

# A Linear Logic Based Approach to Timed Petri Nets

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## 1. Introduction

### 1.1 Relationship between Petri net and linear logic

*Petri nets* were first introduced by Petri in his seminal Ph.D. thesis, and both the theory and the applications of his model have flourished in concurrency theory (Reisig & Rozenberg, 1998a; Reisig & Rozenberg, 1998b).

The relationships between Petri nets and linear logics have been studied by many researchers (Engberg & Winskel, 1997; Farwer, 1999; Hirai, 2000; Hirai 1999; Ishihara & Hiraish, 2001; Kamide, 2004, Kamide, 2006; Kanovich, 1995; Kanovich 1994; Larchey-Wendling & Galmiche, 1998; Larchey-Wendling & Galmiche, 2000; Lilius, 1992; Martí -Oliet & Meseguer, 1991; Okada, 1998; Tanabe, 1997). A category theoretical investigation of such a relationship was given by Martí -Oliet and Meseguer (Martí -Oliet & Meseguer, 1991), purely syntactical approach using Horn linear logic was established by Kanovich (Kanovich, 1995; Kanovich 1994), a naive phase linear logic for a certain class of Petri nets was given by Okada (Okada, 1998), a linear logical view of object Petri nets were studied by Farwer (Farwer, 1999), and various Petri net interpretations of linear logic using quantale models were obtained by Ishihara and Hiraishi (Ishihara & Hiraish, 2001), Engberg and Winskel (Engberg & Winskel, 1997), Larchey-Wendling and Galmiche (Larchey-Wendling & Galmiche, 1998; Larchey-Wendling & Galmiche, 2000), and Lilius (Lilius, 1992).

Petri net interpretations using Kripke semantics for various fragments and extensions of intuitionistic linear logic were studied by Kamide (Kamide, 2004; Kamide, 2006c). In (Kamide, 2004), Petri net interpretations of various fragments of a spatio-temporal soft linear logic were discussed. In (Kamide, 2006c), Petri nets with inhibitor arcs, which were first introduced by Kosaraju (Kosaraju, 1973) to show the limitation of the usual Petri nets, were described using Kripke semantics for intuitionistic linear logic with strong negation. The approaches using Kripke semantics can obtain a very simple correspondence between Petri net and linear logic.

### 1.2 Relationship between timed Petri net and temporal linear logic

A number of formalizations of *timed Petri nets* (Bestuzheva and Rudnev, 1994; Wang, 1998) can be considered since time can be associated with tokens, transitions, arcs and places. In the existing linear logic based approaches including the present paper's one, time was associated to tokens (or markings). In fact, to express the fireability of transitions by

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multisets of tokens in Petri nets, it seems to be a natural extension to do it by multisets of timed tokens in timed Petri nets.

Temporal linear logic based methods for timed Petri nets were introduced and studied by Tanabe (Tanabe, 1997) and Hirai (Hirai, 1999; Hirai, 2000). In (Tanabe, 1997), a relationship between a timed Petri net and a temporal linear logic was discussed based on quantale models with the soundness theorem for this logic. In (Hirai, 1999; Hirai 2000), a reachability problem for a timed Petri net was solved syntactically by extending Kanovich's result (Kanovich, 1994) with an extended temporal intuitionistic linear logic.

In the present paper, a kind of temporal linear logic, called linear-time linear logic, is used to describe timed Petri nets with timed tokens. This logic is formalized using a natural "linear-time" formalism which is widely used in the standard linear-time temporal logic based on the classical logic rather than linear logics.

### 1.3 Linear-time temporal logic

*Linear-time temporal logic* (LTL) has been studied by many researchers, and also been used as a base logic for verifying and specifying concurrent systems (Clarke et al., 1999; Emerson, 1990; Kröger, 1977; Lichtenstein & Pnueli, 2000; Pnueli, 1977; Vardi, 2001; Vardi, 2007) because of the virtue of the "linear-time" formalism (Vardi, 2001). LTL is thus known as one of the most useful modal logics based on the classical logic. Sequent calculi for LTL and its neighbors have been introduced by extending the sequent calculus LK for the classical logic (Kawai, 1987; Baratella and Masini, 2004; Paech, 1988; Pliuškevičius, 1991; Szabo, 1980; Szalas, 1986). A sequent calculus  $LT_{\omega}$  for LTL was introduced by Kawai, and the cut-elimination and completeness theorems for this calculus were proved (Kawai, 1987). A 2-sequent calculus  $2S_{\omega}$  for LTL, which is a natural extension of the usual sequent calculus, was introduced by Baratella and Masini, and the cut-elimination and completeness theorems for this calculus were proved based on an analogy between LTL and Peano arithmetic with  $\omega$ -rule (Baratella and Masini, 2004). A direct equivalence between Kawai's  $LT_{\omega}$  and Baratella and Masini's  $2S_{\omega}$  was shown by Kamide introducing the functions that preserve cut-free proofs of these calculi (Kamide, 2006b). In the present paper, (intuitionistic) linear logic-based versions of  $LT_{\omega}$  and  $2S_{\omega}$  are considered.

### 1.4 Temporal linear logic

*Linear logic*, which was originally introduced by Girard (Girard, 1987), is known as a resource-aware refinement of the classical and intuitionistic logics, and useful for obtaining more appropriate specifications of concurrent systems (Okada, 1998; Troelstra, 1992). In order to handle both resource-sensitive and time-dependent properties of concurrent systems, combining linear logics with temporal operators has been desired, since the (classical) linear logic (as a basis for temporal logics) is more expressive and appropriate than the classical logic. For this purpose, *temporal linear logics* have been proposed by Hirai (Hirai, 2000), Tanabe (Tanabe, 1997), and Kanovich and Ito (Kanovich & Ito, 1998). Hirai's intuitionistic temporal linear logic (Hirai, 2000) is known as useful for describing a timed Petri net (Hirai, 1999) and a timed linear logic programming language (Tamura et al., 2000). Extensions of Hirai's logic were proposed by Kamide (Kamide, 2004; Kamide, 2006a) as certain *spatio-temporal linear logics* combined with the idea of handling spatiality in Kobayashi, Shimizu and Yonezawa's *modal (spatial) linear logic* (Kobayashi et al., 1999). Tanabe's temporal linear logic (Tanabe, 1997) is used as a base logic for timed Petri net

specifications. Kanovich and Ito's temporal linear logics (Kanovich & Ito, 1998) are a result of combining linear logic with linear-time temporal operators.

### 1.5 Linear-time linear logic

*Linear-time (temporal) linear logics* and their usefulness have already been presented by Kanovich and Ito (Kanovich & Ito, 1998). Classical and intuitionistic linear-time linear logics were introduced as cut-free sequent calculi, and the strong completeness theorems for these logics were shown using the algebraic structure of *time phase semantics*. Although in (Kanovich & Ito, 1998), the phase semantic methods for both classical and intuitionistic cases were intensively investigated, other semantic methods and their applications to concurrency theory for the intuitionistic case have yet to be studied sufficiently.

In this paper, an intuitionistic linear-time temporal linear logic, called also here linear-time linear logic, is introduced as cut-free sequent calculi based on the ideas of Kawai's  $LT_{\omega}$  (Kawai, 1987) and Baratella and Masini's  $2S_{\omega}$  (Baratella & Masini, 2004). It is shown that the logic based on these calculi derives intuitive linear-time, informational and Petri net interpretations using Kripke semantics with the completeness theorem. The Kripke semantics presented is introduced based on the existing Kripke semantics by Došen (Došen, 1988), Kamide (Kamide, 2003), Kobayashi, Shimizu and Yonezawa (Kobayashi et al., 1999), Hodas and Miller (Hodas & Miller, 1994), Ono and Komori (Ono & Komori, 1985), Urquhart (Urquhart, 1972) and Wansing (Wansing, 1993a; Wansing, 1993b).<sup>1</sup>

### 1.6 Organization of this paper

This paper is organized as follows.

In Section 2, the linear-time linear logic is introduced as two cut-free Gentzen-type sequent calculi LT and 2LT, and show their equivalence using the method posed in (Kamide, 2006b). The sequent calculi LT and 2LT are regarded as the linear logic based versions of Kawai's  $LT_{\omega}$  and Baratella and Masini's  $2S_{\omega}$ , respectively.

In Section 3, Kripke semantics with a natural timed Petri net interpretation is introduced for LT, and the completeness theorem w.r.t. the semantics is proved as the main result of this paper. The completeness theorem is the basis for obtaining a natural relationship between LT and a timed Petri net.

In Section 4, a timed Petri net with timed tokens is introduced as a structure, and the correspondence between this structure and Kripke frame for LT is observed. An illustrative example for verifying the reachability of timed Petri nets is also addressed based on LT.

In Section 5, this paper is concluded, and some remarks are given.

## 2. Linear-time linear logic

### 2.1 LT

Before the precise discussion, the language used in this paper is introduced. *Formulas* are constructed from propositional variables,  $\mathbf{1}$  (multiplicative constant),  $\rightarrow$  (implication),  $\wedge$  (conjunction),  $*$  (fusion),  $!$  (exponential), temporal operators X (next) and G (globally). Lower-case letters  $p, q, \dots$  are used for propositional variables, Greek lower-case letters  $\alpha, \beta,$

<sup>1</sup> For a historical overview of Kripke semantics for modal substructural logics, see. e.g. (Kamide, 2002).

