A Real Options Approach to Valuing the Risk Transfer in a Multi-Year Procurement Contract

Scot A. Arnold and Marius S. Vassiliou
The Institute for Defense Analyses
The United States of America

1. Introduction

The purpose of this paper is to develop methods to estimate the option value inherent in a multi-year government procurement (MYP), in comparison to a series of single-year procurements (SYP). This value accrues to the contractor, primarily in the form of increased revenue stability. In order to estimate the value, we apply real options techniques.

The United States government normally procures weapons systems in single annual lots, or single year procurements (SYP). These procurements are usually funded through a Congressional Act (the annual National Defense Authorization Act or NDAA) one fiscal year at a time. This gives Congress a great deal of flexibility towards balancing long and short term demands. For defense contractors, however, the Government’s flexibility results in unique difficulties forecasting future sales when demand is driven by both customer needs and global politics.

Defense contractors face risks and advantages that set them apart from commercial businesses. Within a contract, the contractor faces a range of execution cost risk: from none in a cost plus fixed fee contract to high risk in a firm fixed price contract. The government also provides interest-free financing that can greatly reduce the amount of capital a contractor must raise through the capital markets. Additionally the government provides direct investment and profit incentives to contractors to invest in fixed assets. The net effect is that defense contractors can turn profit margins that may appear low when compared to other commercial capital goods sectors, into relatively high return on invested capital.

However, contractors have always faced high inter-contract uncertainty related to the short term funding horizon of the government. While the United States Department of Defense (DoD) has a multiyear business plan, in any given year, generating a budget entails delaying acquisition plans to accommodate changing demands and new information. At the end of the cold war, defense firms were allowed unprecedented freedom to consolidate. The resulting industrial base is composed of five surviving government contractors: Boeing, General Dynamics, Lockheed, Northrop Grumman, and Raytheon. By diversifying across a large number of government customers, these giants with thousands of contracts each have taken a giant step towards reducing inter-contract risk—no one contract is large enough to...

1 E.g., Amram & Howe (2003)


www.intechopen.com
seriously harm the companies if it were canceled for convenience. However, the uncertainty around the likelihood of getting the next contract or how large it will be is still there and it is particularly important for large acquisition programs. For example, while Lockheed is the sole source for the F-22A, they always faced uncertainty in the number of units they will sell in the future. For example both the F-22A and the B-2 were originally expected to sell many more airplanes to the government than the actual number the government eventually purchased.

Under Title 10 Subtitle A Part IV Chapter 137 § 2306b, the military services can enter into multi-year procurement (MYP) contracts upon Congressional approval. There are six criteria that must be satisfied, listed in Table 1. The chief benefit for the government has been the “price break”, criterion 1, afforded through the operating efficiencies of a long term contract. This benefit is readily passed to the government because it funds the necessary working capital investments needed to optimize production. It is still possible for the government to cancel the MYP contract; however, significant financial barriers such as a cancellation or termination liability that make it undesirable to do so.

<table>
<thead>
<tr>
<th>Criteria Descriptions</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 That the use of such a contract will result in substantial savings of the total anticipated costs of carrying out the program through annual contracts.</td>
</tr>
<tr>
<td>2 That the minimum need for the property to be purchased is expected to remain substantially unchanged during the contemplated contract period in terms of production rate, procurement rate, and total quantities.</td>
</tr>
<tr>
<td>3 That there is a reasonable expectation that throughout the contemplated contract period the head of the agency will request funding for the contract at the level required to avoid contract cancellation.</td>
</tr>
<tr>
<td>4 That there is a stable design for the property to be acquired and that the technical risks associated with such property are not excessive.</td>
</tr>
<tr>
<td>5 That the estimates of both the cost of the contract and the anticipated cost avoidance through the use of a multiyear contract are realistic.</td>
</tr>
<tr>
<td>6 In the case of a purchase by the Department of Defense, that the use of such a contract will promote the national security of the United States.</td>
</tr>
</tbody>
</table>

Table 1. The Six Criteria for a Multi-Year Procurement

The government reaps operational savings by negotiating a lower up-front procurement price. These savings are achieved through more efficient production lot sizes and other efficiencies afforded through better long-term planning not possible with SYP contracts. The government can explicitly encourage additional savings by using a cost sharing contract. It can implicitly encourage additional savings with a fixed price contract. In the latter case the longer contract encourages the contractor to seek further efficiencies since it does not share the savings with the government. In fact some might propose this last reason is the best reason for a contractor to seek an MYP.

In addition to the cost savings achieved through more stable production planning horizon, we see that the MYP provides the contractor with intrinsic value through the stabilization of its medium term revenue outlook. Thus an MYP is also coveted by defense contractors because it provides lower revenue risk. What about the possibility that a longer term firm

---

2 United States Code, Title 10, Subtitle A, Part IV, Chapter 137, Section 2306b
fixed price contract exposes the contractor to higher cost risk? This risk is often eliminated through economic pricing adjustment (EPA) clauses that provide a hedge against unanticipated labor and material inflation. Furthermore, from the criteria in Table 1, MYP contracts are only allowed for programs with stable designs that have low technical risk. As stated above, it is more likely that the MYP offers the contractor the opportunity to exploit the principle-agent information asymmetry and make further production innovations unanticipated at contract signing.

