Two-Wave Mixing in Broad-Area Semiconductor Amplifier

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1. Introduction

Two-wave mixing (TWM) is an interesting area in nonlinear optics and has been intensively investigated in the past three decades. TWM can take place in many different nonlinear media, such as second-order nonlinear media like photorefractive materials (Marrakchi et al., 1981; Huignard & Marrakchi, 1981; Yeh, 1983; 1989; Garrett et al., 1992), third-order nonlinear materials like Kerr media (Silberberg & Bar-Joseph, 1982; 1984; Yeh, 1986; 1989; Grandclément et al., 1987; McGraw, 1992), and in gain media like YAG (Brignon & Huignard, 1993). The microcosmic physical process for TWM in different nonlinear media is different. But in general the TWM process can be explained as: two coherent beams are incident on a nonlinear medium and an interference pattern is formed in the medium, such a pattern is characterized by a periodic spatial variation of the intensity; thus a refractive index and/or a gain (absorption) periodic variations will be induced because of the nonlinear response of the medium, and these refractive index and gain variations are usually called volume refractive index (or phase) grating and gain (or absorption) grating; the two beams propagate through the volume gratings formed by them and they undergo Bragg scattering (the Bragg condition is satisfied automatically); one beam scatters into the other and vice versa, so the energy and phase exchanges may occur between these two beams, i.e., the TWM takes place.

Nonlinear four-wave mixing in narrow-stripe and broad-area semiconductor lasers and amplifiers is of interest as a method to obtain high phase conjugate reflectivity (Nakajima & Frey, 1985; 1986; Frey, 1986; Agrawal, 1987; Kürz et al., 1996). The nonlinear four-wave mixing can also be used to measure carrier dynamics and gain behaviour directly in the devices, as well as for understanding device physics and application (Lucente et al., 1988a; 1988b; Zhu, 1997a; 1997b; 1997c). The gain and refractive index gratings created in broad-area semiconductor lasers by coherent four-wave mixing are very interesting nonlinear interactions which may be applied to realize high brightness semiconductor lasers as well as to study the carrier dynamics and the physics of the devices (Petersen et al., 2005). But no work on TWM was done in broad-area semiconductor amplifiers previously.

In this chapter, we present both the theoretical and experimental results of TWM in broad-area semiconductor amplifier. For the generality, we assume that the frequencies of the
pump beam and the signal beam are different, i.e., a moving gain grating and a moving refractive index grating are induced in the broad-area semiconductor amplifier. The coupled-wave equations of TWM are derived based on Maxwell’s wave equation and rate equation of the carrier density. The analytical solutions of the coupled-wave equations are obtained in the condition of small signal when the total light intensity is far below the saturation intensity of the amplifier. The results show that the optical gain of the amplifier is affected by both the moving phase grating and the moving gain grating. The different contributions from both the refractive index grating and the gain grating to the TWM gain are analyzed. Depending on the moving direction of the gratings and the anti-guiding parameter, the optical gain of the amplifier may increase or decrease due to the TWM.

As a special case, the degenerate TWM (the frequencies of the pump beam and the signal beam are the same, i.e., a static gain grating and a static refractive index grating are induced in the semiconductor amplifier) in an 810 nm, 2 mm long and 200 μm wide GaAlAs broad-area amplifier is investigated experimentally. In this case, the theoretical results show that when the amplifier is operated below transparency the optical gain of both beams is increased due to the induced gain grating, and when the amplifier is operated above transparency the optical gain of both beams is decreased due to the gain grating. The refractive index grating does not affect the optical gain of both beams; and there is no energy exchange between the pump and the signal beams. The dependence of the TWM gain on the output power of the pump and angle between the two beams is measured. The experimental results show good agreement with the theory. A diffusion length of 2.0 μm for the carrier is determined from the experiment.

