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Formation Control for Non-Holonomic Mobile Robots: A Hybrid Approach

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1. Introduction

Many cooperative tasks in real world environments, such as exploring, surveillance, search and rescue, transporting large objects and capturing a prey, need the robots to maintain some desired formations when moving. Formation control refers to the problem of controlling the relative position and orientations of robots in a group, while allowing the group to move as a whole. Problems in formation control that have been investigated include assignment of feasible formations, moving into formation, maintenance of formation shape (Desai et al., 2001) and switching between formations (Desai et al., 1999; Fierro et al., 2002). The work in (Das et al., 2002) is a very good example of the state of the art in robot formation control, in which it is presented a complete framework to achieve a stable formation for car-like and unicycle-like mobile robots. A feasible solution to address these problems is by using hybrid control systems in formation control. In fact, several papers can be found in the literature using hybrid control systems: including a discrete event system at the supervisory level and continuous controllers to give the control actions (Desai et al., 1999; Ogren & Leonard, 2003; Chio & Tarn, 2003; Ogren, 2004; Shao et al., 2005).
In this chapter, a hybrid approach for the autonomous navigation of a mobile robots team in a specified formation is developed considering a centralized leader-follower controller (Gava et al., 2007) (see for example (Shao et al., 2005) for a review on the leader-following method) and the non-holonomic constraint of the unicycle-like mobile robots (Gulec & Unel, 2005). In this last paper, the authors state that a complicated coordinated task can be interpreted in terms of simpler coordinate tasks that are to be manipulated sequentially. The leader robot of the team, which navigate independently according to its own control laws, has a laser range-finder, odometry sensors and an omnidirectional camera, whereas the followers have odometry and collision (sonar) sensors. The laser range-finder and odometry sensors of the leader robot are used to implement the leader robot controller (Toibero et al., 2007); and the omnidirectional camera is used to identify the follower postures relative to the leader coordinate system needed in the implementation of the centralized formation controller (Gava et al., 2007). The existence of such a relative sensor is not a constraint since the leader could get access to these positions using another absolute position sensor such as, for instance, a GPSs or odometry and then convert them to the framework attached to the leader robot. In addition, the centralized control architecture, where the control actions for all the followers are generated by the leader, could be decentralized by allowing the

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followers to estimate the leader movements (angular and translational velocities) and performing a minimal communication between the robots (Fredslund & Mataric, 2002). Therefore, a decentralized control scheme could also be supported by this strategy.
The focus of this chapter is not in the formation control framework, but in the way that a hybrid system can improve the performance of the formation controller in many applications by adding a few simple behaviors and a supervisor which generates switching signals while guaranteeing the asymptotic stability of the hybrid formation control system.
The hybrid control strategy developed along this chapter involves mobile robot formation control when considering obstacles. Its main objectives are: *i*) place the follower robots at the desired positions in the given formation before starting the leader navigation, this is the so-called static formation problem (Antonelli et al., 2006); *ii*) reduce the temporary large formation errors during the autonomous navigation of the complete robot team; *iii*) avoid unknown obstacles while maintaining the formation geometry, instead of changing the formation geometry as in (Das et al., 2002). For this last objective, it is considered the obstacle contour-following strategy for the leader robot as presented in (Toibero et al., 2006). Regarding the others two major objectives, a hybrid approach based on a formation controller is proposed.
The rest of the chapter includes: Section 2 presents a review of the stable leader-based formation controller. Then, in Section 3 it is described the hybrid control system including simulations results and stability considerations. In Section 4 some comparative simulation results are presented. Finally, in Section 5 experimental results are reported to state conclusions in Section 6.

2. Stable Formation Control

The kinematics model employed in this paper considers formation errors with respect to a Cartesian mobile coordinate system over the leader robot, which Y-axle coincides with the heading of this robot (Fig.1.) The movement of each robot in the world coordinate system (with upper index *w*) is ruled by the well-known unicycle-like mobile robot kinematics: for the leader in (1) and for the *i*-th follower in (2)

$$\begin{aligned} {}^{w}\dot{x} &= v\cos\left({}^{w}\theta\right) \\ {}^{w}\dot{y} &= v\sin\left({}^{w}\theta\right) \\ {}^{w}\dot{\theta} &= \omega \end{aligned} \tag{1}$$

$$\begin{aligned} {}^{w}\dot{x}_i &= v_i\cos\left({}^{w}\theta_i\right) \\ {}^{w}\dot{y}_i &= v_i\sin\left({}^{w}\theta_i\right) \\ {}^{w}\dot{\theta}_i &= \omega_i \end{aligned} \tag{2}$$

The leader movement is controlled through its absolute velocities: v and ω. The formation controller objective is to find the values of the velocities v_i and ω_i for the follower robots in such a way that the formation errors decay asymptotically to zero. A third kinematics model must be considered in order to obtain the *i*-th follower coordinates relative to the leader ($O\ {}^{L}\mathrm{X}\ {}^{L}\mathrm{Y}$) coordinate system which moves at a linear velocity v and angular velocity ω

$$^{L}\dot{x}_i = v_i \cos\left({}^{L}\theta_i\right) + \omega l \sin(\varsigma_i) \tag{3}$$

$$^{L}\dot{y}_i = v_i \sin\left({}^{L}\theta_i\right) - \omega l \cos(\varsigma_i) - v \tag{4}$$

$$^{L}\dot{\theta}_i = \omega_i - \omega \tag{5}$$

here, l is the distance between the robot centre and the origin of the mobile coordinate system and ς_i is the angle between the ^{L}X axis and l (Fig.2.) Note that for a static leader, these equations reduce to

$$\begin{aligned} {}^{L}\dot{x}_i &= v_i \cos\left({}^{L}\theta_i\right) \\ {}^{L}\dot{y}_i &= v_i \sin\left({}^{L}\theta_i\right) \\ {}^{L}\dot{\theta}_i &= \omega_i \end{aligned} \tag{6}$$

which describe the i-th follower movement on the leader coordinate system. The consideration of the postures of the followers relative to the leader coordinate system allows managing the entire formation without knowing the absolute positions of the followers.

