Type Design of Decoupled Parallel Manipulators with Lower Mobility

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1. Introduction

A typical parallel mechanism consists of a moving platform, a fixed base, and several kinematical chains (also called the legs or limbs) which connect the moving platform to its base. Only some kinematical pairs are actuated, whose number usually equals to the number of degrees of freedom (dofs) that the platform possesses with respect to the base. Frequently, the number of legs equals to that of dofs. This makes it possible to actuate only one pair per leg, allowing all motors to be mounted close to the base. Such mechanisms show desirable characteristics, such as large payload and weight ratio, large stiffness, low inertia, and high dynamic performance. However, compared with serial manipulators, the disadvantages include lower dexterity, smaller workspace, singularity, and more noticeable, coupled geometry, by which it is very difficult to determine the initial value of actuators while the end effector stands at its original position.

In an engineering point of view, it is always important to develop a simple and efficient original position calibration method to determine initial values of all actuators. This calibration method usually becomes one of the key techniques that a type of mechanism can be simply and successfully used to the precision applications. Accordingly, few have been reported that the parallel manipulators being applied to high precision situations except micro-movement ones.

The study of movement decoupling for parallel manipulators shows an opportunity to simply the original position calibration and to improve the precision of parallel manipulators in a handy way. One of the most important things in the study of movement decoupling of parallel manipulators is how to design a new type with decoupled geometry. Decoupled parallel manipulators with lower mobility (LM-DPMs) are parallel mechanisms with less than six dofs and with decoupled geometry. This type of manipulators has attracted more and more attention of academic researchers in recent years. Till now, it is difficult to design a decoupled parallel manipulator which has translational and rotational movement simultaneously (Zhang et al., 2006a, 2006b, 2006c). Nevertheless, under some rules, it is relatively easy to design a decoupled parallel manipulator which can produce pure translational (Baron & Bernier, 2001; Carricato, & Parenti-Castelli, 2001a; Gao et al., 2005; Hervé, & Sparacino, 1992; Kim & Tsai, 2003; Kong & Gosselin, 2002; Li et al., 2005a, 2005b, 2006a; Tsai, 1996; Tsai et al., 1996; Zhao & Huang, 2000) or rotational (Carricato & Parenti-Castelli, 2001b, 2004; Gogu, 2005; Li et al., 2006b, 2007a, 2007b) movements.

This chapter attempts to provide a unified frame for the type design of decoupled parallel manipulators with pure translational or rotational movements. The chapter starts with the introduction of the LM-DPMs, and then, introduce a general idea for type design. Finally, divide the specific subjects into two independent aspects, pure translational and rotational. Each of them is discussed separately. Special attention is paid to the kinds of joints or pairs, the limb topology, the type design, and etc.

2. The general idea for decoupled parallel manipulators with lower mobility

The general idea for the type design of decoupled parallel manipulators with lower mobility can be expressed as the following theory.

**Theory:** A movement is independent with others if one of the following conditions is satisfied:

1. To the pure translational mechanisms, the translational actuator is orthogonal with the plane composed of other translational actuators.
2. To the pure rotational mechanisms (spherical mechanisms), the translational actuator is parallel with the axis of rotational actuator.

Depend on part (1) of the theory, we can design some kinds of 3-dofs pure translational decoupled parallel manipulators. Also we can get some kinds of 2-dofs spherical mechanism based on part (2) of the theory.

For the convenience, first, let us define some letters to denote the joints (or pairs). They are the revolute joint (R), the spherical joint (S), the prismatic pair (P), and the planar pair or flat pair (F). They possess one revolute dof, three revolute dofs, one translational dof and three dofs (two translational and one revolute) respectively. Then the theory can be expressed by figure 1 and figure 2 separately.

Figure 1 illustrates the limb topology. The actuator should be installed with the prismatic pair. The flat pair can be composed in different ways. Using this kind of limb, we can design some kinds of 3-dofs pure translational decoupled parallel manipulators.

