \text{sat}(x_3) \\
\dot{x}_3 &= -x_3 + u \\
y &= x_1
\end{align*}
\] (16)

The step respond of close-loop control system is shown in Fig.2 in which we can see the controller could regulate the plant and the close-loop system is stable. The step respond of the control system in situation, where \( \gamma \) has different values, is shown in Fig.2.b. From this we can see that the quality of the transient process depends on \( \gamma \).
4.2 Adaptive Control of DC Motor Containing Nonlinear Friction

The high-quality servos are largely described by nonlinear models. Their performance is often limited by nonlinear phenomena such as friction and backlash. Therefore, we consider a problem of this type, namely, a servo with nonlinear friction, which causes difficulties and gives rise to poor performance in precision servos.

Friction compensation has been considered before. In order to address better the demands of high fidelity control, adaptive friction compensation algorithms have recently appeared in the literature. The use of a recursive least-squares algorithm to estimate the parameters in a nonlinear friction model was described by Canudas et al. (Canudas et al., 1987). Friedland and Park (Friedland B. et al., 1987) presented another adaptive friction compensation scheme which was based upon a Lyapunov-like argument involving the position error. Many other studies on friction compensation are reported in a survey paper (Armstrong-Helouvy et al., 1994). However, these methods are based on the characteristics of the nonlinearity and knowledge of some of the parameters, in contrast to the adaptive methods proposed here.
The overall scheme of adaptive fuzzy logic control system, which is considered in this paper, is shown in Fig.3. A DC motor with a permanent magnet was used in our control system. Such a motor is used in robots and precision servos. The motor with nonlinear friction can be described by the following model:

\[
\begin{align*}
\dot{x}_1 &= -\frac{1}{T_m}x_1 + \frac{k_m}{T_m}[x_2 - M_d - F(x_1)] \\
\dot{x}_2 &= -\frac{1}{T_e}x_2 + \frac{k_e u}{T_e},
\end{align*}
\]  

(17)

where \(x_1 = \omega\) is the velocity of the motor shaft, \(M_d\) is the load disturbance, \(k_e\) and \(k_m\) are the gains, \(T_e\) and \(T_m\) are the time coefficients. The friction model is:

\[
M_f = F(\omega) = \alpha_0 \text{sgn}(\omega) + \alpha_1 \exp(-\alpha_2 |\omega|)\text{sgn}(\omega)
\]  

(18)

Figure 3. Adaptive control system of DC-motor
We apply the adaptive RBF-network controller developed in this chapter to control the DC motor with nonlinear friction. The computer simulation of control system is done by the language MATLAB.

The simulation result of the DC motor with adaptive RBF-network controller is shown in Fig.4, in which we can see that the developed controller could achieve the reference velocity of DC motor. We also investigate the motor velocity with the sinusoidal reference signal. In Fig.5 is showed the respond of velocity depending on load changing. From this we can see that the adaptive RBF-network controller achieves good trajectory. Figure 4. The velocity of DC motor and load.
4.3 Adaptive Control of Robot-Manipulator

Robot manipulators are complicated nonlinear dynamical systems with inherent unmodeled dynamics and unstructured uncertainties. These uncertainties make the controller design for robot manipulators a difficult task in the framework of classical adaptive and unadaptive control.

The traditional PID control with a simple structure and implementation (Koditchek D.E., 1984) has been the predominant method used for industrial manipulators controllers. CTM (Paul R.C., 1972) and ACM (Slotine J.E. et al., 1987) good performance, if manipulator dynamics are exactly known. However, they suffer from following difficulties: a) they require explicit a priory knowledge of individuals manipulators, which is very difficult to acquire in most practical applications; b) uncertainties existing in real manipulators seriously devalue the performance of both methods; c) the computational load of both methods is high.

In recent years, much attention has been paid to neural-network (NN) based controller. The nonlinear mapping and learning properties of NN a key factor for their use in the control field. These controllers take advantage of the capability of a NN for learning nonlinear functions and of the massive parallel computation, required in the implementation of advanced control algorithms.

This section deals with a neural-network based controller developed in this chapter for motion dynamic control of robot manipulator. The uncertainties on the robot dynamic parameters have motivated the design adaptive controller.

