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C-Field Cosmological Model for Barotropic Fluid Distribution with Variable Gravitational Constant

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1. Introduction

The importance of gravitation on the large scale is due to the short range of strong and weak forces and also to the fact that electromagnetic force becomes weak because of the global neutrality of matter as pointed by Dicke and Peebles [1965]. Motivated by the occurrence of large number hypothesis, Dirac [1963] proposed a theory with a variable gravitational constant (G). Barrow [1978] assumed that $G \propto t^{-n}$ and obtained from helium abundance for $-5.9 \times 10^{-13} < n < 7 \times 10^{-13}$, $\left| \frac{\dot{G}}{G} \right| < (2 \pm .93)x10^{-12}\text{yr}^{-1}$ by assuming a flat universe.

Demarque et al. [1994] considered an ansatz in which $G \propto t^n$ and showed that $|n| < 0.1$ corresponds to $\left| \frac{\dot{G}}{G} \right| < 2 \times 10^{-1} \text{yr}^{-1}$. Gaztanga et al. [2002] considered the effect of variation of gravitational constant on the cooling of white dwarf and their luminosity function and concluded that $\left| \frac{\dot{G}}{G} \right| < 3 \times 10^{-1} \text{yr}^{-1}$.

To achieve possible verification of gravitation and elementary particle physics or to incorporate Mach's principle in General Relativity, many attempts (Brans and Dicke [1961], Hoyle and Narlikar [1964]) have been made for possible extension of Einstein's General Relativity with time dependent G.

In the early universe, all the investigations dealing with physical process use a model of the universe, usually called a big-bang model. However, the big-bang model is known to have the short comings in the following aspects.

i. The model has singularity in the past and possibly one in future.

ii. The conservation of energy is violated in the big-bang model.

iii. The big-bang models based on reasonable equations of state lead to a very small particle horizon in the early epochs of the universe. This fact gives rise to the 'Horizon problem'.

iv. No consistent scenario exists within the frame work of big-bang model that explains the origin, evolution and characteristic of structures in the universe at small scales.

v. Flatness problem.
Thus alternative theories were proposed from time to time. The most well known theory is the 'Steady State Theory' by Bondi and Gold [1948]. In this theory, the universe does not have any singular beginning nor an end on the cosmic time scale. For the maintenance of uniformity of mass density, they envisaged a very slow but continuous creation of matter in contrast to the explosive creation at \( t = 0 \) of the standard FRW model. However, it suffers the serious disqualifications for not giving any physical justification in the form of any dynamical theory for continuous creation of matter. Hoyle and Narlikar [1966] adopted a field theoretic approach introducing a massless and chargeless scaler field to account for creation of matter. In C-field theory, there is no big-bang type singularity as in the steady state theory of Bondi and Gold [1948]. Narlikar [1973] has explained that matter creation is a accomplished at the expense of negative energy C-field. He also explained that if overall energy conservation is to be maintained then the primary creation of matter must be accompanied by the release of negative energy and the repulsive nature of this negative reservoir will be sufficient to prevent the singularity. Narlikar and Padmanabhan [1985] investigated the solution of modified Einstein's field equation which admits radiation and negative energy massless scalar creation field as a source. Recently Bali and Kumawat [2008] have investigated C-field cosmological model for dust distribution in FRW space-time with variable gravitational constant.

In this chapter, we have investigated C-field cosmological model for barotropic fluid distribution with variable gravitational constant. The different cases for \( \gamma = 0 \) (dust distribution), \( \gamma = 1 \) (stiff fluid distribution), \( \gamma = 1/3 \) (radiation dominated universe) are also discussed.

Now we discuss Creation-field theory (C-field theory) originated by Hoyle and Narlikar [1963] so that it may be helpful to readers to understand Creation-field cosmological model for barotropic fluid distribution with variable gravitational constant.

