Application of Particle Swarm Optimization Algorithm in Smart Antenna Array Systems

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1. Introduction

In wireless applications the antenna pattern is shaped so as to cancel interfering signals (placing nulls) and produce or steer a strong beam towards the wanted signal according to signal direction of arrival (DOA). Such antenna system is called smart antenna array. This chapter presents the efficiency of Particle Swarm Optimization algorithm (PSO) compared to Genetic algorithm (GA) in solving antenna array pattern synthesis problem. Also PSO is applied to determine optimal antenna elements feed that provide null (minimum power) in the directions of the interfering signals while to maximize of radiation in the direction of the useful signal. Application for PSO algorithm in Direct Data Domain Least Squares (D3LS) approach that is used to estimate incoming signal is illustrated. Due to environment changing the target goal is changing so modification in the algorithm is proposed to provide optimal solution for varying real time target (to track the desired users and reject interference sources). The problem is formulated and solved by means of the proposed algorithm. Examples are simulated to demonstrate the effectiveness and the design flexibility of PSO in the framework of electromagnetic synthesis of linear arrays.

2. Smart Antenna Array System Overview

The ability to communicate with people on the move has evolved remarkably since Marconi first demonstrated radio’s ability to provide continuous contact with ships sailing the English Channel in 1897. There onwards, new wireless methods and services have been adopted. Smart antenna system represents one of the valuable parts that support the increasing requirement and needs to higher quality wireless services. Smart antenna systems processes signals arriving from different directions to detect (estimate) desired signal direction of arrival DOA. Biased on the estimated DOA the beamformer optimize antenna elements weights such that the radiation pattern of the antenna array is adjusted to minimize a certain error function or to maximize a certain reward function derived by the adaptive algorithm. Figure 1. Presents block diagram for Smart antenna system. Smart antenna processing core is represented in three areas the adaptive algorithms the DOA estimation algorithm and the beamformer control. One of the simplest geometries for an array is a linear array in which the centers of the antenna elements are aligned along a straight line. For simplicity consider the uniformly
spaced linear array of N elements and that there is M signals received. We assume that K samples are observed by the array then output vector $X(n)$ is

$$X(n) = A(\theta)S(n) + O(n), \quad n = 1, 2, \ldots$$  \hspace{1cm} (1)$$

$X(n)$ is $(N \times K)$ matrix of array output signals at any given instant (sampling time) $n$, $A(\theta)$ is $(N \times M)$ steering matrix, $S(n)$ is $(M \times K)$ signal matrix, $O(n)$ is noise matrix. The array steering matrix (array manifold) $A(\theta)$ is

$$A(\theta) = [a(\theta_1), a(\theta_2), \ldots, a(\theta_N)]$$  \hspace{1cm} (2)$$

Where

$$a(\theta_i) = \left[1, \exp\left(\frac{2\pi d \sin \theta_i}{\lambda}\right), \ldots, \exp\left(\frac{2\pi d(N-1) \sin \theta_i}{\lambda}\right)\right], \quad i = 1, 2, \ldots, M$$  \hspace{1cm} (3)$$

$a(\theta_i)$ is the response of the linear array to the $i^{th}$ source arriving from direction $(\theta_i)$. The array manifold is defined as the one-dimensional manifold composed of all the steering vectors as $\theta$ ranges over all possible angles i.e. $\theta \in [0, 2\pi]$.

Figure 1. Block diagram of smart antenna array system and linear array signal model

The array manifold used to calibrate the array for direction finding estimation. Each element output is multiplied by a complex weight $w^*_i$, suggested by the adaptive algorithm then the beamformer update the phase and amplitude relation between the branches, and sum them to give information signal $Y(n)$

$$Y(n) = W(n)^T X(n)$$  \hspace{1cm} (4)$$

$W = [w_1 \ w_2 \ w_3 \ \ldots \ w_N]$  

3. PSO for Smart Antenna System

The smart antenna changes their directional pattern with the help of few adjustable parameters in according to the estimation and analysis to received signal, environment and
pre-known information to improve the performance and capacity of the system. A promising way for the determination of a suitable parameter configuration for the antenna is the application of heuristic optimization procedures.