... are used for formulas, and Greek capital letters  $\Gamma, \Delta, \dots$  are used for finite (possibly empty) multisets of formulas. For any  $\sharp \in \{!, X, G\}$ , an expression  $\sharp\Gamma$  is used to denote the multiset  $\{\sharp\gamma \mid \gamma \in \Gamma\}$ . The symbol  $\equiv$  is used to denote equality as sequences (or multisets) of symbols. The symbol  $\omega$  or  $N$  is used to represent the set of natural numbers. An expression

$X^i\alpha$  for any  $i \in \omega$  is used to denote  $\overbrace{XX \cdots X}^i \alpha$ , e.g.  $(X^0\alpha \equiv \alpha)$ ,  $(X^1\alpha \equiv X\alpha)$  and  $(X^{n+1}\alpha \equiv X^n X\alpha)$ . An expression  $\Gamma^*$  means  $\gamma_1 * \cdots * \gamma_n$  ( $0 < n$ ) if  $\Gamma \equiv \{\gamma_1, \dots, \gamma_n\}$  and means  $\emptyset$  if  $\Gamma \equiv \emptyset$ . An expression  $\Delta^*$  means  $\gamma_1 * \cdots * \gamma_n$  if  $\Delta \equiv \{\gamma_1, \dots, \gamma_n\}$  ( $0 < n$ ) and means 1 if  $\Delta$  is empty. Lower-case letters  $i, j$  and  $k$  are used to denote any natural numbers. A sequent is an expression of the form  $\Gamma \Rightarrow \gamma$  (the succedent of the sequent is not empty). It is assumed that the terminological conventions regarding sequents (e.g. antecedent, succedent etc.) are the usual ones. If a sequent  $S$  is provable in a sequent system  $L$ , then such a fact is denoted as  $L \vdash S$  or  $\vdash S$ . The parentheses for  $*$  is omitted since  $*$  is associative, i.e.  $\vdash \alpha * (\beta * \gamma) \Rightarrow (\alpha * \beta) * \gamma$  and  $\vdash (\alpha * \beta) * \gamma \Rightarrow \alpha * (\beta * \gamma)$  for any formulas  $\alpha, \beta$  and  $\gamma$ .

In the following, the linear-time linear logic LT is introduced as a sequent calculus. This is regarded as a linear logic version of Kawai's  $LT\omega$  (Kawai, 1987).

**Definition 1 (LT)** The initial sequents of LT are of the form:

$$X^i\alpha \Rightarrow X^i\alpha \quad \Rightarrow X^i\mathbf{1}.$$

The cut rule of LT is of the form:

$$\frac{\Gamma \Rightarrow X^i\alpha \quad X^i\alpha, \Delta \Rightarrow \gamma}{\Gamma, \Delta \Rightarrow \gamma} \text{ (cut)}.$$

The logical inference rules of LT are of the form:

$$\frac{\Gamma \Rightarrow \gamma}{X^i\mathbf{1}, \Gamma \Rightarrow \gamma} \text{ (1we)}$$

$$\frac{\Gamma \Rightarrow X^i\alpha \quad X^i\beta, \Delta \Rightarrow \gamma}{X^i(\alpha \rightarrow \beta), \Gamma, \Delta \Rightarrow \gamma} \text{ (\rightarrow left)} \quad \frac{X^i\alpha, \Gamma \Rightarrow X^i\beta}{\Gamma \Rightarrow X^i(\alpha \rightarrow \beta)} \text{ (\rightarrow right)}$$

$$\frac{X^i\alpha, \Gamma \Rightarrow \gamma}{X^i(\alpha \wedge \beta), \Gamma \Rightarrow \gamma} \text{ (\wedge left1)} \quad \frac{X^i\beta, \Gamma \Rightarrow \gamma}{X^i(\alpha \wedge \beta), \Gamma \Rightarrow \gamma} \text{ (\wedge left2)}$$

$$\frac{\Gamma \Rightarrow X^i\alpha \quad \Gamma \Rightarrow X^i\beta}{\Gamma \Rightarrow X^i(\alpha \wedge \beta)} \text{ (\wedge right)}$$

$$\frac{X^i\alpha, X^i\beta, \Gamma \Rightarrow \gamma}{X^i(\alpha * \beta), \Gamma \Rightarrow \gamma} \text{ (*left)} \quad \frac{\Gamma \Rightarrow X^i\alpha \quad \Delta \Rightarrow X^i\beta}{\Gamma, \Delta \Rightarrow X^i(\alpha * \beta)} \text{ (*right)}$$

$$\frac{X^i\alpha, \Gamma \Rightarrow \gamma}{X^i!\alpha, \Gamma \Rightarrow \gamma} \text{ (!left)} \quad \frac{X^i!\Gamma \Rightarrow X^k\alpha}{X^i!\Gamma \Rightarrow X^k!\alpha} \text{ (!right)}$$

$$\frac{X^i!\alpha, X^i!\alpha, \Gamma \Rightarrow \gamma}{X^i!\alpha, \Gamma \Rightarrow \gamma} \text{ (!co)} \quad \frac{\Gamma \Rightarrow \gamma}{X^i!\alpha, \Gamma \Rightarrow \gamma} \text{ (!we)}$$

$$\frac{X^{i+k}\alpha, \Gamma \Rightarrow \gamma}{X^iG\alpha, \Gamma \Rightarrow \gamma} \text{ (Gleft)} \quad \frac{\{\Gamma \Rightarrow X^{i+j}\alpha\}_{j \in \omega}}{\Gamma \Rightarrow X^iG\alpha} \text{ (Gright)}.$$

It is remarked that (Gright) has infinite premises. It is noted that the cases for  $i = k = 0$  in LT derive the usual inference rules for the intuitionistic linear logic.

Although a proof is not given in this paper, the following cut-elimination theorem can be proved by a phase semantic method (Kamide, 2007).

**Theorem 2** (Cut-elimination for LT) *The rule (cut) is admissible in cut-free LT.*

An expression  $\alpha \Leftrightarrow \beta$  means the sequents  $\alpha \Rightarrow \beta$  and  $\beta \Rightarrow \alpha$ . Then, the following sequents are provable in LT for any formulas  $\alpha, \beta$  and any  $i \in \omega$ :

$$\begin{aligned} X^i \mathbf{1} &\Leftrightarrow \mathbf{1}, \\ X^i(\alpha \circ \beta) &\Leftrightarrow X^i \alpha \circ X^i \beta \quad (\circ \in \{\rightarrow, \wedge, *\}), \\ X^i !\alpha &\Leftrightarrow !X^i \alpha, \\ !G\alpha &\Rightarrow G! \alpha, \\ G\alpha &\Rightarrow X\alpha, \\ G\alpha &\Rightarrow \alpha, \\ G\alpha &\Rightarrow GG\alpha, \\ G\alpha &\Rightarrow XG\alpha, \\ !\alpha, !G(\alpha \rightarrow X\alpha) &\Rightarrow !G\alpha. \end{aligned}$$

The last sequent above corresponds to the linear logic version of the temporal induction axiom:  $\alpha \rightarrow (G(\alpha \rightarrow X\alpha) \rightarrow G\alpha)$ , and an LT-proof of this sequent is as follows. .

$$\frac{\begin{array}{c} \vdots \\ \{ \alpha, !G(\alpha \rightarrow X\alpha) \Rightarrow X^k \alpha \}_{k \in \omega} \end{array} \text{ (Gright)}}{\frac{\frac{\alpha, !G(\alpha \rightarrow X\alpha) \Rightarrow G\alpha}{!\alpha, !G(\alpha \rightarrow X\alpha) \Rightarrow G\alpha} \text{ (!left)}}{!\alpha, !G(\alpha \rightarrow X\alpha) \Rightarrow !G\alpha} \text{ (!right)}}$$

where  $\vdash \alpha, !G(\alpha \rightarrow X\alpha) \Rightarrow X^k \alpha$  for any  $k \in \omega$  is shown by mathematical induction on  $k$  as follows. The base step, i.e.  $k = 0$ , is obvious using (!we). The induction step can be shown using (!co) as follows.

$$\frac{\begin{array}{c} \vdots \text{ ind.hyp.} \\ \alpha, !G(\alpha \rightarrow X\alpha) \Rightarrow X^k \alpha \quad X^{k+1} \alpha \Rightarrow X^{k+1} \alpha \end{array} \text{ (}\rightarrow\text{left)}}{\frac{\frac{\alpha, !G(\alpha \rightarrow X\alpha), X^k(\alpha \rightarrow X\alpha) \Rightarrow X^{k+1} \alpha}{\alpha, !G(\alpha \rightarrow X\alpha), G(\alpha \rightarrow X\alpha) \Rightarrow X^{k+1} \alpha} \text{ (Gleft)}}{\frac{\alpha, !G(\alpha \rightarrow X\alpha), !G(\alpha \rightarrow X\alpha) \Rightarrow X^{k+1} \alpha}{\alpha, !G(\alpha \rightarrow X\alpha) \Rightarrow X^{k+1} \alpha} \text{ (!left)}} \text{ (!co).}$$

## 2.2 2LT

A 2-sequent calculus 2LT for the linear-time linear logic is introduced below. This calculus is a linear logic version of Baratella and Masini's 2-sequent calculus 2S $\omega$  (Baratella & Masini, 2004). The language of 2LT and the notations used are almost the same as those of LT.