2. Hoyle-Narlikar creation-field theory

Hoyle's approach (1948) to the steady state theory was via the phenomena of creation of matter. In any cosmological theory, the most fundamental question is "where did the matter (and energy) we see around us originate?" by origination, we mean coming into existence by primary creation, not transmutation from existing matter to energy or vice-versa. The Perfect Cosmological Principle (PCP) deduces continuous creation of matter. In the big-bang cosmologies, the singularity at \( t = 0 \) is interpreted as the primary creation event. Hoyle's aim was to formulate a simple theory within the framework of General Relativity to describe such a mechanism.

Now I discuss this method since it illustrates the power of the Action-principle in a rather simple way.

The action principle

The creation mechanism is supposed to operate through the interaction of a zero rest mass scalar field \( C \) of negative energy with matter. The action is given by

\[
A = -\frac{1}{16\pi G} \int R \sqrt{-g} \, d^4x - \sum_a ma \int da - \frac{1}{2} f \int C_i C^i \sqrt{-g} \, d^4x + \sum_a \int C_i \, da
\]  

(2.1)

where \( C_i = \frac{\partial C}{\partial x^i} \) and \( f > 0 \), is a coupling constant between matter and creation field.
The variation of a stretch of the world line of a typical particle 'a' between the world points \(A_1\) and \(A_2\) gives

\[
\delta A = \int_{A_1}^{A_2} ma \left\{ \frac{d^2 \delta a^i}{d a^2} + \Gamma^i_{kl} \frac{d x^k}{d a} \frac{d \delta a^l}{d a} \right\} g_{ik} \delta a^k da - \left\{ ma \frac{d a^i}{d a} g_{ik} - C_k \right\} \cdot \delta a^k \bigg|_{A_1}^{A_2}
\]

(2.2)

Now suppose that the world-line is not endless as it is usually assumed but it begins at \(A_1\) and the variation of the world line is such that \(\delta a^k \neq 0\) at \(A_1\). Thus for arbitrary \(\delta a^k\) which vanish at \(A_2\), we have

\[
\frac{d^2 \delta a^i}{d a^2} + \Gamma^i_{kl} \frac{d x^k}{d a} \frac{d \delta a^l}{d a} = 0
\]

(2.3)

along \(A_2A_2\) while at \(A_1\),

\[
ma \frac{d a^i}{d a} g_{ik} = C_k
\]

(2.4)

The equation (2.3) tells us that C-field does not alter the geodesic equation of a material particle. The effect of C-field is felt only at \(A_1\) where the particle comes into existence. The equation (2.4) tells us that the 4-momentum of the created particle is balanced by that of the C-field. Thus, there is no violation of the matter and energy-momentum conservations law as required by the action principle. However, this is achieved because of the negative energy of the C-field. The variation of C-field gives from \(\delta A = 0\),

\[
C_{ij} = \frac{1}{f} n
\]

(2.5)

where \(n\) = number of creation events per unit proper 4-volume. By creation event, we mean points like \(A_1\), if the word line had ended at \(A_2\) above, we would have called \(A_2\) an annihilation event. In \(n\), we sum algebraically (i.e. with negative sign for annihilation events) over all world-line ends in a unit proper 4-volume. Thus the C-field has its sources only in the end-points of the world-lines.

Finally, the variation of \(g_{ik}\) gives the Einstein's field equation

\[
R^{ik} - \frac{1}{2} R g^{ik} = -8\pi G \left[ T_{(m)}^{ik} + T_{(C)}^{ik} \right]
\]

(2.6)

Here \(T_{(m)}^{ik}\) is the energy-momentum of particles \(a, b, ...\) while

\[
T_{(C)}^{ik} = -f \left\{ C^i C^k - \frac{1}{2} g^{ik} C^j C_j \right\}
\]

(2.7)

is due to Hoyle and Narlikar (1964).

A comparison with the standard energy-momentum tensors of scalar fields shows that the C-field has negative energy. Thus, when a new particle is created then its creation is accompanied by the creation of the C-field quanta of energy and momentum. Since the C-
field energy is negative, it is possible to have energy momentum conservation in the entire process as shown in (2.4).

