Exploring the bee hive metaphor as a model for problem solving: search, optimisation, and more

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1. Introduction

One of the great challenges attracting researchers of various disciplines has been the question: how to solve it? Problem solving has been an important topic of research e.g. in artificial intelligence and many remarkable results have been achieved there. However, the challenge is too complex that only one discipline, or one paradigm, or one approach can prevail.

Within artificial intelligence, there has been a natural tendency to relate somehow artificial problem solvers to human ones: either in the way they behave, or in the way they actually do it. However, nature seems to offer a much broader wealth of inspiration. Among other sources of inspiration, social insects seem to play a distinct role. Indeed, their behaviour is interesting not only individually, but also in a collective, or a colony, or a swarm.

From among the kinds of social insects, ants have gained perhaps the biggest attraction of problem solving researchers. Their ability to solve classes of optimisation tasks with a help of pheromones has become infamous. Only relatively recently, other kinds of social insects began to draw attention of researchers. We shall devote our attention to honey bees (apis mellifera). Honey bees are relative newcomers, although the very idea of taking inspiration from a bee hive model to represent knowledge for a knowledge based system can be traced back to 1986 (Bullock, 1986). We are interested in taking inspiration from honey bees for devising of approaches to problem solving and in particular to automated, or computer based problem solving.

Behaviour of honey bees is a subject of study of other disciplines, in particular of biologists. Their studies have proven to be extremely useful (Beekman, 2007), (Biesmeijer, 2001), (Bonabeau, 1996), (Camazine, 1991), (Selley, 1991), (Zhang, 2006). Without them, we would not have the underlying knowledge on how honey bees behave in nature. From the problem solving perspective, much of the research has been concentrated on the optimisation task, perhaps in reference to the success of ant colonies (Karaboga, 2005), (Pham, 2006), (Teodorovic, 2005), (Tovey, 2005), (Wong, 2008).
We venture to identify one additional new dimension for the problem solving: the web. It brings new challenges, since it poses new kinds of problems. We had a great wealth of information before the web. But now we have a great, and probably even a greater wealth of information instantly retrievable. Therefore, we identify web problem solving as another grand theme of research.

The rest of this chapter is structured as follows. First, we take a look at self-organization in biological systems. Next, we deal briefly with approaches to bee hive modelling. We mention at least some approaches in related work. Then we present our bee hive model. As a special issue, we devote a section to web page evaluation. We continue with presenting our experiments with web story tracking and with optimisation. Another concept we experiment with is hierarchy. We conclude with some comments on possible future works.

2. Self-Organization in Biological Systems

Social insects rank a key position in the field of artificial life, especially because of relative simplicity of behaviour of an individual as opposed to their complex collective behaviour. Colonies of social insects are able to develop means for solving problems collectively. Complexity of these tasks exceeds any individual problem solving ability. They can solve tasks without central direction, without existence of ahead given fixed structures, although the existence of expressive internal whirrs.

Ants are among the social insects that have been studied most intensively in both the social biology and computing literature. Ants have an ability to produce structures, which change dynamically. They can find the shortest way from a source of food to their nest without using the sight. They can adapt to changes of environment. For example they can find a new shortest way in case that the old one cannot be used any more because it has become barricaded off by a barrier. The main means that ants use to form and maintain the connection is a pheromone mark. Ants secrete certain amount of pheromones. When moving, they prefer direction, which is marked by a higher number of them. In this case, ants which had chosen the shorter path reconstruct the new path more quickly than the ones which had chosen the longer one. The shorter path obtains more pheromones per unit of time.

The apprehension of emergent abilities of ants' colony, in particular of the collective behaviour of a colony was an inspiration for proposing new distributive computing methods. This is the case not only for ants, but for other kinds of social insects, too. Bees in case of searching the nectar due to using air lines do not have the possibility to mark their way as ants. So bees pass the information to each other about the source distance of nectar by dance.

In 1973, Austrian zoologist Karl Ritter von Frisch was awarded the Nobel prize for discovery of the language of dancing bees (Gadakar, 1996). If the source is situated near the bee hive, a bee dances a circular dance, which does not include information about the direction of nectar. If the source is in a greater distance, bees encode the information about direction and distance of the source into dance, too. The dance begins to elongate and acquire the shape of the number eight.

