Identification of Dynamic Systems & Selection of Suitable Model

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1. Introduction

Process Industry is growing very rapidly. To tackle this fast growth, current control methods need to be replaced to produce product with compatible quality & price. Normally the systems are described by suitable mathematical models. These models are replaced by actual process later on. Actually controllers are designed on behalf of suitable models to control the process effectively. So suitable models are very crucial. Different purposes demand for different types of models where the objective could be: (Bjorn Sohlberg, 2005)

- Construction of controllers to control the process.
- Simulation of control system to analyze the effect of changing reference
- Simulate the behaviour of system during different production situations.
- Supervise different parts of process which properties change due subjected to wear or changing product quality.

The exact model of any system will reflect detailed description. A simple feedback controller demands a simple process description than a process description which is going to be used for supervision of wear. Often a more advanced application, demand for a more complex model. The relation between the purpose of the model and its complexity is shown below.

<table>
<thead>
<tr>
<th>Feedback Control</th>
<th>Feedforward Control</th>
<th>Process Simulation</th>
<th>Process Supervision</th>
</tr>
</thead>
</table>

Fig. 1. Model Complexity

The development of information technology has opened new prospective in modelling and simulation of processes used in different scientific applications. There are different types of models which will be discussed in next section. In this chapter we will discuss different types of models, Identification techniques using matlab identification toolbox & different examples. Several aspects on experimental design for identification purposes will be also discussed. In a nutshell this chapter will be useful especially for those who want to do linear black box identification. For any given system/process modelling & identification techniques would be useful to apply after proper understanding of this chapter.
1.1 Types of models

To describe a process or a system we need a model of system. This is nothing new, since we use models daily, without paying this any thoughts. For example, when we drive a car and approaching a road bump, we slow down because we fell intuitively that when this speed is too high we will hit the head in the roof. So from experiences we have developed a model of car driving. We have a feeling of how the car will behave when reach the bump and how we will be affected. Here the model of situation can be considered as a mental model. We can also describe the model by linguistic terms. For example if we drive the car faster than 110km/h then we will hit the head at the roof. This is linguistic model, since the model uses words to describe what happens. (Bjorn Sohlberg, 2005)

A third way of describing the systems is to use scientific relations to make a mathematical model, which describes in what way output signals respond due to changes in input signal. There are different types of models to represent the systems.

1. **White Box Modelling:** When a model is developed by modelling, we mean that model is constructed completely from mathematical scientific relations, such as differential equations, difference equations, algebraic equations and logical relations. The resulting model is called white box or a simulation model.

   **Example:** For example a model of electrical network using Kirchhoff’s laws and similar theorems:

   ![Fig. 2. RC Circuit](image)

   In above RC-circuit where the relation between the input signal $u(t)$ and output signal $y(t)$ is given by Ohm’s law. The resulting model would be a linear differential equation with the unknown parameter $M=RC$, which can be estimated form an experiment with the circuit or formal nominal values of the resistor and the capacitor. A mathematical model is given by:

   $$M\dot{y}(t) + y(t) = u(t)$$

   Similarly other processes can be modelled using scientific relations.

2. **Black Box Modelling:** When a model is formed by means of identification, we consider the process completely unknown. The process is considered black box with inputs and outputs. Thus it is not necessary to use any particular model structure which reflects the physical characteristics of the system. Normally we use a model which given from a group of standard models. Unknown model parameters are estimated by using measurement data which is achieved from an experiment with the process. In this way model shows input-output relation.
Identification using black box models have been used for industrial, economic, ecological and social systems. Within industry, black box models have been used for adaptive control purposes.

**Example:** Consider a standard model given by equation 2. The process consists of one input signal \( u(k) \) and one output signal \( y(k) \). Here there are two unknown parameters \( a \) and \( b \). These parameters are estimated using identification from measured data of process.

\[
y(k) + a \cdot y(k-1) = b \cdot u(k-1)
\]

(2)

We want the model output to look like the process output as good as possible. The difference between the process and model outputs, the error \( e(t) \) will be a measure to minimise to find the values of the parameters that is \( a \) and \( b \).

![Fig. 3. Black Box Identification](image)

3. **Grey Box Modelling:** For many processes there is some but incomplete knowledge about the process. The amount of knowledge varies from one process to another. Between the white box and black box models there is grey zone.