![Fig. 1. The idea for limb which can be used to compose decoupled translational mechanisms](image)

Figure 2(a) illustrates the general one geometry of a decoupled 2-dofs spherical mechanism. The moving platform is anchored to the base by two legs. A leg consists of two revolute joints, \(R_1\) and \(R_2\), whose axes, \(z_1\) and \(z_2\), intersect at point \(o\) and connect to each other perpendicularly to form a universal joint; so the value of \(\alpha\) is \(\pi/2\). The other leg consists of a revolute joint, \(R_3\), a flat pair, \(F\), and a prismatic pair \(P\), in which the moving direction of \(P\) is perpendicular to the working plane of \(F\) and the axis of \(R_3\). The revolute joints \(R_2\) and \(R_3\) are mounted on the moving platform in parallel. The prismatic pair \(P\) and the revolute joint \(R_3\) are assembled to the base, in which the moving direction of \(P\) is parallel to the axis of \(R_1\).
Suppose that the input parameters, \( q_1 \) and \( q_2 \), represent the positions of the revolute joint \( R_1 \) and the prismatic pair \( P \), which are driven by a rotary actuator and a linear actuator separately. The pose of the moving platform is defined by the Euler angles \( \theta_1 \) and \( \theta_2 \) of the platform. When the value of \( q_1 \) changes and \( q_2 \) holds the line, only \( \theta_1 \) alters. On the other hand, when the value of \( q_2 \) changes, only \( \theta_2 \) changes. So, \( \theta_1 \) and \( \theta_2 \) are independently determined by \( q_1 \) and \( q_2 \) respectively, i.e., one output parameter only relates to one input parameter. In other words, the platform rotations around two axes are decoupled.

Figure 2(b) is an improved idea of figure 2(a). Using this idea, we can get a decoupled 2-dofs spherical mechanism with a hemi-sphere work space.

![Diagram of decoupled 2-dof spherical mechanisms](image)

**Fig. 2.** The idea for decoupled 2-dof spherical mechanisms

### 3. Design of 3-dofs translational manipulators with decoupled geometry

#### 3.1 Type design

The Type design of 3-dofs translational manipulators is based on the analysis of limb topology shown in figure 1.

![Flat pair and prismatic pair](image)

**Fig. 3.** The substitutes for the flat pair and the prismatic pair

Firstly, we construct deferent structures to replace the flat pair and the prismatic pair. Some substitutes for the flat pair and the prismatic pair are shown in figure 3. Then, using the pairs to form variational kinds of limbs. Figure 4 shows three examples. Finally, we can constitute the 3-dofs translational manipulators by installing the specified limbs in orthogonal as shown in figure 5, 6 and 7.
Fig. 4. The examples of limbs

Fig. 5. 3-PPP manipulator

Fig. 6. 3-7R manipulator
3.2 Kinematics

The forward and inverse kinematic analyses for the 3-PPP manipulator shown in figure 5 are trivial since there exists a one-to-one correspondence between the moving platform position and the input pair displacements. So the velocity jacobian matrix is a $3 \times 3$ identity matrix.

The kinematics of 3-7R manipulator can be analysed as follows. Referring to figure 6(b), each limb constrains point $P$ to lie on a plane which passes through points $M_{j2}, M_{j3},$ and $B_j$, and is perpendicular to the axis of $x, y,$ and $z$, respectively. The position of $j^{th}$ plane is determined only by $\theta_j$ whenever the length $l_{j1}$ is given. Consequently, the position of $P$ is determined by the intersection of three planes, i.e., the intersection of $\theta$ for $j=1,2,3$. If the distance from $M_{j1}$ to $M_{j2}$ is $m_{0j}$, then a simple kinematic relation can be written as

$$\begin{bmatrix}
  a_{01} + m_{01} + l_{j1} \sin \theta_j \\
  a_{02} + m_{02} + l_{j1} \sin \theta_j \\
  -a_{03} + m_{03} + l_{j1} \sin \theta_j
\end{bmatrix} + \begin{bmatrix}
  a_{01} \\
  a_{02} \\
  -a_{03}
\end{bmatrix} \theta_j = \begin{bmatrix}
  p_x \\
  p_y \\
  p_z
\end{bmatrix}$$

where $p=[p_x, p_y, p_z]^T$ denotes the position vector of the end-effector. Taking the time derivative of equation (1) yields

$$\begin{bmatrix}
  \dot{\theta}_1 \\
  \dot{\theta}_2 \\
  \dot{\theta}_3
\end{bmatrix} = J^{-1} \begin{bmatrix}
  \dot{p}_x \\
  \dot{p}_y \\
  \dot{p}_z
\end{bmatrix}$$

where $J$ is a diagonal matrix that holds
The kinematics of the modified 3-7R manipulator are the same.