3. The metric and field equations

We consider FRW space time in the form

$$ds^2 = dt^2 - R^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right]$$

(3.1)

where $k = 0, -1, 1$

The modified Einstein's field equation in the presence of C-field is due to Hoyle and Narlikar [1964]) is given by

$$R^i_j - \frac{1}{2} R g^i_j = -8\pi G \left[ T^i_j + T^{(C)}_i_j \right]$$

(3.2)

where $\bar{R} = g^{ij}R_{ij}$, is the scalar curvature, $T^i_j$ is the energy-momentum tensor for matter and

$T^{(C)}_i_j$ the energy-momentum tensor for C-field are given by

$$T^i_j = (\rho + p) v^i v^j - pg^i_j$$

(3.3)

and

$$T^{(C)}_i_j = -f \left[ C_i C^i - \frac{1}{2} g^{ij} C_i C_j \right]$$

(3.4)

$p$ being isotropic pressure, $\rho$ the matter density, $f > 0$. We assume that flow vector to be comoving so that $v^1 = 0 = v^2 = v^3$, $v^4 = 1$ and $C_i = \frac{\partial C}{\partial x^i}$.

The non-vanishing components of energy-momentum tensor for matter are given by

$$T^1_{(m)} = (\rho + p) \cdot 0 - p = -p$$

(3.5)

Similarly

$$T^2_{(m)} = -p = T^3_{(m)}$$

(3.6)

$$T^4_{(m)} = (\rho + p) \cdot 1 - p = \rho$$

(3.7)

The non-vanishing components of energy-momentum tensor for Creation field are given by

$$T^1_{(C)} = -f \left[ 0 - \frac{1}{2} \cdot \dot{C}^2 - 4C^2 \right] = \frac{1}{2} f \dot{C}^2$$

(3.8)
Similarly

\[ T^2 (c) = \frac{1}{2} f \dot{\mathcal{C}}^2 = T^3 (c) \]  

(3.9)

\[ T^4 (c) = - f \left[ g^{44} c^2 - \frac{1}{2} \cdot 1 \cdot g^{44} c^2 \right] = - f \left[ c^2 - \frac{1}{2} c^2 \right] = - \frac{1}{2} f \dot{\mathcal{C}}^2 \]  

(3.10)

where \( c_4 \equiv \dot{\mathcal{C}} \)

The modified Einstein field equation (3.10) in the presence of C-field for the metric (3.1) for variable \( G(t) \) leads to

\[
\frac{3 \ddot{R}^2}{R^2} + \frac{3 k}{R^2} = 8 \pi G(t) \left[ \rho - \frac{1}{2} f \dot{\mathcal{C}}^2 \right] 
\]  

(3.11)

\[
\frac{2 \dddot{R}}{R} + \frac{\dddot{R}}{R^2} + k = - 8 \pi G(t) \left[ p - \frac{1}{2} f \dot{\mathcal{C}}^2 \right] 
\]  

(3.12)

### 4. Solution of field equations

The conservation equation

\[
\left( 8 \pi G T^4 \right)_{;ij} = 0 
\]  

(4.1)

leads to

\[
\frac{\partial}{\partial x^j} \left( 8 \pi G T^4 \right) + 8 \pi G T^4 \Gamma^i_{ij} - 8 \pi G T^4 \Gamma^i_{ij} = 0 
\]

which gives

\[
\frac{\partial}{\partial t} \left( 8 \pi G T^4 \right) + 8 \pi G \left[ T_4^1 \left( \Gamma^1_{11} + \Gamma^1_{12} + \Gamma^1_{13} \right) + T_2^3 \left( T_3^3 \right) + T_3^3 \left( 0 \right) \right] 
\]

\[
T_4^4 \left( \Gamma^4_{14} + \Gamma^4_{24} + \Gamma^4_{34} \right) - 8 \pi G \left[ T^1_1 \left( \Gamma^1_{14} + \Gamma^1_{11} \right) + T^2_2 \left( \Gamma^2_{14} + \Gamma^2_{24} \right) + T^3_3 \left( \Gamma^3_{14} + \Gamma^3_{24} + \Gamma^3_{34} \right) + T^4_4 \left( 0 \right) \right] 
\]