We believe that the lower risk MYP contract will allow investors to discount contractors’ cash flow with a lower cost of capital creating higher equity valuations. From the contractors’ perspective, the MYP contract provides a hedge against revenue risk. We can estimate the incremental value of the MYP versus the equivalent SYP sequence using option pricing methods. Presently the government does not explicitly recognize this risk transfer in its contracting profit policy. The government profit policy is to steadily increase the contract margin as cost risk is transferred to the contractor. For example a cost plus fixed fee contract might have a profit margin of 7% while a fixed price contract, where the contractor is fully exposed to the cost-risk, of similar content could have a margin of 12%.

By limiting some of the contractor’s cost-risk exposure, an EPA clause might result in a lower profit margin; however, the profit policy makes no mention of an MYP contract, which reduces the contractor’s inter-contract risk. And while most of the profit policy is oriented towards compensating the contractor for exposing its capital to intra-contract risk and entrepreneurial effort, there are provisions designed to provide some compensation for exposing capital to inter-contract risk—e.g. the facilities capital markup. The implication is that as long as the government does not explicitly price the reduction in cost-risk going from a fixed price SYP contact to an MYP contract, the contractor is able to keep the “extra” profit. In this paper we present a method to estimate the value an MYP creates for a defense contractor in its improved revenue stability. The contractor can use this information in two ways. First, the information provides guidance for how much pricing slack the contractor can afford as it negotiates an MYP with the government whether or not the latter recognizes that better revenue stability has discernable value. Second, if the government tries to reduce the contractor’s price based on this transfer of risk, the contractor has a quantitative tool to guide its negotiation with the government.

2. Financial structure and valuation of an MYP

In this paper, we will present how to estimate the value imbedded in the risk transfer from the contractor to the government in an MYP contract using real options analysis. Table 2 lists recent MYP contracts. Note that while the table mostly shows aircraft the contract type can be applied to other acquisitions. Since FY2000, MYP contracts have declined from about 18 percent of defense procurement to about 10 percent; however, over this period they have totaled to about $10 billion per year. These contracts are 3 to 5 times larger than SYP contracts and can represent an important portion of the contractor’s revenue.

---


4 Generally the project with a cost plus contract has higher technical uncertainty than the project with the fixed price contract. The government does not expect contractors to accept high technical risk projects using a fixed price contract.
As an acquisition programmatures, the contractor implicitly receives an option on an MYP that is not executable until authorized by the Congress and negotiated by the relevant military service. If conditions are met and the option is exercised, the contractor transfers the SYP revenue risk to the government, which commits to buying the predetermined number of units. There are two financial instruments that approximate this transaction: a put and a cash flow swap or exchange option. Both structures provide the protection buyer, i.e. the contractor, insurance against losses in the underlying asset, i.e. the net present value of the cash flow derived from the sales. For the duration of the MYP contract, the contractor receives predictable revenue while the government forgoes the flexibility to defer or cancel the procurement by agreeing to pay substantial cost penalties for canceling the MYP contract. To value the MYP, we will employ the exchange option of Margrabe. From this analysis the government will be able to estimate the contractor’s value of transferring revenue risk to the government as a function of the size of the contract and the volatility of the contract’s value. Since the option is not actively traded, the ultimate negotiated price could be heavily influenced by the government and contractor attitudes towards risk.

3. Real options

A put option is a common financial contract that gives the owner the right to sell an asset, such as a company’s stock, for a pre-determined price on or before a predetermined date. Non-financial contingent pay-offs that behave like financial options, but are not traded as separate securities are called real options. Real options provide the holder of the asset similar risk management flexibility though they are not yet sold separately from the underlying asset. For example, oil drilling rights give the holder the option, but do not require, exploring, drilling, or

<table>
<thead>
<tr>
<th>Program</th>
<th>Period</th>
<th>Amount ($ Billions)</th>
<th>Type of System</th>
</tr>
</thead>
<tbody>
<tr>
<td>Virginia Class</td>
<td>2009-2013</td>
<td>$14.0</td>
<td>Submarine</td>
</tr>
<tr>
<td>CH-47F</td>
<td>2008-2013</td>
<td>4.3</td>
<td>Aircraft</td>
</tr>
<tr>
<td>V-22</td>
<td>2007-2012</td>
<td>10.1</td>
<td>Aircraft</td>
</tr>
<tr>
<td>F-22A</td>
<td>2007-2010</td>
<td>8.7</td>
<td>Aircraft</td>
</tr>
<tr>
<td>F-18 E/F</td>
<td>2005-2009</td>
<td>8.8</td>
<td>Aircraft</td>
</tr>
<tr>
<td>DDG-51</td>
<td>2002-2005</td>
<td>5.0</td>
<td>Ship</td>
</tr>
<tr>
<td>AH-1 Apache</td>
<td>2001-2005</td>
<td>1.6</td>
<td>Aircraft</td>
</tr>
<tr>
<td>C-17A</td>
<td>1997-2003</td>
<td>14.4</td>
<td>Aircraft</td>
</tr>
</tbody>
</table>

Table 2. Recent Major Multi-Year Procurement Contracts

---

5 Internal publication from Northrop Grumman, “Navy Awards $14 Billion Contract for Eight Virginia Class Submarines”, Currents, January 5-9, 2009


9 Second of two multi-year contracts.

10 Margrabe, W., Journal of Finance, 33, 177-86 (1978)
marketing the oil to customers. Patents are another example that can be viewed the same way: the holder of the patent has the option but is not obliged to deploy the technology. Usually these investment flexibilities come into play as contingent pay-offs: they allow the investor to delay committing cash until positive pay-off is better assured. Real options capture the capability of investors or managers to make valuable decisions in the future.