Pattern synthesis problem (beamforming) is a continuous varying target real-time problem that needs fast optimal solution to adjust array pattern and support for the service required. Also the controlling parameters are limited due to practical design and cost aspects. Consequently, Enhancement to PSO algorithm is proposed to support for these two major needs.

3.1 PSO and Dynamic Real Environment Optimization

For real-time dynamic environment problem, the goal value changes, original PSO algorithm has no method for detecting this change and the particles are still influenced by their memories of the original goal position. If the change in the goal is small, this problem is self-correcting. Subsequent fitness evaluations will result in positions closer to the new goal location replacing earlier position $\mathbf{X}$ vectors, and the swarm should follow, and eventually intersect the moving goal.

However, if the movement of the goal is more pronounced, it moves too far from the swarm for subsequent fitness evaluations to return values better than the current personal best $\mathbf{P}_i^g$ vector, and the particles do not track the moving goal. A proposed attempt to rectify this problem by having the particles periodically replaces their $\mathbf{P}_i^g$ vector with their current $\mathbf{X}_i^g$ vector, thus “forgetting” their experiences to that point. This differs from a restart, in that the particles, in retaining their current location, have retained the profits from their previous experiences, but are forced to redefine their relationship to the goal at that point. Figure 2 presents the flow chart for the proposed PSO algorithm to support for varying dynamic target optimization problem.

Figure 2. Dynamic Particle Swarm Algorithm
3.2 PSO and bounded search space
Constraint is usually set to the array parameters these constraints may be spatial, for example, that the interelement spacing be greater than a prescribed value or that the element positions be within specified limits. Other type of design constraint is the excitation where it may require that the elements feed is phase only or amplitude and that the current-taper ratio be less than or equal to a prescribed value.

Introducing constraint to the PSO will decrease degree of freedom. Search time will also increase if the concept of accept and after the each particle movement for each iteration according to boundaries. However, if we can convert the problem to an unconstrained one initially through using suitable transformations of the constraint parameter this will eliminate time lost in explore and probability of rejecting the particle movement. Illustration for such solution will be clear in next section while simulation.

4. PSO use for pattern synthesis
This section objects to reformulate and define antenna array adaptive beamforming in term of an optimization problem. Problem Search Space represented by array pattern controlling parameters is identified. Fitness function that measures the deviation of the optimal proposed solution from the target is defined.

Let us consider the linear array of $M$ non-uniformly spaced point source isotropic elements located along a straight line at the positions $x_k$, where $k = 0, \ldots, M - 1$. The beam pattern function $P(u)$ of the array, is defined as follows,

$$P(u) = \left| \sum_{k=0}^{M-1} w_k e^{j\frac{2\pi}{\lambda} x_k u} \right|$$

$$w_k = \alpha_k \exp(j\beta_k)$$

$$p(u) = \left| \sum_{k=0}^{M-1} \alpha_k e^{j\frac{2\pi}{\lambda} x_k u + \beta_k} \right|$$

(5)

Where $\alpha_k$ is the weight coefficient of the $k^{th}$ element, $\lambda$ is the background wavelength, $u = \sin \theta - \sin \theta_o$, being $\theta$ and $\theta_o$ the incident angle of the impinging plane wave and the steering angle of the array, respectively. In order to generate a beam pattern (BP) that attain specific characteristics e.g., sidelobes level (SLL) lower than a fixed threshold or reproduces a
desired shape $P_{dB}^{ref}(u)$, initially we have to identify the array designing parameters and their boundaries i.e. The particle search space in PSO algorithm.

Let vector $\zeta$ be defined as follow,

$$\zeta = [M, x_0, \ldots, x_{M-1}; w_0, \ldots, w_{M-1}; D]^T;$$

Where, $M$ is number of array elements, $[x_0, \ldots, x_{M-1}]$ is array elements spacing vector, $[w_0, \ldots, w_{M-1}]$ is array elements feed vector generally represented as $w_k = \alpha_k \exp (\beta_k)$, finally $D$ is array length. $\zeta$ boundary limits has to be taken in account when solving the problem to facilitate practical and cost design needs.