### 3.3 Original position calibration

The calibration of 3-PPP manipulator is the same as a pure translational 3-dofs serial manipulator. So we just consider the manipulator of 3-7R and modified 3-7R, they can be expressed in the same way as shown in figure 8(a).

![Fig. 8. Original position calibration](image)

For convenience, we suppose,

1. The input $\theta_j$ ($j=1,2,3$) is within $[-\theta_{jm}, \theta_{jm}]$, where $\theta_{jm} > 0$, and $\theta_{jm}$ denotes the initial position of the $j^{th}$ limb;

2. In the initial position (see figure 8), the angle between the link $l_j$ ($j=1,2,3$) and the axis $u(u=x,y,z)$ is $\theta_{0j}$ ($j=1,2,3$).

Then the initial value $\theta_{jm}$ of $\theta_j$ can be determined as

$$\theta_{jm} = \frac{\pi}{2} - \theta_{0j} \quad (4)$$

So we can determine $\theta_{0j}$ first, then $\theta_{jm}$, the steps of the calibration can be as follows. From the initial position $\theta_{0j}$ of the arm in figure 8, rotate the driving arm twice in a specified angle $\theta_s$, which satisfies

$$2\theta_s + \theta_{0j} \leq \pi \quad (5)$$

During the process, record the two moving distances $l_0$ and $l$ of the platform in the direction of axis $u(u=x,y,z)$, they satisfy

$$\begin{align*}
J &= \begin{bmatrix}
l_{j1} \cos \theta_1 \\
l_{j1} \cos \theta_2 \\
l_{j1} \cos \theta_3 
\end{bmatrix} \\
\begin{bmatrix}
l_{j1} \cos \theta_1 - l_j \cos (\theta_{0j} + \theta_s) = l_0 \\
l_{j1} \cos \theta_2 - l_j \cos (\theta_{0j} + 2\theta_s) = l_0 + l 
\end{align*} \quad (6)
$$
expand $\cos(\theta_0 + \theta_1)$ and $\cos(\theta_0 + 2\theta_1)$ in equation (6). and eliminate $\sin \theta_0$, we get

$$\cos \theta_0 = \frac{l_0 + l - 2l_0 \cos \theta_1}{2l_1(l_1 - \cos \theta_1)} \quad (7)$$

If $0 \leq \pi / 3$ and let $\theta = \pi / 3$, then equation (7) yields

$$\cos \theta_0 = \frac{l}{l_1} \quad (8)$$

The geometric signification of the equation (8) is shown in figure 8(b), which is very sententious and convenient to industrial applications. $\theta_{m}$ can be get from equation (4).

### 3.4 Singularity

The 3-PPP manipulator has no singularity, so we just discuss the manipulator of 3-7R and modified 3-7R, they can be expressed in the same.

From equation (2) we can find out that the rotational actuator speed is nonlinear to the velocity of the end-effector. Moreover, if $\theta_j = \pm 90^\circ$, then $\det|J| = 0$, for any expected velocity of the end-effector, the rotational speed of the actuator will be infinite. When $\theta_j$ is not equal but close to $\pm 90^\circ$, then $\det|J| \to 0$, the required rotational speed of the actuator may be still too high to reach. So the value of the $\theta_j$ must be designed in an appropriate range whenever the speed limit of the end-effector is given.