More generally, real options analysis captures some of the value of management’s capability to make dynamic programmatic changes, based on new and better information, within the levers and construct of a given business project. The real-options approach explicitly captures the value of management’s ability to limit downside risk by stopping poorly performing programs. It also captures the value inherent in the possibility that management will exploit unexpected successes.

An MYP contract contains a real option allowing the contractor a choice to abandon the uncertainty associated with relying on sequential SYP contracts to implement the government’s acquisition strategy for a weapon system. For example an aircraft manufacturer who is the single source for an air vehicle, such as the F-16 or F/A-18, has the exclusive option to negotiate an MYP contract to sell the next four lots to the Air Force or Navy. Given that most weapons acquisition programs buy fewer units than planned, the contractor will exercise the option by entering into an MYP contract.

The contractor implicitly owns the MYP option as the sole source for the procurement. Unlike a financial option which the buyer can choose from a selection of the strike prices and tenors, an MYP option does not explicitly exist until the government and contractors negotiate the terms of the contract. In negotiating the terms of the MYP, the contractor and government are negotiating the option’s strike price—and up to that point it appears as though the contractor received the option for free. Once negotiated it is usually executed which is like exercising an at-the-money put option. We will define the option parameters below, recognizing that they may not be explicitly defined until the option is exercised.

There are a number of techniques that may be used to value a real option. One way is to adapt the framework developed by Black and Scholes\textsuperscript{11} (BS) for financial options. Real-options investments are not often framed as neatly as puts and calls on corporate equities traded on the Chicago Board Options Exchange. However, if we can describe the real options embedded in an MYP contract along the lines of the appropriate standard options framework, we can try to employ the BS option pricing framework. Other alternatives include the binomial method\textsuperscript{12}, dynamic programming, simulation, and other numerical methods to name a few.

4. Are real options really used by managers?

Real options have been a topic of vigorous academic research for decades. The published literature abounds with theoretical papers, and with applications to a wide variety of domains. These domains include, for example: the aerospace\textsuperscript{13,14}, telecommunications\textsuperscript{15},

\begin{itemize}
  \item \textsuperscript{11}Black & Scholes (1973)
  \item \textsuperscript{12}E.g., Copeland & Tufano (2004)
  \item \textsuperscript{13}Richard L. Shockley, \textit{J. of Applied Corporate Finance}, 19(2), Spring 2007
  \item \textsuperscript{14}Scott Matthews, Vinay Datar, and Blake Johnson, \textit{J. of Applied Corporate Finance}, 19 (2), Spring 2007
  \item \textsuperscript{15}Charnes et al. (2004)
\end{itemize}
oil\textsuperscript{16}, mining\textsuperscript{17}, electronics\textsuperscript{18}, and biotechnology\textsuperscript{19} industries; the valuation of new plants and construction projects\textsuperscript{20}; real estate\textsuperscript{21}; the analysis of outsourcing\textsuperscript{22}; patent valuation\textsuperscript{23}; the analysis of standards\textsuperscript{24}; and the valuation of R&D and risky technology projects\textsuperscript{25}.

There is some evidence that real-options thinking has permeated the real world in some niches. The technique does appear to be used seriously in the oil industry, for example,\textsuperscript{26} to analyze new ventures. Perhaps one reason is that it is easier to track the value of the underlying asset in that industry than in others. Reportedly, real options analysis has been used at Genentech in all drug development projects since 1995, and Intel has used it to value plant expansion\textsuperscript{27}. Hewlett-Packard reportedly uses a set of risk management tools, including real options analysis, in its procurement practices\textsuperscript{28}. It is perhaps not surprising that real options analysis has taken root in engineering and R&D-intensive industries engaging in large and risky capital expenditures. The fact that many of these companies have relatively high proportions of engineers and scientists in their management structures may also be a contributing factor. There appears to be a perception that real options analysis is inherently more “difficult” than other valuation methods, although this is not necessarily the case\textsuperscript{29}.

Real-options analysis is not as pervasive as conventional discounted cash flow analysis in most corporate and government capital budgeting decisions. This alone does not invalidate the analysis; it takes decades for analytical tools to take hold or to be changed. Financial engineering has become entrenched in the financial services and consulting industries\textsuperscript{30}. As these tools evolve it will be natural to apply them to non-financial business problems. Indeed the tools are not unique to the financial sector but were adapted from the mathematical sciences. The relatively slow penetration of real-options analysis reflects the difficulty for most organizations in articulating the risks faced in capital decisions.

The remainder of this paper will focus on explaining and applying options pricing methods to valuing the portion of the MYP contract this is a risk management proposition.

5. Options theory

We will use closed form BS-type option pricing methods to estimate the contractor’s value in an MYP contract. Financial options fit into the larger domain of derivatives or contingent

\textsuperscript{16} Cornelius et al. (2005)
\textsuperscript{17} Colwell et al. (2003)
\textsuperscript{18} Duan et al. (2003)
\textsuperscript{19} Ekelund (2005); Remer et al. (2001)
\textsuperscript{20} Ford et al. (2004); Rothwell (2006)
\textsuperscript{21} Fourt (2004); Oppenheimer (2002)
\textsuperscript{22} Nembhard et al. (2003)
\textsuperscript{23} Laxman & Aggarwal (20030
\textsuperscript{24} Gaynor & Bradner (2001)
\textsuperscript{25} Paxson (2002); MacMillan et al. (2006)
\textsuperscript{26} Cornelius et al. (2005); IOMA (2001)
\textsuperscript{27} IOMA (2001)
\textsuperscript{28} Maumo (2005)
\textsuperscript{29} Amram & Howe (2003); Copeland & Tufano (2004)
\textsuperscript{30} Although with mixed results in structured finance and credit default swap applications.
claims: financial instruments whose value derives from claims on pay-offs from event-driven changes in the value of an underlying asset. There are two types of derivatives buyers: hedgers who are naturally exposed to the underlying asset volatility and speculators who seek exposure to this risk.