2. Theory of TWM in broad-area semiconductor amplifier

Fig. 1. Configuration of the TWM in a broad-area semiconductor amplifier with moving gratings, K shows the direction of the grating vector.

The TWM geometry is shown in Fig. 1, the pump beam of amplitude A₁ and the signal beam of amplitude A₂ are coupled into the broad-area amplifier. Both beams are linearly polarized along the Y direction, and the frequencies are ω₁ and ω₂ respectively. The two beams
interfere in the medium to form a moving interference pattern, and a moving modulation of the carrier density in the active medium is caused, thus both a moving gain and a moving phase gratings are created. The nonlinear interaction in the gain medium is governed by Maxwell’s wave equation:

\[ \nabla^2 E - \frac{n^2}{c^2} \frac{\partial^2 E}{\partial t^2} = \frac{1}{\varepsilon_0 c^2} \frac{\partial^2 P}{\partial t^2}, \]

where \( n \) is the refractive index of the semiconductor material at transparency, \( c \) is the velocity of light in vacuum, and \( \varepsilon_0 \) is the vacuum permittivity. The total electric field is given by (Agrawal, 1987; Chi et al., 2006; 2008):

\[ E = A_1 e^{i(K_1 \cdot r - \omega_1 t)} + A_2 e^{i(K_2 \cdot r - \omega_2 t)}, \]

where \( K_1 \) and \( K_2 \) are the wave vectors of the pump and the signal in the amplifier. \( P \) is the induced polarization in the semiconductor amplifier. It is given by (Agrawal, 1987; Chi et al., 2006; 2008):

\[ P = \varepsilon_0 \chi(N) E, \]

where the susceptibility \( \chi \) is given by (Agrawal, 1987; Chi et al., 2006; 2008):

\[ \chi(N) = -\frac{n c}{\omega} (\beta + i) g(N), \]

the quantity \( \beta \) is the anti-guiding parameter accounting for the carrier-induced refractive index change in semiconductor amplifier, and \( g(N) \) is the gain for the light intensity that is assumed to vary linearly with the carrier density \( N \), i.e., \( g(N) = \Gamma a(N - N_0) \) where \( a \) is the gain cross-section, \( \Gamma \) is the confinement factor, and \( N_0 \) is the carrier density at transparency. The carrier density \( N \) is governed by the following rate Eq. (Petersen et al., 2005; Chi et al., 2006; 2008):

\[ \frac{dN}{dt} = \frac{I}{qV} - \frac{N}{\tau} + D \nabla^2 N - g(N) \left| \frac{E}{\hbar \omega} \right|^2, \]

where \( I \) is the injected current, \( q \) is the electron charge, \( V \) is the active volume, \( \tau \) is the spontaneous recombination lifetime and \( D \) is the ambipolar diffusion constant. In the TWM process the origin of the gain and the refractive index gratings is the modulation of the carrier density due to the interference between \( A_1 \) and \( A_2 \). Thus the carrier density that leads to the formation of the moving gratings may be written as:

\[ N = N_B + \Delta N \exp\left[i(-Kx + \delta t)\right] + \Delta N' \exp\left[i(Kx - \delta t)\right], \]

where \( N_B \) is the average carrier density, \( \Delta N \) is the induced carrier modulation. \( K = |K_2 - K_1| = 4\pi \sin((\theta_1 - \theta_2)/2)/\lambda_1 \) is the grating vector; \( \theta_1 \) is the angle between the pump beam and the Z axis, and \( \theta_2 \) is the angle between the signal beam and the Z axis; we assume \( \theta_1 = -\theta_2 \), thus the direction of the grating vector is in the X direction; \( \lambda_1 \) is the wavelength of the pump beam in the amplifier (where we assume that \( \lambda_1 \equiv \lambda_2 \), since usually the frequency
difference of the signal and the pump beams is less than a few gigahertz. \( \delta = \omega_s - \omega_p \) is the frequency difference between the signal and pump beams. In the following perturbation analysis it is assumed that \( \Delta N << N_B \). Inserting Eqs. (2) and (6) into Eq. (5), we find after some simple calculations that the average carrier density \( N_B \) and the carrier modulation \( \Delta N \) are given by:

\[
N_B = \frac{1\pi/qV + N_0 |E_0|^2/P_s}{1 + |E_0|^2/P_s},
\]

\[
\Delta N = -\frac{(N_B - N_0) A_1 A_2^* / P_s}{1 + D\tau K^2 + |E_0|^2/P_s + i\delta},
\]

where \( |E_0|^2 = |A_1|^2 + |A_2|^2 \) is the average intensity, and \( P_s = (h\omega)/\Gamma\alpha \) is the saturation intensity of the amplifier. Inserting Eqs. (2) and (3) into Eq. (1), and using the obtained results of the average carrier density \( N_B \) and the carrier modulation \( \Delta N \), after some calculations, the coupled-wave equations for TWM with moving gratings in a broad-area semiconductor amplifier are obtained:

\[
\cos\theta_1 \frac{\partial A_1}{\partial z} - i \left[ \alpha(\beta + i) \frac{1}{1 + |E_0|^2/P_s} \right] \left[ 1 - \frac{|A_2|^2/P_s}{1 + D\tau K^2 + |E_0|^2/P_s + i\delta} \right] A_1 = 0,
\]

\[
\cos\theta_2 \frac{\partial A_2}{\partial z} - i \left[ \alpha(\beta + i) \frac{1}{1 + |E_0|^2/P_s} \right] \left[ 1 - \frac{|A_1|^2/P_s}{1 + D\tau K^2 + |E_0|^2/P_s - i\delta} \right] A_2 = 0,
\]

where \( \alpha = \Gamma a(1\pi/qV - N_B)/2 \) is the small-signal gain coefficient of the amplifier. Since the refractive index of the semiconductor material is high, normally the angles \( \theta_1 \) and \( \theta_2 \) are less than 2º in experiment (Chi et al., 2006); so the cosine factor in Eqs. (9) and (10) is neglected below.

In the small signal approximation, and if we assume that the total intensity of the two beams is much less than the saturation intensity, i.e., \( |A_2|^2 \ll |A_1|^2 \ll P_s \), the terms accounting for saturation in the denominator and the term accounting for the coupling in Eq. (9) may be neglected. Thus the coupled-wave equations can be solved analytically. The solutions are:

\[
A_1 = A_{10} \exp[(1 - i\beta)\alpha z],
\]

\[
A_2 = A_{20} \exp\left\{ (1 - i\beta) \left[ \alpha z - \gamma_1(e^{i\alpha \gamma_1} - 1)/2 \right] \right\},
\]

where \( A_{10} \) and \( A_{20} \) are the amplitudes of the pump and the signal beams at the front facet of the amplifier. \( \gamma_1 \) is a parameter defined as:

\[
\gamma_1 = \frac{|A_{20}|^2}{P_s} \left[ 1 + \frac{1}{1 + D\tau K^2 - i\delta} \right].
\]
The first term in Eq. (13) is for the saturation effect, the second term is for the beam coupling.

Define the TWM gain of the signal beam $g_{\text{TWM}}$ as the natural logarithm of the ratio of the output intensity of signal with the coherent pump to that with the non-coherent pump:

$$g_{\text{TWM}} = \ln \left( \frac{\left| A_2(z_0)_{\text{coherent pump}} \right|^2}{\left| A_2(z_0)_{\text{non-coherent pump}} \right|^2} \right) = -\frac{\left| A_1(z_0) \right|^2 - \left| A_{10} \right|^2}{P_s} \frac{1 + DrK^2 + \beta\delta\tau}{(1 + DrK^2)^2 + (\delta\tau)^2},$$