<table>
<thead>
<tr>
<th>Type of Model</th>
<th>Application Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>Black Box Models</td>
<td>Process Control</td>
</tr>
<tr>
<td>Grey Box Models</td>
<td>Economical Systems, Hydrological Systems</td>
</tr>
<tr>
<td>White Box Models</td>
<td>Electronic Circuits</td>
</tr>
</tbody>
</table>

Table. 1. Grey Box Models

The other two common terms in modelling are deterministic and stochastic models. In deterministic models we neglect the influence of disturbance. It is not realistic to make a perfect deterministic model of a real system. The model would be too expensive to develop and would probably be too complex to use. Therefore it is good idea to divide the model into two parts; one deterministic part and one stochastic.

2. **Linear black box identification**

Black box identification deals with identification of a system using linear models from a family of standard models. The tentative black box model consists of unknown parameters, needed to be estimated from measured data. Some linear models are ARX, ARMAX and
similar types of other models. Prerequisite for black box identification is measured data, which are achieved from an experiment with the system. Experimental design will be discussed in next section. During the identification of the model procedure, we normally let three models ARX, ARMAX and OUTPUT-ERROR to find the best model.

2.1 ARX models:
The most common black box model identification is named as ARX-model (Bjorn Sohlberg, 2005). ARX stands for auto regression exogenous. By using the shift operator \( q^{-1} \), the model is reformulated in the following form:

\[
A(q^{-1})y(k)=B(q^{-1})u(k)+e(k)
\]  
(3)

The following polynomials \( A(q^{-1}) \) and \( B(q^{-1}) \) are given by equations (4) and (5), where \( n_a \) and \( n_b \) are positive number which define the order of the polynomials.

\[
A(q^{-1}) = 1 + a_1q^{-1} + \ldots + a_{na}q^{-na}
\]  
(4)

\[
B(q^{-1}) = b_1q^{-1} + \ldots + b_{nb}q^{-nb}
\]  
(5)

Observations while using ARA Model:
- Easy to use
- Models the disturbance as an regression process (output is non-white even when input=0)
- Better disturbance models than that in Output-Error
- Poles of the dynamic model and poles of the disturbance model coincide; as a result, modelling is not very flexible.

2.2 ARMAX -model:
The model given by equations (6) can be augmented to include a model of the disturbance. ARMAX stands for auto regression moving average exogenous model. Mathematically, this can be introducing a polynomial \( C(q^{-1}) \):

\[
A(q^{-1})y(k) = B(q^{-1})u(k) + C(q^{-1})e(k)
\]  
(6)

\[
A(q^{-1}) = 1 + a_1q^{-1} + \ldots + a_{na}q^{-na}
\]  
(7)

\[
B(q^{-1}) = b_1q^{-1} + \ldots + b_{nb}q^{-nb}
\]  
(8)

\[
C(q^{-1}) = 1 + c_1q^{-1} + \ldots + c_{nc}q^{-nc}
\]  
(9)

Observations while using ARMAX Model:
- More complex than ARX model
- Poles of dynamic model and disturbance model are same, as in ARX, but provides extra flexibility with an MA model of disturbance
2.3. Output-Error model

When the disturbances mainly influence the measurements of the output signal, the general model can be transformed to the output error model:

\[ F(q^{-1})y(k) = B(q^{-1})u(k) + e(k) \]  

(10)

The simulation and use of these models will be shown in case study section.

3. Parameter estimation

There are different estimators available. To estimate the parameters one should keep following in the mind:

- The model is never an exact representation of the system
- Undesirable noise always contaminates the measured data
- The system itself may contain sources of disturbance
- Error between the measured output(s) and the model output(s) is unavoidable
- A good identification is one that minimizes this error

![Fig. 4. Difference between the process and model output](image)

3.1 Least squares estimation

Parameter estimation using lease minimization is an early applied method to estimate unknown parameters in mathematical models. The theory was developed in the beginning of 1800 century by Gauss and Legendre. The parameters are estimated such that the sum of the squares of errors is minimized. For an error vector \([e]_{N \times 1}\), the LSE minimizes the following sum

\[ V = \frac{1}{N} \sum_{i=1}^{N} e_i^2 = \frac{1}{N} e^T e \]  

(11)

This sum is also known as the Loss Function. Next, we shall generalize the least squares estimation problem for a system with any arbitrary relationship between input & output. Relation exists between the response (dependent variable) of the system under test and regressor (independent variables) via some function. This is represented by linear regression model as:

\[ y = f(\varphi_1, \varphi_2, ..., \varphi_p, \theta) + v \]  

(12)

The relationship is known except for the constants or coefficients \(\theta\) called parameters and a possible disturbance \(v\). The term \(\varphi_i\) could be taken as regressor. An important special case for the function \(f\) is linear regression based on the model:
\[ y = \varphi \theta + v \]  

(13)