A simple example is an equipment manufacturer with occasional large foreign exchange exposures when its machines are exported. The manufacturer could hedge the foreign exchange risk by buying put options on the foreign currency he expects to receive upon the sales transaction. The put option allows the manufacturer to exchange foreign currency for dollars at a predetermined date and exchange rate and thus eliminates profit volatility. The manufacturer is the hedger and the bank could be a speculator31.

Insurance is another example where the insurer (the speculator), sells coverage to insureds (hedgers) for a premium. The insurer mitigates its position through many risk management tools: setting up loss reserve accounts which are based on detailed loss histories; diligent underwriting (i.e. pricing the coverage according to specific risks); avoiding certain risks (i.e. correlated high exposure risks such as asbestos, floods, or mold damage); limiting correlated risks (i.e. wind damage in Florida or earthquakes in California); hedging through reinsurance; etc. The government is actually one of the largest insurers providing many types of coverage against risks that many private insurers avoid: flood, nuclear; commercial space launch, terrorism, aviation war and hijacking, etc.

Compared to most risks to which the government is exposed, absorbing a few years of SYP volatility is a relatively tame risk transfer particularly in the context of the statutory “underwriting” that must occur before Congress will authorize such a contract. In the MYP contract, the defense contractor is the hedger, while the government is “speculating” that by meeting the MYP criteria it should be able to benefit by accepting the contractor’s risk. The MYP criteria in Table 1 are an effective underwriting tool for the government. By passing the criteria, the government is actually absorbing little risk since by criteria 2 and 3 they would have acquired all of the units even without the MYP.

It is important to note that not all hedges make good business sense. The rules as whether or not to hedge are based entirely on the cost and benefits to shareholders who are free to diversify some of the idiosyncratic risk away from their investment portfolio. The options pricing models will not discern this trade-off for the contractors but it is likely to be the basis for the contractor’s perspective in negotiating with the government. Regardless of the contractors’ risk aversion, our goal is to elucidate the value created by the risk transfer. The government is taking on new risk by entering into the MYP contract—this risk transfer creates a significant benefit for the contractor counterparty whether or not they want to pay for it.

6. MYP option analysis

A put option has the desired insurance-like structure of an MYP contract: with the embedded risk transfer component of the MYP contract the contractor gains the right to sell a fixed number of units at a pre-set price. However, the MYP, like many real options, does not strictly eliminate the SYP risk; there is some risk that the government could cancel the

31 The bank may also hedge its foreign exchange exposure.
contract or change the number of units. Thus an exchange option, which gives the holder the right to exchange one cash flow for another on or before a given date, has advantages over a put option since its cash flow corresponds more closely to the way an MYP would be structured. The put and exchange options are closely related.

The key difference between the put and the exchange option is that on exercise, a put buyer receives a certain cash settlement while with an exchange option the buyer obtains a “cash flow” with different volatility. This property is ideal when in fact the MYP contract usually has a flexibility clause for variations in quantity (VIQ).

Consider a put option for the sake of the simplicity of its properties. A put provides a payoff to the option holder when it is exercised before the expiry and the exercise price is greater than the market or spot price of the underlying asset. An option holder can buy the asset at spot price \( S \) and sell it at the strike price \( X \) and receive a payoff \( X - S \). Alternatively, an option holder having a long position in an asset can hedge against losses with puts, much like an insurance policy.

Figure 1 depicts the payoff of a put option on or prior to the expiry. Once exercised, options are zero-sum contracts: the writer “loses” and the holder gains or vice versa. If the option expires unexercised, the holder’s only loss is the premium paid to the writer. If the put option is held as a 1:1 hedge against a long position in the underlying stock, however, the net pay-off is nil, or negative once the option premium is included. In the same way a contractor with an MYP contract is hedging against the uncertainty in the government’s procurement decisions. The contractor net gain is neutral since the payoff depicted in Figure 1 is offset by the underlying losses in sales that would have happened if there were no MYP. The MYP option pay-off is the protection against losses and the contractor will only observe

---

32 Canceling the contract usually would come with considerable cost to the government.
that it has stable, predictable cash flows. However, more predictable cash flows allow investors to value the contractor’s equity higher. The government, on the other side, faces the risk that it will be forced to manage future budget uncertainties by increasing taxes or debt, cutting programs other than the MYP, or paying a higher termination fee if it cuts the MYP.

7. Extending financial options to the MYP option

Ideally we would like to be able to use a formula, such as that of Black and Scholes, to estimate the value of a MYP contract option. However, this is only reasonable if the contingent pay-offs behave within the constraints and assumptions behind the BS model. Though the basic BS formula applies to dividend protected European options in an arbitrage free market, it could be applied to a real option if its value depends on: the underlying asset value (S); the asset’s volatility (σ); and whether the option time frame resembles that of a European option. The worth of the MYP contract option depends on the value of the underlying asset—i.e. the net present value of future cash flow implied by the procurements. The uncertainty around the size of these cash flows is also a key value driver: low risk SYP contracts have less risk to be transferred to the government and lower the contractor’s need for an MYP. Later we will discuss in more detail how to assess the volatility (the standard deviation of the market price of an asset) of the value of a series SYP contracts. Unlike equity stocks, currencies, and other traded securities, volatility in the case of a real option is difficult if not impossible to observe so we need to find a suitable tracking asset. The option pricing models can still be used to value the real option using the tracking asset’s volatility if there is sufficient correlation between the tracking asset and the real option underlying asset valuation fluctuations. The time frame of the MYP contract option is reasonably close to a European option, since it can be exercised only when the contract is executed. Also inherent in the BS model is that the return process of the underlying asset follows a Brownian motion process where the returns have a lognormal distribution.