**Definition 3** *An expression  $\alpha^i$  ( $\alpha$  is a formula and  $i \in \omega$ ) is called an indexed formula. Let  $\gamma$  be an indexed formula and  $\Gamma$  be finite (possibly empty) multiset of indexed formulas. Then an expression  $\Gamma \Rightarrow^2 \gamma$  is called a 2-sequent.*

An expression  $\Gamma^i$  is used to denote the multiset of  $i$ -indexed formulas.

**Definition 4 (2LT)** The initial sequents of 2LT are of the form:

$$\alpha^i \Rightarrow^2 \alpha^i \quad \Rightarrow^2 \mathbf{1}^i.$$

The cut rule of 2LT is of the form:

$$\frac{\Gamma \Rightarrow^2 \alpha^i \quad \alpha^i, \Delta \Rightarrow^2 \gamma}{\Gamma, \Delta \Rightarrow^2 \gamma} \text{ (cut2)}.$$

The logical inference rules of 2LT are of the form:

$$\begin{array}{c} \frac{\Gamma \Rightarrow^2 \gamma}{\mathbf{1}^i, \Gamma \Rightarrow^2 \gamma} \text{ (1we2)} \\ \frac{\Gamma \Rightarrow^2 \alpha^i \quad \beta^i, \Delta \Rightarrow^2 \gamma}{(\alpha \rightarrow \beta)^i, \Gamma, \Delta \Rightarrow^2 \gamma} \text{ (\rightarrowleft2)} \quad \frac{\alpha^i, \Gamma \Rightarrow^2 \beta^i}{\Gamma \Rightarrow^2 (\alpha \rightarrow \beta)^i} \text{ (\rightarrowright2)} \\ \frac{\alpha^i, \Gamma \Rightarrow^2 \gamma}{(\alpha \wedge \beta)^i, \Gamma \Rightarrow^2 \gamma} \text{ (\wedgeleft12)} \quad \frac{\beta^i, \Gamma \Rightarrow^2 \gamma}{(\alpha \wedge \beta)^i, \Gamma \Rightarrow^2 \gamma} \text{ (\wedgeleft22)} \\ \frac{\Gamma \Rightarrow^2 \alpha^i \quad \Gamma \Rightarrow^2 \beta^i}{\Gamma \Rightarrow^2 (\alpha \wedge \beta)^i} \text{ (\wedgeright2)} \\ \frac{\alpha^i, \beta^i, \Gamma \Rightarrow^2 \gamma}{(\alpha * \beta)^i, \Gamma \Rightarrow^2 \gamma} \text{ (*left2)} \quad \frac{\Gamma \Rightarrow^2 \alpha^i \quad \Delta \Rightarrow^2 \beta^i}{\Gamma, \Delta \Rightarrow^2 (\alpha * \beta)^i} \text{ (*right2)} \\ \frac{\alpha^i, \Gamma \Rightarrow^2 \gamma}{(!\alpha)^i, \Gamma \Rightarrow^2 \gamma} \text{ (!left2)} \quad \frac{(!\Gamma)^i \Rightarrow^2 \alpha^k}{(!\Gamma)^i \Rightarrow^2 (!\alpha)^k} \text{ (!right2)} \\ \frac{(!\alpha)^i, (!\alpha)^i, \Gamma \Rightarrow^2 \gamma}{(!\alpha)^i, \Gamma \Rightarrow^2 \gamma} \text{ (!co2)} \quad \frac{\Gamma \Rightarrow^2 \gamma}{(!\alpha)^i, \Gamma \Rightarrow^2 \gamma} \text{ (!we2)} \\ \frac{\alpha^{i+k}, \Gamma \Rightarrow^2 \gamma}{(G\alpha)^i, \Gamma \Rightarrow^2 \gamma} \text{ (Gleft2)} \quad \frac{\{ \Gamma \Rightarrow \alpha^{i+j} \}_{j \in \omega}}{\Gamma \Rightarrow^2 (G\alpha)^i} \text{ (Gright2)} \\ \frac{\alpha^{i+1}, \Gamma \Rightarrow^2 \gamma}{(X\alpha)^i, \Gamma \Rightarrow^2 \gamma} \text{ (Xleft)} \quad \frac{\Gamma \Rightarrow^2 \alpha^{i+1}}{\Gamma \Rightarrow^2 (X\alpha)^i} \text{ (Xright)}.$$

An expression  $L \vdash \Gamma \Rightarrow^2 \gamma$  is used to denote the fact that  $\Gamma \Rightarrow^2 \gamma$  is provable in a 2-sequent calculus  $L$ .

**Definition 5** Let  $\mathcal{L}_1$  be the set of formulas of LT and  $\mathcal{L}_2$  be the set of indexed formulas of 2LT.

A function  $f$  from  $\mathcal{L}_1$  to  $\mathcal{L}_2$  is defined by  $f(X^i \alpha) := \alpha^i$  for any formula  $\alpha$ .

A function  $g$  from  $\mathcal{L}_2$  to  $\mathcal{L}_1$  is defined by  $g(\alpha^i) := X^i \alpha$  for any formula  $\alpha$ .

It is remark that  $fg(\alpha^i) = \alpha^i$  and  $gf(X^i \alpha) = X^i \alpha$  hold for any formula  $\alpha$ .

**Theorem 6 (Equivalence between LT and 2LT)** (1) for any 2-sequent  $\Gamma \Rightarrow^2 \gamma$ , if  $2LT \vdash \Gamma \Rightarrow^2 \gamma$ , then  $LT \vdash g(\Gamma) \Rightarrow g(\gamma)$ . (2) for any sequent  $\Gamma \Rightarrow \gamma$ , if  $LT - (\text{cut}) \vdash \Gamma \Rightarrow \gamma$ , then  $2LT - (\text{cut2}) \vdash \Gamma \Rightarrow^2 \gamma$ .

**Proof** We show only (1) by induction on a proof  $P$  of  $\Gamma \Rightarrow^2 \gamma$  in 2LT. We show only the following case.

Case (Xleft): The last inference of  $P$  is of the form:

$$\frac{\alpha^{i+1}, \Sigma \Rightarrow^2 \gamma}{(X\alpha)^i, \Sigma \Rightarrow^2 \gamma} \text{ (Xleft)}.$$

By the hypothesis of induction, we obtain  $\text{LT} \vdash g(\alpha^{i+1}), g(\Sigma) \Rightarrow g(\gamma)$ , and hence obtain  $\text{LT} \vdash g((X\alpha)^i), g(\Sigma) \Rightarrow g(\gamma)$  by  $g(\alpha^{i+1}) = X^{i+1}\alpha = X^i(X\alpha) = g((X\alpha)^i)$ . Q.E.D.

By Theorems 2 and 6, the following theorem is obtained.

**Theorem 7 (Cut-elimination for 2LT)** *The rule (cut2) is admissible in cut-free 2LT.*

**Proof** Suppose  $2\text{LT} \vdash \Gamma \Rightarrow^2 \gamma$  for a 2-sequent  $\Gamma \Rightarrow^2 \gamma$ . Then we have  $\text{LT} \vdash g(\Gamma) \Rightarrow g(\gamma)$  by Theorem 6 (1). By Theorem 2, we obtain  $\text{LT} - (\text{cut}) \vdash g(\Gamma) \Rightarrow g(\gamma)$ . We thus obtain  $2\text{LT} - (\text{cut2}) \vdash fg(\Gamma) \Rightarrow^2 fg(\gamma)$  by Theorem 6 (2). Therefore  $2\text{LT} - (\text{cut2}) \vdash \Gamma \Rightarrow^2 \gamma$ .

Conversely, by Theorem 7 and an appropriate modification of Theorem 6, a proof of Theorem 2 is also derived. Q.E.D.

### 3. Kripke semantics

#### 3.1 Kripke model and soundness

The following definition (except the existence of  $N$ ) of the Kripke frame is the same as that for the (fragment of) intuitionistic linear logic (Kamide, 2003).

**Definition 8** A Kripke frame for LT is a structure  $\langle M, N, \cdot, \dagger, \varepsilon, \geq \rangle$  satisfying the following conditions:

1.  $N$  is the set of natural numbers,
2.  $\langle M, \cdot, \varepsilon \rangle$  is a commutative monoid with the identity  $\varepsilon$ ,
3.  $\langle M, \geq \rangle$  is a pre-ordered set,
4.  $\dagger$  is a unary operation on  $M$  such that

$$\begin{aligned} \text{C0: } & \varepsilon \geq \dagger\varepsilon, \\ \text{C1: } & \dagger x \geq x \text{ for all } x \in M, \\ \text{C2: } & \dagger x \geq \dagger\dagger x \text{ for all } x \in M, \\ \text{C3: } & \dagger x \cdot \dagger y \geq \dagger(x \cdot y) \text{ for all } x, y \in M, \\ \text{C4: } & \dagger x \geq \dagger x \cdot \dagger x \text{ for all } x \in M, \\ \text{C5: } & \dagger y \cdot x \geq x \text{ for all } x, y \in M, \end{aligned}$$

5.  $\cdot$  is monotonic with respect to  $\geq$ , i.e.

$$\text{C6: } y \geq z \text{ implies } x \cdot y \geq x \cdot z \text{ for all } x, y, z \in M.$$

**Definition 9** A valuation  $\models$  on a Kripke frame  $\langle M, N, \cdot, \dagger, \varepsilon, \geq \rangle$  for LT is a mapping from the set of all propositional variables to the power set of  $M \times N$  and satisfying the following hereditary condition:  $(x, i) \in \models (p)$  and  $y \geq x$  imply  $(y, i) \in \models (p)$  for any propositional variable  $p$ , any  $i \in N$  and any  $x, y \in M$ . An expression  $(x, i) \models p$  will be used for  $(x, i) \in \models (p)$ . Each valuation  $\models$  can be extended to a mapping from the set of all formulas to the power set of  $M \times N$  by

1.  $(x, i) \models \mathbf{1}$  iff  $x \geq \varepsilon$ ,
2.  $(x, i) \models \alpha \rightarrow \beta$  iff  $(y, i) \models \alpha$  implies  $(x \cdot y, i) \models \beta$  for all  $y \in M$ ,
3.  $(x, i) \models \alpha \wedge \beta$  iff  $(x, i) \models \alpha$  and  $(x, i) \models \beta$ ,
4.  $(x, i) \models \alpha * \beta$  iff  $(y, i) \models \alpha$  and  $(z, i) \models \beta$  for some  $y, z \in M$  with  $x \geq y \cdot z$ ,
5.  $(x, i) \models !\alpha$  iff  $y \models \alpha$  for some  $(y, i) \in M$  with  $x \geq \dagger y$ ,

6.  $(x, i) \models X\alpha$  iff  $(x, i + 1) \models \alpha$ ,
7.  $(x, i) \models G\alpha$  iff  $(x, l) \models \alpha$  for all  $l \in N$  with  $l \geq i$ .