which leads to

\[
\frac{\partial}{\partial t} \left[ 8 \pi G \left( \rho - \frac{1}{2} f \dot{\mathcal{C}}^2 \right) \right] + 8 \pi G \left[ \left( \frac{1}{2} f \dot{\mathcal{C}}^2 - p \right) \left( \frac{kr}{1 - kr^2} + \frac{1}{r} \right) \right] + \left( \frac{1}{2} f \dot{\mathcal{C}}^2 - p \right) \cot \theta + \left( \rho - \frac{1}{2} f \dot{\mathcal{C}}^2 \right) \frac{3 \dot{R}}{R} - 8 \pi G \left[ \left( \frac{1}{2} f \dot{\mathcal{C}}^2 - p \right) \right] 
\]
\[ \left( \frac{\dot{R}}{R} + \frac{k r}{1 - kr^2} \right) + \left( \frac{1}{2} f' \dot{C}^2 - p \right) \left( \frac{\dot{R}}{R} + \frac{1}{r} \right) + \left( \frac{1}{2} \dot{C}^2 - p \right) \left( \frac{\dot{R}}{R} + \frac{1}{r} + \cot \theta \right) = 0 \]  

(4.3)

which gives

\[ 8\pi G \left( \rho - \frac{1}{2} f' \dot{C}^2 \right) + 8\pi G \left( \dot{\rho} - f' \dot{C} \dot{C} \right) + 8\pi G \left( \frac{3\dot{R}}{R} \rho + \frac{3\dot{R}}{R} \rho - \frac{3\dot{R}}{R} f' \dot{C}^2 \right) = 0 \]  

(4.4)

which yields \( \dot{C} = 1 \) when used in source equation. Using \( \dot{C} = 1 \) in (3.11), we have

\[ 8\pi G \rho = \frac{3R^2}{R^2} + \frac{3k}{R^2} + 4\pi G f \]  

(4.5)

We assume that universe is filled with barotropic fluid i.e. \( p = \gamma \rho \ (0 \leq \gamma \leq 1) \), \( \rho \) being the isotropic pressure, \( \rho \) the matter density. Now using \( p = \gamma \rho \) and \( \dot{C} = 1 \) in (3.12), we have

\[ \frac{2\dot{R}}{R} + \frac{\dot{R}^2}{R^2} + \frac{k}{R^2} = -8\pi G(t) \left[ \gamma \rho \frac{1}{2} f' \right] \]  

(4.6)

Equations (4.5) and (4.6) lead to

\[ \frac{2\dot{R}}{R} + (1 + 3\gamma) \frac{\dot{R}^2}{R^2} = (1 - \gamma)4\pi G f - (1 + 3\gamma) \frac{k}{R^2} \]  

(4.7)

To obtain the deterministic solution, we assume that

\[ G = R^n \]  

(4.8)

where \( R \) is scale factor and \( n \) is a constant. From equations (4.7) and (4.8), we have

\[ 2\dot{R} + (3\gamma + 1) \frac{\dot{R}^2}{R} = (1 - \gamma)AR^{n+1} - \frac{k}{R}(3\gamma + 1) \]  

(4.9)

where

\[ A = 4\pi f \]  

(4.10)

To find the solution of (4.9), we assume that

\[ \dot{R} = F(R) \]  

(4.11)

Thus

\[ \dot{R} = \frac{d\dot{R}}{dt} = \frac{dF}{dt} = \frac{dF}{dR} \frac{dR}{dt} = F' \]  

(4.12)

where

\[ F' = \frac{dF}{dR} \]
Using (4.11 and (4.12) in (4.9), we have

\[
\frac{dF^2}{dR} + \frac{(3\gamma + 1)F^2}{R} = (1 - \gamma) A R^{n+2} - \frac{k(3\gamma + 1)}{R} \tag{4.13}
\]