We shall assume that the waggle dance is a means of communicating a food source. The dancer lets know not only distance and direction of the food source. Duration of dancing is influenced by the quality of the source.
3. Bee hive modelling

In (Selley, 1991), authors have demonstrated experimentally that a decision of a bee in a process of searching for nectar is based on limited information acquired from the visited sources. Despite the simplicity of bee’s individual behaviour, the hive is able to select the best source of nectar. It is selected by following certain way of dancing for a better source and by possibly abandoning the source of a lower quality.

The authors performed an interesting experiment designed to investigate how a hive (a system of bees) chooses between the sources. Experiment was performed in a desert. In the vicinity of a colony, there were placed two nectar sources, one of them 400 m to the north, the other one 400 m to the south. 12 bees were trained to fly to the north source, 15 other bees to the south source. The sources were of different quality and the experiment lasted from 8.00 am till 4.00 pm. The south one was better (sugar concentration of 2.5 units) than the north one (1.0 units) initially. At noon, however, the sources were swapped so the concentration of sugar was changed, too.

Empirical observation showed that number of bees foraging for the better source was increasing in time, whereas number of bees foraging for the worse source remained low. After having swapped the sources, the situation changed, too. The north one – it is the one that has become the better one in the afternoon – started to be visited by more and more bees. The south one – the worse one now - started to lose visitors as more and more foragers were abandoning it.

The result of the simulation of the experiment is showed in Fig. 1.

Fig. 1. Result of the experiment aimed at investigating how colonies choose among nectar sources (Seeley, 1991).

4. Related work

In (Camazine, 2003) authors deal with description of self organization of a honey bee system. Bees select the best source of nectar with a help of simple rules. They fly out to their surround and look for nectar, which is essential for their survival.

When a bee finds a food source, it flies back to the bee hive and brings the information about the source.
Having returned to the hive, a bee has the following options:
- to stay with the food source and keep on retrieving it without attempting to allure other bees,
- to attempt to allure other bees for the food source, to pass information about it, doing all that by a dance that encodes the direction, quality and distance from the source,
- to abandon the source and expose herself to be allured by some other bee, which propagates presumably a better source (with a higher concentration of sugar).

Next, we briefly present and comment a selection of works in which authors describe behaviour of bees and attempt to use the metaphor of bees for acquiring information. In many of such works, a swarm of bees is viewed symbolically as a multiagent system. One of the main problems in these systems is the way how agents communicate with each other. In case of bees, it is hypothesized that dance is the primary, if not the only way of communication. Agents have neither a global view of the state of the world nor in particular a view of the state of the problem being solved. They see only locally. The system, however, is able to take into account also constraints, which single agents do not apprehend because of local nature of their view. The agents are forced to cooperate in a rational way. One of the possible ways of cooperation among agents without an explicit communication is making use of the swarm intelligence (Bonabeau, 1999), (Vries, 1998).

In (Vries, 1998), authors deal with modelling of bees collectively looking for collecting nectar. Individually oriented simulation is constructed to simulate collective behaviour of bees in time of foraging. Each bee follows the same set of behavioural rules. The aim was to build up a simulation model, which would achieve similar results as those presented in (Selley, 1991).

In (Camazine, 1991), a mathematical simulation describing dynamical interaction among bees in the process of carrying nectar from two sources is presented. On the ground of the mathematical model of (Camazine, 1991), a prototype of multi-agent recommending system was proposed in (Lorenzi, 2005A) and presented in (Lorenzi, 2005B), (Lorenzi, 2005C). They make use of the bee hive metaphor. They built up also on works of (Schafer, 2001) and (Resnick, 1994) applying different approaches to solving problems. This question was elaborated in (Lorenzi, 2004), too. They generalized the model of (Camazine, 1991) by allowing more than two sources of food. However, their model assumes there are as many bees as there are sources. In the next Section, we present our enhancement of this model.

5. Bee hive model

We took an initial inspiration from the model of (Lorenzi, 2005A, B) who in turn were inspired by (Camazine, 1991).

The way bees communicate among themselves in nature contributes to formation of their collective intelligence, called also swarm intelligence. We believe that swarm intelligence has a potential to nurture new ideas that could ameliorate research approaches to various open problems.