**Proposition 10** Let  $\models$  be a valuation on a Kripke frame  $\langle M, N, \cdot, \dagger, \varepsilon, \geq \rangle$  for LT. Then the following hereditary condition holds:  $(x, i) \models \alpha$  and  $y \geq x$  imply  $(y, i) \models \alpha$  for any formula  $\alpha$ , any  $i \in N$ , and any  $x, y \in M$ .

**Proof** By induction on the complexity of  $\alpha$ . Q.E.D.

**Definition 11** A Kripke model for LT is a structure  $\langle M, N, \cdot, \dagger, \varepsilon, \geq, \models \rangle$  such that

1.  $\langle M, N, \cdot, \dagger, \varepsilon, \geq \rangle$  is a Kripke frame for LT,
2.  $\models$  is a valuation on  $\langle M, N, \cdot, \dagger, \varepsilon, \geq \rangle$ .

A formula  $\alpha$  is true in a Kripke model  $\langle M, N, \cdot, \dagger, \varepsilon, \geq, \models \rangle$  for LT if  $(\varepsilon, 0) \models \alpha$ , and valid in a Kripke frame  $\langle M, N, \cdot, \dagger, \varepsilon, \geq \rangle$  for LT if it is true for any valuation  $\models$  on the Kripke frame. A sequent  $\alpha_1, \dots, \alpha_n \Rightarrow \beta$  (or  $\Rightarrow \beta$ ) is true in a Kripke model  $\langle M, N, \cdot, \dagger, \varepsilon, \geq, \models \rangle$  for LT if the formula  $\alpha_1 * \dots * \alpha_n \rightarrow \beta$  (or  $\beta$  respectively) is true in it, and valid in a Kripke frame for LT if the formula  $\alpha_1 * \dots * \alpha_n \rightarrow \beta$  (or  $\beta$  respectively) is valid in it.

The Kripke model  $\langle M, N, \cdot, \dagger, \varepsilon, \geq, \models \rangle$  defined has a natural informational interpretation due to Urquhart (Urquhart, 1972) and Wansing (Wansing, 1993a; Wansing, 1993b).  $M$  is a set of information pieces,  $\cdot$  is the addition of information pieces,  $\dagger$  is the infinite addition of information pieces, and  $\varepsilon$  is the empty piece of information. Then the forcing relation  $(x, i) \models \alpha$  can read as “the resource  $\alpha$  is obtained at the time  $i$  by using the information piece  $x$ .”

**Theorem 12 (Soundness)** Let  $C$  be a class of Kripke frames for LT,  $L := \{S \mid \text{LT} \vdash S\}$  and  $L(C) := \{S \mid S \text{ is valid in all frames of } C\}$ . Then  $L \subseteq L(C)$

**Proof** It is sufficient to prove the following: for any sequent  $S$ , if  $S$  is provable, then  $S$  is valid in any frame  $F := \langle M, N, \cdot, \dagger, \varepsilon, \geq \rangle \in C$ . This is proved by induction on a proof  $P$  of  $S$ . We distinguish the cases according to the last inference rules and initial sequents in  $P$ . Let  $\models$  be a valuation on  $F$ . In the following, we sometimes use implicitly the fact that  $\geq$  is a pre-order,  $\langle M, \cdot, \varepsilon \rangle$  is a commutative monoid with the identity  $\varepsilon$ ,  $\cdot$  is monotonic, and  $\models$  has the hereditary condition (Proposition 10). We show some cases.

Case (!left): It is shown that  $L(C)$  is closed under (!left), i.e. for any formula  $\alpha$  and any multiset  $\Gamma$  of formulas, if  $X^i\alpha, \Gamma \Rightarrow \gamma$  is valid in  $F$  then so is  $X^{i!}\alpha, \Gamma \Rightarrow \gamma$ . In the following, we consider only the case that  $\Gamma$  is nonempty (the empty case can be shown similarly). Suppose that (1)  $(\varepsilon, 0) \models X^i\alpha * \Gamma^* \rightarrow \gamma$  and (2)  $(x, 0) \models X^{i!}\alpha * \Gamma^*$  for any  $x \in M$ . We will show  $(x, 0) \models \gamma$ . By (2), there exist  $x_1, x_2 \in M$  such that (3)  $x \geq x_1 \cdot x_2$ , (4)  $(x_1, i) \models !\alpha$  and (5)  $(x_2, 0) \models \Gamma^*$ . By (4), there exists  $x'_1 \in M$  such that (6)  $x_1 \geq \dagger x'_1$  and (7)  $(x'_1, i) \models \alpha$ . By (6), the frame condition C1 and the transitivity of  $\geq$ , we have (8)  $x_1 \geq x'_1$ . Moreover, by (8) and the monotonicity of  $\cdot$ , we have (9)  $x_1 \cdot x_2 \geq x'_1 \cdot x_2$ . By (9), (3) and the transitivity of  $\geq$ , we have (10)  $x \geq x'_1 \cdot x_2$ . Thus, by (10), (7) and (5), we obtain the following: there exist  $x'_1, x_2 \in M$  such that  $x \geq x'_1 \cdot x_2$ ,  $(x'_1, 0) \models X^i\alpha$  and  $(x_2, 0) \models \Gamma^*$ . Hence, by (1) we have  $(x, 0) \models \gamma$ .

Case (!right): It is shown that  $L(C)$  is closed under (!right), i.e. for any formula  $\alpha$  and any multiset  $\Gamma$  of formulas, if  $X^i\Gamma \Rightarrow X^i\alpha$  is valid in  $F$  then so is  $X^{i!}\Gamma \Rightarrow X^{i!}\alpha$ .

We only show the case that  $\Gamma$  is nonempty (the empty case can easily be shown using the frame condition C0). Suppose (1)  $(x, 0) \models (X^{i!}\Gamma)^*$  ( $\Gamma \equiv \{\gamma_1, \dots, \gamma_n\}$  ( $0 < n$ )) for any

$x \in M$  and (2)  $(\varepsilon, 0) \models (X^i! \Gamma)^* \rightarrow X^i \alpha$ . We will show  $(x, 0) \models X^i \alpha$ . By (1), we have that there exist  $x_1, \dots, x_n \in M$  such that (3)  $x \geq x_1 \cdots x_n$ , and (4)  $(x_1, i) \models !\gamma_1, \dots, (x_n, i) \models !\gamma_n$ . Then, by (4), we have that for any  $j \in \{1, \dots, n\}$ , there exists  $x'_j \in M$  such that (5)  $x_j \geq \dagger x'_j$  and (6)  $(x'_j, i) \models \gamma_j$ . By (6), the frame condition C1 and the hereditary condition of  $\models$ , we obtain (7)  $(\dagger x'_j, i) \models \gamma_j$ . Thus we have that there exists  $\dagger x'_j \in M$  (because  $M$  is closed under  $\dagger$ , and there exists  $x'_j \in M$ ) such that  $\dagger x'_j \geq \dagger \dagger x'_j$  (by the frame condition C2) and  $(\dagger x'_j, i) \models \gamma_j$  (by (7)). This means that (8)  $(\dagger x'_j, i) \models !\gamma_j$ , i.e.  $(\dagger x'_j, 0) \models X^i !\gamma_j$  for any  $j \in \{1, \dots, n\}$ . Further we have (9)  $\dagger x'_1 \cdots \dagger x'_n \geq \dagger x'_1 \cdots \dagger x'_n$  since  $\geq$  is reflexive. Hence we have that there exist  $\dagger x_1, \dots, \dagger x_n \in M$  such that (8) and (9). This means  $(\dagger x'_1 \cdots \dagger x'_n, 0) \models X^i !\gamma_1 * \cdots * X^i !\gamma_n$ , i.e. (10)  $(\dagger x'_1 \cdots \dagger x'_n, 0) \models (X^i ! \Gamma)^*$ . By the hypothesis (2) and the fact (10), we have (11)  $(\dagger x'_1 \cdots \dagger x'_n, 0) \models X^i \alpha$ , i.e.  $(\dagger x'_1 \cdots \dagger x'_n, i) \models \alpha$ . By the facts (3), (5), the monotonicity of  $\cdot$  and the frame conditions C2, C3, we have (12)  $x \geq x_1 \cdots x_n \geq \dagger x'_1 \cdots \dagger x'_n \geq \dagger \dagger x'_1 \cdots \dagger \dagger x'_n \geq \dagger (\dagger x'_1 \cdots \dagger x'_n)$ .