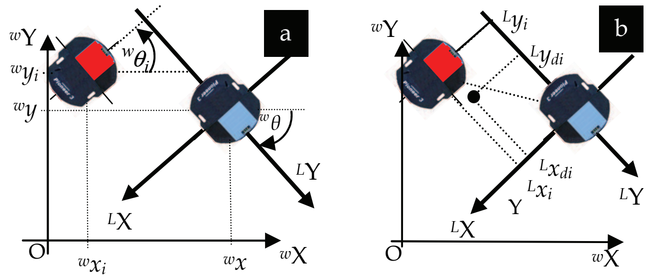

Figure 1. a) Reference systems: world (absolute) reference coordinate system (O ^{w}X ^{w}Y), and a second coordinate system attached to the leader robot (O ^{L}X ^{L}Y) where the desired positions for each of the followers are defined. b) i-th follower robot positioned at coordinates (${}^{L}x_i$, ${}^{L}y_i$) on the leader Cartesian reference where the reference position is given by (${}^{L}x_{di}$, ${}^{L}y_{di}$)

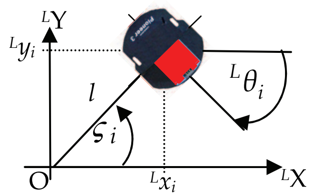

Figure 2. i-th follower in the leader coordinate system

This relative posture can be obtained using a sensor system (for example a cathadioptric vision system) mounted on one of the robots (Fig.3). However, if the absolute postures information is available, it can be easily converted to the leader coordinate system with the transformation:

$$^{L}x_i = \left({}^{w}x_i - {}^{w}x\right)\sin\left({}^{w}\theta\right) - \left({}^{w}y_i - {}^{w}y\right)\cos\left({}^{w}\theta\right) \tag{7}$$

$$^{L}y_i = \left({}^{w}x_i - {}^{w}x\right)\cos\left({}^{w}\theta\right) + \left({}^{w}y_i - {}^{w}y\right)\sin\left({}^{w}\theta\right) \tag{8}$$

$$^{L}\theta_i = {}^{w}\theta_i - {}^{w}\theta + \pi/2 \tag{9}$$

which gives the relation between the absolute (${}^{w}x_i$, ${}^{w}y_i$) and the relative (${}^{L}x_i$,${}^{L}y_i$) coordinates. On the other hand, the i-th follower absolute heading angle can be transformed to the leader coordinate system by using equation (9).

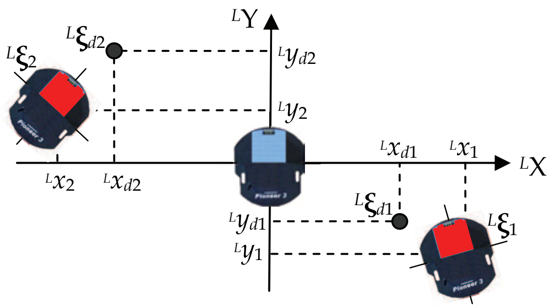

Figure 3. Representation of the vectors ${}^{L}\xi_i$ and ${}^{L}\xi_{di}$

In Fig.4 it can be seen the block diagram of the proposed controller. From this figure, it must be noted the formation controller independence on the leader motion generation, that is, the leader navigates according to its own motion laws and the formation controller only needs the commands computed by the leader controller.

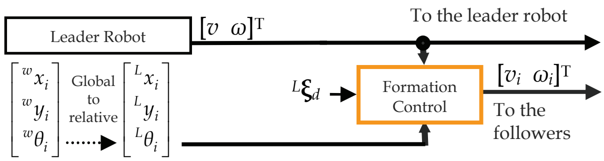

Figure 4. Formation control block diagram

In order to calculate an error indicator between the current and the desired positions of the robots in the formation, let consider that (10) is the position vector of the i-th follower robot, and that (11) denotes the i-th follower desired position, with i=1, 2,..., n.

$$^{L}\xi_i = \left[{}^{L}x_i \quad {}^{L}y_i\right]^{\mathrm{T}} \tag{10}$$

$$^{L}\xi_{di} = \left[{}^{L}x_{di} \quad {}^{L}y_{di}\right]^{\mathrm{T}} \tag{11}$$

Both vectors are defined on the framework attached to the leader robot (Fig.3.) The n individual position vectors (10) and (11) can be arranged in the global position vectors:

$$ {}^{L}\boldsymbol{\xi} = \begin{bmatrix} {}^{L}\boldsymbol{\xi}_1^{\mathrm{T}} & {}^{L}\boldsymbol{\xi}_2^{\mathrm{T}} & \ldots & {}^{L}\boldsymbol{\xi}_n^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} \tag{12} $$

$$ {}^{L}\boldsymbol{\xi}_d = \begin{bmatrix} {}^{L}\boldsymbol{\xi}_{d1}^{\mathrm{T}} & {}^{L}\boldsymbol{\xi}_{d2}^{\mathrm{T}} & \ldots & {}^{L}\boldsymbol{\xi}_{dn}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} \tag{13} $$