Then a quantized measure for the solution distance from the target required should be defined, this value will be function of the search space parameter vector $\zeta$. Generally for antenna array pattern synthesis most of the well known target consideration is the main beam $f_{MB}$, total pattern $f_{BP}$, sidelobe level $f_{SLL}$, number, location and width of nulls $f_{null}$, number of array elements $f_N$ then we can us define global antenna array fitness function $f$, as follows:

$$f(\zeta) = \frac{1}{c_1f_{BP}(\zeta) + c_2f_{MB}(\zeta) + c_3f_{SLL}(\zeta) + c_4f_{null}(\zeta) + c_5f_N(\zeta)}$$  \hspace{1cm} (6)

Where

$$f_{BP}(\zeta) = \int_{u \in B} \left( \frac{P_{dB}(u)}{Q} - P_{dB}^{ref}(u) \right) du$$  \hspace{1cm} (7)

$$f_{MB}(\zeta) = \sum_{i=1}^{mb} \left( \int_{u \in MB} \left( \frac{P_{dB}(u)}{Q} - P_{dB}^{ref}(u) \right) du \right)$$  \hspace{1cm} (8)

$$f_{SLL}(\zeta) = \frac{q}{\max(P_{dB}(u))} \text{ for } u_{start} \leq u \leq 1$$  \hspace{1cm} (9)

$$f_{null}(\zeta) = \sum_{i=1}^{nl} \left( \int_{u \in BN_i} \left( \frac{P_{dB}(u)}{Q} - P_{dB}^{ref}(u) \right) du \right)$$  \hspace{1cm} (10)

$$f_N(\zeta) = M;$$  \hspace{1cm} (11)

Where $u_{start}$ being a value that allows excluding the main lobe from the calculation of the SLL. Moreover, $Q$ is a normalizing constant, $B$ represents visible region; while $P_{dB}^{ref}(u)$ represents the desired BP shape. $MB$ represents the range of values covering the Main beam, $mb$ number of beams in the pattern, $BN$ corresponds to the nulls locations and $nl$ is number of nulls required. Finally, $c_i$ are coefficients that identify each criteria value.

It is often necessary to impose a constraint on the interelement spacing to minimize the mutual coupling effects. For and array with an even number of elements the constraint may be expressed as follow

$$x_1 \geq \frac{d}{2}, \quad x_i - x_{i-1} \geq d \quad i = 2, 3, \ldots, M \hspace{1cm} (12)$$

The above constrain can be represented using the following transformation:

$$x_1 = \frac{d}{2} + (x'_1)^2$$

$$x_2 = \left( \frac{d}{2} + d \right) + (x'_1)^2 + (x'_2)^2$$
Generally

\[(x_i) = \left(i - \frac{1}{2}\right) d + \sum_{k=1}^{i} \left(x_k^2\right), \quad i = 1, 2, ..., M\]

For odd elements number array

\[(x_i) = (i - 1)d + \sum_{k=1}^{i-1} \left(x_k^2\right), \quad i = 2, ..., (M - 1)/2 \quad (13)\]

Solving using equation 10 allows minimization to be carried out with the new primed variables, and it is readily seen that the constraints are always satisfied.

Another type of constraint on spacing's usually imposed is the one requiring the elements to lie within a specified range mainly required to avoid unacceptable practical array dimensions. Stated mathematically in the following form:

\[a_i \leq x_i \leq b_i \quad i = 1, 2, ..., M \quad (14)\]

the transformation to be used in this case is

\[x_i = a_i + (b_i - a_i) \sin^2 \hat{x}_i \quad (15)\]

It is sometimes necessary to constrain the current taper to be within specified limits. That is,

\[I_i \leq I \pm C, \quad i = 1, 2, ..., (16)\]

It is easily verified that the transformation of the form in equation (15) will transform the constrained space into an unconstrained one

\[I_i = I + C \sin I_i \quad (17)\]

Next section will investigate the efficiency of the PSO for solving linear array configuration compared to other algorithms.