where $z_0$ is the length of the semiconductor amplifier. The non-coherent pump means the pump beam is not coherent with the signal beam, but the intensity is the same as the coherent pump, thus the term accounting for saturation in Eq. (14) vanishes. In experiment, the coherent pump and the non-coherent pump can be achieved by changing the polarization direction of the pump beam (Chi et al., 2006). Eq. (14) shows that $g_{\text{TWM}}$ changes linearly with the output intensity (power) of the pump, and it decreases quickly when the angle between the two beams increases because the diffusion of carriers washes out the gratings as the angle between the two beams increases. Eq. (14) also shows that depending on the detuning frequency $\delta$, the TWM gain can be positive or negative no matter the amplifier is operated above or below the transparency (i.e., $\left| A_1(z_0) \right|^2 = \left| A_{10} \right|^2$). These phenomena will be discussed below.

3. Experiment of the degenerate TWM in a broad-area amplifier

In order to verify the theory described in Section 2, a special case of TWM, i.e., degenerate TWM, in a broad-area semiconductor amplifier is investigated experimentally. For this case, the frequencies of the pump and the signal are the same, i.e., $\delta = 0$; thus a static refractive index grating and a static gain grating are induced in the amplifier. The coupled-wave equations (9) and (10) in this case are changed to:

$$\frac{\partial A_1}{\partial z} - i \left[ -\frac{\alpha(\beta + i)}{1 + \left| E_0 \right|^2 / P_s} \right] \left( 1 - \frac{\left| A_1 \right|^2 / P_s}{1 + DrK^2 + \left| E_0 \right|^2 / P_s} \right) A_1 = 0 ,$$

$$\frac{\partial A_2}{\partial z} - i \left[ -\frac{\alpha(\beta + i)}{1 + \left| E_0 \right|^2 / P_s} \right] \left( 1 - \frac{\left| A_1 \right|^2 / P_s}{1 + DrK^2 + \left| E_0 \right|^2 / P_s} \right) A_2 = 0 .$$

The equations show that the coupling term between the two beams decreases the optical gain (above transparency) or absorption (below transparency) for both beams simultaneously. This is different to the situation in photorefractive materials, where one beam is amplified and the other is decreased at the same time (Marrakchi et al., 1981; Huignard & Marrakchi, 1981; Yeh, 1983; 1989). This is also different to the situation in Kerr media, where the intensity of one beam is not affected by the other beam in the degenerate TWM case (Yeh, 1986; 1989; Chi et al., 2009).

To clarify this phenomenon, the relative position of the intensity pattern, the carrier density grating, the refractive index grating and the gain grating is shown in Fig. 2 when the amplifier is operated above the transparency. Because of the spatial hole-burning effect, the
carrier density grating is $\pi$ out of phase with the intensity pattern. Since the gain varies linearly with the carrier density, the gain grating is also $\pi$ out of phase with the intensity pattern. The refractive index grating is in phase with the interference intensity pattern due to the anti-guiding effect. The refractive index grating has no contribution to the energy coupling between the two beams, when it is in phase or $\pi$ out of phase with the interference pattern (Yeh, 1989). The gain grating will decrease the optical gain of both beams simultaneously because it is $\pi$ out of phase with the interference pattern. Reversely, the gain grating will decrease the absorption of both beams when the amplifier is operating below the transparency since the gain grating is in phase with the interference pattern in that case (Chi et al., 2009).

Fig. 2. The relative position of the interference pattern, the carrier density grating, the refractive index and the gain gratings formed in the broad-area amplifier.

