On the Adaptive Tracking Control of 3-D Overhead Crane Systems

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1. Introduction

For low cost, easy assembly and less maintenance, overhead crane systems have been widely used for material transportation in many industrial applications. Due to the requirements of high positioning accuracy, small swing angle, short transportation time, and high safety, both motion and stabilization control for an overhead crane system becomes an interesting issue in the field of control technology development. Since the overhead crane system is underactuated with respect to the sway motion, it is very difficult to operate an overhead traveling crane automatically in a desired manner. In general, human drivers, often assisted by automatic anti-sway system, are always involved in the operation of overhead crane systems, and the resulting performance, in terms of swiftness and safety, heavily depends on their experience and capability. For this reason, a growing interest is arising about the design of automatic control systems for overhead cranes. However, severely nonlinear dynamic properties as well as lack of actual control input for the sway motion might bring about undesired significant sway oscillations, especially at take-off and arrival phases. In addition, these undesirable phenomena would also make the conventional control strategies fail to achieve the goal. Hence, the overhead crane systems belong to the category of incomplete control system, which only allow a limited number of inputs to control more outputs. In such a case, the uncontrollable oscillations might cause severe stability and safety problems, and would strongly constrain the operation efficiency as well as the application domain. Furthermore, an overhead crane system may experience a range of parameter variations under different loading condition. Therefore, a robust and delicate controller, which is able to diminish these unfavorable sway and uncertainties, needs to be developed not only to enhance both efficiency and safety, but to make the system more applicable to other engineering scopes.

The overhead crane system is non-minimum phase (or has unstable zeros in linear case) if a nonlinear state feedback can hold the system output identically zero while the internal dynamics become unstable. Output tracking control of non-minimum phase systems is a highly challenging problem encountered in many practical engineering applications such as aircraft control [1], marine vehicle control [2], flexible link manipulator control [3], inverted pendulum system control [4]. The non-minimum phase property has long been recognized to be a major obstacle in many control problems. It is well known that unstable zeros cannot
be moved with state feedback while the poles can be arbitrarily placed (if completely controllable). In most standard adaptive control as well as in nonlinear adaptive control, all algorithms require that the plant to be minimum phase. This chapter presents a new procedure for designing output tracking controller for non-minimum phase systems (The overhead crane systems).

Several researchers have dealt with the modeling and control problems of overhead crane system. In [5], a simple proportional derivative (PD) controller is designed to asymptotically regulate the overhead crane system to the desired position with natural damping of sway oscillation. In [6], the authors propose an output feedback proportional derivative controller that stabilizes a nonlinear crane system. In [7], the authors proposed an indirect adaptive scheme, based on dynamic feedback linearization techniques, which was applied to overhead crane systems with two control input. In [8], Li et al attacked the under-actuated problem by blending four local controllers into one overall control strategy; moreover, experimental results delineating the performance of the controller were also provided. In [9], a nonlinear controller is proposed for the trolley crane systems using Lyapunov functions and a modified version of sliding-surface control is then utilized to achieve the objective of cart position control. However, the sway angle dynamics has not been considered for stability analysis. In [10], the authors proposed a saturation control law based on a guaranteed cost control method for a linearized version of 2-DOF crane system dynamics. In [11], the authors designed a nonlinear controller for regulating the swinging energy of the payload. In [12], a fuzzy logic control system with sliding mode Control concept is developed for an overhead crane system. Y. Fang et al. [13] develop a nonlinear coupling control law to stabilize a 3-DOF overhead crane system by using LaSalle invariance theorem. However, the system parameters must be known in advance. Ishide et al. [14] train a fuzzy neural network control architecture for an overhead traveling crane by using back-propagation method. However, the trolley speed is still large even when the destination is arrived, which would result in significant residual sway motion, low safety, and poor positioning accuracy. In the paper [15], a nonlinear tracking controller for the load position and velocity is designed with two loops: an outer loop for position tracking, and an inner loop for stabilizing the internally oscillatory dynamics using a singular perturbation design. But the result is available only when the sway angle dynamics is much faster than the cart motion dynamics. In the paper [16], a simple control scheme, based on second-order sliding modes, guarantees a fast precise load transfer and swing suppression during the load movement, despite of model uncertainties. In the paper [17], it proposes a stabilizing nonlinear control law for a crane system having constrained trolley stroke and pendulum length using the Lyapunov’s second method and performs some numerical experiments to examine the validity of the control law. In the paper [18], the variable structure control scheme is used to regulate the trolley position and the hoisting motion towards their desired values. However the input torques exhibit a lot of chattering. This chattering is not desirable as it might shorten the lifetime of the motors used to drive the crane. In the paper [19], a new fuzzy controller for anti-swing and position control of an overhead traveling crane is proposed based on the Single Input Rule Modules (SIRM). Computer simulation results show that, by using the fuzzy controller, the crane can be smoothly driven to the
destination in a short time with low swing angle and almost no overshoot. D. Liu et al. [20] present a practical solution to analyze and control the overhead crane. A sliding mode fuzzy control algorithm is designed for both X-direction and Y-direction transports of the overhead crane. Incorporating the robustness characteristics of SMC and FLC, the proposed control law can guarantee a swing-free transportation. J.A. Mendez et al. [21] deal with the design and implementation of a self-tuning controller for an overhead crane. The proposed neurocontroller is a self-tuning system consisting of a conventional controller combined with a NN to calculate the coefficients of the controller on-line. The aim of the proposed scheme is to reduce the training-time of the controller in order to make the real-time application of this algorithm possible. Ho-Hoon Lee et al. [22] proposes a new approach for the anti-swing control of overhead cranes, where a model-based control scheme is designed based on a V-shaped Lyapunov function. The proposed control is free from the conventional constraints of small load mass, small load swing, slow hoisting speed, and small hoisting distance, but only guarantees asymptotic stability with all internal signals bounded. This paper also proposes a practical trajectory generation method for a near minimum-time control, which is independent of hoisting speed and distance. In this paper [23], robustness of the proposed intelligent gantry crane system is evaluated. The evaluation result showed that the intelligent gantry crane system not only has produced good performances compared with the automatic crane system controlled by classical PID controllers but also is more robust to parameter variation than the automatic crane system controlled by classical PID controllers. In this paper [24], the I-PD and PD controllers designed by using the CRA method for the trolley position and load swing angle of overhead crane system have been proposed. The advantage of CRA method for designing the control system so that the system performances are satisfied not only in the transient responses but also in the steady-state responses, have also been confirmed by the simulation results.