### 4.1 PSO and GA for Pattern Synthesis

To validate the PSO approach, initially we apply PSO, to find the optimized element weight to achieve the Chebyshev pattern for 10 equispaced isotropic elements with \(\lambda/2\) interelement spacing antenna array of minimum SLL of 26dB, and compare its performance to GA, for solving the same problem. The sample points, are chosen 300 equally distributed points over \(\theta\) on a personal computer with a Pentium IV processor running at 1GHz. The target beam will be \(P_{db}^{ref}\)

\[P_{db}^{ref} = 2.79 \cos u + 2.49 \cos 3u - 0.97 \cos 5u + 1.35 \cos 7u + \cos 9u\]

We consider 10 elements symmetric array with amplitude excitation only i.e. \(\beta_i = 0\) then

\[
\zeta = \begin{bmatrix} w_0 & \ldots & w_{(M/2)-1} \end{bmatrix}^T; \quad M = 10
\]

\[w_{k+(M/2)} = w_{(M/2)-(k+1)} \quad k = 0, \ldots, M/2 \]

\[f_{BP}(\zeta) = 2 \int_{u \in B} \left( \frac{P_{db}(u)}{Q} - P_{db}^{ref}(u) \right) du; \quad \text{for } 0 \leq u \leq 1 \]
\[ f_{\text{SLL}}(\zeta) = \max_{u_{\text{start}} \leq u} \frac{26}{P_{\text{dB}}(u)} \quad u_{\text{start}} = 0.25 \]

Figure 4 presents the output pattern explored over the optimization process by one particle until it reaches the optimum solution. Corresponding proposed elements weigh for these local minima is as listed in Table 1.

<table>
<thead>
<tr>
<th>Iteration No</th>
<th>( w_0, w_9 )</th>
<th>( w_1, w_8 )</th>
<th>( w_2, w_7 )</th>
<th>( w_3, w_6 )</th>
<th>( w_4, w_5 )</th>
<th>Max. SLL dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1 (5)</td>
<td>0.3292</td>
<td>0.5337</td>
<td>0.7030</td>
<td>0.9883</td>
<td>1</td>
<td>-20</td>
</tr>
<tr>
<td>P2 (30)</td>
<td>0.3543</td>
<td>0.3243</td>
<td>0.5679</td>
<td>1</td>
<td>0.7044</td>
<td>-15</td>
</tr>
<tr>
<td>P3 (78)</td>
<td>0.3521</td>
<td>0.4688</td>
<td>0.7158</td>
<td>0.8378</td>
<td>1</td>
<td>-12</td>
</tr>
<tr>
<td>P4 (122)</td>
<td>0.3574</td>
<td>0.4850</td>
<td>0.7055</td>
<td>0.8921</td>
<td>1</td>
<td>-26</td>
</tr>
</tbody>
</table>

Table 1. Optimum proposed weight corresponding to one particle

Figure 4, 5 shows behavior of the fitness values for solutions explored versus the number of iterations for one particle. Dotted curve represents the gbest fitness value where it intersects with the particles. Note that although the particle has achieved good fitness value in its exploring journey it was not trapped at these local minima at P1, P2, P3, P4.

Figure 6 presents comparisons between the fitness error per iteration for GA and PSO algorithms solving the above problem with same initial random feed using PSO and Genetic algorithm. It can be noticed the performance difference in reaching optimum solution is not big only difference comes for the time per iteration in each algorithm. According to output in Table 1 that the optimized proposed element feeds is the same for both algorithms.
Figure 5. Behavior of the fitness function per iteration for one particle, dotted curve represent behavior of gbest fitness value per iteration

Figure 6. Fitness per iteration behavior for PSO algorithm and GA algorithm

Next section will search the capabilities of the PSO for solving array configuration. A simulation for steering single beam, introducing multiple beams in DOA and introducing nulls in the imposed directions by controlling the excitations of the array elements feed or the elements spacing represented in term of $\lambda$. also the adaptive ability of PSO for changing the problem target in runtime is presented such feature is to be useful in digital beamforming.
4.2 PSO and Pattern Synthesis Phase Control

The phase-only null synthesizing is attractive since in a phased array the required controls are available at no extra cost [Steyskal, H., 1986]. This section will illustrate different scenarios for pattern shaping using PSO to search suitable phase feed to fulfill required pattern. Initially consider it is required to introduce single null at direction $\theta = 50^\circ$ and SLL < 30 dB with same mainbeam. PSO evaluated element weighting which fulfilled the requirements of the design using fitness function equation 6.

Figure 7, shows the output pattern after 200 iteration notice that the SLL criterion is not achieved.