Hence we obtain the following: there exist  $\dagger x'_1 \cdots \dagger x'_n \in M$  (because  $M$  is closed under  $\cdot$ ) such that  $x \geq \dagger (\dagger x'_1 \cdots \dagger x'_n)$  (by (12) and the transitivity of  $\geq$ ) and  $(\dagger x'_1 \cdots \dagger x'_n, i) \models \alpha$  (by (11)). This means  $(x, i) \models !\alpha$ , i.e.  $(x, 0) \models X^i !\alpha$ .

Case (!co): It is shown that  $L(C)$  is closed under (!co), i.e. for any formulas  $\alpha, \gamma$  and any multiset  $\Gamma$  of formulas, if  $X^i !\alpha, X^i !\alpha, \Gamma \Rightarrow \gamma$  is valid in  $F$  then so is  $X^i !\alpha, \Gamma \Rightarrow \gamma$ . In the following we consider only the case that  $\Gamma$  is nonempty (the empty case can be shown similarly). Suppose (1)  $(x, 0) \models X^i !\alpha * \Gamma^*$  for any  $x \in M$  and (2)  $(\varepsilon, 0) \models X^i !\alpha * X^i !\alpha * \Gamma^* \rightarrow \gamma$ . We will show  $(x, 0) \models \gamma$ . By (1), there exist  $x_1, x_2 \in M$  such that (3)  $x \geq x_1 \cdot x_2$ , (4)  $(x_1, i) \models !\alpha$  and (5)  $(x_2, 0) \models \Gamma^*$ . By (4), we have that there exists  $x'_1 \in M$  such that (6)  $x_1 \geq \dagger x'_1$  and (7)  $(x'_1, i) \models \alpha$ . By (3), (6) and the monotonicity of  $\cdot$ ; we have (8)  $x \geq x_1 \cdot x_2 \geq \dagger x'_1 \cdot x_2$ . On the other hand, we have that there exists  $\dagger x'_1 \in M$  such that  $\dagger x'_1 \geq \dagger \dagger x'_1$  (by the frame condition C4),  $(\dagger x'_1, i) \models !\alpha$  (because, by (7), the frame conditions C1, C2 the hereditary condition of  $\models$ , we have that there exists  $\dagger x'_1 \in M$  such that  $\dagger x'_1 \geq \dagger \dagger x'_1$  and  $(\dagger x'_1, i) \models \alpha$ ). This means (9)  $(\dagger x'_1, 0) \models X^i !\alpha * X^i !\alpha$ . Further we have that there exist  $\dagger x'_1, x_2 \in M$  such that  $x \geq \dagger x'_1 \cdot x_2$  (by (8) and the transitivity of  $\geq$ ),  $(\dagger x'_1, 0) \models X^i !\alpha * X^i !\alpha$  (by (9)) and  $(x_2, 0) \models X^i \Gamma^*$  (by (5)). This means (10)  $(x, 0) \models X^i !\alpha * X^i !\alpha * \Gamma^*$ . By the hypothesis (2) and the fact (10), we obtain  $(x, 0) \models \gamma$ .

Case (!we): It is shown that  $L(C)$  is closed under (!we), i.e. for any formulas  $\alpha, \gamma$  and any multiset  $\Gamma$  of formulas, if  $\Gamma \Rightarrow \gamma$  is valid in  $F$  then so is  $X^i !\alpha, \Gamma \Rightarrow \gamma$ . In the following we consider only the case that  $\Gamma$  is nonempty (the empty case can be shown similarly). Suppose (1)  $(x, 0) \models X^i !\alpha * \Gamma^*$  for any  $x \in M$  and (2)  $(\varepsilon, 0) \models \Gamma^* \rightarrow \gamma$ . We will show  $(x, 0) \models \gamma$ . By (1), we have that there exist  $x_1, x_2 \in M$  such that (3)  $x \geq x_1 \cdot x_2$ , (4)  $(x_1, i) \models !\alpha$  and (5)  $(x_2, 0) \models \Gamma^*$ . By (4), we have that there exists  $x'_1 \in M$  such that (6)  $x_1 \geq \dagger x'_1$  and  $(x'_1, i) \models \alpha$ . Then we obtain (7)  $x \geq x_2$  since we have  $x \geq x_1 \cdot x_2 \geq \dagger x'_1 \cdot x_2 \geq x_2$  by (3), (6), the monotonicity of  $\cdot$ , the transitivity of  $\geq$  and the frame condition C5. Hence, by (7), (5) and the hereditary condition of  $\models$ , we obtain (8)  $(x, 0) \models \Gamma^*$ . Thus we obtain  $(x, 0) \models \gamma$  by the hypothesis (2) and the fact (8).

Case (Gleft): It is shown that  $L(C)$  is closed under (Gleft), i.e. for any formulas  $\alpha$  and multiset  $\Gamma$  of formulas, if  $X^{i+k} \alpha, \Gamma \Rightarrow \gamma$  is valid in  $F$  then so is  $X^i G\alpha, \Gamma \Rightarrow \gamma$ . In the following, we consider only the case that  $\Gamma$  is nonempty (the empty case can be shown

similarly).

Suppose  $(\varepsilon, 0) \models X^{i+k}\alpha * \Gamma^* \rightarrow \gamma$ , i.e.,  $\forall y \in M [ \exists y_1, y_2 \in M ( y \geq y_1 \cdot y_2 \text{ and } (y_1, 0) \models X^{i+k}\alpha \text{ and } (y_2, 0) \models \Gamma^* ) \text{ implies } (y, 0) \models \gamma ]$ . We will show  $(\varepsilon, 0) \models X^i G\alpha * \Gamma^* \rightarrow \gamma$ , i.e.,  $\forall y \in M [ \exists y_1, y_2 \in M ( y \geq y_1 \cdot y_2 \text{ and } (y_1, 0) \models X^i G\alpha \text{ and } (y_2, 0) \models \Gamma^* ) \text{ implies } (y, 0) \models \gamma ]$ . It is thus enough to show that  $(y_1, 0) \models X^i G\alpha$  implies  $(y_1, 0) \models X^{i+k}\alpha$ . Suppose  $(y_1, 0) \models X^i G\alpha$ . Then  $(y_1, 0) \models X^i G\alpha$  iff  $(y_1, i) \models G\alpha$  iff  $\forall j \geq i [ (y_1, j) \models \alpha ]$  iff  $\forall l \in N [ (y_1, i+l) \models \alpha ]$ . Thus we obtain  $(y_1, 0) \models X^{i+k}\alpha$ .

Case (Gright): It is shown that  $L(C)$  is closed under (Gright), i.e. for any formula  $\alpha$  and multiset  $\Gamma$  of formulas, if  $\Gamma \Rightarrow X^{i+j}\alpha$  (for all  $j \in \omega$ ) are valid in  $F$  then so is  $\Gamma \Rightarrow X^i G\alpha$ . We consider here only the case that  $\Gamma$  is nonempty (the empty case can be shown similarly). Suppose  $(\varepsilon, 0) \models \Gamma^* \rightarrow X^{i+j}\alpha$  for all  $j \in \omega$ , i.e. for all  $j \in \omega, \forall y \in M [ (\varepsilon, 0) \models \Gamma^* \text{ implies } (y, 0) \models X^{i+j}\alpha ]$ . We will show  $(\varepsilon, 0) \models \Gamma^* \rightarrow X^i G\alpha$ , i.e.,  $\forall y \in M [ (\varepsilon, 0) \models \Gamma^* \text{ implies } (y, 0) \models X^i G\alpha ]$ . It is thus enough to show that  $(y, 0) \models X^{i+j}\alpha$  for all  $j \in \omega$  implies  $(y, 0) \models X^i G\alpha$ . Suppose  $(y, 0) \models X^{i+j}\alpha$  for all  $j \in \omega$ , i.e.  $(y, i+j) \models \alpha$  for all  $j \in \omega$ . Then we obtain  $(y, 0) \models X^i G\alpha$  as follows:  $\forall j \in \omega [ (y, i+j) \models \alpha ]$  iff  $\forall k \geq i [ (y, k) \models \alpha ]$  iff  $(y, i) \models G\alpha$  iff  $(y, 0) \models X^i G\alpha$ . Q.E.D.

### 3.2 Completeness

In order to prove the completeness theorem, constructing a canonical model is needed, and the resulting canonical model will be used to show the relationship between a timed Petri net and LT.