Our model (Navrat, 2006A) uses a preset number of bees to find the best of the sources by evaluating them and using social interaction to agree upon the best source. The mechanism of interaction is shown in Fig. 2.
Fig. 2. The bee hive comprises of the dispatch room, dance floor and auditorium.

We enhanced the model by a dispatch room, bringing additional flexibility to it. The dispatch room is a place where addresses of the food sources are available. The information exchange among individual bees is the most important part of the collective knowledge sharing. Communication among bees about the quality of nectar sources takes place in a dancing area, i.e. the dance floor by performing and observing the waggle dance. The auditorium is a place where bees are able to watch dancers on the dance floor. Parameters of our model are the following:

- **N (BIOR+BISB)** - Number of bees in the hive (BIOR - bees in the observing room - observers, BISB - bees in the source base - foragers),
- **MDT** - Maximal dancing time - maximal time the bee can stay on the dance floor,
- **OT** - Observing time - maximal time a bee can spend in the auditorium,
- **ERR** - Error of source quality evaluation.

### 5.1 Mechanisms of the model

When the user inputs a search query, the bees leave the dispatch room and they are randomly assigned to the sources. After a bee has collected enough data at the source to be able to evaluate it, she returns to the hive and makes a decision whether to stay with the source, or to abandon it. The probability of staying with the source is set to be equal to the quality q of the visited source, assuming that q is expressed as a number within <0, 1>.

If a bee decides to stay with the source, it makes another decision, whether to keep on foraging, or to start dancing for the source. Again, the probability of deciding to dance is set to be equal to the quality of the source. If the bee decides to start dancing for the source, it moves to the dance floor and starts dancing. Length of a dance depends on the parameter (MDT) and quality q of the source (MDT*q). The better the source is, the longer time the bee...
dances. The bees that decide not to dance, return to the same sources they visited before and continue foraging.

If a bee decides to abandon the source, she moves into the auditorium to watch the dancing bees, for a period of time that depends on the parameter OT. She then considers the sources being propagated at the dance floor by the dancing bees. The chance of choosing a propagated source is equal to the number of bees dancing for the propagated source divided by the total number of dancing bees. If a bee fails to choose from the propagated sources within the observation time, she transfers into the dispatch room and receives some randomly assigned source. This is important, because if we have fewer bees than sources, we need a mechanism to find and subsequently propagate sources without the need of bees assigned to them in the beginning.

A more detailed elaboration of the mechanism of our model can be found in (Navrat, 2006B).

5.2 Experiments with parameters of model

Various experiments were accomplished alike in (Selley, 1991), with monitoring of actual settings of system new parameters.

Fig. 3. Experiments with parameter MDT.

By increasing the parameter MDT, dynamics of the system was observed to decrease. When bees dance longer, expectancy of alluring other bees becomes higher. From among the allured bees inevitably some bees will later go dancing, too. As a consequence, with a constant total number of bees in the hive, there remain less bees to amass around the best source. The hive’s ability to yield a decisive outcome gets slower – see Fig. 3.

By decreasing the parameter OT dynamics of the system increases. When the time is very short (i.e., values are near the zero), frequent behaviour variances of system occur, because bees often take off at random sources, see Fig. 4.
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Fig. 4. Experiment with parameter OT, x axis represents time, y axis represents number of bees.

More experiments with parameters of the model were documented in (Navrat, 2007 A) and (Navrat, 2008).

6. Web pages evaluation

Relevance of particular web page is subjective from the user’s point of view. Each user has different interests and knowledge. There are algorithms to determine relevancy of a particular web page based on properties of the web represented as a graph. In this graph, vertices represent web pages and edges represent interconnections (links) between pages. Page rank algorithm (Page, 1998) can by considered a simulation of an imaginary user that accidentally chooses various links on the web. After each click the user decides if he continues. The probability of requesting another random page is called damping factor d. Many studies discussed the problem of appropriate values of damping factor. Generally this factor is assumed to have value approximately 0.85 (Page, 1998).

The universal formula of PageRank calculation according to (Page, 1998) is:

$$PR(p_i) = 1 - d + d \sum_{p_j \in M(p_i)} \frac{PR(p_j)}{L(p_j)}$$  \hspace{1cm} (1)
where \( p_i \) is page, of which PageRank value is computed, \( M(p_i) \) is set of pages referring to page \( p_i \) and \( L(p_j) \) is number of outgoing links from page \( p_j \).