Although most of the control schemes mentioned above have claimed an adaptive stabilizing tracking/regulation for the crane motion, the stability of the sway angle dynamics is hardly taken into account. Hence, in this chapter, a nonlinear control scheme which incorporates both the cart motion dynamics and sway angle dynamics is devised to ensure the overall closed-loop system stability. Stability proof of the overall system is guaranteed via Lyapunov analysis. To demonstrate the effectiveness of the proposed control schemes, the overhead crane system is set up and satisfactory experimental results are also given.

2. Dynamic Model of Overhead Crane

The aim of this section is to drive the dynamic model of the overhead crane system. The model is dived using Lagrangian method. The schematic plotted in Figure 1 represents a three degree of freedom overhead crane system. To facilitate the control development, the following assumptions with regard to the dynamic model used to describe the motion of overhead crane system will be made. The dynamic model for a three degree of freedom (3-DOF) overhead crane system (see Figure 1) is assumed to have the following postulates.

A1: The payload and the gantry are connected by a mass-less, rigid link.
A2: The angular position and velocity of the payload and the rectilinear position and...
velocity of the gantry are measurable.

**A3:** The payload mass is concentrated at a point and the value of this mass is exactly known; moreover, the gantry mass and the length of the connecting rod are exactly known.

**A4:** The hinged joint that connects the payload link to the gantry is frictionless.

Fig. 1. 3-D Overhead Crane System

The 3-D crane system will be derived based on Lagrange-Euler approach. Consider the 3-dimensional overhead crane system as shown in Figure 1. The cart can move horizontally in x-y plane, in which the moving distance of the cart along the X-rail is denoted as \( x(t) \) and the distance on the Y-rail measured from the initial point of the construction frame is denoted as \( y(t) \). The length of the lift line is denoted as \( l \). Define the angle between the lift line and its projection on the y-z plane as \( \alpha(t) \) and the angle between the projection line and the negative axis as \( \beta(t) \). Then the kinetic energy and potential energy of the system can be found in Equation (1.1) and (1.2), respectively and be expressed as the following equations.

\[
K = \frac{1}{2}m_1\dot{x}^2 + \frac{1}{2}(m_1 + m_2)\dot{y}^2 + \frac{1}{2}m_c(\dot{x}_c^2 + \dot{y}_c^2 + \dot{z}_c^2)
\]  

(1)

\[
V = -mgl \cos \alpha \cos \beta
\]  

(2)

where \( x_c, y_c \) are the related positions of the load described in the Cartesian coordinate, which can be mathematically written as

\[
x_c = x + l \sin \alpha
\]  

(3)
\[ y_c = y + l \cos \alpha \sin \beta \]  
\[ z_c = -l \cos \alpha \cos \beta \]  

The following equations express the velocities by taking the time derivative of above equations

\[ \dot{x}_c = \dot{x} + l \dot{\alpha} \cos \alpha \]  
\[ \dot{y}_c = \dot{y} - l \dot{\alpha} \sin \alpha \sin \beta + l \dot{\beta} \cos \alpha \cos \beta \]  
\[ \dot{z}_c = -l \dot{\alpha} \sin \alpha \cos \beta - l \dot{\beta} \cos \alpha \sin \beta \]  

By using the Lagrange-Euler formulation,

\[ \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} = \tau_i, i = 1, 2, 3, 4. \]  

where \( L = K - V \), \( q_i \) is the element of vector \( q = [x \ y \ \alpha \ \beta]^T \) and \( \tau_i \) is the corresponding external input to the system, we have the following mathematical representation which formulates the system motion

\[ M(q)\ddot{q} + C(q, \dot{q}) + G(q) = \tau \]  

where \( M(q) \in \mathbb{R}^{4\times4} \) is inertia matrix of the crane system, \( C(q, \dot{q}) \in \mathbb{R}^{4\times1} \) is the nonlinear terms coming from the coupling of linear and rotational motion, \( G(q) \in \mathbb{R}^{4\times1} \) is the terms due to gravity, and \( \tau = [u_x \ u_y \ 0 \ 0]^T \) is the input vector.