![Figure 7. Pattern proposed after 200 iteration for null at 50°](image)

Now let us consider the target is moved. Assume it is required to steer the mainbeam to be at $\theta = 110^\circ$ and presence of interference at $\theta = 150^\circ$. PSO evaluated antenna array elements’ phase which fulfill these requirements of the design output proposed pattern as Figure 8a shows the output pattern after 50 iteration as can be noticed although that the SLL < 20 dB was not achieved as the maximum level is 18 dB. Assume surrounding environment is stable so the algorithm is to continue search for better feeding solution Figure 8b shows the proposed pattern corresponding after 500 iteration maximum SLL of -22 dB was achieved and also the null width and is increased. Figure 9 shows the total fitness value per iteration curve corresponding to Figure 7 and Figure 8.

### Table 2. Optimum proposed weight corresponding to PSO, GA algorithm

<table>
<thead>
<tr>
<th>Algorithm/Normalized weight</th>
<th>$w_0, w_9$</th>
<th>$w_1, w_8$</th>
<th>$w_2, w_7$</th>
<th>$w_3, w_6$</th>
<th>$w_4, w_5$</th>
<th>No. of Iteration</th>
<th>Total time min.</th>
</tr>
</thead>
<tbody>
<tr>
<td>PSO</td>
<td>0.3574</td>
<td>0.4850</td>
<td>0.7055</td>
<td>0.8921</td>
<td>1.000</td>
<td>115</td>
<td>2</td>
</tr>
<tr>
<td>GA</td>
<td>0.3563</td>
<td>0.4845</td>
<td>0.7055</td>
<td>0.891</td>
<td>1.000</td>
<td>122</td>
<td>9</td>
</tr>
</tbody>
</table>
Figure 8. a) Pattern proposed for mainbeam steered to 110° and null at 150° after 50 iterations

Figure 8. b) Pattern proposed for mainbeam steered to 110° and null at 150° after 500 iteration

Figure 9. Fitness per iteration curve corresponding to figure 7, 8
4.3 PSO and Pattern Synthesis Phase – Position Control

The phase-only synthesis with equal element spacing requires a large number of elements compared to the amplitude only arrays. Controlling the inter element space and elements phases feed we can have the potential to circumvent this design challenge. Theoretically, the unequal spacing of antenna elements corresponds to nonuniform sampling of signals in the time domain,[H. Unz, 1960].

The PSO is applied to search for the optimum element phases and positions of the uniform amplitude linear arrays to achieve target pattern and minimum side lobe level. We only consider symmetric arrays for the next results however same can be applied for non symmetric array. Synthesis of an unequally spaced array is carried out separately for the position-only and the position-phase cases for various limits in the distance between the elements. The number of elements considered for the PSO-based synthesis is 32; hence the number of parameters to be optimized is 16 for the position-only synthesis and 32 for the phase-position synthesis.

The PSO synthesis results of positions and phases for the cases when \( d_{\text{max}} = 0.6\lambda \) and \( d_{\text{max}} = \lambda \) array patterns are shown in Figure. 10 and 11, respectively. From Figure 10, we can see that the maximum SLL for the position-phase synthesis is lower than that for the position-only synthesis. In Figure 11 When \( d_{\text{max}} = \lambda \), the maximum SLL of the position-phase synthesis and position-only synthesis is 23.34 and 22.53 dB, respectively.

For the case \( d_{\text{max}} = \lambda \), the time taken to reach -20 dB SLL was about 10 min, and the total time taken for 300 iterations was about 23 min for a swarm of 320 agents. The simulations were carried out on a PC based on an Intel Pentium-IV 3-GHz processor.

We can conclude that for smaller, \( d_{\text{max}} \) the element phases have a larger effect in lowering the SLL of an unequally spaced array with no significant difference in the directivity from Figures 10–11.

![Figure 10. Array patterns for the PSO-based position-only (dashed line) the position-phase (solid line) for \( d_{\text{max}} = 0.6\lambda \) ](image-url)
We have seen that the unequally spaced array derived using the position-phase synthesis has lowered SLL compared to that of the unequally spaced arrays derived using the position-only synthesis. Let us consider the PSO-based position-phase synthesis and phase-only synthesis for designing a pencil beam array.