Equation (4.13) leads to

\[
F^2 = \left(\frac{dR}{dt}\right)^2 = \frac{A(1 - \gamma) R^{n+2}}{(n + 3\gamma + 3)} - k \tag{4.14}
\]

which leads to

\[
\frac{dR}{\sqrt{R^{n+2} - \frac{k(n + 3\gamma + 3)}{A(1 - \gamma)} dt}} = \frac{A(1 - \gamma)}{(n + 3\gamma + 3)} dt \tag{4.15}
\]

To get determinate value of \(R\) in terms of cosmic time \(t\), we assume \(n = -1\). Thus equation (4.15) leads to

\[
\frac{dR}{\sqrt{R - \frac{k(3\gamma + 2)}{A(1 - \gamma)} dt}} = \frac{A(1 - \gamma)}{(3\gamma + 2)} dt \tag{4.16}
\]

From equation (4.16), we have

\[
R = (at + b)^2 + \frac{k(3\gamma + 2)}{A(1 - \gamma)} \tag{4.17}
\]

where

\[
a = \frac{1}{2} \sqrt{\frac{A(1 - \gamma)}{(3\gamma + 2)}}, \quad b = \frac{N}{2} \tag{4.18}
\]

where \(N\) is the constant of integration.

Therefore, the metric (3.1) leads to

\[
ds^2 = dt^2 - \left[ (at + b)^2 + \frac{k(3\gamma + 2)}{A(1 - \gamma)} \right]^2 \left[ \frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right] \tag{4.19}
\]

where

\[
\gamma \neq 1.
\]

Taking \(a = 1\), \(b = 0\), the metric (4.19) reduces to

\[
ds^2 = dt^2 - \left[ t^2 + \frac{k(3\gamma + 2)}{A(1 - \gamma)} \right]^2 \left[ \frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right] \tag{4.20}
\]
5. Physical and geometric features

The homogeneous mass density (\(\rho\)), the isotropic pressure (\(p\)) for the model (4.19) are given by

\[
8\pi p = \frac{12a^2 (at + b)^2 + 3k}{(at + b)^2 + \frac{k(3\gamma + 2)}{A(1-\gamma)}} + A \tag{5.1}
\]

\[
8\pi \rho = 8\pi \rho = \frac{12a^2 \gamma(at + b)^2 + 3k\gamma}{(at + b)^2 + \frac{k(3\gamma + 2)}{A(1-\gamma)}} + A\gamma \tag{5.2}
\]

\[
G = R^{-1} = \frac{1}{(at + b)^2 + \frac{k(3\gamma + 2)}{A(1-\gamma)}} \tag{5.3}
\]

\(q = \text{Deceleration parameter}\)

\[
q = -\frac{\dot{R}}{\ddot{R}} = -\frac{R^2}{\dddot{R}}
\]

where \(R\) is scale factor given by (4.17). Thus

\[
q = -\frac{\left[2a^2 (at + b)^2 + \frac{2ka^2 (3\gamma + 2)}{A(1-\gamma)}\right]}{4a^2 (at + b)^2} + A \tag{5.4}
\]

To find C (creation field)

Using \(p = \gamma \rho\), (5.1), (5.3) and (4.17) in (4.4), we have

\[
\frac{d\dot{C}^2}{dt} + \frac{10t}{t^2 + \frac{k(3\gamma + 2)}{A(1-\gamma)}} \dot{C}^2 = \frac{4}{A} \left[6t^3 (3\gamma + 2) + \frac{3k(3\gamma + 1)t + 6kt(3\gamma + 2)}{2} \right] \tag{5.5}
\]

\[
+ \frac{A(3\gamma + 2)t}{2} \left\{t^2 + \frac{k(3\gamma + 2)}{A(1-\gamma)}\right\}
\]

Equation (5.5) is linear in \(\dot{C}^2\). The solution of (5.5) is given by
\[ \dot{c}^2 = \frac{4(3\gamma + 2)}{A(1 - \gamma)} \] (5.6)

which gives

\[ \dot{C} = 1 \] (5.7)

which agrees with the value used in source equation. Here \( \frac{(3\gamma + 2)}{\pi f (1 - \gamma)} = 1 \) which gives \( \gamma = \frac{\pi f - 2}{\pi f + 3} \). Equation (5.7) leads to

\[ C = t \] (5.8)

Thus creation field increases with time.