Since $\delta = 0$, the TWM gain for the degenerate TWM is changed to:

$$
\Delta z_{TWM} = \ln \left( \frac{|A_2(z_0)_{\text{coherent pump}}|^2}{|A_2(z_0)_{\text{non-coherent pump}}|^2} \right) = -\frac{|A_1(z_0)|^2 - |A_{10}|^2}{(1 + D\tau K^2)P_z}.
$$  \hspace{1cm} (17)

Eq. (17) shows that the $\Delta z_{TWM}$ is negative when the amplifier is operated above the transparency, is positive when it is operated below the transparency, and is zero when it is operated at transparency. It agrees with the analyse above. Eq. (17) also shows that the $\Delta z_{TWM}$ decreases linearly with the output intensity (power) of the pump, and it decreases quickly when the angle between the two beams increases because the diffusion of carriers washes out the gratings as the angle between the two beams increases. These analyses will be verified by experiments of TWM in a semiconductor amplifier below.

The experimental set-up is shown in Fig. 3. The set-up is arranged like a Mach-Zehnder interferometer. The pump beam $A_1$ and the signal beam $A_2$ are derived from a tunable diode laser system based on a tapered amplifier (Chi et al., 2005). The wavelength used in the experiment is 813.5 nm. We use the same method as Goldberg (Goldberg et al., 1993) to
couple the two beams into the broad-area amplifier. In each arm, a combination of a cylindrical lens of 150 mm focal length and an aspherical lens of 8.0 mm focal length with a N.A. of 0.5 (this lens is shared by the two arms) is used as an afocal telescope to inject the two beams into the amplifier. The input coupling efficiency of this setup is around 50%. The two cylindrical lenses can be translated in the arrow direction to vary the injection angle in the junction plane. After the amplifier, a cylindrical lens of 5.0 mm focal length is used to collimate the output beam in the fast axis. A half-wave plate is inserted in the pump arm to change the polarization direction of the pump. All the components are antireflection coated for the near infrared wavelength.

Fig. 3. Experimental set-up for the TWM in a broad-area amplifier. M: mirror, BSs: beam splitter, HWP: half-wave plate, BAA: broad-area amplifier (the units are in mm).

The broad-area amplifier is an 810 nm, 2 mm long and 200 µm wide GaAlAs amplifier. It was grown by the Metallorganic Chemical Vapor Phase Deposition (MOCVD) technique on a GaAs substrate by Alcatel Thales III-V Lab. The structure contains a Large Optical Cavity (LOC), which has a thickness of approximately 1 µm, and which consists of a tensile-strained GaInP quantum well, two GaInP barriers and two AlGaInP cladings. Both facets of the amplifier are antireflection coated; the reflectivity is less than 0.1%.

First, the dependence of the $g_{TWM}$ on the output power of the pump is measured. The input powers of the pump and the signal measured before the aspherical lens are 21.0 and 4.1 mW. The angle between the two beams is around 4°. The output power of the signal was measured at different injected current of the amplifier with a co-polarized pump (the polarization direction of both beams is perpendicular to the chip of the amplifier) and an orthogonally-polarized pump. The output power of the pump was measured when it is coherent with the signal. The experimental results are shown in Fig. 4. It is clearly seen that the $g_{TWM}$ decreases linearly with the output pump power. Fitting the experimental data with Eq. (17), the two parameters: the input power of the pump $A_{10}^2$ and the $1/(1 + DτK^2) P_0$ are obtained. The $A_{10}^2$ is round 9.1 mW, corresponding to a coupling efficiency of 43%; and using the result of $Dτ$ obtained later, the saturation power $P_s$ is found to be around 220 mW, which is much larger than the output power of the pump in this experiment. Using the value of $A_{10}^2$ and Eq. (11), the optical gain of 1.7 is obtained for the highest output power of the pump. The $g_{TWM}$ is about 5% of the optical gain.