The difference between the actual and the desired robot position is (14); and the formation error is defined in (15) as follows (Kelly et al., 2004; Carelli et al., 2006)

$$ {}^{L}\tilde{\boldsymbol{\xi}} = {}^{L}\boldsymbol{\xi}_d - {}^{L}\boldsymbol{\xi} \tag{14} $$

$$ \tilde{\mathbf{h}} = \mathbf{h}_d - \mathbf{h} = \mathbf{h}\left({}^{L}\boldsymbol{\xi}_d\right) - \mathbf{h}\left({}^{L}\boldsymbol{\xi}\right) \tag{15} $$

$$ \mathbf{h} = \mathbf{h}\left({}^{L}\boldsymbol{\xi}\right) = \mathbf{h}\left({}^{L}\boldsymbol{\xi}_d - {}^{L}\tilde{\boldsymbol{\xi}}\right) \tag{16} $$

where $\mathbf{h}$ is a suitable selected output variable representing the formation parameters, which captures information about the current conditions of the group of robots; $\mathbf{h}_d$ represents the desired output variable. For instance, $\mathbf{h}$ can be selected as the xy-position of each follower robot. Function $\mathbf{h}\left({}^{L}\boldsymbol{\xi}\right)$ must be defined in such a way to be continuous and differentiable, and the Jacobian matrix J has full rank.

$$ \dot{\mathbf{h}} = \mathbf{J}(\boldsymbol{\xi})\, {}^{L}\dot{\boldsymbol{\xi}} = \frac{\partial\, \mathbf{h}\left({}^{L}\boldsymbol{\xi}\right)}{\partial\, {}^{L}\boldsymbol{\xi}}\, {}^{L}\dot{\boldsymbol{\xi}} \tag{17} $$

Vector ${}^{L}\dot{\boldsymbol{\xi}}$ (robot translational velocities in the leader reference system) has two different components,

$$ {}^{L}\dot{\boldsymbol{\xi}} = {}^{L}\dot{\boldsymbol{\xi}}_s - {}^{L}\dot{\boldsymbol{\xi}}_l \tag{18} $$

where ${}^{L}\dot{\boldsymbol{\xi}}_s$ is the time variation of ${}^{L}\boldsymbol{\xi}$ produced by the velocities of the follower robots $\left\| {}^{L}\dot{\boldsymbol{\xi}}_{si} \right\| = v_i$; and ${}^{L}\dot{\boldsymbol{\xi}}_l$ is the time variation of ${}^{L}\boldsymbol{\xi}$ produced by the velocities of the leader robot. Now, (17) can be written as:

$$ \dot{\mathbf{h}} = \mathbf{J}\left({}^{L}\boldsymbol{\xi}\right)\left({}^{L}\dot{\boldsymbol{\xi}}_s - {}^{L}\dot{\boldsymbol{\xi}}_l\right) \tag{19} $$

The control objective is to guarantee that the mobile robots will asymptotically achieve the desired formation, that is, $\lim_{t\to\infty} \tilde{\mathbf{h}}(t) = 0$. To this aim, it is first defined a reference velocities vector as:

$$ {}^{L}\dot{\boldsymbol{\xi}}_r = \mathbf{J}^{-1}\left({}^{L}\boldsymbol{\xi}\right)\left[\dot{\mathbf{h}}_d + \mathbf{K}\,\mathbf{f}_{\tilde{\mathbf{h}}}\left(\tilde{\mathbf{h}}\right)\right] + {}^{L}\dot{\boldsymbol{\xi}}_l \tag{20} $$

where K is a symmetric and positive definite gain matrix; $\mathbf{f}_{\tilde{\mathbf{h}}}\left(\tilde{\mathbf{h}}\right)$ is a saturation function applied to the output error, such that $\mathbf{x}^{\mathrm{T}}\,\mathbf{f}_{\tilde{\mathbf{h}}}(\mathbf{x}) > \mathbf{0} \quad \forall \mathbf{x} \neq \mathbf{0}$. This function could be selected for example as $\mathbf{f}_{\tilde{\mathbf{h}}}(\mathbf{x}) = \tanh(\mathbf{x})$. ${}^{L}\dot{\boldsymbol{\xi}}_r$ represents the velocities of the followers robots on the framework attached to the leader robot that allow them to reach (and to maintain) the desired formation while following the leader. Assuming perfect velocity servoing

$$^{L}\dot{\xi}_{s} \equiv {}^{L}\dot{\xi}_{r} \tag{21}$$

then from (19),(20) and (15) the following closed loop equation can be obtained:

$$\dot{\tilde{\mathbf{h}}} + \mathbf{f}_{\tilde{\mathbf{h}}}\left(\tilde{\mathbf{h}}\right) = \mathbf{0} \tag{22}$$

Now, in order to consider the formation errors analysis under the perfect servoing assumption the following Lyapunov candidate function (Slotine & Li, 1991) is introduced

$$V = \tilde{\mathbf{h}}^{T}\,\tilde{\mathbf{h}}/2 \tag{23}$$

with its time-derivative along system trajectories

$$\dot{V} = \tilde{\mathbf{h}}^{\mathrm{T}}\,\dot{\tilde{\mathbf{h}}} = -\tilde{\mathbf{h}}^{\mathrm{T}}\,\mathbf{K}\,\mathbf{f}_{\tilde{\mathbf{h}}}\left(\tilde{\mathbf{h}}\right) < 0 \tag{24}$$

It is clear that if $^{L}\dot{\xi}_{s} \equiv {}^{L}\dot{\xi}_{r} \Rightarrow \tilde{\mathbf{h}}(t) \to 0$ asymptotically.

Remark 1. This condition is verified for the ideal case in which the robots follow exactly the reference velocity (21). However, for a real controller this velocity equality will eventually be reached asymptotically. The convergence of the control error to zero under this real condition will be analyzed at the end of this section.