We will show its implementation in our research work later on. In case of colour noise affecting the process we use pseudo least square method. **Example:** Estimating the parameters of a 2nd order ARX model of the following order:

\[ y(k) + a_1 y(k-1) = b_1 u(k-1) + b_2 u(k-2) + e(k) \]  

(14)

Using matlab system identification toolbox we can do it in following way:

\[
\begin{align*}
\text{>> z} & \quad = \text{iddata(y u)} & \% \text{From measured data} \\
\text{>> mn} & \quad = [1 2 1] & \% \text{Configure the order of the model} \\
\text{>> m} & \quad = \text{arx(z, mn)} & \% \text{Estimate unknown parameters} \\
\text{>> present(m)} & \quad \% \text{Present values and accuracy of estimates.}
\end{align*}
\]

### 4. Model analysis

After the model parameters have been estimated by using measured data, the model has to be analysed. It is important to investigate the quality of model and how well the model is adapted to measured data. By model analysis we will study in what way the model describes the static and dynamic characteristics of the process. Further we will study if the parameter estimates are reproducible. This is done by using two or more different measurement sequences and comparing the estimates by each of them. It is also interesting to calculate residual. The value of the loss function is also used when we are going to choose between different model candidates. Usually the model having lower loss function is preferred. Moreover we check the frequency characteristics. Below is short summary of steps:

#### 4.1 Simulation

The model is simulated using the inputs from the experiment and the outputs from the model and process are plotted in the same diagram and study if the curves are about the same. In short dynamics of the curves should be same and follow the same trajectory. A systematic difference in the levels is possible to compensate by using regulator. **Example:** Below is one example for simulation of model and real process. Both curves are matched in this case.

![Fig. 5. Simulation](www.intechopen.com)
4.2 Statistical analysis
There are several tests which can be used to study whether the residual sequence is white noise. The most important are autocorrelation of the residuals, cross correlation between the residuals. A simple and fast way to get an opinion about the residual is to make a plot in time diagram. Trends in signal will get clear overview. In short we take care of following points while doing statistical analysis.
- Autocorrelation
- Cross correlation
- Normal Distribution
- Residual Plot

4.3 Model structure analysis
When a model is constructed, it should describe the behaviour of the system as perfect as possible. As a measure of perfectness of the model we can use the loss function, since a better model will generate smaller residuals than a worse model. It is observed from experiments that loss function will decrease with the increase in number of parameters. This means accuracy of estimated parameters will decrease.

4.4 Parameter analysis
If possible, the experiment is repeated so we will have two different measurements sequences. The circumstances around the experiments should be as similar as possible. During these conditions, we investigate whether it is possible to reproduce the same value of the estimates.
The results from estimation can also be presented by a pole/zero plots. We can find whether the model is over determined and too many parameters are estimated. In case of overlapping the two poles and zeros upon each other, the order of system should be reduced.
Example: Pole Zero Diagram of system which is not over determined.

Fig. 6. Pole Zero Diagram
4.5 Frequency analysis
The frequency analysis is a complement to the analysis in time space. It is used to give information about whether the frequency relation between inputs and outputs is covered by model.
The method gives possibility to investigate whether the model can describe the characteristics of the process within a specific interesting frequency range. The frequency plot based on an estimated model and spectrum from measured input/output signals is performed by using Matlab system identification tool box. This is shown in Fig. 16.

5. Model appraisal
During the model appraisal, the model is evaluated based on the purpose of model. This model will be used in some way in feedback control, Feedforward control, model predictive control supervision or failure detection. Following points could be useful during model appraisal.

- When the model is going to be used for designing PID-controllers, then dynamical properties are most important. This can be analysed by plotting the outputs from the process and the model in same diagram. It is important that the model outputs should follow the variations in the process outputs while it is not necessary that both are same.
- When the model is going to be used for simulation purposes, then statistical properties of the model and process are important.
- When the model is going to be used for supervision of parameters within the process which are not possible to measure by a transducer it is necessary to have a model which contains white box parts.

6. Case study: active control of tall structure building
Considerable attention has been paid to active structural control research in recent years; with particular emphasis on alleviation of wind and seismic response. There have been numerous investigations, both analytical and experimental, into the area of passive vibration control of tall buildings in previous decades. Passive vibration control devices such as tuned mass dampers (TMD) have proven to be effective for certain applications but they are limited in the magnitude of motion reduction they can achieve. These limitations have led to the development of active control devices. This device uses a control algorithm which analyses the dynamic structural feedback to create a control force which drives a mass. The theory for active control has been extensively investigated for the past two decades and it has been found to be a superior method of vibration control.