Suppose the desired velocity of the end-effector is $v_e$, and the permissible rotational speed of the actuator is $n_r$, then the absolute maximum value of the $\theta_j$ for $j = 1, 2, 3$ can be obtained from equation (2), that is

$$\cos|\theta_j| = \frac{v_e}{l_j n_r} \quad (9)$$

Let

$$\theta_{m j} = \arccos \left( \frac{v_e}{l_j n_r} \right) \quad (10)$$

Then $\theta_j$ should satisfy

$$-\theta_{m j} \leq \theta_j \leq \theta_{m j} \quad (11)$$

Whenever the mechanism design satisfies equation (11), no singularity will exist.

### 4. Design of 2-dofs spherical manipulators with decoupled geometry

#### 4.1 Type Design

The Type design of 2-dofs spherical manipulators is based on the general idea shown in figure 2. Using the 3R and 4R pairs in figure 3 to replace the F and P pairs separately, a new
structure (2R&8R manipulator) for figure 2(a) is constructed as shown in figure 9. Similarly, figure 10 shows the improved configuration of figure 2(b), a 2R&PRR manipulator, but distinguishingly, additional modification is that a through hole is added to the center of the revolute joint $R_1$, so the prismatic pair $P$ can be set in the center of the hole and rotates with $R_1$. As a result, the workspace of $\theta_1$ can reach $2\pi$.

**4.2 Kinematics**

Firstly, the 2R&8R manipulator in figure 9 will be discussed. Let $e$ be the distance between the axes of $R_2$ and $R_3$, $m$ be the distance between the axes of $R_8$ and $R_{10}$ (or $R_7$ and $R_9$). Also, suppose that, when the moving platform is on the initial position, the axis of $R_1$ is perpendicular to the plane consisting of the axes of $R_2$ and $R_3$. Then the displacement relationships between input and output for the 2R&8R manipulator are:
\[
\begin{cases}
q_1 = \theta_1 \\
m \sin q_2 = e \sin \theta_2
\end{cases}
\] (12)

In the structure design, it is easy to set the length \(m\) of \(\overline{R_2R_3}\) and \(\overline{R_4R_{10}}\) equal to the distance \(e\) between the axes of \(R_2\) and \(R_3\) so as to get the one-to-one input-output mapping. Let \(m = e\), it follows that:

\[
q_1 = \theta_1 \\
q_2 = \theta_2
\] (13)

This implies that the direct linear one-to-one input-output correlation, so the velocity jacobian matrix becomes an identity one.

Now we discuss the the 2R&PRR manipulator shown in figure 10. Suppose that the input parameters, \(q_1\) and \(q_2\), represent the angular displacement of the revolute joint \(R_1\) and the distance between the axes of \(R_2\) and \(R_4\) separately. They are driven by a rotary actuator and a linear actuator. The pose of the moving platform is defined by the Euler angles \(\theta_1\) and \(\theta_2\) of the platform. Let \(e\) be the distance between the axes of \(R_2\) and \(R_3\), \(m\) be the distance between the axes of \(R_3\) and \(R_4\). Also suppose that, axis \(z_3\) is through the point \(o\) and always perpendicular to the plane of \(z_1z_2\) and moreover, define the value of \(\theta_2\) is zero whenever the axis of \(R_3\) is on the plane of \(z_1z_2\). Then the coordinates of \(R_4\) and \(R_3\) for the axes \(z_1\) and \(z_3\) are

\[
\begin{cases}
R_4(z_1,z_3) = R_4(q_2,0) \\
R_3(z_1,z_3) = R_3(e \cos \theta_2, e \sin \theta_2)
\end{cases}
\] (14)

The displacement relationship between input and output is:

\[
\begin{cases}
q_1 = \theta_1 \\
(q_2 - e \cos \theta_2)^2 + e^2 \sin^2 \theta_2 = m^2
\end{cases}
\] (15)

Taking the derivative of equation (15), it follows that

\[
\begin{bmatrix}
\dot{q}_1 \\
\dot{q}_2
\end{bmatrix} = J^{-1} \begin{bmatrix}
\dot{\theta}_1 \\
\dot{\theta}_2
\end{bmatrix}
\] (16)