Vector $^{L}\dot{\xi}_{l}$ in (20) is computed using the knowledge of linear and angular velocities of the leader robot, and the relative positions of the follower robots:

$$r_1 = v/\omega \tag{25}$$

$$r_{2i} = \sqrt{(r_1 + x_i)^2 + y_i^2} \tag{26}$$

$$\beta_i = \arctan\left(y_i/(r_1 + x_i)\right) \tag{27}$$

$$\left\| {}^{L}\dot{\xi}_{li} \right\| = \omega\, r_{2i} \;\text{ and }\; \begin{bmatrix} {}^{L}\dot{\xi}_{lxi} \\ {}^{L}\dot{\xi}_{lyi} \end{bmatrix} = \begin{bmatrix} -\left\| {}^{L}\dot{\xi}_{li} \right\| \sin(\beta_i) \\ \left\| {}^{L}\dot{\xi}_{li} \right\| \cos(\beta_i) \end{bmatrix} \tag{28}$$

where r_1 and r_{2i} are virtual turning radius (Fig.5) and subscript i denotes the i-th follower.

Remark 2. In the case $\omega = 0$, vector $^{L}\dot{\xi}_{li}$ is calculated as $^{L}\dot{\xi}_{li} = [0 \quad v]^{\mathrm{T}}$.

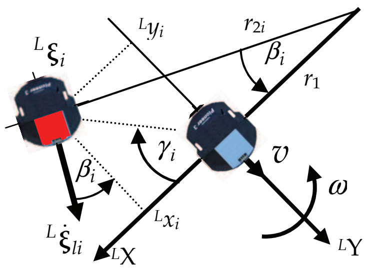

Figure 5. Velocity computation

The commands for the linear and angular velocities of each robot are computed in order to secure that the robots will reach asymptotically the velocity reference ($^{L}\dot{\xi}_{s} \to {}^{L}\dot{\xi}_{r}$).

The proposed control law for heading control is:

$$\omega_i = k_{\omega i} f\left({}^L\tilde{\theta}_i\right) + {}^L\dot{\theta}_{ri} + \omega \qquad (29)$$

where ${}^L\tilde{\theta}_i = {}^L\theta_{ri} - {}^L\theta_i$ is the angular error between the i-th follower robot heading ${}^L\theta_i$ and the angle of its reference velocity ${}^L\theta_{ri} = \angle\left(\dot{\boldsymbol{\xi}}_{ri}\right)$; consequently ${}^L\dot{\theta}_{ri}$ is the time derivative of this reference velocity heading for the i-th robot; ω is the angular velocity of the mobile framework attached to the leader robot; and $f\left({}^L\tilde{\theta}_i\right)$ is a saturation function applied to the angular error, which has the same properties of function $\mathbf{f}_{\tilde{\mathbf{h}}}\left(\tilde{\mathbf{h}}\right)$ included in (20) ; and $k_{\omega i}$ is a positive constant. Next, by equating (29) and (5), the following closed-loop equation can be obtained:

$${}^L\dot{\tilde{\theta}}_i + k_{\omega i} f\left({}^L\tilde{\theta}_i\right) = 0 \qquad (30)$$

Now, in order to analyze the stability for the heading control, it is introduced the following Lyapunov candidate (Slotine & Li, 1991)

$$V = {}^L\tilde{\theta}_i^2 / 2 \qquad (31)$$

and its time derivative

$$\dot{V} = {}^L\tilde{\theta}_i \; {}^L\dot{\tilde{\theta}}_i = -k_{\omega i} \; {}^L\tilde{\theta}_i \; f\left({}^L\tilde{\theta}_i\right) < 0 \qquad (32)$$

which implies that ${}^L\tilde{\theta}_i(t) \to 0$ as $t \to \infty$. That is, the robot orientation on the leader Cartesian coordinate system tends asymptotically to the desired reference orientation, which guarantees maintaining the desired formation. Once it was proved that ${}^L\theta_i(t) \to {}^L\theta_{ri}(t)$, it must now be proved that the same occurs for $\left\|{}^L\dot{\boldsymbol{\xi}}_{si}\right\| = v_i \to \left\|{}^L\dot{\boldsymbol{\xi}}_{ri}\right\|$. To this aim, the following control law for the linear velocity is proposed:

$$v_i = \left\|{}^L\dot{\boldsymbol{\xi}}_{ri}\right\| \cos\left({}^L\tilde{\theta}_i\right) \qquad (33)$$

which obviously produces that $v_i \to \left\|{}^L\dot{\xi}_{ri}\right\|$, since it has been proved that ${}^L\tilde{\theta}_i(t) \to 0$. The factor $\cos\left({}^L\tilde{\theta}_i\right)$ has been added to prevent high control actions when a large angular error exists. Now, we have proved that ${}^L\dot{\xi}_r - {}^L\dot{\xi}_s = \boldsymbol{\rho}$ with $\boldsymbol{\rho}(t) \to 0$, which is a more realistic assumption than (21). Then, formation errors are considered again in order to analyze its stability under the new condition. So, (22) can be written as :

$$\dot{\tilde{\mathbf{h}}} + \mathbf{K}\,\mathbf{f}_{\tilde{\mathbf{h}}}\left(\tilde{\mathbf{h}}\right) = \mathbf{J}\boldsymbol{\rho} \qquad (34)$$

Let us consider the same Lyapunov candidate (23) but now with its time derivative:

$$\dot{V} = \tilde{\mathbf{h}}^{\mathrm{T}}\dot{\tilde{\mathbf{h}}} = -\tilde{\mathbf{h}}^{\mathrm{T}}\left(\mathbf{K}\,\mathbf{f}_{\tilde{\mathbf{h}}}\left(\tilde{\mathbf{h}}\right) - \mathbf{J}\boldsymbol{\rho}\right) \qquad (35)$$

A sufficient condition for (35) to be negative definite is

$$\left\| \mathbf{f}_{\tilde{\mathrm{h}}}\left(\tilde{\mathbf{h}}\right) \right\| > \frac{\|\mathbf{J}\|\|\boldsymbol{\rho}\|}{\|\mathbf{K}\|} \tag{36}$$

As $\boldsymbol{\rho}(t) \to \mathbf{0}$ and taking into account the properties of function $\mathbf{f}_{\tilde{\mathrm{h}}}\left(\tilde{\mathbf{h}}\right)$, it can be concluded that $\left\|\tilde{\mathbf{h}}(t)\right\| \to 0$ as $t \to \infty$.