As mentioned previously, the dynamic equation of motion described the overhead crane system also have the same properties as follows

**P1:** The inertia matrix \( M(q) \) is symmetric and positive definite for all \( q \in \mathbb{R}^n \).

**P2:** There exists a matrix \( B(q, \dot{q}) \) such that \( C(q, \dot{q}) = B(q, \dot{q})\dot{q} \), and \( \forall x \in \mathbb{R}^4 \) \( x^T(M - 2B)x = 0 \), i.e., \( M - 2B \) is skew-symmetric. \( B(q, \dot{q}_x)\dot{q}_y = B(q, \dot{q}_y)\dot{q}_x \).

**P3:** The parameters of the system can be linearly extracted as

\[ M(q)\ddot{q} + C(q, \dot{q}) + G(q) = W_f(q, \dot{q}, \ddot{q})\Phi_f \]  

where \( W_f(q, \dot{q}, \ddot{q}) \) is the regressor matrix and \( \Phi_f \) is a vector containing the system parameters.

**Dynamic Model of Overhead Crane**

In this section, an adaptive control scheme will be developed for the position tracking of an overhead crane system.
2.1 Model formulation

For design convenience, a general coordinate is defined as follows

\[ q^T = [q_p^T \ q_\theta^T] \]

where

\[ q_p^T = [x \ y], \ q_\theta^T = [\alpha \ \beta] \]

and using the relations in P2, the dynamic equation of an overhead crane (10) is partitioned in the following form

\[
\begin{bmatrix}
M_{pp} & M_{p\theta} \\
M_{p\theta} & M_{\theta\theta}
\end{bmatrix}
\begin{bmatrix}
\ddot{q}_p \\
\ddot{q}_\theta
\end{bmatrix}
+
\begin{bmatrix}
B_{pp} & B_{p\theta} \\
B_{\theta p} & B_{\theta\theta}
\end{bmatrix}
\begin{bmatrix}
\dot{q}_p \\
\dot{q}_\theta
\end{bmatrix}
+
\begin{bmatrix}
G_p(q) \\
G_\theta(q)
\end{bmatrix}
=
\begin{bmatrix}
u_p \\
0
\end{bmatrix}
\]

(12)

where \( M_{pp}, M_{p\theta}, M_{\theta\theta}, B_{pp}, B_{p\theta}, B_{\theta p}, B_{\theta\theta} \) are 2×2 matrices partitioned from the inertia matrix \( M(q) \) and the matrix \( B(q, \dot{q}) \), respectively, \( G_p, G_\theta \) are 2×1 vectors, and \( u_p^T = [u_x \ u_y] \). Before investigating the controller design, let the error signals be defined as

\[ e = q - q_d = [e_p^T \ e_\theta^T]^T \]

(13)

and the stable hypersurface plane is defined as

\[ s = \dot{e} + Ke = \begin{bmatrix}
\dot{e}_p + K_p e_p \\
\dot{e}_\theta + K_\theta e_\theta
\end{bmatrix} = \begin{bmatrix}
s_p \\
s_\theta
\end{bmatrix} \]

(14)

where

\[ e_p = q_p - q_{pd} = [x - x_d \ y - y_d]^T \equiv [e_x \ e_y]^T, \]

\[ e_\theta = q_\theta - q_{\theta d} = [\alpha - \alpha_d \ \beta - \beta_d]^T \equiv [e_\alpha \ e_\beta]^T, \]

\[ K_p = \begin{bmatrix}
k_1 & 0 \\
0 & k_2
\end{bmatrix}, \quad K_\theta = \begin{bmatrix}
k_3 & 0 \\
0 & k_4
\end{bmatrix} \]

and \( x_d, y_d, \alpha_d, \beta_d \) are defined trajectories of \( x, y, \alpha \) and \( \beta \) respectively, and \( K_p, K_\theta \) are some arbitrary positive definite matrices.

Then, after a lot of mathematical arrangements, the dynamics of the newly defined signal vectors \( s_p, s_\theta \) can be derived as
\[
\begin{bmatrix}
M_{pp} & M_{p\theta} \\
M_{p\theta}^{T} & M_{\theta\theta}
\end{bmatrix}
\begin{bmatrix}
\dot{\theta}_p \\
\dot{s}_\theta
\end{bmatrix}
+ \begin{bmatrix}
B_{pp} & B_{p\theta} \\
B_{p\theta} & B_{\theta\theta}
\end{bmatrix}
\begin{bmatrix}
s_p \\
s_\theta
\end{bmatrix}
= \begin{bmatrix}
\tau_p + u_p \\
\tau_\theta
\end{bmatrix}
\tag{15}
\]

where
\[
\tau_p = M_{pp}(-\ddot{q}_p + k_p \dot{e}_p) + M_{p\theta}(-\ddot{q}_\theta + k_\theta \dot{e}_\theta) + B_{pp}(-\dot{q}_p + k_p e_p) + B_{p\theta}(-\dot{q}_\theta + k_\theta e_\theta)
\tag{16}
\]
\[
\tau_\theta = M_{p\theta}(-\ddot{q}_p + k_p \dot{e}_p) + M_{\theta\theta}(-\ddot{q}_\theta + k_\theta \dot{e}_\theta) + B_{p\theta}(-\dot{q}_p + k_p e_p) + B_{\theta\theta}(-\dot{q}_\theta + k_\theta e_\theta)
\tag{17}
\]