The dependence of the $g_{TWM}$ on the grating vector is also measured by changing the angle between the two beams. The direction of the pump beam is fixed during the experiment; the angle is changed by changing the direction of the signal beam. The injected powers of the pump and the signal measured before the aspherical lens are 21.0 and 4.1 mW; the output
power of the pump is around 35 mW. The experimental results are shown in Fig. 5. Fitting the experimental data with Eq. (17), $D \tau$ is obtained to be $4.1 \mu m^2$, leading to a diffusion length $L = \sqrt{D \tau}$ of 2.0 $\mu m$. Assuming that $\tau$ is 5 ns (Marcianete & Agrawal, 1996), $D$ is calculated to be 8.2 $cm^2/s$. This is in good agreement with the direct measured value of 9.5 $cm^2/s$ (Lucente et al., 1988b). We should mention that the output power of the pump beam is decreased a little when the polarization direction of it is changed from perpendicular to the chip to parallel to the chip. We do not know the reason of this decrease but the effect of this decrease on the measured $g_{TWM}$ is small.

To obtain the coupled-wave equations of TWM, three assumptions are made. Here we should discuss the validity of these assumptions in our experiment. The first is the plane-
wave assumption. In the experiment, since the two beams are coupled into the amplifier from an external laser, the mode of the two beams in the slow axis is determined by the external laser and the focusing optics. The two beams are nearly Gaussian beams in the slow axis, they are collimated by the aspherical lens and the width of the beams is around 140 μm. We believe the plane-wave assumption is a good approximation for these two beams in this direction. The wave guiding mode of the field distribution in the fast axis does not affect the derivation of the equations (Marciane & Agrawal, 1996). The second is the linear variation of the material gain $g(N)$ on the carrier density. The transparent current of the amplifier used here is around 1.1 A, and the highest current used in our experiment is 1.8 A, according to Eq. (7), the carrier density $N_0$ is calculated to be around 1.5 $N_0$, not much higher than the transparent carrier density. The third assumption is the small population modulation in Eq. (6). With the injected current of 1.8 A, according to Eq. (8), $|\Delta N|$ is calculated to be around 2% of $N_0$ (∼1.3% of $N_0$), it is much less than the average carrier density $N_0$. Therefore, we believe the assumptions made in the theory are valid in our experiment.

In conclusion, the degenerate TWM in broad-area semiconductor amplifier is investigated experimentally. The experimental results show good agreement with the theory. The validity of the theory is discussed.

4. Calculations and discussion

Unlike the condition of degenerate TWM, where only static gratings are generated; the coupling term between the two beams has different contribution to the optical gain of these two beams for the nondegenerate TWM (Chi et al., 2008). The nondegenerate TWM may increase the power of one beam and decrease the power of another beam in this case, i.e., energy exchange occurs.

According to Eq. (14), the dependence of $g_{TWM}$ on the frequency difference $\delta$ with different anti-guiding parameter $\beta$ is calculated; here we assume that the amplifier is operated above the transparent current. The calculated results are shown in Fig. 6. In the calculation, we use the same parameters used in and obtained from the TWM experiment in a GaAlAs broad-area semiconductor amplifier with static gratings described in Section 3 (Chi et al., 2006); i.e., $|A_1(z_0)|^2 = 48.8$ mW, $|A_{10}|^2 = 9.1$ mW, $P_s = 220$ mW, $D\tau = 4.1$ μm², $K = 0.51$ μm⁻¹ (the $K$ value corresponds to a 4.0° angle between the two beams). Assuming that $\tau$ is 5 ns (Marciane & Agrawal, 1996). From Fig. 6 we can find that when $\delta = 0$, the $g_{TWM}$ is negative and independent of $\beta$; if $\beta = 0$, the $g_{TWM}$ is always negative and the curve of the TWM gain versus $\delta$ is symmetric around the axis of $\delta = 0$. If $\beta \neq 0$, however, the $g_{TWM}$ is negative when $\delta > 0$, and the $g_{TWM}$ can be negative or positive when $\delta < 0$. These properties can be explained by analyzing the different contributions from the refractive index grating and the gain grating formed in the broad-area semiconductor amplifier to the TWM gain.