3. Hybrid Formation Control

The interaction between the leader and the follower controllers must be in such a way that the followers always maintain their desired positions independently of the leader manoeuvres. This allows preserving the formation and therefore the involved robots can perform a cooperative task. Our approach is based on the detection of leader movements that will significantly increase transitory formation errors. In Fig.6 we present a hybrid formation control strategy where it can be appreciated the inclusion of a supervisor which generates switching signals at both levels: leader (σ_L) and followers (σ_{Fi}) based on: *i*) the follower posture, *ii*) the leader absolute posture and *iii*) the leader control actions. Besides, it was also included a new orientation controller, that corrects the followers heading accordingly to a given logic (next, in Fig.9).

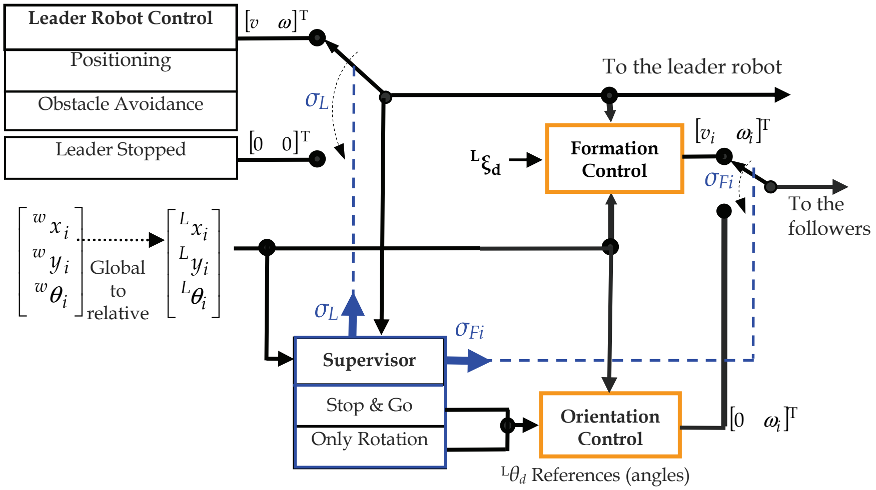

Figure 6. Hybrid formation control block diagram

The main idea is to detect leader movements that will immediately produce formation errors. These errors will arise due to the non-holonomic constraint of the unicycle-like wheeled mobile robots (mostly due to different robot headings.) Hence, the headings of the followers are set to values that prevent these initial errors and only after this correction is done, the leader is allowed to continue with its planned movement. These leader movements (which are detected directly from the leader control commands) are namely: *i*) "stop & go" (a step in the forward velocity) and *ii*) "only-rotation movements" (a step in the angular velocity command with null forward velocity).

The leader motion control is based on the results of (Toibero et al., 2007) and gives the robot the capability to get a desired posture $[{}^{w}x_{DES}, {}^{w}y_{DES}, {}^{w}\theta_{DES}]^{T}$ in the world coordinate system while avoiding obstacles. This motion could only be stopped by the supervisor ("leader stopped" in the block diagram of Fig.6). This strategy allows separating completely the control analysis into the leader motion control analysis and the follower motion control analysis. For the leader this analysis is trivial since its motion control is asymptotically stable, then the new control system which is assumed to include the possibility to stop the leader during a finite time, will also be asymptotically stable. Now, regarding the follower robots, the inclusion of the orientation controller must be considered into the stability analysis (Section 3.2). Note the existence of a switching signal σ_{Fi} for each follower, and consequently, an orientation controller available for each follower robot.

3.1 Follower Robots: Heading Control

In this section it is introduced a proportional only-bearing controller that allows the follower robots to set their headings to desired computed values that will provide good initial heading conditions for the future formation evolution. The position of each follower robot in the formation is defined by its coordinates $({}^{L}x_{id}, {}^{L}y_{id})$ regardless of its orientation. Taking advantage of the unicycle kinematics, it will be assumed from here on that the robots can rotate without distorting the formation (allowing change the "formation heading"). In other words, for instance, if the robots are transporting an object, they must be able to turn freely over its own centers without changing the transported object orientation. It is proposed a proportional controller for the heading error

$$ {}^{L}\tilde{\theta}_i = {}^{L}\theta_d - {}^{L}\theta_i \tag{37} $$

where the desired value ${}^{L}\theta_d$ is computed according to Section 3.2. Then, proposing the following Lyapunov function (38) with the control action (39) the asymptotic stability of this control system could be immediately proved by (40)

$$ V_{ori} = {}^{L}\tilde{\theta}_i^{2} / 2 \tag{38} $$

$$ \omega_i = -k_{\omega i} \tanh\left({}^{L}\tilde{\theta}_i\right) \tag{39} $$

$$ \dot{V}_{ori} = {}^{L}\tilde{\theta}_i \; {}^{L}\dot{\tilde{\theta}}_i = -k_{\omega i} \; {}^{L}\tilde{\theta}_i \tanh\left({}^{L}\tilde{\theta}_i\right) < 0 \tag{40} $$