**Remark 1:** The desired trajectories \(x_d, y_d, \alpha_d\) and \(\beta_d\) should be carefully chosen so as to satisfy the internal dynamics, as shown in the lower part of equation (15), when the control objective is achieved, i.e.,

\[
M_{p\theta}(q_d) \dot{\alpha}_d + M_{\theta\theta}(q_d) \dot{\beta}_d + B_{p\theta}(q, \dot{q}) \dot{x}_d + B_{\theta\theta}(q, \dot{q}) \dot{y}_d + G_\theta(q) = 0
\tag{18}
\]

Without loss of generality, we always choose an exponentially-convergent trajectories with final constant values for \(x_d, y_d\) and zero for \(\alpha_d, \beta_d\).

### 2.2 Adaptive Controller Design

In this subsection, an adaptive nonlinear control will be presented to solve the tracking control problem.

As indicated by property P3 in section 1.2, the dynamic equations of an overhead crane have the well-known linear-in-parameter property. Thus, we define
\[ \omega_1 \dot{\phi}_1 = M_{pp} (\ddot{q}_{pd} + k_p \dot{e}_p) + M_{p\theta} (-\ddot{q}_{d\theta} k_\theta \dot{e}_\theta) + B_{pp} (\ddot{q}_{pd} + k_p e_p) + B_{p\theta} k_p e_\theta \]  

(19)

\[ \omega_2 \dot{\phi}_2 = M_{q} (\ddot{q}_{pd} + k_p \dot{e}_p) + M_{q\theta} (k_\theta \dot{e}_\theta) + B_{q} (\ddot{q}_{pd} + k_p e_p) + B_{q\theta} k_p e_\theta \]  

(20)

where \( \omega_1, \omega_2 \) are regressor matrices with appropriate dimensions, and \( \phi_1, \phi_2 \) are their corresponding vectors of unknown constant parameters, respectively. As a majority of the adaptive controller, the following signal is defined

\[ \dot{Z}_x = \begin{cases} 
2(\sqrt{Z_x} a_x (t) + b_x (t)), & Z_x (t) > 0 \\
2b_x (t), & Z_x (t) = 0, \\
\delta_x, & Z_x (t) < 0 
\end{cases} \]  

(21)

where \( \delta_x \) is some small positive constant and

\[ a_x (t) = \frac{\|s_p\|^2 + \varepsilon}{\|s_p\|^2} (-s_\theta^T \omega_2 \dot{\phi}_2 - s_\theta^T K_{v\theta} s_\theta) \]  

(22)

\[ b_x (t) = \frac{\varepsilon}{\|s_p\|^2 + \varepsilon} (-s_\theta^T \omega_2 \dot{\phi}_2 - s_\theta^T K_{v\theta} s_\theta) \]  

(23)

**Remark 2:** Note that (21) is simply to define a differential equation of which its variable \( Z_x (t) \) remains positive. Let another signal \( k(t) \) be defined to be its positive root, i.e., \( k = \sqrt{Z_x} \). It can be shown that

\[ \dot{k}(t) = \frac{1}{k(t)} \left( \frac{k\|s_p\|^2 + \varepsilon}{\|s_p\|^2 + \varepsilon} \right) \left( -s_\theta^T \omega_2 \dot{\phi}_2 - s_\theta^T K_{v\theta} s_\theta \right) \quad k \neq 0 \]  

(24)

In the sequel, we will first assume that there exists a measure zero set of time sequences \( \{t_i\}_{i=1}^{\infty} \) such that \( Z(t_i) = 0 \) or \( k(t_i) = 0 \), \( i = 1, 2, 3, \ldots, \infty \), and then, verify the existence assumption valid.

Now let the adaptive control law be designed as

\[ u_p = -\omega_1 \dot{\phi}_1 - \tau_v - K_{vp} s_p \]  