For calculation of sufficiently exact PageRank value of each page, several iterations are required. During each iteration the PageRank of every page in the collection is recomputed. Number of iterations depends on the number of pages in the collection and on the complexity of interconnections among particular pages.

In Fig. 5, web pages are represented by vertices denoted A, B, etc and links between them are represented by orientated edges.

Fig. 5. A simple graph expressing interconnections of web pages.

PageRank values of pages must be computed iteratively. For example, value of the page E cannot be determined without knowing values of the pages B, D, F, and H. Of course, in order to determine values of these pages, values of still other pages may need to be known (i.e., computed). Conversely, value of the page E is needed to compute values of the pages A, C, G, I.

Let us contemplate about using bees to perform the above described evaluations. The bees would benefit from a slight modification of their behaviour. When a bee flies to the source E (page or vertex E), she evaluates quality of the source, i.e., she determines a preliminary PageRank value of E denoted as PR(E). At the same time she knows that to compute PR(E) more precisely, values of PR(D), PR(B), PR(F) and PR(H) must be known.

If the source E has a high quality, the bee probably goes in the dance floor and dances there. Our idea is that she will not dance attempting to allure other bees for the source E. She dances to allure other bees for one of the sources that refer to source E. The choice of the source will be random with an even assignment of choice probability to each of the pages that are sources of links pointing to E. In this case the bee would try to attract other bees to come to sources, for which she needs to calculate PR of the vertex she visited. The bee would
fly back to the source E after finishing dancing, whereby if she would succeed to attract the other bees to fly to source D, B, F or H, the bee on the source E could determine PR(E) more accurately. There is a high probability for the bee to fly to the dance floor again and dancing there for one of the sources D, B, F or H, in the case that the quality of the source is high enough also after its actualization. The whole cycle is repeated and PR (E) gradually approaches its theoretical value.

Computing PageRank values of all the pages (represented by vertices) proceeds in a similar way. Experiments published in (Navrat, 2008) show that employing bees to compute PageRank values is comparable, in many cases even faster than a standard iterative procedure.

7. Web story
Finding and reading most relevant and up to date articles requires continuously observing all the new sources for updates of stories one is interested in. It also includes discovering new data sources. All this can be problematic if not impossible for a human, so a system capable of doing these tasks might be helpful.

We propose to use a focused crawler to download relevant pages. We took an inspiration for constructing the crawler from nature, particularly from the social behaviour of honey bees. The field of focused crawlers is not new. Early concepts of such crawlers include best-first, fish search (De Bra, 1994) and shark search (Hersovici, 1998) algorithms. In late 90’s the term focused crawler was introduced in (Chakraborti, 1999).

Even the use of focused crawler for online search is not new. In the system called Fetuccino (Ben-Shaul, 1999) the authors tried to solve the classic problem of web search with an offline database, viz. the problem that the pages returned may have changed since they were indexed into the database. They called the classical web search as static search and enhanced it by dynamic search. A dynamic search was an approach to revisit the pages at the time of searching after the results from static search had been obtained. The results were then updated according to the dynamic search and provided to the user.

There have been attempts to propose nature inspired algorithm for focused crawling. For example, the focused Ant Crawling Algorithm (Dziwinski, 2008), for hypertext graph crawling is claimed to be better than the Shark-Search crawling algorithm.

Another example of using online search is the agent InfoSpider (Menczer, 1999). The authors based this agent on previous work on adaptive agents (Menczer, 1998). In (Pant, 2004) and (Menczer, 2000) the authors used adaptive agents, too. Another area related to our work is story tracking. In (Pouliquen, 2008) there is published an approach of handling information overflow by clustering similar articles into stories.

7.1 Modified bee hive model
We chose the model (Fig. 2) and specified the behaviour of the bee outside the hive (Navrat, 2007 B). The web page was used as the source and the aim of the hive was to find the most relevant pages and thus focus the search for new pages into the more promising areas. When a bee flies outside the hive to a source (web page), she estimates its quality (relevancy) as q. With the probability q she stays with the current page, or with the probability 1−q she follows one of the links on the page to visit some new source. Then she will with the probability q fly back to the hive with her current source or with the
complementary probability stay outside the hive and search for better sources. The bee cannot stay outside the hive forever, therefore we used the concept of energy taken from (Menczer, 1999). Every time the bee visits some source, the energy will increment by the quality of the source (non relevant source has zero quality) and decrement by the specified parameter. If the bee has no more energy (energy <= 0) she will return to the hive regardless of other conditions.