The importance of introducing the heading controller can be appreciated from Fig.7, starting with a null error formation (Fig.7.a) the leader develops a pure-rotational evolution (Fig.7.b), and the follower tries to keep the formation with a significant transition error. It is clear that this error could be avoided if the starting orientation of the follower robot is set to ψ before the leader starts its rotation. This angle is the orientation of the first velocity reference vector ${}^{L}\dot{\boldsymbol{\xi}}_r$ and is computed depending of the sign of the leader angular velocity according to:

$$ \psi_i = \gamma_i + \mathrm{sgn}(w)\pi / 2 \tag{41} $$

where the angle γ_i is given by

$$ \gamma_i = \tan^{-1}\left({}^{L}y_{id} / {}^{L}x_{id}\right) \tag{42} $$

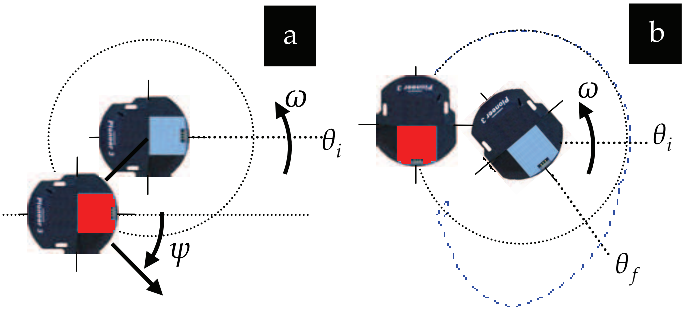

Figure 7. Formation control without orientation control for a leader (at the centre) and a follower: a) initial configuration; b) the path described by the follower (dotted line)

Moreover, depending on the leader angular velocity, formation errors could be greater or even produce follower backward movements. This ψ-angle correction avoids transitory formation errors improving the whole control system performance. The same analysis could be done for the leader "stop & go" movement with heading errors on the follower robots. In this case, the leader attempts to start its translational motion and it is easy to see that the robot configuration that will present minimal formation error at this transitory will be the formation in which all robots have the same heading angle.

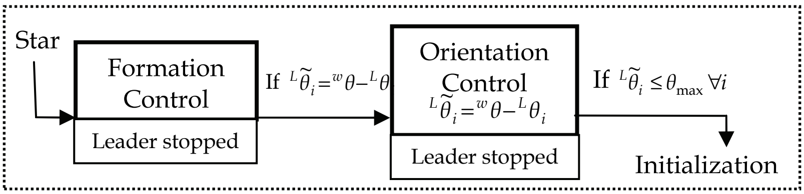

Figure 8. Initialization logic: static formation

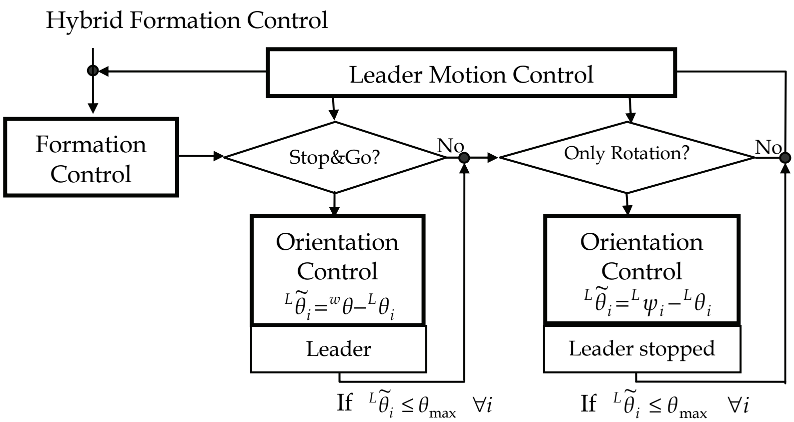

Figure 9. General hybrid formation logic

3.2 Stability Analysis

The supervisor logics were divided into two cases: an initialization case (or static formation case of Fig.8) that corrects followers' initial postures to a new posture with null formation error and with the same leader heading; and the general case that allows keeping the formation geometry (Fig.9) when the leader starts moving. The orientation control is used in two situations: one related to the leader only-bearing movement that corrects the ψ-angle for each follower; and the other related to the leader "stop & go" movement, that equals all the followers headings to the leader heading. In both cases the objective is to minimize the heading errors before starting the leader movement. Accordingly to the exposed logics, it is considered a switching between the formation controller of Section 2 and an orientation controller of Section 3.1 for each follower which stability at switching times must be analyzed. This is done by considering Multiple Lyapunov Functions (Liberzon, 2003): It must be guaranteed that the sequence associated to the discontinuous Lyapunov Functions (when are active) be decreasing for all the controllers involved and furthermore, it must be guaranteed also that the switching is not arbitrarily fast. In Fig.10 it is depicted a typical switching instant (at <t_1>) for a three robot formation. At this point, the leader is stopped, and the orientation controllers compute their references (note the existence of different values for each robot); then, at <t_2> follower 1 has achieved the maximum acceptable error θ_{max}, however the formation controller will not start with its movement after instant <t_3> when the second follower has achieved its maximum heading error.

In consequence the logics secure: *i*) that the switching from the orientation control back to the formation control is slow enough to allow the followers to achieve its desired postures avoiding the undesirable *chattering* effect; *ii*) that the value of (23) is the same before and after the switching since it does not depend on the follower' headings because the formation error is defined only as a function of the followers' positions ${}^{L}\xi$. This fact can be seen in Fig.10 where $V(t_1) = V(t_3)$. This way, the asymptotic stability proved for the formation controller (and its performance) will not be affected by the proposed switching.