**Definition 13** A canonical model is a structure  $\langle M, N, \cdot, \dagger, \varepsilon, \geq, \models \rangle$  such that

1.  $N$  is the set of natural numbers,
2.  $M := \{ \Gamma \mid \Gamma \text{ is a finite multiset of formulas} \}$ ,
3. for any  $\Gamma, \Delta \in M$ ,  $\Gamma \cdot \Delta := \Gamma \cup \Delta$  (the multiset union), where  $\Gamma \cdot \Delta$  will also be denoted as  $\Gamma, \Delta$ ,
4. for any  $\Gamma \in M$ ,  $\dagger \Gamma := !\Gamma =$  the multiset  $\{ !\gamma \mid \gamma \in \Gamma \}$ ,
5.  $\varepsilon$  is the empty multiset,
6. for any  $\Gamma, \Delta \in M$ ,  $\Gamma \geq \Delta$  is defined by  $\vdash \Gamma \Rightarrow \Delta^*$  where  $\Delta^* \equiv \gamma_1 * \dots * \gamma_n$  if  $\Delta \equiv \{ \gamma_1, \dots, \gamma_n \}$  ( $0 < n$ ), and  $\Delta^* \equiv \mathbf{1}$  if  $\Delta$  is empty,
7. a valuation  $\models$  on  $\langle M, N, \cdot, \dagger, \varepsilon, \geq \rangle$  is a mapping from the set PROP of all propositional variables to the power set of  $M \times N$  defined by

$$(\Gamma, i) \in \models (p) \text{ iff } \vdash \Gamma \Rightarrow X^i p, \text{ for any } p \in \text{PROP}, \text{ any } \Gamma \in M \text{ and any } i \in N.$$

It can then be shown that  $\langle M, N, \cdot, \dagger, \varepsilon, \geq \rangle$  is a Kripke frame for LT. It is remarked that the condition C0 corresponds to  $\vdash \Rightarrow \mathbf{1}$  since  $\varepsilon > \dagger \varepsilon$  or  $\varepsilon > \varepsilon$  is defined by  $\{ \} \Rightarrow \{ \}^*$  where  $\{ \}$  is the empty multiset. It is also remarked that the sequent  $\Rightarrow$  is not true in this canonical model.  $\{ \} \geq \{ \}$  is interpreted as  $\vdash \Rightarrow \mathbf{1}$  (i.e.,  $\{ \} \Rightarrow \{ \}^*$ ) but  $\{ \} \Rightarrow \{ \}$  does not correspond to  $\Rightarrow \mathbf{1}$ . Also  $\Rightarrow$  is not true in any Kripke model, because  $\Rightarrow$  is interpreted as  $\{ \} \Rightarrow \{ \}^*$  (i.e.,  $\{ \} \Rightarrow \{ \}$ ).

Further it will be proved that  $\langle M, N, \cdot, \dagger, \varepsilon, \geq, \models \rangle$  is a Kripke model for LT. To show this fact is essentially to show the completeness theorem. To achieve the completeness theorem, the following lemma is needed.

**Lemma 14** Let  $\langle M, N, \cdot, \dagger, \varepsilon, \geq, \models \rangle$  be the canonical model defined in Definition 13. Then, for any formula  $\gamma$ , any  $\Gamma \in M$  and any  $i \in N$ ,

$$(\Gamma, i) \models \gamma \text{ if and only if } \text{LT} \vdash \Gamma \Rightarrow X^i \gamma.$$

**Proof** This lemma is proved by induction on the complexity of  $\gamma$ . We show some cases.

(Case  $\gamma \equiv p$ ): By the definition of  $\models$ .

(Case  $\gamma \equiv \mathbf{1}$ ): Suppose  $(\Gamma, i) \models \mathbf{1}$ . Then we have  $(\Gamma, i) \models \mathbf{1}$  iff  $\Gamma \geq \{\}$  iff  $\vdash \Gamma \Rightarrow \{\}^*$  iff

$\vdash \Gamma \Rightarrow \mathbf{1}$ . Thus we obtain  $\vdash \Gamma \Rightarrow X^i \mathbf{1}$ :

$$\frac{\frac{\Gamma \Rightarrow \mathbf{1} \quad \mathbf{1} \Rightarrow X^i \mathbf{1}}{\Gamma \Rightarrow X^i \mathbf{1}} \text{ (cut)}}{\Gamma \Rightarrow X^i \mathbf{1}} \text{ (1we)}$$

Conversely, suppose  $\vdash \Gamma \Rightarrow X^i \mathbf{1}$ . Then we have

$$\frac{\frac{\Gamma \Rightarrow X^i \mathbf{1} \quad X^i \mathbf{1} \Rightarrow \mathbf{1}}{\Gamma \Rightarrow \mathbf{1}} \text{ (cut)}}{\Gamma \Rightarrow \mathbf{1}} \text{ (1we)}$$

and hence  $(\Gamma, i) \models \mathbf{1}$

(Case  $\gamma \equiv \alpha_1 \rightarrow \alpha_2$ ): First we show that  $(\Gamma, i) \models \alpha_1 \rightarrow \alpha_2$  implies  $\vdash \Gamma \Rightarrow X^i \alpha_1 \rightarrow \alpha_2$ . Suppose  $(\Gamma, i) \models \alpha_1 \rightarrow \alpha_2$  i.e.  $(\Delta, i) \models \alpha_1$  implies  $(\Gamma \cup \Delta, i) \models \alpha_2$  for any  $\Delta \in M$ . We take  $\{X^i \alpha_1\}$  for  $\Delta$ . We have  $(\{X^i \alpha_1\}, i) \models \alpha_1$  by the induction hypothesis, and  $(\Gamma \cup \{X^i \alpha_1\}, i) \models \alpha_2$  by the hypothesis. Thus we have  $\vdash \Gamma, X^i \alpha_1 \Rightarrow X^i \alpha_2$  by the induction hypothesis, and hence  $\vdash \Gamma \Rightarrow X^i(\alpha_1 \rightarrow \alpha_2)$  by ( $\rightarrow$ right). Conversely, suppose  $\vdash \Gamma \Rightarrow X^i(\alpha_1 \rightarrow \alpha_2)$  and  $(\Delta, i) \models \alpha_1$  for any  $\Delta \in M$ . Then we have  $\vdash \Delta \Rightarrow X^i \alpha_1$  by the induction hypothesis. We obtain:

$$\frac{\frac{\frac{\Delta \Rightarrow X^i \alpha_1 \quad \Gamma \Rightarrow X^i(\alpha_1 \rightarrow \alpha_2)}{\Gamma, X^i \alpha_1 \Rightarrow X^i \alpha_2} \text{ (cut)}}{\Gamma, \Delta \Rightarrow X^i \alpha_2} \text{ (cut)}}{\Gamma, \Delta \Rightarrow X^i \alpha_2} \text{ (}\rightarrow\text{left)}$$

and hence  $(\Gamma \cup \Delta, i) \models \alpha_2$  by the induction hypothesis.

(Case  $\gamma \equiv \alpha_1 \wedge \alpha_2$ ): First we show that  $(\Gamma, i) \models \alpha_1 \wedge \alpha_2$  implies  $\vdash \Gamma \Rightarrow X^i(\alpha_1 \wedge \alpha_2)$  for any  $\Gamma \in M$ . Suppose  $(\Gamma, i) \models \alpha_1 \wedge \alpha_2$ . Then we have  $(\Gamma, i) \models \alpha_1$  and  $(\Gamma, i) \models \alpha_2$ . We obtain  $\vdash \Gamma \Rightarrow X^i \alpha_1$  and  $\vdash \Gamma \Rightarrow X^i \alpha_2$  by the hypothesis of induction. Thus we have  $\vdash \Gamma \Rightarrow X^i(\alpha_1 \wedge \alpha_2)$  by ( $\wedge$ right). Conversely, suppose  $\vdash \Gamma \Rightarrow X^i(\alpha_1 \wedge \alpha_2)$ . Then we have

$$\frac{\frac{\Gamma \Rightarrow X^i(\alpha_1 \wedge \alpha_2) \quad X^i \alpha_1 \Rightarrow X^i \alpha_1}{\Gamma \Rightarrow X^i \alpha_1} \text{ (cut)}}{\Gamma \Rightarrow X^i \alpha_1} \text{ (}\wedge\text{left1)}$$

$$\frac{\frac{\Gamma \Rightarrow X^i(\alpha_1 \wedge \alpha_2) \quad X^i \alpha_2 \Rightarrow X^i \alpha_2}{\Gamma \Rightarrow X^i \alpha_2} \text{ (cut)}}{\Gamma \Rightarrow X^i \alpha_2} \text{ (}\wedge\text{left2)}$$

Thus we have  $(\Gamma, i) \models \alpha_1$  and  $(\Gamma, i) \models \alpha_2$  by the induction hypothesis, and hence  $(\Gamma, i) \models \alpha_1 \wedge \alpha_2$ .

(Case  $\gamma \equiv !\beta$ ): First, we show that  $(\Gamma, i) \models !\beta$  implies  $\vdash \Gamma \Rightarrow X^i !\beta$  for any  $\Gamma \in M$ . Suppose  $(\Gamma, i) \models !\beta$ . Then there exists  $\Delta \in M$  such that  $\vdash \Gamma \Rightarrow (!\Delta)^*$  and  $(\Delta, i) \models \beta$ . By the hypothesis of induction, we obtain  $\vdash \Delta \Rightarrow X^i \beta$ . Thus we obtain  $\vdash \Gamma \Rightarrow X^i !\beta$ :

$$\frac{\begin{array}{c} \Delta \Rightarrow X^i \beta \\ \vdots \text{ (!left)} \\ !\Delta \Rightarrow X^i \beta \\ !\Delta \Rightarrow X^i !\beta \text{ (!right)} \\ \vdots \text{ (*left) or (!we)} \\ \Gamma \Rightarrow (!\Delta)^* \quad (!\Delta)^* \Rightarrow X^i !\beta \end{array}}{\Gamma \Rightarrow X^i !\beta} \text{ (cut).}$$

Conversely, suppose  $\vdash \Gamma \Rightarrow X^i !\beta$ . We will show  $(\Gamma, i) \models !\beta$ , i.e. there exists  $\Delta \in M$  such that  $\vdash \Gamma \Rightarrow (!\Delta)^*$  and  $(\Delta, i) \models \beta$ . We take  $\{X^i \beta\}$  for  $\Delta$ . Then we obtain  $(\{X^i \beta\}, i) \models \beta$  by the induction hypothesis. Using the hypothesis  $\vdash \Gamma \Rightarrow X^i !\beta$ , we obtain  $\vdash \Gamma \Rightarrow !X^i \beta$ :

$$\frac{\begin{array}{c} X^i \beta \Rightarrow X^i \beta \\ X^i !\beta \Rightarrow X^i \beta \text{ (!left)} \\ \Gamma \Rightarrow X^i !\beta \quad X^i !\beta \Rightarrow !X^i \beta \text{ (!right)} \end{array}}{\Gamma \Rightarrow !X^i \beta} \text{ (cut).}$$

Thus, we obtain  $\vdash \Gamma \Rightarrow (!\Delta)^*$ .