7.2 Bees scouts or recruits
While performing the experiments with this behaviour we encountered a problem with discovering of few relevant sources where the bees could start the search. We again found inspiration in nature (Biesmeijer, 2001) and in the failed follower hypothesis (Beekman, 2007). The foraging bees fall within one of two categories – scouts or recruits. Scouts search for food independently regardless of the other bees. Recruits are bees that have been allured by a dance of some other bee. Under the failed follower hypothesis the scouts are failed followers. It means that if a bee does not find a dancing bee to follow, she will become a scout and search for food on her own. As a result, if the food is scarce, the probability of finding a dancing bee is low and more bees become scouts. If there is plenty of food, there will be more dancing bees and consequently more bees become recruits. We managed to accommodate this hypothesis with our original model without even the need of modifying it.

7.3 Story tracking
We assume that aim of on-line search is not to retrieve some single information, the aim is to find a relevant set of pages which would create a story. It is supposed to be used on sites containing frequently changing or added information.

One of excellent applications is for headline stories. An example of such a headline story are elections. We present a case study of tracking the story of the recent presidential elections in Slovakia.

The aim of the case study was to explore if the algorithm can track a story being in development. We chose the second round of presidential elections in Slovakia.

The search started from two Slovak news portals www.sme.sk and www.pravda.sk. During the experiment our system discovered 4615 different pages from various domains, 742 of them had above zero quality. 217 pages could be marked as relevant to its content. From the 217 pages marked as relevant, only 85 had informative character.

Our system was able to track the story back to its immediate origins and beyond, the oldest article was published in February 2009.

We divided the real story of presidential elections in Slovakia into five parts and inspected how many pages the bees were able to find (Table 1).

We can conclude that the system was able to follow the story on the day of the elections. Moreover, it was able to track the story back.
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We took an inspiration from a popular model described in (Pham, 2006). It has been successfully applied to optimisation of mathematical functions. This algorithm has good results in terms of number of evaluations of the optimized function needed to achieve the required result. Another bee inspired algorithm is described in (Wong, 2008). The authors successfully applied their algorithm to the travelling salesman problem. Other algorithms from the area of optimisation using the bee metaphor are Bee Colony Optimisation (Teodorovic, 2005) and Abstract Bee Colony (Karaboga, 2005). In this part we will describe a specialization of our model (Navrat, 2006A) to optimisation of mathematical functions.

This model is very general and does not define the behaviour of the bee outside the hive. It means that before applying the model to specific problems this behaviour has to be defined. However, the generality brings an opportunity to fine-tune the behaviour of the algorithm to the specific needs of the problem domain without the need to modify the basic behaviour of the hive. The bee hive metaphor can be thus used for such different tasks as on-line web search or function optimisation.

We proposed a behaviour of a bee outside the hive for the case of optimizing mathematical functions. Sources in this case are different vectors of values of function arguments.

### 8.1 New parameters of the model

We introduce two new parameters for the model to suit the optimisation task.

<table>
<thead>
<tr>
<th>Story part</th>
<th>Number of found pages</th>
</tr>
</thead>
<tbody>
<tr>
<td>First leg of elections</td>
<td>12</td>
</tr>
<tr>
<td>Campaign before the second leg</td>
<td>28</td>
</tr>
<tr>
<td>The day of elections (second leg)</td>
<td>30</td>
</tr>
<tr>
<td>Announcement of the results</td>
<td>9</td>
</tr>
<tr>
<td>The reactions</td>
<td>6</td>
</tr>
</tbody>
</table>

Table 1. Parts of coverage of presidential elections in Slovakia 2009

This is a simple system for tracking a developing story that is based on a model of a bee hive. We performed a case study that demonstrates the way how our proposed system works. From the case study we can conclude the following:
- the system is able to collect relevant pages,
- it can monitor the story being developed during the search,
- it can reconstruct the story backwards in time.

### 8. Optimisation based on a social behaviour of honey bees

The bee behaviour inspired various researches. Their interests are basically: modelling of bees’ behaviour, or constructing algorithms inspired by bees’ behaviour.