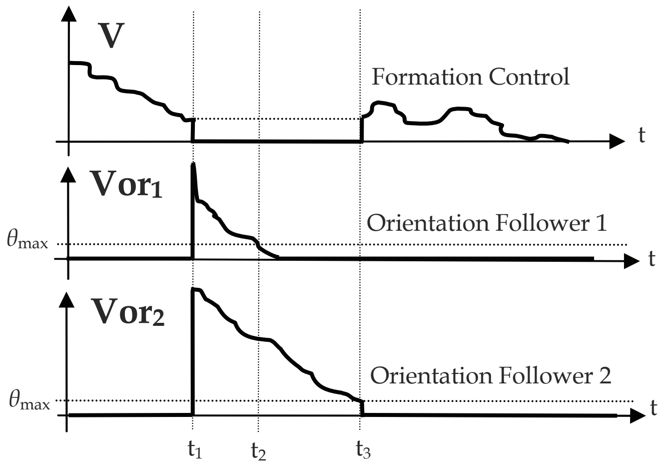

Figure 10. Multiple Lyapunov Function approach

4. Simulation Results

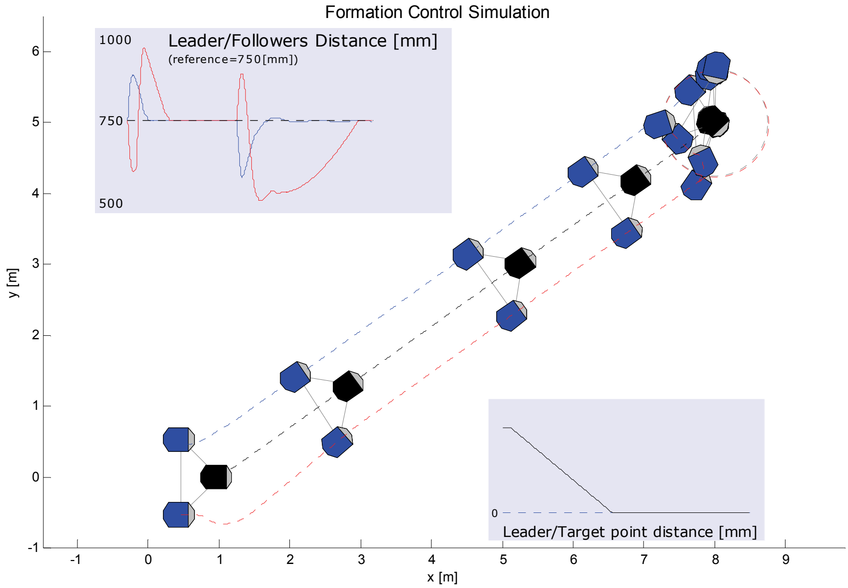

Figure 11. Formation control simulation

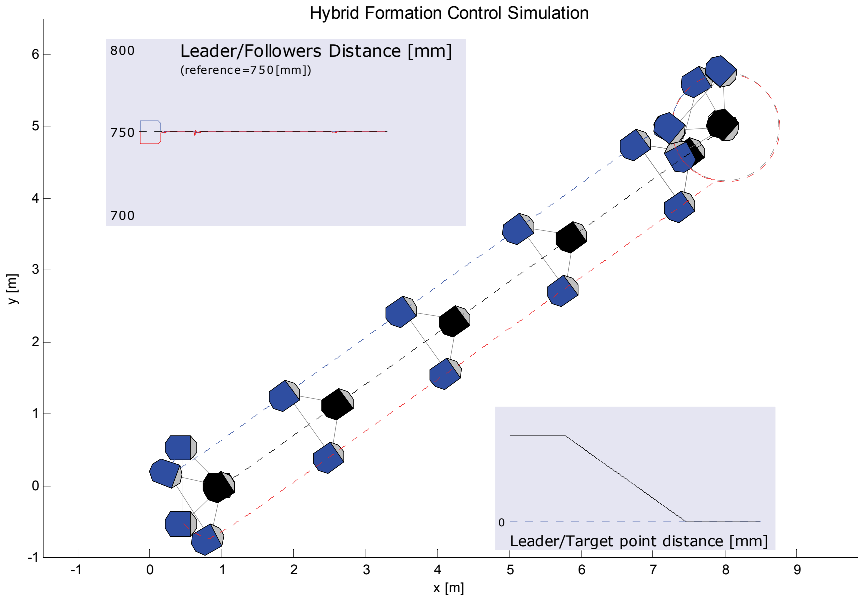

Figure 12. Hybrid formation control simulation

Before introducing the experimental results obtained with real robots we present some comparative simulation results. The strategies of Sections 2 and 3 are compared with the aim of highlight the improvement on the performance achieved with the hybrid formation proposal.
It is considered the same simulation experiment for a three-robot triangle formation under both controllers. The task consists of a simple free obstacle navigation between two points. The experiment includes both situations mentioned along this chapter: “stop & go” and “only rotation”.
Figure 11 shows the simulation results for the formation controller of Section 2 where large transitory formation errors can be appreciated. These formation errors appear due to the “stop & go” situation at the beginning of the experiment and due to the leader “only heading” movement when the leader robot achieve the goal point. In spite of the stability of this controller (the formation errors tend asymptotically to zero), those transitory errors could be unacceptable for many applications.
On the other hand, Fig.12 presents the results for the same simulation experiment but using the hybrid formation controller. It can be noted that this hybrid strategy is able to deal better with the “stop & go” and “only heading” movement, considerably reducing transitory formation errors.

5. Experimental Results

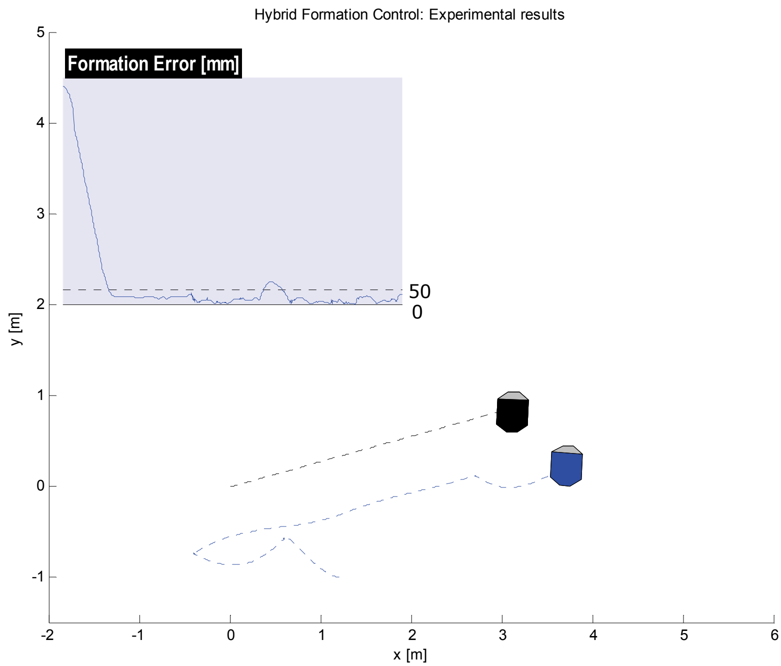

Figure 13. Experimental results: formation control without obstacles including initial formation error