6.1 Background
In recent years, innovative means of enhancing structural functionality and safety against natural and man-made hazards have been in various stages of research and development. By and large, they can be grouped into three broad areas: (i) base isolation; (ii) passive damping; and (iii) active control (Y. Fujino et al., 1996). Of the three, base isolation can now be considered a more mature technology with wider applications as compared with the other two. Implementation of passive energy dissipation systems, such as tuned mass dampers (TMDs), to reduce vibration response of civil engineering structures started in the
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U.S.A. in the 1970s and in Japan in the 1980s. In parallel, research and development of active control progressed greatly during the 80’s in both the U.S.A. and Japan (R.J Facian et al., 1995).

It has been shown in field studies that tall buildings that are subjected to wind induced oscillations usually oscillate at the fundamental frequency of the building. In some cases this is coupled with torsion motion, when the torsion and lateral oscillation frequencies are close. One of the most common control schemes used to correct these oscillations is a TMD system. Basically, TMD consists of a mass attached to a building, such that it oscillates at the same frequency of the structure but with a phase shift. The mass is attached to the building via a spring-dashpot system and the energy is dissipated by the dashpot as relative motion develops between the mass and structure (R.J Facian et al., 1995).

In the mid 1960s it was studies by Banning and others that the dynamic characteristics of sloshing liquid which eventually initiated the development of a series of natural dampers. The rotation dampers have some unique advantages such as low cost, easy installation and adjustment of liquid frequency, and little maintenance etc. which are unmatched by the traditional TMD system. The rotation dampers work by absorbing and dissipating energy through the sloshing or oscillating mechanisms of liquid inside a container. Two of the major devices developed in this category include the tuned liquid damper (TLD) and the tuned liquid column damper (J.T.P.Yao, 1972). Both these devices provide excellent overview in the development and application.

Dynamic loads that act on large civil structures can be classified into two main types: environmental, such as wind, wave, and earthquake; and man-made, such as vehicular and pedestrian traffic and those caused by reciprocating and rotating machineries. The response of these structures to dynamic loads will depend on the intensity and duration of the excitation, the structural system, and the ability of the structural system to dissipate the excitations energy. The shape of the structure also has a significant effect on the loading and resulting response from wind excitation. The advent of high strength, light and more flexible construction materials has created a new generation of tall buildings. Due to the smaller amount of damping provided by these modern structures, large deflection and acceleration responses result when they are subjected to environmental loads. Such large responses, in turn, can cause human discomfort or illness and some times, unsafe conditions. Passive, semi active, and active vibration control schemes are becoming an integral part of the system of the next generation of tall buildings (Mohsin Jamil et al., 2007).

6.2 Selection of strategy:
The available strategies are:

- Active Tuned Mass Damper (ATMD)
- Sinusoidal Reference Strategy (SRS)
- Mass dampers and their optimal designs

Comparing all the above strategies, most results are similar with very few differences. The efficiency and robustness of SRS strategy and ATMD are similar to that of LQG (linear quadratic Gaussian) sample controller. Due to lack of help from the passive method, the control forces are much larger than that using the ATMD actuation system. So for this experiment LQG controller is suitable to apply and easy to develop. In the case of active tuned control devices, an actuator is required; the installation cost of the actuator is more. So
the operation cost of the active tuned mass dampers is more than the LQG controller. Here we selected active tuned mass damper for implementation.

6.3 Process identification and selection of suitable model
The knowledge about the dynamic characteristics of a system is one of the most important aspects of control system design. An accurate mathematical model of the system determines whether a controller works properly or becomes unstable. Because of the practical unavailability of wind force measurement, the system identification in this study is processed without them. The active control force and wind forces are the actual exciting input forces to the structure.

A linear black box model shall be developed for this process. The steps followed to develop the model are:

- Make an experiment
- Data processing
- Model selection
- Parameter estimation
- Model analysis
- Model appraisal

6.3.1 Experimental setup
A model of a flexible tall building structure is designed in the laboratory (see fig 7) in order to study and design of a semi active control system with ATMD i.e. active tune mass damper. Dynamic parameters collected through system identification as described in the following section, and controller performance is simulated through MATLAB. An accelerometer is placed on top of the model to measure the response of the structure with nominal dimensions 1m x 0.05m.

Fig. 7. Experimental Setup
In the fig 7, we can see the mass damper attached on the top of the system to reduce the vibration of the structure. The mass damper having the variable mass but we can change the mass in the initial stage i.e. before the system to run. This mass damper i.e. (TMD) tune mass damper operate with the help of a linear servo motor. For more broad view of the mass damper, servo motor and accelerator meter is shown in the fig.8 below.