Where,

\[
J = \begin{bmatrix}
1 & 0 \\
0 & e \cos \theta_2 - q_2
\end{bmatrix}
\] (17)

### 4.3 Singularity and workspace

The 2R&8R manipulator shown in figure 9 has two legs. The first leg (\(R_1\) to \(R_2\)) produces the Euler angle \(\theta_1\) of the platform by the input of \(q_1\); while the second one (\(R_{10}\) to \(R_3\)) produces \(\theta_2\) by \(q_2\). To illustrate the motional relationship, let us introduce a transition parameter \(z\) to equation (12), it follows that:
\[
\begin{align*}
q_1 &= \theta_1 \\
\sin m q_2 &= z = e \sin \theta_2
\end{align*}
\]  
(18)

where, \(z\) is the displacement of F-pair \((R_4 \text{ to } R_6)\) in the direction of \(z_1\).

From equation (18), it is seen that the Euler angle \(\theta_1\) is produced from the input of \(q_1\) directly by the first leg; while \(\theta_2\) is produced from \(q_2\) by the second leg through two transformations, which include (1) rotary to linear motion \(q_2 \Rightarrow z\) using \(m \cdot \sin q_2 = z\), and (2) linear to rotary motion \(z \Rightarrow \theta_2\) using \(z = e \cdot \sin \theta_2\). In the second transformation, there exists a limitation related to friction circle. Let \(\rho\) denote the radius of the friction circle of \(R_2\), which is determined by the product of the radius \(r\) of the revolute joint’s axis and the equivalent friction coefficient \(\mu\) as follows.

\[
\rho = \mu r
\]  
(19)

\[\begin{align*}
\gamma &= \angle z_1 R_3 R_2 , \text{ and decompose the force } F \text{ into two parts, the radial component } F_r \text{, and the tangent component } F_t \text{ (see figure 11). Then the force } F \text{ acts on } R_2 \text{ is equivalent to a force } Q \text{ and a torque } M, \text{ which can be calculated from the following equations.}
\end{align*}\]

\[
\begin{align*}
Q &= F_r = F \cdot \cos \gamma \\
M &= F_t \cdot e = F \cdot e \cdot \sin \gamma
\end{align*}
\]  
(20)

As a basic law in mechanics, the effect of a force \(Q\) and a torque \(M\) acting on a rigid body is equivalent to a force \(Q_b\) with an offset \(h\), which is shown in figure 12 and can be calculated as follows

\[
\begin{align*}
Q_b &= Q \\
h &= M / Q = e \cdot \tan \gamma
\end{align*}
\]  
(21)

where, \(h\) is the distance between the action lines of force \(Q_b\) and \(Q\).
Fig. 12. Force couple equivalent

There exist three instances for the different relationship between $h$ and $\rho$, which are (1) $h < \rho$, the revolute joint $R_2$ will never rotate regardless the value of $Q_h$; (2) $h > \rho$, revolute joint $R_2$ can rotate; and (3) $h = \rho$, the critical condition. In the critical condition of $h = \rho$, using equation (21), it follows that:

$$\gamma = \arctan\left(\frac{\rho}{e}\right)$$

Then the workspace of $\theta_2$ satisfies:

$$-(\pi/2 - \gamma) < \theta_2 < \pi/2 - \gamma$$

On the other hand, the angle $\theta_1$ produced by the first leg is limited only by the structure design of the F-pair and the base, so the workspace of $\theta_1$ can reach a designated area through proper design. Assume that the workspace of $\theta_1$ is from $-\pi/2$ to $\pi/2$, then the workspace of the spherical mechanism can be depicted by the reachable range of the point $P$ as shown in figure 13. The workspace is smaller than a hemisphere, so it would be limited in some applications.

When the mechanism is running, the direction of axis $z_1$ keeps unchanged, while the direction of axis $z_2$ alters according to $\theta_1$. So the workspace represented by spherical surface in figure 13 can be interpreted as follows: point $P$ draws latitude lines when only $\theta_1$ changes and draws longitude lines while only $\theta_2$ alters.