Taking \( a = 1, b = 0 \) in equations (5.1) — (5.4), we have

\[ 8\pi \rho = 12 + 4\pi f \] (5.9)

\[ 8\pi p = 8\pi \gamma f = 12\gamma + 4\pi f \] (5.10)

\[ G = \frac{1}{t^2 + \frac{k}{4}} \] (5.11)

\[ q = -\left(\frac{1}{2} + \frac{k}{8t^2}\right) \] (5.12)

6. Discussion

The matter density (\( \rho \)) is constant for the model (4.20). The scale factor (R) increases with time. Thus inflationary scenario exists in the model (4.20). \( \frac{\dot{G}}{G} \equiv \frac{1}{t} = H \) where H is Hubble constant. \( G \rightarrow \infty \) when \( t \rightarrow 0 \) and \( G \rightarrow 0 \) when \( t \rightarrow \infty \). The deceleration parameter (\( q \)) < 0 which indicates that the model (4.20) represents an accelerating universe. The creation field \( C \) increases with time and \( \dot{C} = 1 \) which agrees with the value taken in source equation. The matter density \( \rho = \text{constant} \) as given by (5.9). This result may be explained as: Referring to Hoyle and Narlikar [2002], Hawking and Ellis [1973], the matter is supposed to move along the geodesic normal to the surface \( t = \text{constant} \). As the matter moves further apart, it is assumed that more matter is continuously created to maintain the matter density at constant value. For \( k = 0, \gamma = 0 \) and for \( k = \pm 1, \gamma = 0 \), we get the same results as obtained by Bali and Tikekar [2007], Bali and Kumawat [2008] respectively.

The coordinate distance to the horizon \( r_H \) is the maximum distance a null ray could have travelled at a time \( t \) starting from the infinite past i.e.

\[ r_H(t) = \int_{-\infty}^{t} \frac{dt}{R^3(t)} \] (5.13)
We could extend the proper time $t$ to $(-\infty)$ in the past because of non-singular nature of the space-time. Now

$$r_{H}(t) = \int_{0}^{t} \frac{dt}{\alpha t^{3}}$$

(5.14)

where $\alpha = \sqrt{\frac{4\pi f(1-\gamma) - k(3\gamma + 1)}{3\gamma + 1}}$

This integral diverges at lower time showing that the model (4.20) is free from horizon.

**Special Cases:**

i. Dust filled universe i.e. $\gamma = 0$, the metric (4.20) leads to

$$ds^2 = dt^2 - \left( t^2 + \frac{k}{2\pi f} \right)^2 \left[ \frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right]$$

(5.15)

For $k = 0$, the metric (5.15) leads to the model obtained by Bali and Tikekar (2007).

ii. For $k = \pm 1$, $\gamma = 0$, the model (5.15) leads to the model obtained by Bali and Kumawat (2008).

iii. For $\gamma = 1/3$ (Radiation dominated universe), the model (4.20) leads to

$$ds^2 = dt^2 - \left( t^2 + \frac{9k}{8\pi f} \right) \left[ \frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right]$$

(5.16)

For $\gamma = 1$ (stiff fluid universe), the model (4.20) does not exist.

### 7. References

This book presents some aspects of the cosmological scientific odyssey that started last century. The chapters vary with different particular works, giving a versatile picture. It is the result of the work of many scientists in the field of cosmology, in accordance with their expertise and particular interests. It is a collection of different research papers produced by important scientists in the field of cosmology. A sample of the great deal of efforts made by the scientific community, trying to understand our universe. And it has many challenging subjects, like the possible doomsday to be confirmed by the next decade of experimentation. May be we are now half way in the life of the universe. Many more challenging subjects are not present here: they will be the result of further future work. Among them, we have the possibility of cyclic universes, and the evidence for the existence of a previous universe.

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