![Mass damper, Stepparmotor, Accelerometer](image)

**Fig. 8. Closer View of Setup**

In the fig.8, we can see the steel rod of one meter height attached in the base of the steel structure stands for the height of the building. The steel rod attached is quite light weight and flexible to allow it vibrate with the impact of the external force. The motion of the structure is just like a pendulum. It vibrates with its natural frequency whose acceleration in terms of voltages is roundabout (+0.02) to (~ 0.03) voltages if even without any external force. As it is described previously that voltages output represent the acceleration of the system. The natural movement graph is given below in fig. 9.

![Natural frequency graph of the structure model](image)

**Fig. 9. Natural frequency graph of the structure model.**
So as this vibration of the system is due to the natural frequency or natural movement, we can’t reduce this vibration more than this low frequency or acceleration. The length of steel rod made our system model high degree of robust and fast vibrating. So reduction of the vibration is harder and required more time to stop to natural frequency as compare to the actual system. As shown in the fig.7, the plate for wind damping is used to create the effect of linearization of the wind effect. It also tries to create real conditions as in our steel structure rods having between empty spaces to pass the air as compare to the high structure building. So to create the filled effect of the building, we use this board sheet as show in fig.7. It also creates the linear effect for the air force.

PCI-1710, I/O card (Input and output) is used to get the output of the model structure acceleration in term of the analogue voltages. With the A/D converter (analogue to digital converter), voltages from the accelerometer is converted to digital form to communicate with the matlab. This voltages to use for control, i.e., for taking the measurements feedback, computing the control command and then sending out back the voltages to the servo motor through the PCI I/O card. D/A conversion are used for the voltages coming out from the computer to the structure model’s servo motor. The schematic diagram of the PCI-1710(12/16 bit multifunction card) is shown in appendix. In PCI-1710 series four pins i.e. 57, 58, 60 and 68 are used. Pin57 and 60 are grounded, Pin58 (D/A) is the output from the system and Pin 68(A/D) is the output from the setup.

The basic idea behind this control of the structure model against the external forces is to operate the mass attached on the top (tuned mass damper) in opposite direction to the motion of the structure

![Uncontrolled vibration of the system under external force](image)

**Fig. 10. Uncontrolled vibration due to single external of the structure model.**

The figure 10 shows the behavior of the tall structure prototype model under external force without controlling it in open loop. This shows that the system vibrates like a pendulum and even after 100 seconds the vibration is more than 0.1 units of acceleration. This figure provides good reference for us to compare between the controlled and uncontrolled system. It also shows that after 30 seconds system comes to high vibration level as 1.00 units of acceleration. And after 60 seconds the vibration comes in the range of 0.5 voltages.

### 6.3.2 Experiment design

The goal of experiment is to affect all frequencies of interest with as much as possible of the available input energy. When a process model is constructed by using identification
methods, measured data is necessary when unknown model parameters are estimated. This means both the input and the output variables have to be collected when process is running. Several aspects have to be considered when designing an experiment to achieve informative measurements from the process. First of all there are practical considerations in what way we can affect the process. We have to realise when the process is affected by external signal, the process will be disturbed. Hence the output of the process will deviate from a desired result. This situation will limit the amplitude of input signal and the length of the experiment. When designing the signal is that the inputs shall influence the process in a way so the interesting frequencies are affected. Further to keep the relative error constant, we need a signal with constant signal/noise ratio (Bjorn Sohlberg, 2005). Much commercial process cannot be exposed by an open loop experiment. It is not possible to run the experiment without proper controller. There may also be problem with running the process safely, which means it is necessary to keep important process variables within specified limits. From the discussion, we have that a suitable wave form is a pseudo random binary signal.

Generation of PRBS Signal:
This kind of signal has the lowest crest factor and is easy to implement. This signal has also the advantages to be piece wise constant, which makes it suitable to identify discrete time linear models. The PRBS signal is generated from the matlab routine named makeprbs. PRBS signal is applied at the input of the system. For this purpose it is necessary to define its parameters. (Bjorn Sohlberg, 2004)

```matlab
>> PRBS = makeprbs(tstop, ts, tmin, tmax, umin, umax);
```

**Stop time:** the experiment have to take place during a time long enough to achieve estimation of the unknown parameters. The given process is vibration process so 30 seconds more enough to estimation of unknown parameters. So we take: tstop=30 seconds;

**Sample time:** when the sample time is long, we will have high variance values of the estimated parameters. When the sample time is too short, the change of the outputs may be small compared to the measurement disturbances. When the sample time is h=0.05 seconds, we will have three samples during the transport time.