The number of elements has to increase to meet beam requirement we consider symmetric array of 60 elements. For the position-phase synthesis, the prior limits assumed in the minimum and maximum distance between the elements are $d_{\text{min}} = 0.5\lambda$ and $d_{\text{max}} = 0.7\lambda$, respectively. For phase-only synthesis, the uniform distances between the elements are...
assumed to be 0.5\(\lambda\). Figure 12 shows the corresponding array patterns shows the phases and positions derived using the PSO-based phase-only synthesis and position-phase synthesis. We can see that for the position-phase synthesis, the SLL is lower compared to that of the phase-only synthesis.

5. PSO Application in Smart Antenna Array Signal Estimation

Conventional adaptive beamforming algorithms are based on a stationary environment. Assume that the desired signal and interferers are not correlated. Using statistical theory, one requires several successive snapshots of the data to form a covariance matrix of the interference with independent identically distributed secondary data [B. D. Van, IEEE 1986]. The snapshots accumulation is quite time consuming. Thus when the environment becomes nonstationary, an inaccurate covariance matrix is derived, which results in that the interference cannot be rejected. Therefore the adaptive processing using a single snapshot [Markus E. Ali] is more suitable for a dynamic environment. A direct data domain least squares (D3LS) algorithm [T. K. Sarkar, 2000] has been developed to analyze the received data using a single snapshot. Although the D3LS algorithm has certain advantages, it has some drawbacks such that the degrees of freedom are limited to nearly half. Furthermore it is shown by simulations that while the jammers can be rejected, the main lobe of the antenna beam pattern is often deviated from the direction of the desired signal and the sidelobe level is relative high.

5.1 Algorithm Formulation

Consider an array composed of \(N\) sensors separated by a distance as shown in Figure 1. We assume that narrowband signals consisting of the desired signal plus possibly coherent multipath and jammers with center frequency \(f\) are impinging on the array from various angles, with the constraint. For sake of simplicity, we assume that the incident fields are coplanar and that they are located in the far field of the array. Each received signal \(x_m(k)\) includes additive, zero mean, Gaussian noise. Time is represented by the \(k^{th}\) time sample. Thus, for \(X(t) = [x_1(k) \quad x_2(k) \quad x_N(k)]^T\)

\[
x(k) = [\tilde{a}(\theta_1) \quad \tilde{a}(\theta_2) \quad ... \quad \tilde{a}(\theta_M)].\begin{bmatrix} s_1(k) \\
\vdots \\
s_M(k) \end{bmatrix} + \tilde{n}(k) \tag{19}
\]

\(\tilde{a}(\theta_i)\) is \(M\)-elements array steering vector for the \(\theta_i\) direction of arrival, \(\lambda\) wavelength and \(d\) is the elements interspacing distance. \(\tilde{s}(k)\) is the vector of incident signals at time \(k\) and \(\tilde{n}(k)\) is noise vector at each array element \(m\), zero mean, variance. Then for

\[
\tilde{A} = [\tilde{a}(\theta_1) \quad \tilde{a}(\theta_2) \quad ... \quad \tilde{a}(\theta_D)]_{M \times D}
\]

matrix of steering vectors \(\tilde{a}(\theta_i)\)

\[
\tilde{X} = \tilde{A}.\tilde{s}(k) + \tilde{n}(k) \tag{20}
\]

Thus, each of the D-complex signals arrives at angles \(\theta_i\) and is intercepted by the \(M\) antenna elements. It is assumed the number of arriving signals \(D < M\). It is understood that the arriving signals are time varying and thus our calculations are based upon time snapshots of
the incoming signal. Obviously if the transmitters are moving, the matrix of steering vectors is changing with time and the corresponding arrival angles are changing. Let $S_n$ be the complex voltage induced in the $n$th array element at a particular instance of time due to a signal of unity amplitude coming from a direction $\theta_s$,

$$S_n = \exp\left[j2\pi \left(\frac{(n-1)d}{\lambda}\sin(\theta_s)\right)\right]$$

(21)

Let $x_n$ be the complex voltages that are measured at the $n$th element due to the actual signal, jammers and thermal noise

$$x_n = \alpha_s S_n + \text{Interference + Noise}$$

(22)

$$x_n = \alpha_s S_n + \sum_{p=1}^{b-1} \left(A_p \exp\left(j\frac{2\pi(n-1)d}{\lambda}\sin(\theta_p)\right)\right) + n_n$$