Fig. 13. The workspace denoted by the locus of point $P$
Now we examine the 2R&PRR manipulator in figure 10. The only limitation of this mechanism is caused by the friction circle of $R_2$. This limitation can be described by figure 14, from which we can see that the workspace of $\theta_2$ satisfies

$$\theta_{2\min} < \theta_2 < \theta_{2\max}$$  \hspace{1cm} (24)$$

Where $\theta_{2\min}$ and $\theta_{2\max}$ are the minimum and the maximum boundaries, which can be simply calculated based on figure 14 as follows

$$\theta_{2\min} = \arcsin \frac{m\rho}{e\sqrt{\rho^2 + (\sqrt{e^2 - \rho^2} + m)^2}} > 0$$ \hspace{1cm} (25)$$

$$\theta_{2\max} = \arctan \frac{m - \sqrt{e^2 - \rho^2}}{\rho} + \arctan \frac{\sqrt{e^2 - \rho^2}}{\rho} < 2\pi$$  \hspace{1cm} (26)$$

Fig. 14. Workspace of $\theta_2$ limited by friction circle of $R_2$

It means that the workspace of the mechanism can not reach a hemisphere. Clearly, this is not desirable.

In fact, because the workspace of $\theta_1$ is $[0, 2\pi]$, the mechanism workspace can reach a hemisphere only if the workspace of $\theta_2$ is chosen $[0, \pi/2]$ or $[\pi/2, \pi]$. So there exist two methods to get a hemisphere workspace.

Figure 15 shows the critical instances for both of them; each one uses the similar technique to offset the axis of $R_4$ from the axis $z_1$. Let $n$ denotes the axis offset of $R_4$ (or the length of $AR_4$), and $n_c$ is the special value of $n$ for the critical configurations as shown in figure 15, then $n$ should be chosen equation (27). Using this technique, a hemisphere work space can be obtained.

$$n > n_c = \frac{m\rho}{e}$$ \hspace{1cm} (27)$$
Fig. 15. Two methods to modify the boundaries of $\theta_2$: (a) $\theta_{2\text{min}} = 0$, (b) $\theta_{2\text{max}} = 2\pi$

Fig. 16. The improved mechanism for $\theta_2 \in [0, \pi / 2]$

The improved architectures are shown in figure 16 and figure 17, in which the workspace of $\theta_2$ includes the area of $[0, \pi/2]$ or $[\pi/2, \pi]$ separately.

A prototype model of the mechanism for the condition of $\theta_2 \in [0, \pi / 2]$ is designed. Figure 18 shows the outline picture of this model. In this design, one leg is actuated by a servo motor through a tooth belt; while the other leg is actuated by the other servo motor through
a ball screw, which converts the rotational movement into the translational one. Both motors are fixed on the base. Besides, another revolute joint \( R_5 \) is added to connect the prismatic pair with the nut of the ball screw.

Fig. 17. The improved mechanism for \( \theta_2 \in \left[ \pi/2, \pi \right] \)

Fig. 18. The prototype model for \( \theta_2 \in [0, \pi/2] \)
5. Conclusions

A general idea for type design of decoupled parallel manipulators with lower mobility is introduced. A unified frame for the type design is provided and divided into two independent aspects. Some kinds of decoupled parallel manipulators with 3-dofs pure translational and 2-dofs pure rotational movements are obtained.

6. Acknowledgement

The author gratefully acknowledges the financial support of the National Natural Science Foundation of China (No. 50475055).

7. References


Parallel manipulators are characterized as having closed-loop kinematic chains. Compared to serial manipulators, which have open-ended structure, parallel manipulators have many advantages in terms of accuracy, rigidity and ability to manipulate heavy loads. Therefore, they have been getting many attentions in astronomy to flight simulators and especially in machine-tool industries. The aim of this book is to provide an overview of the state-of-art, to present new ideas, original results and practical experiences in parallel manipulators. This book mainly introduces advanced kinematic and dynamic analysis methods and cutting edge control technologies for parallel manipulators. Even though this book only contains several samples of research activities on parallel manipulators, I believe this book can give an idea to the reader about what has been done in the field recently, and what kind of open problems are in this area.

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