Thus we select: ts=0.05 seconds;

**Time at the same level of the input signal:** the input signal must affect the process longer than a shortest time period which influences the outputs considerable. The signal length on the same level must be long enough to influence the dynamic of the process. Long duration on the same level give no more information about the behaviour of the process. Hence between the shortest and longest time periods the signal change levels with a random distribution of the time duration on the same level. The longest time of the input signal can be chosen as 0.7seconds, if the value more than this the output of the process is not follow the input, means when the signal is constant at that moment is also system is oscillating. The shortest time of the input signal at the same level can be chosen as 0.5 seconds, if the value less than this the output of the process is slow than the input of the process, means when the input signal varying the output is varied slowly.

`tmin =0.5 seconds, tmax =0.7 seconds;`
Amplitude values of the PRBS: large amplitudes may also increase the influence of process non-linearity’s, which makes the system identification, based on a linear model, more difficult. We should avoid using values of the inputs at the ends of its range. In our process, the input signal can be affected within the range [2.4 – 4.6]. The input signal must not produce non-linear characteristics. The system is to be controlled in a steady state; hence, the small oscillations in transient time must be avoided. For this reason we have limitations in

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control signal, $U_{max}=4.3$ volts and $U_{min}=3.3$ volts; the change in the input signal should not be large because linear model would turn to be nonlinear model. The parameters are estimated from the different experimental trials and then decide the values are suitable for this process. Finally we have:

$$\text{PRBS= makeprbs (30, 0.05, 0.5, 0.7, 3.3, 4.3);}$$

Let the input signal be PRBS; the sample time in all appropriate blocks in the simulink window has to be changed at the start of the experiment. The stop time for the simulink window must be same as the experiment stop time. After the experimentation the system will record the measured data through sensors. For this data will help to make mathematical model.

Figure. 13. Input and Output of the Process

The results from the experiment are shown in fig.13. It shows both the input and output of the process. From the experiment it can be concluded that the variation in the amplitude of the output is same as input. It also shows the output follows the input with little delay.

6.3.3 Data processing

After this idealised experiment, there is no need for any data processing. From fig. 13, we can see there are no outliers, or filtering or low and high frequency disturbances needed to be eliminated. The stationary point for the output is 2.24 and input is 3.75. Hence we need to eliminate mean values from the measurements.

$$\text{data_d=dtrend (dat); %removing mean values}$$

$$\text{data_d=dtrend (dat, 1); % removing trend values}$$

6.3.4 Parameter estimation

During the identification of the model procedure, we let three models ARX, ARMAX and Output-Error to find the best model. Details of these models have been already explained earlier. So we just apply and show the research results.
<table>
<thead>
<tr>
<th>Model Parameter</th>
<th>ARX-3</th>
<th>ARX-4</th>
<th>ARX-5</th>
<th>ARMAX-3</th>
<th>ARMAX-4</th>
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<tbody>
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<td>(±0.03516)</td>
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<td>0.0309</td>
<td>0.01499</td>
<td>0.00968</td>
<td>0.02928</td>
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<tr>
<td></td>
<td>(±0.001846)</td>
<td>(±0.0008174)</td>
<td>(±0.001243)</td>
<td>(±0.001587)</td>
<td>(±0.001005)</td>
</tr>
<tr>
<td>Loss fcn</td>
<td>0.00150523</td>
<td>0.000265366</td>
<td>0.00028017</td>
<td>0.00054384</td>
<td>0.0001517</td>
</tr>
<tr>
<td>FPE</td>
<td>0.00152941</td>
<td>0.000270705</td>
<td>0.00028453</td>
<td>0.00056146</td>
<td>0.00016517</td>
</tr>
<tr>
<td>Model Parameter</td>
<td>ARMAX-5</td>
<td>OE-3</td>
<td>OE-4</td>
<td>OE-5</td>
<td></td>
</tr>
<tr>
<td>A1</td>
<td>-3.642</td>
<td>0.2612</td>
<td>0.3144</td>
<td>0.8788</td>
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</tr>
<tr>
<td></td>
<td>(±0.03699)</td>
<td>(±0.7533)</td>
<td>(±0.502)</td>
<td>(±0.1729)</td>
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</tr>
<tr>
<td>A2</td>
<td>5.875</td>
<td>1.395</td>
<td>0.2596</td>
<td>-0.3017</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(±0.1166)</td>
<td>(±0.9609)</td>
<td>(±0.4368)</td>
<td>(±0.1248)</td>
<td></td>
</tr>
<tr>
<td>A3</td>
<td>-5.306</td>
<td>-1.598</td>
<td>0.8884</td>
<td>0.0144</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(±0.16)</td>
<td>(±0.6474)</td>
<td>(±0.3116)</td>
<td>(±0.0684)</td>
<td></td>
</tr>
<tr>
<td>A4</td>
<td>2.667</td>
<td></td>
<td>-1.392</td>
<td>0.5596</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(±0.1132)</td>
<td></td>
<td>(±0.4393)</td>
<td>(±0.1175)</td>
<td></td>
</tr>
<tr>
<td>A5</td>
<td>-0.5745</td>
<td></td>
<td></td>
<td>-1.123</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(±0.03478)</td>
<td></td>
<td></td>
<td>(±0.1684)</td>
<td></td>
</tr>
<tr>
<td>B1</td>
<td>0.01171</td>
<td>-0.8989</td>
<td>-0.8811</td>
<td>-0.952</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(±0.001239)</td>
<td>(±0.0914)</td>
<td>(±0.06313)</td>
<td>(±0.0474)</td>
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<tr>
<td>Loss fcn</td>
<td>0.00015433</td>
<td>1.49666</td>
<td>1.35876</td>
<td>0.740544</td>
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<tr>
<td>FPE</td>
<td>0.00016944</td>
<td>1.52094</td>
<td>1.38649</td>
<td>0.773725</td>
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</tr>
</tbody>
</table>