Since the frequencies of the pump and the signal are different, a moving interference pattern is generated in the amplifier: $|E|^2 = |E_0|^2 + \left[ A_1 A_2 e^{-i(Kx + \delta\tau)} + c.c. \right]$. Inserting Eq. (8) into Eq. (6), the carrier density is obtained:

$$N = N_0 + \left\{ -\frac{(N_0 - N_0) A_1 A_2^*/P_s}{1 + D\tau K^2 + |E_0|^2/P_0 + i\delta\tau} \exp[i(-Kx + \delta\tau)] + c.c. \right\}.$$  (18)
Fig. 6. The calculated TWM gain $g_{\text{TWM}}$ versus $\delta$ with different anti-guiding parameter $\beta$ according to Eq. (14).

The modulation part $N_m$ of the carrier density for the generating of the gain and the phase gratings is:

$$N_m = \frac{A_1 A_2^* (N_n - N_0)/P_s}{\sqrt{1 + D\tau K^2 + |E_0|^2 / P_s^2}} \exp[i(-Kx + \delta t + \pi - \theta)] + \text{c.c.},$$  \hspace{1cm} (19)

where

$$\theta = \arctg \frac{\delta \tau}{1 + D\tau K^2 + |E_0|^2 / P_s^2} (-\pi/2 < \theta < \pi/2).$$  \hspace{1cm} (20)

Eq. (19) shows, because of the hole-burning effect and the finite response time of the broad-area amplifier, there is a phase difference $\pi - \theta$ between the interference pattern and the carrier density grating. Since the gain varies linearly with the carrier density, the gain grating $\Delta g$ is also $\pi - \theta$ out of phase with the intensity pattern, i.e.,

$$\Delta g = \frac{\Gamma a (N_n - N_0)}{2P_s} \frac{A_1 A_2^*}{\sqrt{1 + D\tau K^2 + |E_0|^2 / P_s^2}} \exp[i(-Kx + \delta t + \pi - \theta)] + \text{c.c.}. \hspace{1cm} (21)$$

The refractive index grating is $\pi$ out of phase with the gain grating because of the anti-guiding effect, so the refractive index grating $\Delta n$ is $-\theta$ out of phase with the intensity pattern and proportional to the anti-guiding parameter $\beta$, i.e.,

$$\Delta n = \frac{\Gamma a A_1 (N_n - N_0)}{4\pi P_s} \frac{\beta A_2^*}{\sqrt{1 + D\tau K^2 + |E_0|^2 / P_s^2}} \exp[i(-Kx + \delta t - \theta)] + \text{c.c.}. \hspace{1cm} (22)$$
The relative position of the interference pattern, the carrier density grating, the refractive index grating and the gain grating formed in the broad-area amplifier is shown in Fig. 7.

![Graph showing the relative position of the interference pattern, carrier density pattern, refractive index pattern, and gain pattern.](image)

Fig. 7. The relative position of the interference pattern, the carrier density grating, the refractive index grating and the gain grating formed in the BAA, assuming \(1 + D\tau K^2 + |E_0|^2 / P_s = \delta \tau\), i.e., \(\theta = \pi / 4\).

The TWM gain caused by the gain grating \(g_{\text{gain}}\) is (Chi et al., 2009):

\[
g_{\text{gain}} \propto \frac{\cos(\pi - \theta)}{\sqrt{1 + D\tau K^2 + |E_0|^2 / P_s}^2 + (\delta \tau)^2}}. \tag{23}
\]

Here we should notice that the effect of the gain grating is the same for both beams, i.e., to increase (below transparent current) or decrease (above transparent current) the intensity of the pump and the signal beams simultaneously, thus it will not cause the energy exchange between the two beams. The TWM gain caused by the phase grating \(g_{\text{phase}}\) is (Yeh, 1989; Chi et al., 2009):

\[
g_{\text{phase}} \propto \frac{\beta \sin(-\theta)}{\sqrt{1 + D\tau K^2 + |E_0|^2 / P_s}^2 + (\delta \tau)^2}}. \tag{24}
\]