(23)

Where $\alpha_s$ denotes the complex amplitude of the desired SOI, $A_p$ and $\theta_p$ are the amplitude and direction of arrival of the $p$ jammer signal, $n_n$ is the thermal noise at the $n$th element. There are $D$ jammers and $D < M - 1/2$. With $S_n$ and $X_n$ ($n=0,...,M$) the known received signal data, one can construct the matrix $X$ and $S$ such that

$$X = \begin{bmatrix} x_0 & x_1 & \ldots & x_L \\ x_1 & x_2 & \ldots & x_{L+1} \\ \vdots & \vdots & \ddots & \vdots \\ x_L & x_{L+1} & \ldots & x_{(L+1)} \end{bmatrix}_{(L+1)\times(L+1)}$$

$$S = \begin{bmatrix} S_0 & S_1 & \ldots & S_L \\ S_1 & S_2 & \ldots & S_{L+1} \\ \vdots & \vdots & \ddots & \vdots \\ S_L & S_{L+1} & \ldots & S_{(L+1)} \end{bmatrix}_{(L+1)\times(L+1)}$$

(24)

From equations (21) and (23) the matrix $U = X - \alpha_s S$, represents the contribution due to signal multipaths, interferes, clutter and thermal noise (i.e., all the undesired components of the signals except SOI). In an adaptive beamforming, the adaptive weight vector $w$ is chosen in such a way that the contribution from the jammers and thermal noise are minimized to enhance the output signal to interference plus noise ratio (SINR). Hence, the following generalized eigenvalue problem is obtained.

$$UW = (X - \alpha_s S)W = 0$$

$$\bar{w} = [w_1 \ w_2 \ \ldots \ \ w_N]^T$$

(25)

Note that $U(1,1)$ and $U(1,2)$ elements of the interference plus noise matrix, are given by

$$U(1,1) = X_1 - \alpha S_{d_1}$$

(26)

$$U(1,2) = X_2 - \alpha S_{d_2}$$

(27)

Where $X_1$ and $X_2$ are the voltages received at antenna elements 1 and 2 due to the signal, jammer, clutter and noise where as $S_{d_1}$ and $S_{d_2}$ are the values of the SOI only at those elements due to a signal of unit strength, let us define $Z$ as follow

$$z = \exp\left[j2\pi \left(\frac{d}{\lambda}\sin(\theta_s)\right)\right]$$

(28)

Then $U(1,1) - z^{-1}U(1,2)$ contains no component of the desired signal. In general, the same is true for $U(i,j) - z^{-1}U(i,j+1)$, $(i = 1, \ldots, L + 1, j = 1, \ldots, L)$. Therefore one can form a square matrix $F$ of dimension $L+1$, generated from $U$. Therefore, in such way, one can form a
reduced rank matrix combined with a constraint that the gain of the subarray is $C$ in the direction $\theta$, then one can obtain equation given as follow

$$
\begin{bmatrix}
Z^0 \\
X_0 - Z^{-1}X_1 \\
\vdots \\
f_3 - f_4Z^{-1}
\end{bmatrix}
\begin{bmatrix}
Z^1 \\
\vdots \\
\vdots \\
X_{L} - Z^{-1}X_{L+1}
\end{bmatrix}
\begin{bmatrix}
W_0 \\
W_1 \\
\vdots \\
W_L
\end{bmatrix}
= 
\begin{bmatrix}
C \\
0 \\
\vdots \\
0
\end{bmatrix}
$$

(29)

To obtain the desired signal component, equation (5.14) is represented as

$$
[F][W] = [Y]
$$

(30)

Using any optimization algorithm to solve equation (30) for, optimum weight vector $[W]$ that provide maximum signal gain through minimizing objective function represented as equation (31)

$$
\zeta(W) = \frac{\|F\|W - Y\|}{\|Y\|} \leq 10^{-6}
$$

(31)

Consequently SOI the signal component $\alpha$ may be estimated from

$$
\alpha = \frac{1}{C} \sum_{i=0}^{L} (W_i X_i)
$$

(32)

The algorithm above is referred to as a “forward method” in the literature [8], [6], [11]. note we can reformulate the problem using the same data to obtain independent estimate for the solution. This can be achieved by two methods:

a. By reversing the data sequence and then complex conjugating each term of that sequence (Backward method)
b. By combining the (forward-backwards method) to double the given data and thereby increase the number of weights (degrees of freedom) significantly over that of either the forward or backward method alone. The number of degrees of freedom can reach to $1 + (N - 1)/1.5$.