8. The Black-Scholes model

The value of the put option \( p \) on Company A’s stock at time \( t \) until expiration at time \( T \) can be estimated using the BS model:

\[
p(S, t) = Xe^{-r(T-t)}N(-d_2) - SN(-d_1)
\]

(1)

\( S \) and \( X \) are A’s stock spot price at valuation and strike (at expiry \( T \)) per share respectively. \( N(d_1) \) and \( N(d_2) \) are the cumulative normal distributions of \( d_1 \) and \( d_2 \):

\[
d_1 = (\ln(S/X) + (r + \sigma^2/2)(T-t))/ (\sigma (T-t)^{1/2})
\]

\[
d_2 = d_1 - \sigma (T-t)^{1/2}
\]

---

33 European options can only be exercised on the expiration data while American options can be exercised on or before expiry.
σ is the standard deviation or volatility of A’s stock price over the span of the option life.

\( r \) is the interest rate of a risk-free bond with the tenor of the option expiry. Note that the thin dotted curve in Figure 1 never goes below zero; an option has value until expiry even if it is out of the money (i.e. for a put, \( S>X \)). This value is derived from the “time value” or asymmetric opportunity value of the option which allows the holder the possibility that it will come into the money prior to expiry without any risk of negative payoff.

The BS model assumes that the stock price changes are log-normally distributed, such that over time, the logarithm of the price changes follows a Weiner process. With the use of Ito’s theorem and several more assumptions, the put option price \( p \), as a function of \( S \), is calculated using (1).

In contrast to no-dividend European options assumed in (1), American options can be exercised up to or on the expiry date greatly complicating the mathematics behind their valuation. Most single equity options are American, while options on indices, such as the S&P 500, are European.

Applying (1) to the MYP, \( S \) is the net present value (NPV) of the cash flow expected from a series of SYP contracts; \( X \) is the price of the NPV of the MYP contract cash flows; and \( T \) is last day the final lot could be changed under an SYP. \( \sigma \) would ideally be the volatility of the NPV of the SYP cash flows, but since this volatility is virtually impossible to observe, it will be estimated using the contractor’s stock as a tracking asset.

9. Exchange option

The exchange option allows the holder the right to swap cash flow \( x_2 \) (the risky SYP profit stream) for cash flow \( x_1 \) (the less risky MYP profit stream). This option is more general and better captures some of the flexibility the government has with actual MYP contract terms.

The BS-based formula to value an exchange option is:

\[
w = e^{-r(T-t)}x_1N(d_1) - e^{-r(T-t)}x_2N(d_2)
\]

Again \( r \) is the risk-free rate, \( x_1 \) the strike price of asset 1 (MYP), \( x_2 \) the strike price of asset 2 (SYP), and \( N(d_1) \) and \( N(d_2) \) are the cumulative normal distributions of \( d_1 \) and \( d_2 \):

\[
d_1 = (\ln(x_1/x_2) + (\sigma^2/2)(T-t))/ (\sigma (T-t)^{1/2})
\]

\[
d_2 = d_1 - \sigma (T-t)^{1/2}
\]

\[
\sigma = (\sigma_1^2 + \sigma_2^2 - 2 \rho \sigma_1 \sigma_2)^{1/2}
\]

34 Technically it is the instantaneous volatility - something that is hard to measure.

35 \( p(S,t) \) is found by solving the following partial differential equation:

\[
p_t = \frac{1}{2} \sigma^2 S^2 p_{SS} + rS p_S - rp
\]

The equation is subject to the terminal condition: \( p = \max[0,X-S] \) and to upper and lower boundary conditions: \( p = X e^{-rT} \) for \( S = 0 \) and \( p = 0 \) for \( S \to \infty \). \( S \) follows the Wiener process through the following stochastic differential equation:

\[
dS = \mu dt + \sigma dS.
\]

Here \( \mu \) is the average growth rate; \( \sigma \) is the standard deviation of this growth process; and \( r \) is the risk free interest rate.
Where $\sigma_1^2$ is the variance of $x_1$, $\sigma_2^2$ is the variance of $x_2$, and $\rho$ is the correlation between $x_1$ and $x_2$. Here $\rho$ is likely to be close to 1 since $x_1$ and $x_2$ are essentially the same assets whose risks are derived from the same source. In our base analysis, $x_1$ is assumed to be certain, i.e. the MYP units are fixed in each lot and the government has no flexibility to cancel the MYP. Thus $\sigma' = \sigma_2$ since $\sigma_1 = 0$. If however, the MYP contract has some uncertainty, e.g. from a VIQ clause or a low termination fee, $\sigma_1$ could be adjusted to reflect the relative risk between $x_1$ and $x_2$.