Table 2. Parameter estimation
Percentage of Error:
Now we check percentage of error in estimated models. All the values are given below in table 3.

<table>
<thead>
<tr>
<th>Model Parameter</th>
<th>ARX-3</th>
<th>ARX-4</th>
<th>ARX-5</th>
<th>ARMAX-3</th>
<th>ARMAX-4</th>
<th>OE-3</th>
<th>OE-4</th>
<th>OE-5</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>1.4%</td>
<td>0.6%</td>
<td>1.1%</td>
<td>1.2%</td>
<td>0.7%</td>
<td>1%</td>
<td>288%</td>
<td>159%</td>
</tr>
<tr>
<td>A2</td>
<td>3.1%</td>
<td>1.2%</td>
<td>2%</td>
<td>2.45%</td>
<td>1.5%</td>
<td>2%</td>
<td>68%</td>
<td>168%</td>
</tr>
<tr>
<td>A3</td>
<td>5.6%</td>
<td>1.7%</td>
<td>3%</td>
<td>4.1%</td>
<td>2.14%</td>
<td>3%</td>
<td>40%</td>
<td>35%</td>
</tr>
<tr>
<td>A4</td>
<td>2.08%</td>
<td>4.4%</td>
<td></td>
<td>2.7%</td>
<td>4.2%</td>
<td>31.5%</td>
<td>21%</td>
<td></td>
</tr>
<tr>
<td>A5</td>
<td></td>
<td>6.5%</td>
<td>6.1%</td>
<td>15%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B1</td>
<td>9.86%</td>
<td>2.6%</td>
<td>8.3%</td>
<td>16.4%</td>
<td>3.4%</td>
<td>1.1%</td>
<td>10%</td>
<td>7.2%</td>
</tr>
</tbody>
</table>

Table. 3. Percentage of error of the estimated parameters

It clearly shows which model represents the process. Every parameter is estimated given by its margin of errors as the standard deviation.

Discussion on Loss Function: The loss function decreases relatively slowly between the models ARX-3 and ARX-4. The smallest value is achieved from for the ARX-4 model. However we can see that the smallest value of FPE is achieved for ARX-4. Hence, the results from the loss function and the number of estimated parameters favours model ARX-4. The loss function decreases relatively much between the models ARMAX-3 and ARMAX-5. The smallest value achieved from for the ARMAX-4 model. However we can see that the smallest value of FPE is achieved for ARMAX-4. Hence the results from the loss function and the number of estimated parameters favour model ARMAX-4. The loss function decreases relatively much between the models OE-4 and OE-5. The smallest value achieved from for the OE-5 model. However we can see that the smallest value of FPE is achieved for OE-5. Hence the results from the loss function and the number of estimated parameters more.

Discussion on Parameter Estimation: Every parameter estimate is given by its margin of errors, i.e. the standard deviation. For the model ARX-4, the margin of errors is small compared to the model ARX-3 and ARX-5. For the model ARMAX-4, the margin of errors is small compared to the model ARMAX-3 and ARMAX-5. For the model OE-5, the margin of errors is small compared to the model OE-3 and OE-4. Compare all three models ARX-4, ARMAX-4 and OE-5; among these ARX-4 has less parameter error and less parameters compared to ARMAX-4 model.