(Case  $\gamma \equiv X\alpha$ ):  $(\Gamma, i) \models X\alpha$  iff  $(\Gamma, i+1) \models \alpha$  iff  $\vdash \Gamma \Rightarrow X^{i+1}\alpha$  (by the induction hypothesis) iff  $\vdash \Gamma \Rightarrow X^i X\alpha$ .

(Case  $\gamma \equiv G\alpha$ ): Suppose  $(\Gamma, i) \models G\alpha$ . Then we have  $\forall l \geq i [(\Gamma, l) \models \alpha]$ , and hence  $\forall l \geq i [\vdash \Gamma \Rightarrow X^l \alpha]$  by the induction hypothesis. This means  $\forall k \in \omega [\vdash \Gamma \Rightarrow X^{i+k} \alpha]$ , and thus  $\vdash \Gamma \Rightarrow X^i G\alpha$  by (Gright). Conversely, suppose  $\vdash \Gamma \Rightarrow X^i G\alpha$ . Then we have:

$$\frac{\begin{array}{c} X^{i+k} \alpha \Rightarrow X^{i+k} \alpha \\ \Gamma \Rightarrow X^i G\alpha \quad X^i G\alpha \Rightarrow X^{i+k} \alpha \text{ (Gleft)} \end{array}}{\Gamma \Rightarrow X^{i+k} \alpha} \text{ (cut)}$$

for any  $k \in \omega$ , i.e.  $\forall l \geq i [\vdash \Gamma \Rightarrow X^l \alpha]$ . By the hypothesis of induction, we obtain  $\forall l \geq i [(\Gamma, l) \models \alpha]$  and hence  $(\Gamma, i) \models G\alpha$  Q.E.D.

**Lemma 15** *The canonical model  $\langle M, N, \cdot, \dagger, \varepsilon, \geq, \models \rangle$  defined in Definition 13 is a Kripke model for LT such that*

$$(\varepsilon, 0) \models \gamma \text{ if and only if } \text{LT} \vdash \Rightarrow \gamma$$

for any formula  $\gamma$ .

**Proof** The hereditary condition on  $\models$  is obvious. By taking 0 for  $i$  and taking  $\varepsilon$  for  $\Gamma$  in Lemma 14, the required fact is obtained. Q.E.D.

By using Lemma 15, the following theorem is obtained, because for any sequent  $\Delta \Rightarrow \delta$ , it can take the formula  $\Delta^* \rightarrow \delta$  such that  $\vdash \Delta \Rightarrow \delta$  iff  $\vdash \Delta^* \rightarrow \delta$ .

**Theorem 16** (Completeness) Let  $C$  be a class of Kripke frames for LT,  $L := \{S \mid \text{LT} \vdash S\}$  and  $L(C) := \{S \mid S \text{ is valid in all frames of } C\}$ . Then  $L(C) \subseteq L$ .

#### 4. Timed Petri net interpretation

The following definition of timed Petri net is roughly the same as that in (Tanabe, 1997).

**Definition 17 (Timed Petri net)** A timed Petri net is a structure  $\langle N, P, T, (\cdot)^\bullet, (\cdot)_\bullet \rangle$  such that

8.  $N$  is the set of natural numbers representing liner-time,
9.  $P$  is a set of places,
10.  $T$  is a set of transitions,
11.  $(\cdot)^\bullet$  and  $(\cdot)_\bullet$  are mappings from  $T$  to the set  $S$  of all multisets over  $P \times N$ .

For  $t \in T$ ,  $t^\bullet$  and  $t_\bullet$  are called the pre-multiset and the post-multiset of  $t$  respectively. Each element of  $S$  is called a timed marking.

In this definition,  $i \in N$  indicates the waiting time until the pending tokens which are usable in future become available in a place. Thus, an expression  $(\alpha, i) \in (P \times N)$ , which corresponds to the formula  $X^i \alpha$ , means "A token  $\alpha$  has pending time  $i$ , i.e.,  $\alpha$  will be active after  $i$  time units." In such an expression, a token  $(\alpha, 0)$  is called an *active token*, and a token  $(\alpha, i)$  with  $i \neq 0$  is called a *pending token*.

**Definition 18 (Reachability relation)** A firing relation  $[t]$  for  $t \in T$  on  $S$  is defined as follows: for any  $m_1, m_2 \in S$ ,

$$m_1 [t] m_2 \text{ iff } m_1 = m_3 + t^\bullet \text{ and } t_\bullet + m_3 = m_2 \text{ for some } m_3 \in S.$$

A reachability relation  $\gg$  on  $S$  is defined as follows: for any  $m, m' \in S$ ,

$$m \gg m' \text{ iff } m [t_1] m_1 [t_2] \dots [t_n] m_n = m' \text{ for some } t_1, \dots, t_n \in T, m_1, \dots, m_n \in S \text{ and } n \geq 0.$$

It is remarked that  $\gg$  is transitive and reflexive.

We sometimes have to add certain time passage functions and timing conditions to the definitions of timed Petri net, firing relation and reachability relation, in case-by-case. A time passage function  $\delta_i$ , which means the passage of time by  $i$  time units, is a function on  $S$  such that  $\delta_i(\{(\alpha_j, k)\}_{j \in I}) = \{(\alpha_j, k+i)\}_{j \in I}$ . A firing relation may be extended with respect to such time passage functions such that  $m [\delta_i] m'$  iff  $\delta_i(m) = m'$ . A timing condition TC is a binary relation on  $S$ . Then an extended reachability relation  $\gg$  on  $S$  may have, for example, the following conditions:

1.  $\forall m, m' \in S [\delta_i(m) = m' \text{ implies } m \gg m']$ ,
2.  $\forall m, m' \in S [(m, m') \in \text{TC implies } m \gg m']$ ,
3.  $\forall m, m', m'' \in S [m \gg m' \text{ implies } (m+m'' \gg m'+m'') \text{ and } \delta_i(m) \gg \delta_i(m')]$

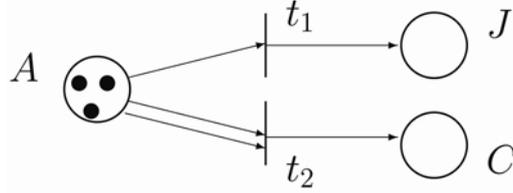
Following (Tanabe, 1997), we give an example of timed Petri nets.

**Example 19 (Apple drinks 1)** Suppose that we have just picked up three apples from an apple tree, and we can choose apple drinks between two options according to the following two rules.

(Rule 1): from an apple of less than one month old (i.e., less than a month has passed since picked from the tree), we can make a glass of apple juice.

(Rule2): from two apples of between 10 and 20 months old, we can make a glass of cider.

We then give a timed Petri net  $\langle N, P, T, (\cdot)^\bullet, (\cdot)_\bullet \rangle$  with two time passage functions  $\delta_1$  and  $\delta_{11}$ , and two timing conditions TC1 and TC2. Let  $P$  be  $\{A, J, C\}$  where  $A, J$  and  $C$  correspond to an apple, a glass of juice and a glass of cider, respectively. Let  $T$  be  $\{t_1, t_2\}$ ,  $t_1^\bullet$  be  $\{(A, i)\}$ ,  $t_{1\bullet}$  be  $\{(J, i)\}$ ,  $t_2^\bullet$  be  $\{(A, i), (A, i)\}$ , and  $t_{2\bullet}$  be  $\{(C, i)\}$ . Let TC1 be  $\{(A, x) \gg (J, 0) \mid (0 \leq x \leq 1)\}$ , and TC2 be  $\{(A, x_1), (A, x_2) \gg (C, 0) \mid (10 \leq x_i \leq 20, i = 1, 2)\}$ . It is remarked that TC1 and TC2 correspond to (Rule1) and (Rule2), respectively. Graphically this becomes the following:



In this net, “•” indicates a timed token  $(A, i)$ .

**Example 20 (Apple drinks 2)** In Example 19, we consider the situation (Tanabe, 1997) that starting from three apples, how can we get a glass of juice and a glass of cider? Going through the stage of getting drinks.

1. We have three fresh apples:  $\{(A, 0), (A, 0), (A, 0)\}$ .
2. One month has passed, i.e., all the apples has become one month old:
3.  $\{(A, 1), (A, 1), (A, 1)\}$ .
4. A glass of juice is made from an apple:  $\{(J, 0), (A, 1), (A, 1)\}$ .
5. More eleven months have passed:  $\{(J, 11), (A, 12), (A, 12)\}$ .
6. We finally have a glass of juice and a glass of cider:  $\{(J, 11), (C, 0)\}$ .