Our hypothesis is that the behaviour the bees show might be an instructive inspiration to develop a model of solving problems from a suitable class. More specifically, having developed a model of a bee hive that can work as a kind of search engine (Navrat, 2008), we propose to investigate a bee hive as a possibly useful metaphor for optimisation.

Abstract Bee Colony (Karaboga, 2005). In this part we will describe a specialization of our model (Navrat, 2006A) to optimisation of mathematical functions.

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We proposed a behaviour of a bee outside the hive for the case of optimizing mathematical functions. Sources in this case are different vectors of values of function arguments.
Parameter Step size: Bees outside the hive can visit more than one source before returning to the hive. When the bee flies from source to source she adds a random number from the interval $<-\text{stepSize}, +\text{stepSize}>$ to each argument of the function.

Parameter Energy: Using this parameter was inspired by (Menczer, 1999). It is the energy of the bee acting outside the hive. When the bee runs off her spare energy, she has to return to the hive without (inspecting, bringing) any source.

8.2 Source quality evaluation
There are two main aspects in the field of evaluating the quality of the source for this model: use of ranking and mapping all values of optimized function from $<-\infty, \infty>$ to $<0, 1>$. Since the proposed model is able to work only with values within range $<0, 1>$ and the functions have their functional values (in general) in range $<-\infty, \infty>$, we need a function which can map every function value into the range. Assume this function is $\text{Map}(x)$. The function must satisfy the three conditions stated in the equations (2), (3) and (4).

$$\forall x, y \in R : x < y \iff \text{Map}(x) < \text{Map}(y)$$  \hspace{1cm} (2)

$$\forall x, y \in R : x > y \iff \text{Map}(x) > \text{Map}(y)$$  \hspace{1cm} (3)

$$\forall x, y \in R : x = y \iff \text{Map}(x) = \text{Map}(y)$$  \hspace{1cm} (4)

For example, the function shown in the formula 5 satisfies these conditions.

$$\text{Map}(x) = \frac{1}{\pi} \left( \arctg(x) + \frac{\pi}{2} \right)$$  \hspace{1cm} (5)

When looking for a global maximum, the formula is used as it is, when looking for a global minimum, the formula $1 - \text{Map}(x)$ is to be used. The function is not linear. However, it is not an issue, because the model uses ranking. The specific formula we used is shown in (6),

$$\text{rank} = \frac{1}{1 - \text{NB}} \ast \left( (1 - \varepsilon) \ast P + \varepsilon - \text{NB} \right) - k$$  \hspace{1cm} (6)

where NB is the number of bees, $\varepsilon$ is a small constant (we have used 0.001), P is position in the ordered list of qualities and k is an empirical constant which safeguards the quality not to be 1. If the quality would be 1 the bee would propagate the corresponding source with 100% probability, i.e. with certainty and the algorithm would be too greedy. We have used $k = 0.2$.

8.3 Results of experiments
For these experiments, we used as the underlying model of the hive the one described in Fig. 2 superimposed by the modification described in 8.1 to optimize a set of benchmarking
functions. These functions’ parameters are stated in Table 2. These data were taken from (Pham, 2006). In Table 3 there are shown results of applying our proposed algorithm to the benchmarking functions as well as results of other commonly used stochastic optimisation algorithms which were previously published in (Pham, 2006).

<table>
<thead>
<tr>
<th>ID</th>
<th>Function name</th>
<th>Interval</th>
<th>Global optima</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Rosenbrock 2D</td>
<td>[-1.2, 1.2]</td>
<td>X(1,1) F=0</td>
</tr>
<tr>
<td>2</td>
<td>Rosenbrock 2D</td>
<td>[-10,10]</td>
<td>X(1,1,1,1) F=0</td>
</tr>
<tr>
<td>3</td>
<td>Goldstein &amp; Price</td>
<td>[-2.2]</td>
<td>X(0,-1) F=3</td>
</tr>
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<td>4</td>
<td>Martin &amp; Gaddy</td>
<td>[0,10]</td>
<td>X(5,5) F=0</td>
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<td>5</td>
<td>Rosenbrock 4D</td>
<td>[-1.2, 1.2]</td>
<td>X(1,1,1,1) F=0</td>
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<tr>
<td>6</td>
<td>De Jong</td>
<td>[-2.048, 2.048]</td>
<td>X(1,1) F=3905.93</td>
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<tr>
<td>7</td>
<td>Branin</td>
<td>[-5, 10]</td>
<td>X(-22/7, 12.275) X(22/7, 2.275) X(66/7, 2.475) F=0.3977272</td>
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<tr>
<td>8</td>
<td>Hyper Sphere</td>
<td>[-5.12, 5.12]</td>
<td>X(0,0,0,0,0,0) F=0</td>
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Table 2. Functions and their parameters as subjected to the experiments