6.3.5 Model analysis
The ARX [4 1 1] model is simulated using the outputs from the process. The output of the model is compared with the output of the process. The result is shown in following figure.
Simulation:

Fig 14: Simulation Results

From the simulation it can be seen that the model ARX [4 1 1] is able to describe the measurements from the process well.

**Auto and Cross Correlation Analysis:** The next step in the model analysis is to figure it out the residuals of auto and cross correlation results.

Fig 15: Auto and Cross Correlation

The curves show that no value of cross correlation and very few values of auto correlation coming out of the defined boundaries of the standard deviation and the residual is independent between two or more samples. The residual is also independent of the control signal. So there is no more information left to be gathered from the process.
Frequency Analysis
The frequency characteristics are investigated by plotting the bode diagrams for the measured data and the models. The Upper bode plot shows the gain (amplitude) and the lower shows the phase of model ARX [4 1 1]

![Frequency Diagram](image)

**Fig 16: Frequency Diagram**

The frequency plot based on an estimated model and spectrum from measured input/output signals is performed by using Matlab SITB, the plot shown in fig 3.6 provides the information, whether the frequency relation between input and outputs is covered by the model.

The above figure is showing that the model and process have similar dynamic curves. So the model is representing the process well.

**Pole-zero analysis:** The following figure shows the poles and zero of the model.

![Pole-Zero Diagram](image)

**Figure 17: Pole-Zero Diagram**
The above figure of poles and zero shows that, the model is not over determined and it is also shows that the pole is far away from the area of the zero. From fig3.9 the uncertainty plot from poles and zeros are not overlapping each other. So there is no pole/zero cancellation. Furthermore the poles/zeros lay inside the unit circle, so the system is stable.

6.3.6 Model appraisal
During the model appraisal, we are going to make an in all analysis concerning which model is best suitable to describe the process. From the model analysis, we have that the model ARX-4 is favoured from the analysis of Loss function, FPE and parameter estimation. The values from the parameter estimate are significant for models ARX-4 and ARMAX-4. But OE-3, OE-4 and OE-5 have overestimated parameters. ARX-4 has fewer margins of errors compared to ARMAX-4. The extra parameters are estimated for the model OE model. Hence some of the parameters for OE are not significant. From the model simulation, it is seen that the models are ARX [4 1 1] are able to describe the measurements from the process. From the correlation, it is obvious that ARX-3, ARX-5, ARMAX-3, ARMAX-4, ARMAX-5, OE_3, OE-4, and OE-5 give a result which is more outside the confidence interval. The ARX-4 fulfils the tests for both the auto and cross correlations. Further, the frequency analysis contradicts the remaining models except ARX-4 model. While pole zero plot shows that ARX-4 is better among the others. It can be conclude that most of the model analysis favours the model ARX [4 1 1]. It is not necessary to expand or use another model type.

6.3.7 Simulated mathematical model of a process
From the chosen mathematical model, the simulink diagram is made in the matlab. In the fig.3.10 the mathematical model is tested for its suitability for the system.

![Fig 18: Selected Mathematical Model in Simulink Window](image)

Generally the system is an oscillating system, so the output must be a sine curve. To check this result a reference signal is given as step input and its value is set to zero, adding step signal at step time 1sec and after 0.1 sec the signal is removed. The above fig. 19 represents the input and output of the mathematical model. From the results the mathematical model
Identification of Dynamic Systems & Selection of Suitable Model

satisfies the original process. Firstly the controller for our mathematical model is designed to replace original process instead of model. Among the various controllers available the pole placement controller and state space pole placement controller are chosen.

![Graph](image)

Fig 19: (a) output signal and (b) input signal

7. Summary

From the results of above experiments, PRBS signal is defined, using this as input to the system and obtaining their corresponding output. For the output obtained, three different matlab scripts were built for model estimation (which calculate the model parameters and represent its behaviour of the system), one for each sort of model (arx, armax, output-error). Using matlab scripts, two of the models are discarded. The order of the chosen one (ARX) was defined by successive limitations of the range and elimination of the least effect parameters. From the analysis of simulation, parameter estimation, pole-zero analysis, frequency diagram, auto and cross correlation, the final model resulted is arx [4 1 1]. It defines the process with 5 parameters with a delay of one sample; the other two models are discarded from the model analysis. All the model analysis is shown in the report. Thus identification of real process was done successfully and then model is selected, it will be used for design of controller in next phase. It has been tested that control design for this model is perfect.

8. References


In this book, a set of relevant, updated and selected papers in the field of automation and robotics are presented. These papers describe projects where topics of artificial intelligence, modeling and simulation process, target tracking algorithms, kinematic constraints of the closed loops, non-linear control, are used in advanced and recent research.

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