When \(\delta \neq 0\), the refractive index grating will cause energy exchange between the two beams, since there is a phase difference \(-\theta\) (\(\theta \neq 0\)) between the intensity pattern and the refractive index grating (Yeh, 1989). The TWM gain \(g_{\text{TWM}}\) is the sum of \(g_{\text{gain}}\) and \(g_{\text{phase}}\). When \(\delta = 0\), (i.e., static gratings are induced in the amplifier), \(\theta\) is equal to zero; thus the gain grating is \(\pi\) out of phase with the interference pattern, and the phase grating is in phase with the interference pattern. According to Eqs (23) and (24), the gain of the phase grating \(g_{\text{phase}}\) is zero; and the \(g_{\text{TWM}}\) equal to \(g_{\text{gain}}\) is negative and independent of \(\beta\) (Chi et al., 2006). If \(\beta = 0\), only the gain grating is generated; according to Eqs. (20) and (23), the TWM gain \(g_{\text{TWM}}\) is always negative and is symmetric around the axis of \(\delta = 0\). If \(\beta \neq 0\), both a gain grating and a
phase grating are generated. When $\delta > 0$ ($\theta > 0$), according to Eqs. (23) and (24), both the $g_{\text{gain}}$ and the $g_{\text{phase}}$ are negative, so the $g_{\text{TWM}}$ is negative; when $\delta < 0$ ($\theta < 0$), the $g_{\text{gain}}$ is negative and the $g_{\text{phase}}$ is positive, so the $g_{\text{TWM}}$ can be positive or negative. The parameters $\beta$ and $\tau$ can be obtained by fitting the measured results of $g_{\text{TWM}}$ versus $\delta$. The optimal $\delta$ to achieve the maximum TWM gain depends on the device parameters $\tau$, $D$, $\beta$ and the grating vector $K$. From Eq. (14), the optimal $\delta$ is $\delta_{\text{opt}} = -(1 + DrK^2)(1 \pm \sqrt{1 + \beta^2})/\beta \tau$.

5. Conclusion

In conclusion, the TWM in broad-area semiconductor amplifier in nondegenerate condition is investigated theoretically. The coupled-wave equations are derived and analytical solutions are obtained when the intensity of the pump is much larger than that of the signal, but much less than the saturation intensity of the amplifier. A special case of TWM, degenerate TWM, is investigated experimentally in a GaAlAs broad-area semiconductor amplifier. The experimental results show good agreement with the theory, and the validity of the theory for this experiment is discussed. A diffusion length of 2.0 $\mu$m is determined from the experiment. The TWM gain in broad-area semiconductor amplifier is calculated as a function of the frequency difference between the pump and the signal based on the data obtained from the degenerate TWM experiment; and the calculated results are discussed based on the different contributions from the refractive index grating and the gain grating to the TWM gain. Depending on $\delta$ and $\beta$, the TWM gain in semiconductor broad-area amplifier can be positive or negative. The energy exchange between the pump and signal beams occurs when $\delta \neq 0$.

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7. References


Lasers and electro-optics is a field of research leading to constant breakthroughs. Indeed, tremendous advances have occurred in optical components and systems since the invention of laser in the late 50s, with applications in almost every imaginable field of science including control, astronomy, medicine, communications, measurements, etc. If we focus on lasers, for example, we find applications in quite different areas. We find lasers, for instance, in industry, emitting power level of several tens of kilowatts for welding and cutting; in medical applications, emitting power levels from few milliwatt to tens of Watt for various types of surgeries; and in optical fibre telecommunication systems, emitting power levels of the order of one milliwatt.

This book is divided in four sections. The book presents several physical effects and properties of materials used in lasers and electro-optics in the first chapter and, in the three remaining chapters, applications of lasers and electro-optics in three different areas are presented.

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