The general equation describing the dynamics of an n-degree of freedom rigid robot manipulator is given by:

$$ M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) + F(q, \dot{q}) = u $$

(19)

where \( q, \dot{q} \in R^n \) are the vector of generalized coordinators and velocities; \( M(q) \in R^{n \times n} \) is the positive inertia matrix; \( C(q, \dot{q}) \in R^n \) are the coriolis and centrifugal torques; \( G(q) \in R^n \) are the gravitational torques; \( F(q, \dot{q}) \) is the unstructured uncertainty of the dynamics including friction and other disturbances.

We will consider robot-manipulator which dynamic model is described in state space (Anna Jadlovska, 2000):

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= \frac{m_b}{m_r} x_1 x_4^2 + \frac{K_1}{m_r} u_1 \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= -\frac{2m_b}{I_{23} + m_b x_1^2} x_2 x_4 + \frac{K_2}{I_{23} + m_b x_1^2} u_2 
\end{align*}
\]
where \( m_b = m_2 + m_i \); \( m_r = m_r + m_b + m_2 \); \( m_g = 35 \) kg is the mass of weight; \( m_i = 52 \) kg is the grasp head and part of the arm; \( m_r = 62.5 \) kg is reduced mass of the gear; \( m_2 = 78 \) kg is the mass of servomotors of the arm; \( K_1 = 281.4 \) Nm, \( K_2 = 291 \) Nm are the constants of the operational values; \( I_{23} = I_r + (m_2 + m_3 + m_4) \cdot r_0^2 \) where \( I_r = 82.5 \) kgm\(^2\) is reduced torque of inertial of the electric servomotor and gear-box; \( m_3 = 90 \) kg; \( m_4 = 125 \) kg; \( r_0 = 250 \) mm.

In this section, we apply the adaptive controller developed in this chapter to control the two-link robot-manipulator of (20). The computer simulation of control system is done by the language MATLAB. The response of close-loop control system is shown in Fig.6, where the given outputs are \( x^1_1 = 1 \) and \( x^1_3 = 0.8 \), \( \gamma_1 = \gamma_2 = 500 \), \( \alpha_1 = \alpha_2 = 1 \), in which we can see that the developed adaptive controller could regulate plant and the close-loop control system is stable.

In Fig.7 is shown the response of the control system, where \( \gamma_1, \gamma_2 \) have different values. From this we can see that the quality of the control system depends on \( \gamma_1, \gamma_2 \ (\alpha_1 = \alpha_2 = 1) \).

Figure 6. The response of the control system

Figure 2. The state variable \( x_1(t) \) \( \gamma_1=\gamma_2=50 \), \( \gamma_1=\gamma_2=100\ldots, \gamma_1=\gamma_2=500 \)
5. Conclusion

In this work, we developed an adaptive fuzzy controller which: 1) does not require an accurate mathematical model of plant under control; 2) uses the error vector as controller’s input, therefore it does not require all that components of state vector to be measurable; and 3) guarantees the global stability of the close-loop system. The simulation results show that the adaptive fuzzy controller could successful control the unknown nonlinear dynamic system.

6. References

R.C. Paul (1972). Modeling, trajectory calculation and servoing of a computer controlled arm. Stanford Artificial Intelligence Laboratory, Stanford University, A.I. Memo 177.
This book is the result of inspirations and contributions from many researchers worldwide. It presents a collection of wide range research results of robotics scientific community. Various aspects of current research in robotics area are explored and discussed. The book begins with researches in robot modelling & design, in which different approaches in kinematical, dynamical and other design issues of mobile robots are discussed. Second chapter deals with various sensor systems, but the major part of the chapter is devoted to robotic vision systems. Chapter III is devoted to robot navigation and presents different navigation architectures. The chapter IV is devoted to research on adaptive and learning systems in mobile robots area. The chapter V speaks about different application areas of multi-robot systems. Other emerging field is discussed in chapter VI - the human-robot interaction. Chapter VII gives a great tutorial on legged robot systems and one research overview on design of a humanoid robot. The different examples of service robots are showed in chapter VIII. Chapter IX is oriented to industrial robots, i.e. robot manipulators. Different mechatronic systems oriented on robotics are explored in the last chapter of the book.

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