The exchange option can also be thought of as a simultaneous call option on asset 1 with strike price $x_2$ and a put option on asset 2 with a strike price $x_1$. A call option is a contract that gives the owner the right to buy an asset at a predetermined price on or before a predetermined time. The main difference between the put and exchange options is that the latter allows both assets to have price volatility. Furthermore the exchange option allows for the upside volatility in the MYP, i.e. that more units than the original plan could be purchased.

10. Estimating option pricing parameters

Consider as an example a major acquisition weapon system, “Program G,”, executed by the contractor Company A. Program G and Company A do not correspond directly to any real-life program or company, although the numbers discussed in this paper are constructed from real examples. Program G’s base SYP net cash flows can be derived from the relevant military service’s Selective Acquisition Report (SAR). Table 1 lists the profits associated with Program G system lots 6 through 10\(^{36}\). Since lot 6 is the first year of both contract scenarios its profits are omitted from the analysis since they will not depend on whether the MYP is executed. The SYP uncertainty is only in lots 7 through 10. The profits are stated in “then year” (nominal) terms and the net present value of the flows is discounted at Company A’s cost of capital.

<table>
<thead>
<tr>
<th>Lot</th>
<th>Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lot 6</td>
<td>$-$</td>
</tr>
<tr>
<td>Lot 7</td>
<td>200</td>
</tr>
<tr>
<td>Lot 8</td>
<td>200</td>
</tr>
<tr>
<td>Lot 9</td>
<td>250</td>
</tr>
<tr>
<td>Lot 10</td>
<td>175</td>
</tr>
<tr>
<td>Total</td>
<td>$825</td>
</tr>
<tr>
<td></td>
<td>Present Value $630</td>
</tr>
</tbody>
</table>

Table 3. Contractor SYP Profit ($ Mils)

The present value total of $630 represents the projected total asset value ($x_2$) of the last four lots of the SYP. We initially restrict $x_2 = x_1$, or that the option be “at the money”\(^{37}\).

\(^{36}\) We assume a dollar for dollar profit cash flow conversion.

\(^{37}\) This is a realistic assumption since the number of units in the MYP and SYP are assumed to be the same in the standard business case analysis.
11. Volatility

For most non-traded assets, such as the profits of Program G, even the historical volatility is difficult to measure\(^\text{38}\). To properly use the BS model to value Program G MYP option it is imperative to find a traded tracking asset whose volatility is highly correlated to the implied volatility of the asset underlying the embedded real option.

Fortunately, Company A’s equity is publicly traded. Company A is a moderately diversified government contractor with two divisions, Defense and Non-Defense, that serve different government sectors. We find from their financial statements that Program G represents a substantial share of the Defense Division’s earnings before interest and tax (EBIT). The EBIT breakout by division is presented in Table 2. The Defense Division has contributed a significant portion of the total profits, particularly in recent years. Comparing Tables 1 and 2 we can see that Program G represents over half of the Defense Division’s historical EBIT.

<table>
<thead>
<tr>
<th>Year</th>
<th>Non-Defense</th>
<th>Defense</th>
<th>Total</th>
<th>Stock Price</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>EBIT</td>
<td>% Total</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2001</td>
<td>$758</td>
<td>24%</td>
<td>$1,000</td>
<td>$13</td>
</tr>
<tr>
<td>2002</td>
<td>564</td>
<td>14%</td>
<td>656</td>
<td>7</td>
</tr>
<tr>
<td>2003</td>
<td>522</td>
<td>20%</td>
<td>650</td>
<td>13</td>
</tr>
<tr>
<td>2004</td>
<td>652</td>
<td>16%</td>
<td>775</td>
<td>20</td>
</tr>
<tr>
<td>2005</td>
<td>679</td>
<td>20%</td>
<td>846</td>
<td>19</td>
</tr>
<tr>
<td>2006</td>
<td>552</td>
<td>32%</td>
<td>809</td>
<td>19</td>
</tr>
<tr>
<td>2007</td>
<td>443</td>
<td>43%</td>
<td>778</td>
<td>21</td>
</tr>
<tr>
<td>2008</td>
<td>742</td>
<td>33%</td>
<td>1,113</td>
<td>26</td>
</tr>
</tbody>
</table>

Table 2. A’s EBIT Breakdown by Division ($ Mils except for Stock Prices - $/Share)

Company A is a large enterprise, and while Program G contributes significant profits towards to total corporate profit, it is not necessarily enough to drive the overall equity performance. Before we can assign Company A’s equity volatility as a tracking asset for Program G, we need to establish a closer linkage. Table 3 shows Company A’s earnings growth and volatility by division as well as the market performance of its equity from 2000 to 2007. We see that the Defense Division tracks the overall stock performance better than the Non-Defense Division, and better than the company as a whole. This may be because Company A is often identified as a defense company and its stock price, which is forward looking, trades on the trends in the overall defense industry.

<table>
<thead>
<tr>
<th>Year</th>
<th>Non-Defense Growth</th>
<th>Defense Growth</th>
<th>Total Growth</th>
<th>Stock Growth</th>
</tr>
</thead>
<tbody>
<tr>
<td>2001</td>
<td>0%</td>
<td>6%</td>
<td>2%</td>
<td>10%</td>
</tr>
<tr>
<td>2002</td>
<td>29%</td>
<td>36%</td>
<td>32%</td>
<td>37%</td>
</tr>
</tbody>
</table>

Table 3. A’s EBIT Growth and Volatility by Division 2001 to 2008

\(^{38}\) A crude estimate could be constructed by collecting the annual Selected Acquisition Report estimates for the number of units funded through the life of the program.
One more indication that Company A’s stock is a good tracking asset for Program G is the correlation between the division’s EBIT and the stock price, as shown in Table 4. Defense Division EBIT has a 72% correlation to the stock price—even higher than the company’s total EBIT. Note that this is not to imply that the stock price drives Program G profit volatility; but rather that the stock price mirrors the EBIT volatility of the Defense division which is strongly driven by the program G business. Since we cannot measure Program G EBIT volatility directly, we will use the stock price volatility as a proxy. We could use the Defense division’s historical EBIT volatility (Table 3) to track Defense division volatility, instead we prefer to use the forward-looking implied volatility estimated in Table 5.