Experimental results were performed by using two Pioneer robots with onboard PCs and wireless internet connection. This way, each sample time (Ts = 100ms) the leader robot asks for the follower position and after computing the control commands, sends them back to the follower. It was considered a reference translational velocity of 150mm/s and a maximum angular velocity of 50º/s for the leader.

In Fig.13 it can be appreciated the formation evolution within a room without obstacles. In the first part, it is considered the static formation problem for a follower initial posture of $[^{L}x_1, {}^{L}y_1, {}^{L}\theta_1]^T$ = [1200, -1000, 180º]^T and a desired formation at $({}^{L}x_{d1}, {}^{L}y_{d1})$ = (600, -600). It can be appreciated the formation error correction according to the initialization logic of Fig.9. After the formation geometry is achieved, the leader robot is allowed to start with its motion towards the goal point at $[^{w}x_{DES}, {}^{w}y_{DES}, {}^{w}\theta_{DES}]^T$ = [3100, 850, 90º]^T from the initial posture at [0, 0, 90º]^T.

Finally, Fig.14 shows the robot trajectories for a similar experiment but considering an obstacle which is detected with the leader laser range finder. In this case the desired formation point was set to $({}^{L}x_{d1}, {}^{L}y_{d1})$ = (0, -600) and the maximum formation error was 105mm while the mean value was of 28mm (that could be compared with the error values for the previous experiment: maximum value of 720mm and mean value of 22mm). From these two previous plots, it can be concluded the low formation error values achieved for the robot formations.

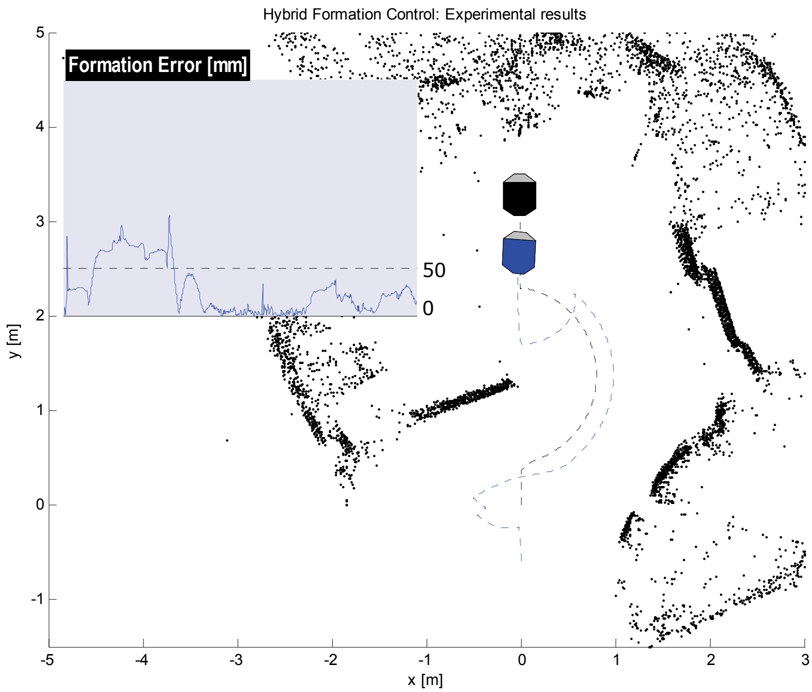

Figure 14. Experimental results: formation control with obstacles

6. Conclusions

In this chapter it has been addressed the problem of the autonomous navigation for a group of non-holonomic mobile robots. In a first stage we considered the classic leader-based formation control problem. In spite of the stability property of this controller, we have detected large transitory errors in some circumstances, being these errors unacceptable for many applications, such as transporting large objects in a cooperative way.
Based on these observations and in order to present a formal solution, we have developed a hybrid approach for the formation problem. The continuous formation controller has been complemented with an orientation controller for each follower, allowing a considerable reduction of formation errors during leader manoeuvres. The resulting hybrid control system presents a switched architecture characterized by the presence of a supervisor which generates a switching signal indicating the active controller at any moment.
Besides, it has been included a formal stability proof for the whole switched system based on the theory of multiple Lyapunov functions.
At the end of this chapter, we exposed simulations results that allow comparing both main strategies. Next, we have included experimental results for a two-robots formation navigating on different settings: without obstacles, and avoiding isolated obstacles by considering a reactive algorithm on the leader robot. Through these experimental results it can be concluded the good performance of the hybrid approach.
Future works on this area will be related to the improvement of the obstacle avoidance capability and to increase the perception abilities of the follower robots (adding new sensors).

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To design a team of robots which is able to perform given tasks is a great concern of many members of robotics community. There are many problems left to be solved in order to have the fully functional robot team. Robotics community is trying hard to solve such problems (navigation, task allocation, communication, adaptation, control, ...). This book represents the contributions of the top researchers in this field and will serve as a valuable tool for professionals in this interdisciplinary field. It is focused on the challenging issues of team architectures, vehicle learning and adaptation, heterogeneous group control and cooperation, task selection, dynamic autonomy, mixed initiative, and human and robot team interaction. The book consists of 16 chapters introducing both basic research and advanced developments. Topics covered include kinematics, dynamic analysis, accuracy, optimization design, modelling, simulation and control of multi robot systems.

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