To investigate the method let us we consider recovering signal using the previous presented algorithm let us consider a single tone signal with specs as table (3) received by liner array of 10 elements linear array.

<table>
<thead>
<tr>
<th></th>
<th>Magnitude in V</th>
<th>Phase</th>
<th>DOA in degree</th>
</tr>
</thead>
<tbody>
<tr>
<td>Signal</td>
<td>1</td>
<td>0</td>
<td>45°</td>
</tr>
<tr>
<td>Jammer #1</td>
<td>1.25</td>
<td>0</td>
<td>75°</td>
</tr>
<tr>
<td>Jammer # 2</td>
<td>2</td>
<td>0</td>
<td>60°</td>
</tr>
<tr>
<td>Jammer #3</td>
<td>0.5</td>
<td>0</td>
<td>0°</td>
</tr>
</tbody>
</table>

Table 3. Incident signal characteristics

The sampling frequency is 10 $f$; Using PSO algorithm as an optimization tools to solve the optimum $W_i$ for the objective equation (31) value for each iteration we get

\begin{align*}
W_1 &= (1.2996248637+j*0.0724160744), \\
W_2 &= (0.9415241429+j*-0.3236468668) \\
W_3 &= (-0.9898155714+j*-0.1071454180), \\
W_4 &= (-1.2513334352+j*0.3583762104)
\end{align*}

Using these weights in equation 32 to get the value of SOI amplitude

The first ten samplings of the signal and the system output are compared as follow

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<table>
<thead>
<tr>
<th>Initial transmitted signal</th>
<th>Estimated signal</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.9999-j0.000003154</td>
</tr>
<tr>
<td>0.809+j0.5877</td>
<td>0.809+j0.5877</td>
</tr>
<tr>
<td>0.309+j0.951</td>
<td>0.309+j0.951</td>
</tr>
<tr>
<td>-0.309+j0.951</td>
<td>-0.309+j0.951</td>
</tr>
<tr>
<td>-0.809+j0.5877</td>
<td>-0.809+j0.5877</td>
</tr>
<tr>
<td>-1</td>
<td>-0.90.999+j0000003154</td>
</tr>
<tr>
<td>-0.809-j0.587</td>
<td>-0.809+j0.5877</td>
</tr>
<tr>
<td>-0.309-j0.951</td>
<td>-0.309-j0.951</td>
</tr>
<tr>
<td>0.309-j0.951</td>
<td>0.309-j0.951</td>
</tr>
<tr>
<td>0.809-j0.5877</td>
<td>0.80.90-j0.5877</td>
</tr>
</tbody>
</table>

Table 4. Output estimated signal using D3LS and PSO algorithm as an optimization method. The total CPU time taken for the above results is 1.19 sec. PSO is less computational operations compared to conjugate gradient method.

6. Conclusion

PSO application for solving different numerical problems in smart antenna is illustrated. Improvement is proposed to the algorithm to support the continuous real time varying target problem. Also a solution is proposed to overcome the case of bounded search space through introducing of transformation function. Simulation for different scenarios is solved with the aid of PSO. Synthesis of an adaptive Beamforming using the phase only control where target is dynamic over time has been presented. PSO was introduced to solve position-only and position-phase synthesis, which is a bounded search space problem. Finally an investigation for using PSO to estimate signal amplitude though D3LS approach is presented.

7. References


Particle swarm optimization (PSO) is a population based stochastic optimization technique influenced by the social behavior of bird flocking or fish schooling. PSO shares many similarities with evolutionary computation techniques such as Genetic Algorithms (GA). The system is initialized with a population of random solutions and searches for optima by updating generations. However, unlike GA, PSO has no evolution operators such as crossover and mutation. In PSO, the potential solutions, called particles, fly through the problem space by following the current optimum particles. This book represents the contributions of the top researchers in this field and will serve as a valuable tool for professionals in this interdisciplinary field.

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