<table>
<thead>
<tr>
<th>Division, Stock Price</th>
<th>18%</th>
<th>72%</th>
<th>59%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-Defense</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Defense</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4. Correlation between Stock Price and Division EBIT from 2001 to 2008

12. Time horizon

We have already hinted at the time horizon for the MYP option. It starts when congress gives the services authority to enter into an MYP with A. It expires at the beginning of the last year or lot of production (assuming one lot per fiscal year) since that would be the last point at which the government could have reduced the number of units in an SYP contract. Assume that the MYP authority is granted six months prior to negotiation. The total life of the MYP is then five years and six months.

13. Interest rate

The risk free interest rate used in the analysis is the rate on a Treasury bill whose maturity ties roughly to the expiry of the MYP option.

14. Option valuation

First we estimate the implied volatility of a Company A call option that expires close to the MYP expiry. Unfortunately options beyond two years are rare, even for established companies like A. Thus we use the Jan ’10 call option to estimate the implied volatility. The parameters to estimate the implied volatility are listed in Table 6. $S^*$, $X^*$, $T^*$, and $c^*$ are the stock price, strike price, expiry, and option price for the A Jan ’10 $25 call. Using these values in the BS call option formula we can calculate the implied asset volatility\(^{39}\). The asset volatilities are then used in (2) to estimate the exchange option price for the MYP.

Table 6 summarizes the valuation of the MYP structured as an at-the-money exchange option. Setting the strike value equal to the spot value gives an option value of $127 million which the contractor would need to pay the government upon executing the MYP contract.

\(^{39}\) We use an algorithm based on the Newton-Raphson method to solve for the implied volatility of a European option.
Much of this value is in the time to expiration or “time premium”. Just to illustrate, if the option were for one month it would be worth $20 million and worth $4 million if it was for one day—all else equal.

<table>
<thead>
<tr>
<th>Risk Free Rate (r)</th>
<th>4.73%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stock Price (S*)</td>
<td>$26.15</td>
</tr>
<tr>
<td>Exercise Price (X*)</td>
<td>$25.00</td>
</tr>
<tr>
<td>Expiry (years) (T*)</td>
<td>1 2/3</td>
</tr>
<tr>
<td>Option Price (c*)</td>
<td>$5.40</td>
</tr>
<tr>
<td>Asset Volatility</td>
<td>29%</td>
</tr>
</tbody>
</table>

Table 5. BS Parameters for A ($/Share)

The analogy between MYP and insurance is a good one because, as anyone who has made a claim might have discovered, the insurance pay-off is not certain. The MYP can have a variation-in-quantity clause that allows the government to reduce the number of units by a pre-determined number. For example, if the EPA clause is activated by unanticipated labor and materials inflation, the government might reduce the quantity purchased to maintain its bottom line budget. Thus there is some uncertainty around the MYP that must be considered in our risk transfer pricing. This is where the exchange option framework has an advantage over the plain put option structure. It can be used to value cash flow trades that have different levels of uncertainty. For the valuation in Table 6 we set $\sigma' = \sigma_1$ and $\sigma_2 = 0$.

Assume now the government and the contractor agree that the former could reduce the number of Program G units by 2 each year or 10% of the number of units in each lot. We use the exchange option structure to value the right to swap the SYP cash flow with volatility $\sigma_1$ for the MYP volatility of volatility $\sigma_2$—see Table 7 for the valuation.

| Present Value SYP (x_2) | $630 |
| Strike Value (x_1) | $630 |
| Real Option Price | $127 |
| Expiry (yr) | 5.0 |

Table 6. MYP Evaluated as an Exchange Option – Risk on SYP Cash Flow Only

The price of the option falls from $127 million to $112 million. It would drop to $84 million with 100% correlation; however, if there were no correlation between the two cash flows, the...
price would have increased to $134 million. This is due to the upside potential of the MYP and SYP. The exchange option is essentially a put option with a stochastic strike price which allows the protection buyer to capture more payoff if the MYP turns out to yield more units. This assumes that the risk of the MYP is symmetric. There is no reason to believe otherwise, since the government can always buy more units than planned, if they are needed.

15. Other real options embedded in an MYP

Within this chapter, we only have the scope to focus on a single real option example within the MYP contract. However, there is at least one other real option available to the contractor with a sole source production franchise such as a major aircraft, missile, ship, etc. This is because defense contracts are incomplete leaving the contractor with residual control of cost reduction innovations. While we will not estimate the value of this real option here, we mention it because in some cases it is potentially worth far more than the revenue stabilization discussed here. Regulatory lag is an incentive concept that emerged from explicitly regulated industries such as utilities. These industries’ profits are regulated directly through rate setting, e.g. $/kWhr, or through rate of return settings by a regulatory authority. Between rate settings, the utility is free to innovate and achieve higher profits. Upon the next regulatory oversight review, the regulator discovers the new cost structure and adjusts the new rate accordingly to a lower profit level—presumably slightly above the weighted average cost of capital for the utility. Longer periods between regulatory oversight periods (i.e. higher regulatory lag), mean greater opportunities for higher profits. Similarly, a defense contractor with a sole source series of production contracts for a weapon system has the incentive to achieve greater than expected efficiency innovations even if the savings are passed on to the government in subsequent contracts. It turns out that there is a substantial regulatory lag in defense contracts due to the length of time it takes for cost reports to be submitted to the government. The regulatory lag increases substantially in a MYP contract.