(25)
\[
\tau_v = \left[ \frac{(k-1)s_p}{s_p^2 + \varepsilon} \right] (s_\theta^T \omega_2 \hat{\phi}_2 - s_\theta^T K_{v\theta} s_\theta)
\]

where

\[
\omega_1 \hat{\phi}_1 = \hat{M}_{pp}(\ddot{q}_{pd} + k_p \dot{e}_p) + \hat{M}_{p\theta}(k_\theta \dot{\theta}_p) + \hat{B}_{pp}(\dot{q}_{pd} + k_p e_p) + \hat{B}_{p\theta} k_p e_p
\]

\[
\omega_2 \hat{\phi}_2 = \hat{M}_{\theta p}(\ddot{q}_{pd} + k_p \dot{e}_p) + \hat{M}_{\theta\theta}(k_\theta \dot{\theta}_p) + \hat{B}_{\theta p}(\dot{q}_{pd} + k_p e_p) + \hat{B}_{\theta\theta} k_\theta e_\theta
\]

and \(\hat{\phi}_1, \hat{\phi}_2\) are the estimates of \(\phi_1, \phi_2\) respectively, then the error dynamics can be obtained as

\[
\begin{bmatrix}
M_{pp} & M_{p\theta} \\
M_{\theta p}^T & M_{\theta\theta}
\end{bmatrix}
\begin{bmatrix}
\dot{s}_p \\
\dot{s}_\theta
\end{bmatrix}
+ \begin{bmatrix}
B_{pp} & B_{p\theta} \\
B_{\theta p} & B_{\theta\theta}
\end{bmatrix}
\begin{bmatrix}
s_p \\
s_\theta
\end{bmatrix}
+ \begin{bmatrix}
K_{vp} & 0 \\
0 & K_{v\theta}
\end{bmatrix}
\begin{bmatrix}
s_p \\
\dot{s}_\theta
\end{bmatrix}
= \begin{bmatrix}
\omega_1 \hat{\phi}_1 - \tau_v \\
\omega_2 \hat{\phi}_2 + K_{v\theta} s_\theta
\end{bmatrix}
\]

or more compactly as

\[
M(q) \dot{s} + h(q, \dot{q}) s + K s = \begin{bmatrix}
\omega_1 \hat{\phi}_1 - \tau_v \\
\omega_2 \hat{\phi}_2 + K_{v\theta} s_\theta
\end{bmatrix}
\]

where

\[
\begin{bmatrix}
\hat{\phi}_1 \\
\hat{\phi}_2
\end{bmatrix}
= \begin{bmatrix}
\hat{\phi}_1 - \phi_1 \\
\hat{\phi}_2 - \phi_2
\end{bmatrix}
\]

Moreover, let the adaptation laws be chosen as

\[
\begin{align*}
\dot{\phi}_1 &= -k_a \omega_1 s_p \\
\dot{\phi}_2 &= -k_b \omega_2 s_\theta
\end{align*}
\]

where \(k_a, k_b\) are some positive definite gain matrices. In what follows we will show that the error dynamics (30) along with the adaptive laws (32) constitutes an asymptotically stable closed-loop dynamic system. This is exactly stated in the following theorem.
Theorem: Consider the 3-D overhead crane system as mathematically described in (10) or (12) with all the system parameters unknown. Then, by applying control laws (25)-(28) and adaptive laws (32), the objective for the tracking control problem can be achieved, i.e., all signals inside the closed-loop system (29) are bounded and $e_x, e_y, e_z, e_\theta \to 0$ asymptotically in the sense of Lyapunov.

Proof: Define the Lyapunov function candidate as

$$V(t) = \frac{1}{2} s^T M(q)s + \frac{1}{2} \tilde{\phi}_1^T k_a^{-1} \tilde{\phi}_1 + \frac{1}{2} \tilde{\phi}_2^T k_b^{-1} \tilde{\phi}_2 + \frac{1}{2} Z_x$$

It is obvious that, due to the quadratic form of system states as well as the definition of $Z_x(t)$, $V(t)$ is always positive-definite and indeed a Lyapunov function candidate. By taking the time derivative of $V$ we get

$$\dot{V}(t) = s^T M(q) \dot{s} + \frac{1}{2} s^T \dot{M}(q)s + \tilde{\phi}_1^T k_a^{-1} \tilde{\phi}_1 + \tilde{\phi}_2^T k_b^{-1} \tilde{\phi}_2 + kk$$

$$= s^T (-B(q, \dot{q})s - K_{vp}s + \begin{bmatrix} -\omega_1 \tilde{\phi}_1 - \tau_v \\ \omega_2 \dot{\phi}_2 + K_{v\theta}s_\theta \end{bmatrix}) + \frac{1}{2} s^T \dot{M}(q)s + s_p^T \omega_1 \tilde{\phi}_1 + s_\theta^T \omega_2 \tilde{\phi}_2$$

$$+ \left( \frac{k\|s_p\|^2}{\|s_p\|^2 + \varepsilon} \right)(-s_\theta^T \omega_2 \dot{\phi}_2 - s_\theta^T K_{v\theta}s_\theta)$$

$$= -sKs - s_p^T \omega_1 \tilde{\phi}_1 - \left( \frac{(k-1)\|s_p\|^2}{\|s_p\|^2 + \varepsilon} \right)(-s_\theta^T \omega_2 \dot{\phi}_2 - s_\theta^T K_{v\theta} s_\theta) + s_\theta^T \omega_2 \phi_2 + s_p^T \omega_1 \tilde{\phi}_1$$

$$+ \left( \frac{k\|s_p\|^2}{\|s_p\|^2 + \varepsilon} \right)(-s_\theta^T \omega_2 \dot{\phi}_2 - s_\theta^T K_{v\theta} s_\theta) + s_\theta^T \omega_2 \tilde{\phi}_2 + s_\theta^T K_{v\theta} s_\theta$$

$$= -s^T Ks \quad (33)$$
It is clear that $\dot{V}(t) < 0$ as long as $K > 0$, which then implies $s, k, \phi_1, \phi_2 \in L_\infty$. Now, assume that $k(t) = 0$ instantaneously at $t_i$. Because the solution $Z_x(t)$ of the equation (21) is well defined and is continuous for all $t \geq 0$, $k(t)$ is continuous at $t_i$, i.e., $k(t_i^-) = k(t_i^+)$. Since $V$ is a continuous function of $k$, it is clear that $V(t)$ remains to be continuous at $t_i$, i.e., $V(t_i^-) = V(t_i^+)$. Form then hypothesis, $\dot{V}(t_i^-) < 0$ and $\dot{V}(t_i^+) < 0$, we hence can conclude that $V$ is nonincreasing in $t$ including $t_i$, which then readily implies that $s, k \in L_\infty$. Therefore, $e, \tau_v$ and $\tau_\theta \in L_\infty$ directly from equation (13) and definitions of $\tau_v$ and $\tau_\theta$. It then follows from (30) that $s \in L_\infty$. On the other hand, if the set of time instants $\{t_i\}_{i=1}^\infty$ is measure zero, then

$$-\int_0^\infty \dot{V} dt = V(0) - V(\infty) < \infty \text{ or equivalently that } -\int_0^\infty \|s\|^2 dt < \infty \text{ so that } s \in L_2.$$ 

Form the error dynamics, we can further conclusion that $s \to 0$ as $t \to \infty$ asymptotically as $t \to \infty$ and therefore, $e, \dot{e} \to 0$ as $t \to \infty$. Note that in the above proof we have used the property $(M(q) - 2B(q, \dot{q}))$ is skew-symmetric. Finally, to complete the proof in theory, we need to show that the above hypothesis that the set of time instants $\{t_i\}_{i=1}^\infty$ is indeed measure zero. However, it is quite straightforward to conclude the result from (21) by simply using the fact that all signals are bounded. This completes our proof.

**Remark 3:** From the robustness point of view, it would be better if additional feedback term $-k_q s_\theta$ is included in the control law (24). With such an inclusion, the sway stabilization result subject to external disturbance can also be maintained as the cart arrived at its destination. This can be easily checked from the stability proof given in the theorem.

**Proof:** Let the Lyapunov function candidate be chosen as

$$V(t) = \frac{1}{2} s^T M(q) s + \frac{1}{2} \phi_1^T k_a^{-1} \phi_1 + \frac{1}{2} \phi_2^T k_b^{-1} \phi_2 + \frac{1}{2} Z_x$$

and take the time derivative of $V$ to get

$$\dot{V}(t) = s^T M(q) \dot{s} + \frac{1}{2} s^T M(q) s + \phi_1^T k_a^{-1} \phi_1 + \phi_2^T k_b^{-1} \phi_2 + k \dot{k}$$

$$= s^T (-B(q, \dot{q}) s - K_{vp} s + \begin{bmatrix} -\omega_1 \phi_1 - \tau_v - k_q s_\theta \\ \omega_2 \phi_2 + K_{v\theta} s_\theta \end{bmatrix}) + \frac{1}{2} s^T M(q) s + s_p^T \omega_1 \phi_1 + s_\theta^T \omega_2 \phi_2$$

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\[
\begin{aligned}
&+ \left( \frac{\|s_p\|^2 + \varepsilon}{\|s_p\|^2 + \varepsilon} \right) (-s_\theta^T \omega_2 \hat{\phi}_2 - s_\theta^T K_\nu s_\theta) \\
&= -s^T K s - k_q s_\theta s_p \\
\dot{V}(t) &= -s^T K s - k_q s_\theta s_p \\
\dot{V}(t) &\leq -\lambda_{\text{min}}(K_v)(s_p^2 + s_\theta^2) + \frac{1}{2} k_q (s_p^2 + s_\theta^2) \\
&= -(\lambda_{\text{min}}(K_v) - \frac{1}{2} k_q)(s_p^2 + s_\theta^2)
\end{aligned}
\]

Thus, the same conclusion can be made as previously if

\[\lambda_{\text{min}}(K_v) > \frac{1}{2} k_q\]

3. Computer Simulation

In this subsection, several simulations are performed and the results also confirm the validity of our proposed controller. The desired positions for \(X\) and \(Y\) axes are 1 m. Figure 3 shows the time response of X-direction. Figure 5 show the time responses of Y-direction. It can be seen that the cart can simultaneously achieve the desired positions in both X and Y axes in approximately 6 seconds with the sway angles almost converging to zero at the same time. Figure 4 and Figure 6 show the response of the sway angle with the control scheme. Figure 7 and Figure 8 show the velocity response of both X-direction and Y-direction. Figure 9 and Figure 10 show the control input magnitude. In Figure 11~14, the parameter estimates are seen to converge to some constants when error tends to zero asymptotically and the time response of the tuning function \(k(t)\) is plotted in Figure 15.

The control gains are chosen to be

\[
k_p = \begin{bmatrix} 1.5 & 0 \\ 0 & 1 \end{bmatrix}, k_\theta = \begin{bmatrix} 2.35 & 0 \\ 0 & 1 \end{bmatrix},
\]
The corresponding adaptive gains are set to be $k_a = k_b = 1$.
Fig. 6. Sway Angle Response $\beta(t)$ with Adaptive Algorithm

Fig. 7. Gantry Velocity Response $\dot{x}(t)$ with Adaptive Algorithm

Fig. 8. Gantry Velocity Response $\dot{y}(t)$ with Adaptive Algorithm
Fig. 9. Force Input $u_x$

Fig. 10. Force Input $u_y$

Fig. 11. Estimated Parameters $\phi_{14}(t)$
Fig. 12. Estimated Parameters

Fig. 13. Estimated Parameters $\phi_{1y}(t)$

Fig. 14. Estimated Parameters $\phi_{2\alpha}(t)$
4. **Experimental Verification**

In this section, to validate the practical application of the proposed algorithms, a three degree-of-freedom overhead crane apparatus, is built up as shown in Figure 16. Several experiments are also performed and indicated in the subsequent section for demonstration of the effectiveness of the proposed controller.

![Experimental setup for the overhead crane system](image)

Fig. 16. Experimental setup for the overhead crane system

The control algorithm is implemented on a xPC Target for use with real time Workshop® manufactured by The Math Works, Inc., and the xPC target is inserted in a Pentium4
2.4GHz PC running under the Windows operating system. The sensing system includes the two photo encoders and two linear position sensors. The cart motion X-direction and Y-direction motion measured by linear potentiometer. Two potentiometers are connected to the travel direction and the traverse direction. An AC servo motor with 0.95 N-m maximum torque and 3.8N-m maximum torque output is used to drive the cart motion X direction and Y direction. The servomotors are set in torque control mode so as to output the desired torques.

In the experimental study, the proposed control algorithms have been tested and compared with the conventional PD controller. From the experimental results, it is found that our proposed algorithms indeed outperform the conventional control scheme in all aspects. A schematic description of the experimental system is draw in Figure 17.

Fig. 17. A Schematic Overview of the Experimental Setup

4.1 Experiments for Conventional PD control as a comparative study
In the experiments, a simple PD control scheme with only position and velocity feedback is first tested for the crane control. Figure 18 and Figure 20 show the control responses. From Figure 19 and Figure 21 it is observed that the sway oscillation cannot be rapidly damped by using only conventional PD control, although the tracking objective is ultimately achieved.
Fig. 18. Gantry Tracking Response $x(t)$ with Conventional PD Control

Fig. 19. Sway Angle Response $\alpha(t)$ with Conventional PD Control

Fig. 21. Sway Angle Response $\beta(t)$ with Conventional PD Control
4.2 Experiments for the Proposed Adaptive Control Method with Set-point Regulation

In the subsection, the developed adaptive controller is applied. The following controlled gains are chosen for experiments.

\[
k_p = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}, \quad k_\theta = \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}, \quad k_{vp} = \begin{bmatrix} 1.5 & 0 \\ 0 & 3 \end{bmatrix}, \quad k_{v\theta} = \begin{bmatrix} 1.35 & 0 \\ 0 & 2 \end{bmatrix}
\]

The corresponding adaptive gains are set to be 1 i.e., \( a = b = 1 \). Figure 22~31 depict the experimental results for the crane system with the adaptive control law. Figure 22 and Figure 24 demonstrate the tracking performance in X and Y directions. It is experimentally demonstrated that the sway angle can be actively damped out by using our proposed adaptive schemes, as shown in Figure 23 and Figure 25 with maximum swing angle about 0.05 rad and 0.06 rad, respectively. Figure 26 and Figure 27 show the input torques from each AC servo motors, whereas Figure 28~30 plot the associated adaptive gain turning trajectories. The trajectory of coupling gain \( k(t) \) is also in Figure 31 with initial value 0.05. The initial values of other state variable are all zero. Apparently the tracking and damping performances by applying the adaptive control algorithm are much better than the ones resulting from the PD control.

Fig. 20. Gantry Tracking Response \( y(t) \) with Conventional PD Control
Fig. 22. Gantry Tracking Response with Adaptive Algorithm $X(t)$

Fig. 23. Sway Angle Response with Adaptive Algorithm $\alpha(t)$

Fig. 24. Gantry Tracking Response with Adaptive Algorithm $Y(t)$
Fig. 25. Sway Angle Response with Adaptive Algorithm $\beta(t)$

Fig. 26. Force Input $U_x$

Fig. 27. Force Input $U_y$
Fig. 28. Estimated Parameters $\phi_{1x}(t)$

Fig. 29. Estimated Parameters $\phi_{1y}(t)$

Fig. 30. Estimated Parameters $\phi_{2\alpha}(t)$ and $\phi_{2\beta}(t)$
Fig. 31. Trajectory of $k(t)$

4.3 Experiments for the Proposed Adaptive Control with Square Wave Tracking

To prove the prevalence of our controllers, experiments on the tracking of square wave, as shown in Figure 6, is also conducted. The gains are kept the same as in the previous experiments. Figure 6(a) and Figure 6(c) demonstrate the tracking performance in X and Y directions, respectively while Figure 6(b) and Figure 6(d) show the suppression results of sway angles. It is found that good performance can still be preserved in spite of the sudden change of desired position.

Fig. 32. Desired Trajectory
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Fig. 33. Tracking Response $x(t)$ with Adaptive Algorithm

Fig. 34. Sway Angle Response $\alpha(t)$ with Adaptive Algorithm

Fig. 35. Tracking Response $y(t)$ with Adaptive Algorithm
5. Conclusion

In this chapter, a nonlinear adaptive control law has been presented for the motion control of overhead crane. By utilizing a Lyapunov-based stability analysis, we can achieve asymptotic tracking of the crane position and stabilization of payload sway angle for an overhead crane system which is subject to both underactuation and parametric uncertainties. Comparative simulation studies have been performed to validate the proposed control algorithm. To practically validate the proposed adaptive schemes, an overhead crane system is built up and experiments are also conducted. Both simulations and experiments show better performance in comparison with the conventional PD control.

6. References

Philippe Martin, Santosh Devasia, and Brad Paden, 1996, “A Different Look at Output


**APPENDIX A**

Mathematical Description of The Dynamic Model

The dynamic equation of the 3D overhead crane system can be derived by using

Largrange-Euler formula and shown in the following

\[ M(q)\ddot{q} + C(q, \dot{q}) + G(q) = \tau \]

where

\[
M(q) = \begin{bmatrix}
m_1 + m_c & 0 & m_c l \cos \alpha & 0 \\
0 & m_1 + m_2 + m_c & -m_c l \sin \alpha \sin \beta & m_c l \cos \alpha \cos \beta \\
m_c l \cos \alpha & -m_c l \sin \alpha \sin \beta & m_c l^2 & 0 \\
0 & m_c l \cos \alpha \cos \beta & 0 & m_c l^2 \cos^2 \alpha
\end{bmatrix}
\]
On the Adaptive Tracking Control of 3-D Overhead Crane Systems

\[ C(q, \dot{q}) = \begin{bmatrix}
-m_c l \dot{\alpha}^2 \sin \alpha \\
-m_c l (\dot{\alpha}^2 + \dot{\beta}^2) \cos \alpha \sin \beta - 2 m_c l \dot{\alpha} \dot{\beta} \sin \alpha \cos \beta \\
m_c l^2 \dot{\beta}^2 \sin \alpha \cos \alpha \\
-2 m_c l^2 \dot{\alpha} \dot{\beta} \sin \alpha \cos \alpha
\end{bmatrix} \]

\[ G(q) = \begin{bmatrix}
0 \\
0 \\
-m_c gl \sin \alpha \cos \beta \\
-m_c gl \cos \alpha \sin \beta
\end{bmatrix} \]

\[ \tau = [u_x \ u_y \ 0 \ 0]^T \]

\[ q = [x \ y \ \alpha \ \beta]^T \]

To satisfy property P2 as stated in section 2 the vector \( C(q, \dot{q}) \) can be re-arranged as \( C(q, \dot{q}) = B(q, \dot{q}) \dot{q} \) where

\[ B(q, \dot{q}) = \begin{bmatrix}
0 & 0 & -m_c l \dot{\alpha}^2 \sin \alpha \\
0 & 0 & -m_c l \dot{\alpha} \cos \alpha \sin \beta - m_c l \dot{\beta} \sin \alpha \cos \beta \\
0 & 0 & 0 \\
0 & 0 & -m_c l^2 \dot{\beta} \sin \alpha \cos \beta \\
0 & -m_c l \dot{\alpha} \cos \beta - m_c l \dot{\beta} \cos \alpha \sin \beta \\
m_c l^2 \sin \alpha \cos \beta \\
- m_c l^2 \dot{\alpha} \sin \alpha \cos \beta
\end{bmatrix} \]

It can be easily checked that
\[
\dot{M} - 2C = \begin{bmatrix}
0 & 0 \\
0 & 0 \\
-m_c \hat{l} \sin \alpha & -m_c \hat{l} \cos \alpha \sin \beta - m_c \hat{l} \hat{\beta} \sin \alpha \cos \beta \\
0 & 0 \\
m_c \hat{l} \sin \alpha & 0 \\
m_c \hat{l} \cos \alpha \sin \beta + m_c \hat{l} \hat{\beta} \sin \alpha \cos \beta & 0 \\
0 & -2m_c \hat{l}^2 \hat{\beta} \sin \alpha \cos \beta \\
2m_c \hat{l}^2 \hat{\beta} \sin \alpha \cos \beta & 0
\end{bmatrix}
\]

which is skew-symmetrical matrix.
Adaptive control has been a remarkable field for industrial and academic research since 1950s. Since more and more adaptive algorithms are applied in various control applications, it is becoming very important for practical implementation. As it can be confirmed from the increasing number of conferences and journals on adaptive control topics, it is certain that the adaptive control is a significant guidance for technology development. The authors the chapters in this book are professionals in their areas and their recent research results are presented in this book which will also provide new ideas for improved performance of various control application problems.

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