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Table 3. Experimental results given as an average from 100 iterations


### 9. Hierarchy of bee hives

There are algorithms which utilize some sort of hierarchy. For example the Hierarchical Subpopulation Particle Swarm Optimisation Algorithm (Chuan, 2007) uses a hierarchy of particles to solve the problem with premature convergence of the algorithm. It isolates better solutions from those worse ones into different hierarchy levels. The concept of hierarchy was studied also in (Cerny, 2000). Cerny used a simple example of a hierarchical problem: there is a hexagon shaped picture composed of other hexagons and the goal is to find a symmetrical shape composed of these pictures. To accomplish this task the author used a simple Simulated Annealing without hierarchy.

Our concept of hierarchy is mostly based on the paper of (Cerny, 2000). Our model of the bee hive is divided into two parts - the hive and the bees. We described two conditions a bee must satisfy to cooperate with the hive. The first condition is that a bee must be able to take a source. The second condition is that a bee must be able to return a source. The fulfilling of the conditions implies that we can consider the hive as a more complex bee, but still the bee.

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The hierarchy is constructed in Fig. 6 as follows: in higher, non ground levels, a hive is a host to other hives, whereas only at the ground level, a hive is host to true bees.

![Diagram of the bee hive hierarchy](image.png)

Fig. 6. An outline of a bee hive hierarchy

Higher level hive is thanks to our design still a hive, since it works as a hive, albeit it hosts other hives, but they in turn are designed to act as bees. The rationale behind our design is to give higher level hives a capability to influence behaviour of lower level hives (without the need to know whether it is a bee or a hive).

### 9.1 Experiments based on hierarchy

Cerny in (Cerny, 2000) contemplated an experiment to find a symmetrical shape. We employed the above described model and actually conducted a series of experiments. Accumulated results of them are in Fig. 7 showing 9 runs of the experiment in a row. As one can see in Fig. 7, each run except of the first one produced symmetrical solutions.

![Experiments aimed to form a symmetrical shape](image.png)

Fig. 7. Experiments aimed to form a symmetrical shape
10. Conclusion and future work

Bee hive metaphor has recently been attracting researchers who investigate methods of problem solving, although early attempts can be traced one or even two decades ago. A bee hive is a complex system composed of bees whose behaviour in turn can by sufficiently approximately described in very simple terms. There are several approaches to do it. We mention some and provide a slightly more detailed treatise of one of them, viz. the one we have proposed and continue to investigate.

However, no matter what is the particular approach, there have been achieved sufficiently solid results to support the claim that the bee metaphor is an attractive alternative that deserves further study. In particular, several works attempt to propose a method of optimisation based on a suitably defined model of a bee hive. Despite the fact that at least in some cases, results comparable to other optimisation approaches have been achieved, it is not clear if optimisation is going to become the kind of application that kills all possible doubts on the potential of the metaphor. Comparing to ants, bees do not work with pheromones, so in effect they do not have available any memory to store even the simplest kinds of data. Although there were some attempts to apply the concept of pheromones to bees, the important thing to be noted is that bees have instead a mechanism of dynamic remembering realised by their dancing. The key question is not to view this as a possible limitation of the bee hive metaphor, but rather as a unique feature.

Leaving the task of optimisation aside, it is not clear if the up to now research has already found the killer application for the bee hive. For example, we have investigated web search, symmetry formation and developing story tracking on the web. The results that we achieved are quite promising, but further research is required before any definitive conclusions can be made.

11. References


This book presents a unique and diversified collection of research work ranging from controlling the activities in virtual world to optimization of productivity in games, from collaborative recommendations to populate an open computational environment with autonomous hypothetical reasoning, and from dynamic health portal to measuring information quality, correctness, and readability from the web.

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