Then this situation is expressed as follows:

$$\begin{aligned} \{(A, 0), (A, 0), (A, 0)\} & \quad [\delta_1] \\ \{(A, 1), (A, 1), (A, 1)\} & \quad [t_1] \\ \{(J, 0), (A, 1), (A, 1)\} & \quad [\delta_{11}] \\ \{(J, 11), (A, 12), (A, 12)\} & \quad [t_2] \\ \{(J, 11), (C, 0)\} & \end{aligned}$$

Thus we can obtain:

$$\{(A, 0), (A, 0), (A, 0)\} \gg \{(J, 11), (C, 0)\}$$

In the next example, this will be verified using LT.

In order to compare timed Petri net and LT, the following definition is considered. It is assumed here that there is no time passage function or timing condition, since these are additional items in case-by-case.

**Definition 21 (Timed Petri net structure)** A timed Petri net structure is a structure  $\langle S, N, +, \emptyset, \gg \rangle$  such that

7.  $N$  is the set of natural numbers representing liner-time,
8.  $S$  is the set of all timed-markings,
9.  $+$  is a multiset union operation on  $S$ ,
10.  $\emptyset$  is the empty multiset,
11.  $\gg$  is a reachability relation on  $S$ .

It is remarked that a timed Petri net structure  $\langle S, N, +, \emptyset, \gg \rangle$  satisfies the following conditions:

1.  $\langle S, +, \emptyset \rangle$  is a commutative monoid,
2.  $\langle S, \gg \rangle$  is a pre-ordered set,
3.  $x_1 \gg x_2$  and  $y_1 \gg y_2$  imply  $x_1 + y_1 \gg x_2 + y_2$  for all  $x_1, x_2, y_1, y_2 \in S$ .

We then have the following basic proposition.

**Proposition 22 (Correspondence: Timed Petri net and Kripke frame)**

A timed Petri net structure  $\langle S, N, +, \emptyset, \gg \rangle$  is just a  $\dagger$ -free reduct of a Kripke frame for LT.

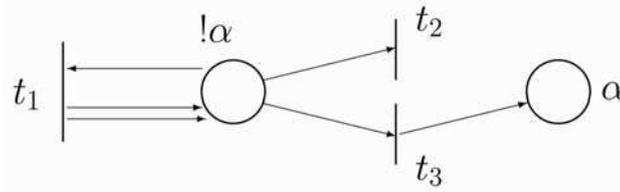
By this proposition and the canonical model defined in Definition 13, a timed Petri net interpretation for LT is obtained.

1. A timed token or place name,  $(\alpha, i)$  or  $(\alpha, 0)$ , corresponds to the formula  $X^i\alpha$  or  $\alpha$ .

2. The reachability of a timed Petri net corresponds to the provability of a sequent in LT, i.e.  $\Gamma \gg \Delta$  corresponds to  $\text{LT} \vdash \Gamma \Rightarrow \Delta^*$ .

Then we have the remained question: "What is the Petri net interpretation of the exponential operator?" The following example is an answer from the idea of Ishihara and Hiraishi (Ishihara & Hiraishi, 2001).

**Example 23 (Exponential operator)** We give a timed Petri net  $\langle N, P, T, (\cdot)^\bullet, (\cdot)_\bullet \rangle$  with  $P := \{!\alpha, \alpha\}$ ,  $T := \{t_1, t_2, t_3\}$ ,  $t_1^\bullet := \{(!\alpha, 0)\}$ ,  $t_{1\bullet} := \{(!\alpha, 0)\}$ ,  $t_2^\bullet := \{(!\alpha, 0)\}$ ,  $t_{2\bullet} := \{\}$ ,  $t_3^\bullet := \{(!\alpha, 0)\}$  and  $t_{3\bullet} := \{(\alpha, 0)\}$ , where all tokens are active tokens, i.e., tokens with  $i = 0 \in N$ . Graphically this becomes the following:



This net corresponds to the facts  $\vdash !\alpha \Rightarrow !\alpha * !\alpha$ ,  $\vdash !\alpha \Rightarrow \mathbf{1}$  and  $\vdash !\alpha \Rightarrow \alpha$ . In this net, the place  $\alpha$  (if it has a timed token) can produce a number of tokens in many-times (i.e. as many as needed).

We now show a LT based expression of the timed Petri net in the apple drink examples discussed before.

**Example 24 (Apple drinks 3)** We reconsider Examples 19 and 20 based on a sequent calculus expression for LT.

The time passage functions  $\delta_1$  and  $\delta_{11}$ , and the timing conditions TC1 and TC2 are expressed as initial sequents (non-logical axioms) for LT.

$\delta_j$  ( $j \in \{1, 11\}$ ):  $\vdash X^i \alpha, X^i \beta, X^i \gamma \Rightarrow X^{i+j} \alpha, X^{i+j} \beta, X^{i+j} \gamma$  for any formulas  $\alpha, \beta$  and  $\gamma$ .

TC1:  $\vdash X^i A \Rightarrow J$  ( $0 \leq i \leq 1$ ).

TC2:  $\vdash X^i A, X^i A \Rightarrow C$  ( $10 \leq i$  ( $j \leq 20$ )).

In the following, we verify  $A, A, A \Rightarrow X^{11} J * C$ , i.e.,  $\{(A, 0), (A, 0), (A, 0)\} \gg \{(J, 11), (C, 0)\}$ .

$$\frac{\frac{\frac{\vdots \text{TC1} \quad \frac{XA \Rightarrow XA \quad XA \Rightarrow XA}{XA \Rightarrow J \quad XA, XA \Rightarrow XA * XA}}{XA, XA, XA \Rightarrow J * XA * XA} \quad J * XA * XA \Rightarrow X^{11} J * C}{XA, XA, XA \Rightarrow X^{11} J * C}}{\frac{\vdots \delta_1}{A, A, A \Rightarrow XA * XA * XA} \quad \frac{XA, XA, XA \Rightarrow X^{11} J * C}{XA * XA * XA \Rightarrow X^{11} J * C}}{A, A, A \Rightarrow X^{11} J * C}$$

where  $J * XA * XA \Rightarrow X^{11} J * C$  is proved by

$$\frac{\frac{\frac{\vdots \delta_{11}}{J, XA, XA \Rightarrow X^{11} J * X^{12} A * X^{12} A} \quad \frac{\vdots \text{TC2} \quad \frac{X^{11} J \Rightarrow X^{11} J \quad X^{12} A * X^{12} A \Rightarrow C}{X^{11} J, X^{12} A * X^{12} A \Rightarrow C}}{X^{11} J * X^{12} A * X^{12} A \Rightarrow X^{11} J * C}}{J, XA, XA \Rightarrow X^{11} J * C}}{J * XA * XA \Rightarrow X^{11} J * C}$$

## 5. Concluding remarks

In this paper, a new logic, called linear-time linear logic, was introduced as two equivalent cut-free sequent calculi LT and 2LT, which are the linear logic versions of Kawai's  $LT_{\omega}$  and Baretella and Masini's  $2S_{\omega}$  for the standard linear-time temporal logic. The completeness theorem w.r.t. the Kripke semantics with a natural timed Petri net interpretation was proved for LT as the main result of this paper. By using this theorem, a relationship between LT and a timed Petri net with timed tokens was clarified, and the reachability of such a Petri net was transformed into the provability of LT and also 2LT. This means that the timed Petri net can naturally be expressed as the proof-theoretic framework by LT.

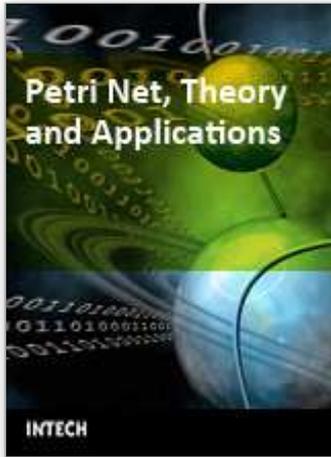
In the following, some technical remarks are given. The Kripke semantics presented is similar to the Kripke semantics (or resource algebras) with location interpretations by Kobayashi, Shimizu and Yonezawa (Kobayashi et al., 1999) and Kamide (Kamide, 2004). The sequent calculi and Kripke semantics for LT can also be adapted to Lafont's (intuitionistic) soft linear logic (Lafont, 2004) by using the framework presented in (Kamide, 2004). The framework posed in this paper can be extended to a rich framework with the first-order universal quantifier  $\forall$ , based on the technique posed in (Kamide, 2004). It is known in (Lilius, 1992) that the linear logic framework with the first-order quantifiers correspond to a high-level Petri net framework.

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Although many other models of concurrent and distributed systems have been developed since the introduction in 1964 Petri nets are still an essential model for concurrent systems with respect to both the theory and the applications. The main attraction of Petri nets is the way in which the basic aspects of concurrent systems are captured both conceptually and mathematically. The intuitively appealing graphical notation makes Petri nets the model of choice in many applications. The natural way in which Petri nets allow one to formally capture many of the basic notions and issues of concurrent systems has contributed greatly to the development of a rich theory of concurrent systems based on Petri nets. This book brings together reputable researchers from all over the world in order to provide a comprehensive coverage of advanced and modern topics not yet reflected by other books. The book consists of 23 chapters written by 53 authors from 12 different countries.

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