These innovations are real options since the contractor is not obligated to make the necessary investment to achieve the cost savings. They can use real options valuation tool to estimate the worth of these options before a program is executed by looking at prior history of achieving cost reduction innovations as well as a forward looking assessment of the opportunities in a specific weapon system. Unlike the revenue stabilization option, there is considerable information asymmetry between the government and contractor with the regulatory lag options. However, the government could look at prior programs and assess the degree of regulatory lag driven innovation that occurred in past programs and roughly estimate the value of this type of incentive on a new program. This valuation can provide important insight into how aggressively contractors will compete to win a large sole source program.

16. Conclusion: the cost implications of the MYP option

Options pricing analysis offers a way to systematically estimate value from the MYP contract earned by the Government for which they have not previously been explicitly compensated. This incremental value is the revenue risk transferred to the Government from the contractor upon signing an MYP. The MYP does not eliminate the revenue risk for
the contractor associated with SYP contracts; rather it transfers it to the government and it becomes budget risk. The Congress clearly values its budget flexibility, as evidenced by the statutory criteria to judge the worth of an MYP proposal. MYP cost savings are usually through operational efficiencies earned through process and purchasing improvements funded by the Government’s “economic order quantity” advance funding. The transfer of revenue risk to the Government is a cash flow hedge that provides real value to the contractor just as any financial hedge does for currency, commodity, or interest rate risks—or property and casualty insurance does for operational risks. Lockheed and Raytheon, for example, carry interest rate swaps that hedge interest rate risk for notional $1 billion and $600 million respectively\(^{40,41}\). General Dynamics reported a currency swap to hedge a Canadian denominated loan with a fair value of $42 million\(^{42}\). It also reported embedded options in the terms of its long term labor and commodity contracts. One can argue that just as public companies are expected to incur expenses as they pay for insurance and financial hedges, they should pay the government when it reduces the contractor’s risk.

The option methodology helps the government objectively quantify some of the cost in relinquishing its budget flexibility with a relatively simple tool that has widespread use in the financial community. We do not try to value the cost of transferring the risk from the Government’s side because there is not a readily available tracking asset to estimate the volatility of the revenue risk. It is possible to estimate the actuarial loss history of certain procurements by looking at the Selected Acquisition Report over the span of past programs. If such data were available, it might be desirable to use it in lieu of the equity volatility of the contractor. One benefit of using the contractor’s volatility, however, is that it is more closely coupled to the risk the contractor might be willing to hedge. The option value of the MYP has not been explicitly paid to the government in the past. Thus any method that helps rationalize the cost of this risk transfer is a benefit to the government. Furthermore, the contractor will likely see the value of the MYP option if it is evaluated in its own financial terms. Strategically, the MYP option value represents a significant reduction in the contractor’s profits. Given the skill and sophistication that contractors employ to manage their government customers, they will likely argue that the MYP real option has limited value as an earnings hedge. They could contend that financial hedges are only appropriate for risks that are outside of managers’ control, such as interest and exchange rates, and cannot be offset within the business. They might also contend that not only is their portfolio of business well diversified among a broad scope of government elements but that they have enough support on Capitol Hill to ensure that they will sell all the units in the SYP plan. They would be arguing that the program is less risky than their business in total (i.e. their equity volatility). This would be a difficult argument for most businesses. However, initially it is unlikely the contractors will proactively volunteer to pay for it.


\(^{42}\) General Dynamics Corporation, Securities and Exchange Commission Form 10-K, Commission file number 1-13671, Fiscal Year December 31, 2006, p. 49.
However, the fact is that the lower earnings risk from an MYP has tangible value whether or not the contractors wish to pay for it. The option has the same value no matter what the contractors’ risk preference. If there is no risk hedge in an MYP, why do the contractors routinely enter into this type of contract? In fact Lockheed readily acknowledged that the value of the MYP is its long term stability.43

The options methodology allows the Government to build a logical business case for reducing the profit on cost paid to contractors when switching from an SYP series to an MYP contract. The exchange option model in particular allows the Government to quickly estimate changes in the value of the contract as the details, e.g. the EPA and VIQ clauses, become more complete.

17. References


---

43 LMT-Q3 2006 Lockheed Martin Earnings Conference Call, Preliminary Transcript, Thompson StreetEvents, Thompson Financial, October 24, 2006, 11:00AM ET
Space technology has become increasingly important after the great development and rapid progress in information and communication technology as well as the technology of space exploration. This book deals with the latest and most prominent research in space technology. The first part of the book (first six chapters) deals with the algorithms and software used in information processing, communications and control of spacecrafts. The second part (chapters 7 to 10) deals with the latest research on the space structures. The third part (chapters 11 to 14) deals with some of the latest applications in space. The fourth part (chapters 15 and 16) deals with small satellite technologies. The fifth part (chapters 17 to 20) deals with some of the latest applications in the field of aircrafts. The sixth part (chapters 21 to 25) outlines some recent research efforts in different subjects.

How to reference
In order to correctly reference this scholarly work, feel free to copy and paste the following:
