The Area Coverage Problem for Dynamic Sensor Networks

Simone Gabriele, Paolo Di Giamberardino
Università degli Studi di Roma "La Sapienza"
Dipartimento di Informatica e Sistemistica "Antonio Ruberti"
Italy

1. Introduction

In this section a brief description of area coverage and connectivity maintenance for sensor networks is given together with their collocation in the scientific literature. Particular attention is given to dynamic sensor networks, such as sensor networks in witch sensing nodes moves continuously, under the assumption, reasonable in many applications, that synchronous or asynchronous discrete time measures are acceptable instead of continuous ones.

1.1 Area Coverage

Environmental monitoring of lands, seas or cities, cleaning of parks, squares or lakes, mine clearance and critical structures surveillance are only a few of the many applications that are connected with the concept of *area coverage*.
Area coverage is always referred to a set, named *set of interest*, and to an action: then, covering means acting on all the physical locations of the set of interest.
Within the several actions that can be considered, such as manipulating, cleaning, watering and so on, sensing is certainly one of the most considered in literature. Recent technological advances in wireless networking and miniaturizing of electronic computers, have suggested to face the problem of taking measures on large, hazardous and dynamic environments using a large number of smart sensors, able to do simple elaborations an perform data exchange over a communication network. This kind of distributed sensors systems have been named, by the scientific and engineering community, *sensor networks*.
Coverage represents a significant measure of the quality of service provided by a sensor network. Considering static sensors, the coverage problem has been addressed in terms of optimal usage of a given set of sensors, randomly deployed, in order to assure full coverage and minimizing energy consumption (Cardei and Wu, 2006, Zhang and Hou, 2005, Stojmenovic, 2005), or in terms of optimal sensors deployment on a given area, such as optimizing sensors locations, as in (Li et al., 2003, Meguerdichian et al., 2001, Chakrabarty et al., 2002, Isler et al., 2004, Zhou et al., 2004).
The introduction of mobile sensors allows to develop networks in which sensors, starting from an initial random deployment condition, evaluate and move trough optimal locations.

In (Li and Cassandras, 2005) coverage maximization using sensors with limited range, while minimizing communications cost, is formulated as an optimization problem. A gradient algorithm is used to drive sensors from initial positions to suboptimal locations.
In (Howard, 2002) an incremental deployment algorithm is presented. Nodes are deployed one-at- time into an unknown complex environment, with each node making use of information gathered by previously deployed nodes. The algorithm is designed to maximize network coverage while ensuring line-of-sight between nodes.
A stable feedback control law, in both continuous and discrete time, to drive sensors to so-called centroidal Voronoy configurations, that are critical points of the sensors locations optimization problem, is presented in (Cortes et al., 2004).
Other interesting works on self deploying or self configuring sensor networks are (Cheng and Tsai, 2003, Sameera and Gaurav S., 2004, Tsai et al., 2004)
The natural evolution of these kind of approaches moves in the direction of giving a greater motion capabilities to the network. And once the sensors can move autonomously in the environment, the measurements can be performed also during the motion (*dynamic coverage*). Then, under the assumption, reasonable in many applications, that synchronous or asynchronous discrete time measures are acceptable instead of continuous ones, the number of sensors can be strongly reduced. Moreover, faults or critical situations can be faced and solved more efficiently, simply changing the paths of the working moving sensors. Clearly, coordinated motion of such *dynamic sensor network*, imposes additional requirements, such as avoiding collisions or preserving communication links between sensors. In order to better motivate why and when a mobile sensor network can be a more successful choice than a static one, some considerations are reported. So, given an area A_{tot} to be measured by a sensor network, and ρ_S the measure range of each sensor (sensors are here supposed homogeneous, otherwise the same considerations should be repeated for all the homogeneous subnets), the number N_{stat} of sensors needed for a static network must satisfy

$$N_{stat} \geq \frac{A_{tot}}{\pi \rho_S^2} \tag{1}$$

When a dynamic network is considered, the area covered by sensors is a time function and, clearly, it not decreases as time passes. A simplified discrete time model of the evolution of the area still uncovered, at (discrete) time $t = k + 1$, by a dynamic sensor network moving with the strategy proposed in this chapter, can be given by the following differences equation

$$A_u(k+1) = \left(1 - \frac{\dot{A}_N}{A_{tot}}\right) A_u(k) \tag{2}$$

where

$$\dot{A}_N = \frac{v_{max}}{2\rho_S} A_{tot} \left(1 - \left(1 - \frac{\pi \rho_S^2}{A_{tot}}\right)^N\right)$$

represents the area covered in the time unit by a number N of mobile sensors subject to the maximum motion velocity v_{max}. Measurements are then modelled as obtained deploying randomly N static sensors on the workspace every $\frac{2\rho_S}{v_{max}}$ seconds. Denoting by

$$A_u(0) = A_{tot}\left(1 - \frac{\pi\rho_S^2}{A_{tot}}\right)^N$$

the initial condition for area to be covered, at each discrete time $t = k$ the fraction of area covered is given by

$$A_{\%}(k) = 1 - \frac{A_u(k)}{A_{tot}} = 1 - \left[\frac{A_u(0)}{A_{tot}}\left(1 - \frac{\dot{A}_N}{A_{tot}}\right)^k\right] \qquad (3)$$

The evolution computed using (3) with $N = 5$, $N = 10$ and $N = 15$ has been compared with the results of simulations where the approach described in the chapter is applied. In Fig. 1 this comparison is reported, showing that (3) is a good model for describing the relationship between the area covered and the time using a dynamic solution.

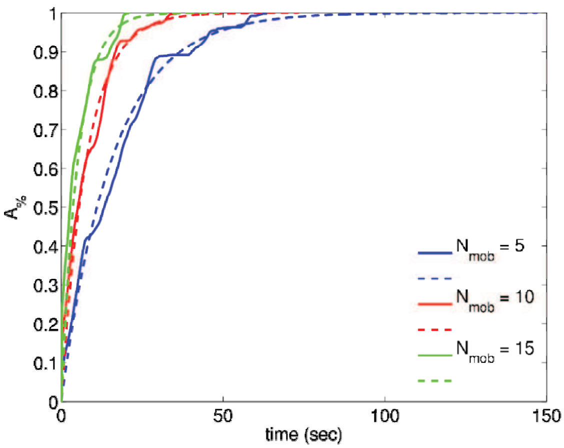


Fig. 1. Comparison between coverage evolution obtained by the model (2) (*dashed*) and simulations of the proposed coverage strategy (*solid*) for different numbers of moving sensors

Then, referring to surveillance tasks, (3) can be used to evaluate the minimum number of sensors (with given ρ_S and v_{max}) required to cover a given fraction $\tilde{A}_{\%}$ of the area of interest according to a given measurement rate. In fact, it is possible to write the relation between the maximum rate at witch the network can cover the $\tilde{A}_{\%}$ fraction of A_{tot} and the number of moving sensors as

$$f = \frac{\log\left(1 - \frac{\dot{A}_N}{A_{tot}}\right)}{\log\left(1 - \tilde{A}_{\%}\right) - N\log\left(1 - \frac{\pi\rho_S^2}{A_{tot}}\right)} \tag{4}$$

Such a relationship between N and f is depicted in Fig. 2, showing, as intuitively expected, almost a proportionality between number of sensors and frequency of measurement at each point of the area A_{tot}.

The motivation and the support of the dynamic solution is evidenced by Fig. (1): lower is the refresh frequency of the measurements at each point (that is higher are the time intervals between measurements) and lower is the number of sensors required, once sensors motion is introduced.

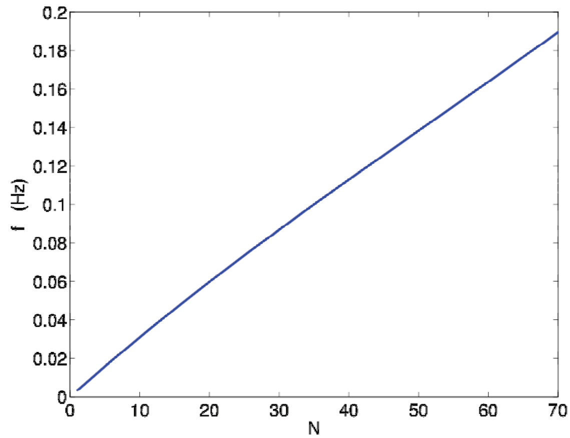


Fig. 2. Maximum measure rate f in function of number of moving sensors. ($A_{tot} = 400\,m^2$, $\rho_S = 1.5\,m$, $v_{max} = 1.5\,\frac{m}{sec}$, $\tilde{A}_{\%} = 0.98$)

Under the assumption of dynamic network, the area coverage problem is posed in terms of looking for optimal trajectories for the N moving sensors in presence of some constraints like communication connection preservation, motion limitations, energetic considerations and so on. In (Tsai et al., 2004, Cecil and Marthler, 2004) the dynamic coverage problem for multiple sensors is studied , with a variational approach, in the level set framework, obstacles occlusions are considered, suboptimal solutions are proposed also in three dimensional environments ((Cecil and Marthler, 2006)). A survey of coverage path planning algorithms for mobile robots moving on the plane is presented in (Choset, 2001). In (Acar et al., 2006) the dynamic coverage problem for one mobile robot with finite range detectors is studied and an approach based on space decomposition and Voronoy graphs is proposed.

In (Hussein and Stipanovic, 2007), a distributed control law is developed that guarantees to meet the coverage goal with multiple mobile sensors under the hypothesis of communication network connection. Collisions avoidance is considered.

Various problems associated with optimal path planning for mobile observers such as mobile robots equipped with cameras to obtain maximum visual coverage in the three-dimensional Euclidean space are considered in (Wang, 2003). Numerical algorithms for solving the corresponding approximate problems are proposed.

In (Gabriele and Di Giamberardino, 2007c, Gabriele and Di Giamberardino, 2007a, Gabriele and Di Giamberardino, 2007b) a general formulation of dynamic coverage is given by the authors, a sensor network model is proposed and an optimal control formulation is given. Suboptimal solutions are computed by discretization. Sensors and actuators limits, geometric constraints, collisions avoidance, and communication network connectivity maintenance are considered.

The approaches introduced up to now, also by the authors ((Gabriele and Di Giamberardino, 2007c, Gabriele and Di Giamberardino, 2007a, Gabriele and Di Giamberardino, 2007b)), are referred to homogeneous sensor networks, that is each node in the network is equivalent to any other one in terms of sensing capabilities (same sensor or same set of sensors over each node). Sensor network nodes were called *heterogeneous* with respect to different aspects. In (Ling Lam and Hui Liu, 2007), the problem of deploying a set of mobile sensor nodes, with heterogeneous sensing ranges, to give coverage is addressed.

In (Lazos and Poovendran, 2006), evaluating coverage of a set of sensors, with arbitrary different shapes, deployed according to an arbitrary stochastic distribution is formulated as a set intersection problem.

In (Hussein et al., 2007) the use of two classes of vehicles are used to dynamically cover a given domain of interest. The first class is composed of vehicles, whose main responsibility is to dynamically cover the domain of interest. The second class is composed of coordination vehicles, whose main responsibility is to effectively communicate coverage information across the network.

The problem of deploying nodes, equipped with different sets of sensors, is studied in (Shih et al., 2007) in order to cover a sensing field in which multiple attributes are required to be sensed.

In this chapter the case of different magnitudes to be measured on a given set of interest is considered. Network nodes are then heterogeneous , like in (Shih et al., 2007), with respect to the set of sensors with witch they are equipped. Moreover different sensors can have different sensing ranges.

1.2 Connectivity Maintenance

Communication aspects are crucial in the design of a multi sensor systems ((Holger Karl, 2005, Stojmenovic, 2005, Santi, 2005)).

Connectivity is obviously necessary for data exchanging and aggregation but also for localization and coordination, in fact, it is often assumed in formation stabilization (Olfati-Saber and Murray, 2002) or consensus problems (Olfati-Saber et al., 2007).

In classical wireless sensor network (Holger Karl, 2005, Stojmenovic, 2005, Akyildiz et al., 2002, Santi, 2005), composed by densely deployed static sensors, a single node has many neighbours with which direct communication would be possible when using sufficiently large transmission power. However high transmission power requires lots of energy, then, it could be useful to deliberately restrict the set of neighbours controlling transmission power, and then communication range, or by simply turning off some nodes for a certain time. For such networks connectivity can then be achieved opportunely deploying nodes or controlling communication power.

For a dynamic sensor network the problem is more challenging, because network topology is, indeed, *dynamic*. Connectivity maintenance became, then, a motion coordination problem. Each sensor is assumed to have a fixed range over which communication is not reliable. Communication network can be modelled as a state dependent dynamic graph; topology, depending from sensors positions, changes while sensors moves.
Then, connectivity maintenance impose to introduce constrains on the instantaneous positions of sensors. The simplest ways to achieve connectivity is to maintain the starting communication graph topology that's assumed to be connected. This can be obtained imposing fixed topology Maintenance as proposed in (Gabriele and Di Giamberardino, 2007a) or flocking (Olfati-Saber, 2006). However, this approaches impose strong constraints to sensors movement and that can affect other aspects as shown in (Gabriele and Di Giamberardino, 2007b) for coverage. Is then more desirable to allow topology to change over time, even though that introduce challenging dynamic graph control problems.

In (Mesbahi, 2004), starting from a class of problems associated with control of distributed dynamic systems, a controllability framework for state-dependent dynamic graphs is considered.
In (Kim and Mesbahi, 2005) the position of a dynamic state-dependent graph vertices are controlled in order to maximize the second smallest eigenvalue of the Laplacian matrix, also named *algebraic connectivity* and that has emerged as a critical parameter that influences the stability and robustness properties of dynamic systems that operate over an information network.
In (Spanos and Murray, 2004) a measure of robustness of local connectedness of a network is introduced that can be computed by local communication only.
K-hop connectivity preservation is considered, in (Zavlanos, 2005), for a network with dynamic nodes. A centralized control framework that guarantees maintenance of this property is developed. Connectivity is modelled as an invariance problem and transformed into a set of constraints on the control variable.
In (Gabriele and Di Giamberardino, 2007b) a centralized approach to connectivity Maintenance, based on preservation of the edges of one *Minimum Spanning Tree* of the communication graph, is proposed by the authors. Connection Maintenance is introduced as a constraint of an optimization problem.

2. General Formulation

In this section definitions are given in order to introduce useful notations. A general model of a dynamic sensor network is given considering heterogeneous sensors. The coverage problem is formulated with respect to multiple magnitudes, connectivity maintenance constraints are considered. In the following sections additional hypothesis are introduced in order to simplify the general problem and to evaluate suboptimal solutions.

2.1 Dynamic Sensor networks

Let be W a specified spatial domain, a compact subset of the real Euclidean space $\mathbb{R}^n$ (n=2,3) called the *set of interest*. The representation of a point $p \in W$ with respect to a given orthonormal basis for $\mathbb{R}^n$ is denoted by $x_p = (x_{p1}, ..., x_{pn})$.
Let be $\Xi = \{\xi_1, \xi_2, \dots\}$ the set of magnitudes of interest defined on W.

A dynamic sensor network can be view a set $\Sigma = \{\sigma_1, \sigma_2, ...\sigma_m\}$ of m mobile sensors. Each *mobile sensor* can be represented by: $\sigma_j =< \mathcal{C}^{(j)}, f^{(j)}, \Xi^{(j)}, \{M_\xi^{(j)} | \xi \in \Xi^{(j)}\}, \kappa^{(j)} >$ where:

- $\mathcal{C}^{(j)}$ is the sensor configuration space.
- $f^{(j)}$ is the sensor dynamic function, that describe the evolution of sensor configuration according to a control input $u^{(j)}$:
$$\dot{q}^{(j)} = f^{(j)}(q^{(j)}, u^{(j)})$$
- $\Xi^{(j)} \subseteq \Xi$ is the set of magnitudes that sensor σ_j can measure.
- $M_\xi^{(j)} = M_\xi^{(j)}(q^{(j)}) \subseteq W$ is the subset of W within sensor σ_j, in configuration q can measure magnitude $\xi \in \Xi^{(j)}$. Let say that sensor σ_j in configuration $q^{(j)}$ $\xi - cover$ the set $M_\xi^{(j)}(q^{(j)})$
- $\kappa^{(j)} : \mathcal{C} \times \mathcal{C} \longrightarrow \mathbb{R}$ is the sensor communication function, such as, σ_j in configuration $q^{(j)}$ can communicate with a sensor σ_h in configuration $q^{(h)}$ if and only if $\kappa^{(j)}\left(q^{(j)}, q^{(h)}\right) > 0$

Looking at the whole network is possible to define generalized configuration and generalized input as:

$$q = \begin{bmatrix} q^{(1)} \\ q^{(2)} \\ \vdots \\ q^{(m)} \end{bmatrix} \qquad u = \begin{bmatrix} u^{(1)} \\ u^{(2)} \\ \vdots \\ u^{(m)} \end{bmatrix}$$

At the same manner the generalized dynamic of the whole network can be written as:

$$\dot{q} = f(q, u) = \begin{bmatrix} f^{(1)}(q^{(1)}, u^{(1)}) \\ f^{(2)}(q^{(2)}, u^{(2)}) \\ \vdots \\ f^{(m)}(q^{(m)}, u^{(m)}) \end{bmatrix}$$

2.2 Coverage

Let indicate with $q^{(j)}(\Theta)$ the evolution of sensor σ_j configuration during a given a time interval $\Theta = [0, t_f]$. It is possible to define the subset of W $\xi - covered$ by σ_j during Θ as:

$$M_\xi^{(j)}(q^{(j)}(\Theta)) = \bigcup_{t \in \Theta} M_\xi^{(j)}(q^{(j)}(t)) \tag{5}$$

Considering generalized configuration $q(\Theta)$ is possible to define the network field of measure, respect to magnitude ξ during Θ, as:

$$M_\xi(q(\Theta)) = \bigcup_{\sigma_j \mid \xi\in\Xi^{(j)}} M_\xi^{(j)}(q^{(j)}(\Theta)) = \bigcup_{\sigma_j \mid \xi\in\Xi^{(j)}} \bigcup_{t\in\Theta} M_\xi^{(j)}(q^{(j)}(t)) \tag{6}$$

Looking at the whole magnitudes set, the subset of W "$\Xi - covered$" by the network can be defined as:

$$M_\Xi(q(\Theta)) = \bigcap_{\xi\in\Xi} M_\xi(q(\Theta)) \tag{7}$$

The area $\Xi - covered$ by the sensor network during Θ is then the measure of $M(q(\Theta))$:

$$A_\Xi(\Theta) = \mu(M_\Xi(q(\Theta))) \tag{8}$$

2.3 Communication

According with their communication capabilities sensors can be view as nodes of a dynamic communication network. This network can be represented by a dynamic graph

$$\mathcal{G}(t) =< N_\mathcal{G}, E_\mathcal{G}(t) >$$

where

$$N_\mathcal{G} = \sigma$$

indicate the nodes set and

$$E_\mathcal{G}(t) = \{(\sigma_j, \sigma_h) \in N_\mathcal{G} \times N_\mathcal{G} \mid \kappa^{(j)}(q^{(j)}(t), q^{(h)}(t)) > 0\}$$

indicate the edges set. As seen the edge set is time varying because it depends from the network generalized configuration q.

An alternative representation of the communication graph can be given using the adjacency matrix $A_\mathcal{G}(t)$:

$$A_\mathcal{G}^{(j,h)}(t) = \kappa^{(j)}(q^{(j)}(t), q^{(h)}(t))$$

2.4 Area Coverage Problem

Making the sensor network to cover the set of interest means evaluating controls that drive the network to measure the value of every magnitude on all the points of W, according with some constrains.

Constrains can be due, for example, from:

- Limitation of sensors motion or measure rate
- Avoiding collisions between sensors

- Maintaining some communication networks features (topology, connectivity,…)

3. Dynamic Sensor Network Model

In this section the general model defined in 2 is specified adding the hypothesis of **L**inear sensors dynamic, **P**roximity based measure model, **P**roximity based communication. Let refer to this particular model as (**LPP**) Model.

3.1 Sensors Dynamics

Each sensor σ_j is modelled, from the dynamic point of view, as a material point of mass m_j moving on $\mathbb{R}^2$. The motion is assumed to satisfy the classical simple equations

$$\ddot{x}^{(j)}(t) = \frac{u^{(j)}(t)}{m_j} \tag{9}$$

where $x^{(j)}$ is the sensor position on $\mathbb{R}^2$. Sensor configuration is represented by:

$$q^{(j)}(t) = \left(\ \dot{x}_1^{(j)}(t), x_1^{(j)}(t), \dot{x}_2^{(j)}(t), x_2^{(j)}(t) \ \right)^T$$

The configuration space is then:

$$\mathcal{C}^{(j)} \subseteq \mathbb{R}^4 \quad \forall j$$

The linearity of 9 allows one to write the dynamics in the form

$$\begin{aligned} &\dot{q}^{(j)}(t) = f^{(j)}(q^{(j)}(t), u^{(j)}(t)) = A_j q^{(j)}(t) + B_j u^{(j)}(t) \\ &x^{(j)}(t) = C_j q^{(j)}(t) \end{aligned} \tag{10}$$

where

$$A_j = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \qquad B_j = \begin{pmatrix} \frac{1}{m_j} & 0 \\ 0 & 0 \\ 0 & \frac{1}{m_j} \\ 0 & 0 \end{pmatrix}$$
$$C_j = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

The evolutions of configuration (state) and position (output) are related with the input's one according with the well known equations:

$$q^{(j)}(t) = \phi_j(q^{(j)}(0), u^{(j)}(t)) = e^{A_j t} q^{(j)}(0) + \int_0^t e^{A_j(t-\tau)} B_j u^{(j)}(\tau) d\tau \tag{11}$$

and

$$x^{(j)}(t) = \psi_j(\phi_j(q^{(j)}(0), u^{(j)}(t))) = C_j \phi_j(q^{(j)}(0), u^{(j)}(t)) \tag{12}$$

In the rest of the chapter *sensor trajectory* will refer to sensor position evolution.
Considering the whole network

$$q(t) = \begin{pmatrix} q^{(1)}(t) & q^{(2)}(t) & \cdots & q^{(m)}(t) \end{pmatrix}^T$$

can be defined to denote the generalized configuration, and the vector

$$x(t) = \begin{pmatrix} x^{(1)}(t) & x^{(2)}(t) & \cdots & x^{(m)}(t) \end{pmatrix}^T$$

to denote the generalized position that is represented, for each t, by m points in the Euclidean space. Evolution of generalized position will be named *generalized network trajectory*.
At the same manner the generalized input is defined as:

$$u(t) = \begin{pmatrix} u^{(1)}(t) & u^{(2)}(t) & \cdots & u^{(m)}(t) \end{pmatrix}^T$$

Generalized dynamics for the whole network can be written as:

$$\begin{aligned} \dot{q}(t) &= Aq(t) + Bu(t) \\ x(t) &= Cq(t) \end{aligned}$$

where:

$$A = \begin{pmatrix} A_1 & 0 & \dots & 0 \\ 0 & A_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & A_m \end{pmatrix} \qquad B = \begin{pmatrix} B_1 & 0 & \dots & 0 \\ 0 & B_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & B_m \end{pmatrix}$$

$$C = \begin{pmatrix} C_1 & 0 & \dots & 0 \\ 0 & C_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & C_m \end{pmatrix}$$

According with 11 and 12, generalized configuration evolution and network generalized trajectory are related with generalized input by:

$$q(t) = \Phi(q(0), q(t)) = \begin{pmatrix} \phi_1(q^{(1)}(0), u^{(1)}(t)) \\ \phi_2(q^{(2)}(0), u^{(2)}(t)) \\ \vdots \\ \phi_m(q^{(m)}(0), u^{(m)}(t)) \end{pmatrix} \tag{13}$$

and

$$x(t) = \Psi(\Phi(q(0), u(t))) = \begin{pmatrix} \psi_1(\phi_1(q^{(1)}(0), u^{(1)}(t))) \\ \psi_2(\phi_2(q^{(2)}(0), u^{(2)}(t))) \\ \vdots \\ \psi_m(\phi_m(q^{(m)}(0), u^{(m)}(t))) \end{pmatrix} \tag{14}$$

3.2 Coverage Model

It is assumed that at every time t sensor σ_j can take measures on magnitude $\xi \in \Xi^{(j)}$ in a circular area of radius ρ_ξ around its current position $x^{(j)}(t)$. The sensor field of measure respect to ξ is then a disk of centre $x^{(j)}(t)$ and radius ρ_ξ:

$$M_\xi^{(j)}(q^{(j)}) = M_\xi(q^{(j)}) = \{p \in W : \|x^{(j)} - x_p\| \leq \rho_\xi \quad \xi \in \Xi^{(j)}\} \tag{15}$$

As seen in 2.2, starting from $M_\xi^{(j)}(q^{(j)})$ is possible to define the area $\xi - covered$ and the area $\Xi - covered$ by the sensor network during a given time interval .

Homogeneous Sensors

A particular case is the one in witch sensors are homogeneous with respect to sensing capabilities:

$$\Xi^{(j)} = \Xi \, \forall j$$

Assuming without loss of generality that there is only one magnitude of interest on W, the set covered by sensor σ_j at every time t can be described by:

$$M(q^{(j)}) = \{p \in W : \|q^{(j)} - x_p\| \leq \rho\} \tag{16}$$

It is possible to define the subset of W covered by the sensor network in a time interval Θ as:

$$M(q(\Theta)) = \bigcup_{\sigma_j \in \Sigma} \left[\bigcup_{t \in \Theta} M(q^{(j)}(t)) \right] \tag{17}$$

3.3 Communication Model

The communication network is modelled as an Euclidean graph. Two mobile sensors at time t are assumed to communicate each other if the distance between them is smaller than a given communication radius ρ_C.

For every sensor σ_j, the communication function is given by:

$$\kappa^{(j)}\left(q^{(j)}, q^{(h)}\right) = \kappa\left(q^{(j)}, q^{(h)}\right) = \rho_C - \left\|x^{(j)} - x^{(h)}\right\| \tag{18}$$

Is easy to see that this communication function makes the network graph $\mathcal{G}$ undirected, in fact:

$$\kappa^{(j)}\left(q^{(j)}, q^{(h)}\right) = \kappa^{(h)}\left(q^{(h)}, q^{(j)}\right) \quad \forall j, h$$

3.4 Coverage Problem Formulation

According with the introduced model is possible to formulate the coverage problem as an optimal control problem. The idea is to maximize the area covered by sensors in a fixed time interval according with some constrains.

3.4.1 Objective Functional

In 2.2 the area $\xi - covered$ by a set of m moving sensors is defined as the union of the measure sets of the sensors, respect to magnitude ξ, at every time t. This quantity is very hard to compute, also for the simple measure set model introduced in 3.2, then an alternative performance measure has to be used.

Defining the distance between a point p of the workspace and a generalized trajectory $x(\Theta)$, within a time interval $\Theta = [0, t_f]$, as

$$d_\xi(x(\Theta), p) = \min_{t \in \Theta, j \in \{1,2,\ldots m\}} \left\| x_p - x^{(j)}(\Theta) \right\| \quad \xi \in \Xi^{(j)} \tag{19}$$

and making use of the function

$$\mathrm{pos}(\chi) = \begin{cases} \chi & \text{if} \quad \chi > 0 \\ 0 & \text{if} \quad \chi \le 0 \end{cases} \tag{20}$$

that fixes to zero any non positive value, the function

$$\hat{d}_\xi(x(\Theta), p, \rho_\xi) = \mathrm{pos}\left(d_\xi(x(\Theta), P) - \rho_\xi\right) \ge 0$$

can be defined. Then, a measure of how the generalized trajectory $q(\Theta)$ produces a good $\xi - coverage$ of the workspace can be given by

$$J_\xi(x(\Theta)) = \int_{p\in W} \hat{d}_\xi(x(\Theta), p, \rho_\xi) \tag{21}$$

Looking at the whole magnitudes of interest set Ξ is possible to introduce a functional that evaluate how a given generalized trajectory $x(\Theta)$ $\Xi - cover$ the set of interest W

$$J_\Xi(x(\Theta)) = \sum_{\xi\in\Xi} J_\xi(x(\Theta)) \tag{22}$$

Smaller is $J_\Xi(x(\Theta)$, better is the $\Xi - coverage$ of W. If $J_\Xi(x(\Theta)) = 0$ then $x(\Theta)$ $\Xi - covers$ completely the workspace.

From 3.1 is possible to see how J_Ξ can be also written as:

$$J_\Xi(q(0), u(\Theta)) = \sum_{\xi\in\Xi} J_\xi(\Psi(\Phi(q(0), u(\Theta)))) \tag{23}$$

Homogeneous Sensors

For the homogeneous sensors case the objective functional is:

$$J(z(0), u(\Theta)) = \int_{p\in W} \hat{d}(\Psi(\Phi(z(0), u(\Theta)))), p, \rho) \tag{24}$$

3.4.2 Geometric Constraints

It is possible to constrain sensors to move inside a box subset of $\mathbb{R}^2$

$$x_{min} \leq x^{(j)}(t) \leq x_{max}$$

If needed is possible to set the staring and/or the final state (positions and/or speeds):

$$q(0) = q_{start}$$
$$q(t_f) = q_{end}$$

A particular case is the periodic trajectories constrain, useful in tasks in which measures have to be repeated continuously:

$$q^{(j)}(0) = q^{(j)}(t_f)$$

Is also necessary to avoid collisions between sensors at every time t

$$\|x^{(j)}(t) - x^{(h)}(t)\| \geq \rho_B$$

for $j \neq h$

3.4.3 Dynamic Constraints

Physical limits on the actuators (for the motion) and/or on the sensors (in terms of velocity in the measure acquisition) suggest the introduction of the following additional constraints

$$|\dot{x}(t)| \leq v_{max}$$
$$|u(t)| \leq u_{max}$$

3.4.4 Communication Constraints

As said communication network connectivity is very important for data exchange and transmission, but also for sensor localization, coordination and commands communication. Under the hypothesis that before sensors start moving the communication network is connected, it is possible to maintain connectivity introducing constraints on the instantaneous position of sensors. More strongly, it is possible to impose a fixed network topology, this can be useful, for example, to fix the level of redundancy on the communication link an then to reach node fault tolerance.

Fixed Network Topology

To maintain a fixed network topology every sensor must maintain direct communication with a subset of its starting neighbors that is fixed in time. Indicating with $\mathcal{G}_d =< V_\mathcal{G}, E_{\mathcal{G}_d}(t) >$ the graph that represents desired topology, where $E_{\mathcal{G}_d}(t) \subseteq E_\mathcal{G}(t)$ $\forall t \in \Theta$.

According with 3.3, for every edge of $\mathcal{G}_d$ a distance constrain between a couple of sensors must be introduced, so maintaining a desired topology $\mathcal{G}_d$ means to satisfy the following constrains set $\forall t \in \Theta$:

$$\|x^{(j)}(t) - x^{(h)}(t)\| \leq \rho_C \quad \forall (\sigma_j, \sigma_h) \in E_{\mathcal{G}_d}(t) \tag{25}$$

Network Connectivity Maintenance

Fixed topology maintenance is, obviously a particular case of connectivity maintenance if the desired topology is connected. Anyway, this approach introduces strong constrains on sensors motion. This constrains can be relaxed in only connectivity is needed, allowing network topology to change over time. That increase coverage performances as shown in (Gabriele and Di Giamberardino, 2007b).

As said before, the communication model introduced in 3.3 makes the communication graph $\mathcal{G}(t)$ to be undirected. A undirected graph is connected if and only if it contain a spanning tree. So it is possible to maintain network connectivity constraining every sensor just to maintain direct communication links that corresponds to the edges of a spanning tree of the communication tree.

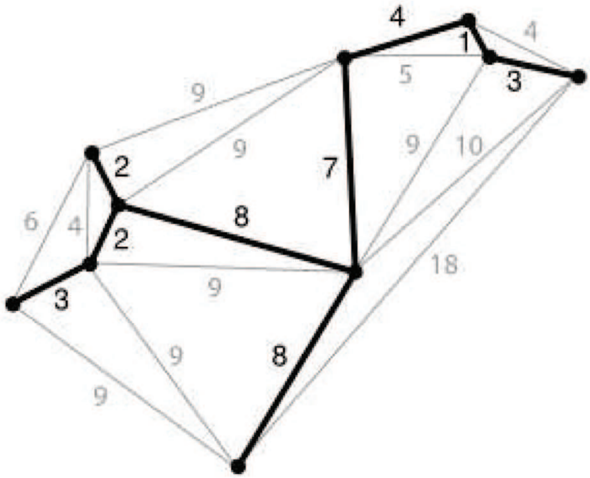


Fig. 3. Minimum Spanning Tree for a planar weighted undirected graph.

Assigning a weight at every edge of $E_{\mathcal{G}}$ is possible to define the *Minimum Spanning Tree* of $\mathcal{G}$ as the spanning tree with minimum weight (Figure 3). In particular being $\mathcal{G}$ an Euclidean graph it come natural to define the edges weights as:

$$w\left(x^{(j)}, x^{(h)}\right) = \left\|x^{(j)} - x^{(h)}\right\|$$

in this case the minimum spanning tree is said *Euclidean* (EMST). The EMST can be easily and efficiently computed by standard algorithms (such as Prim's algorithm or Kruskal's algorithm). Indicating the EMST with $\mathcal{T}(t) =< V_{\mathcal{G}}, E_{\mathcal{T}}(t) >$, where $E_{\mathcal{T}}(t) \subseteq E_{\mathcal{G}}(t)$, maintaining the communication network connection means to satisfy the following constrains $\forall t \in \Theta$:

$$\left\|x^{(j)}(t) - x^{(h)}(t)\right\| \leq \rho_C \quad \forall \left(\sigma_j, \sigma_h\right) \in E_{\mathcal{T}}(t) \tag{26}$$

The minimum spanning tree of the communication network graph changes while sensors moves, so the neighbours set of every node change over time making the network topology dynamic.

3.4.5 Optimal Control Problem

The coverage problem can now be formulated as an optimal control problem:

$$\min_{q(0),u(\Theta)} J_\Xi(q(0),u(\Theta)) = \sum_{\xi\in\Xi} J_\xi(\Psi(\Phi(q(0),u(\Theta))))$$

$$x_{min} \le \Psi(\Phi(q(0),u(\Theta))) \le x_{max} \quad \forall t\in\Theta$$
$$\|\psi(\phi(q^{(j)}(0),u^{(j)}(t))) - \psi(\phi(q^{(h)}(0),u^{(h)}(t)))\| \ge \rho_B \quad i\neq j \quad \forall t\in\Theta$$
$$(q(0) = q_{start})$$
$$(q(t_f) = q_{end})$$

$$|B_j^T\phi(q^{(j)}(0),u^{(j)}(t))| \le v_{max} \quad \forall j \quad \forall t\in\Theta$$
$$|u(t)| \le u_{max} \quad \forall t\in\Theta$$

$$\|\psi(\phi(q^{(j)}(0),u^{(j)}(t))) - \psi(\phi(q^{(h)}(0),u^{(h)}(t)))\| \le \rho_C \quad \forall\,(\sigma_j,\sigma_h)\in E_{\mathcal{T}}(t)$$

$$(\|\psi(\phi(q^{(j)}(0),u^{(j)}(t))) - \psi(\phi(q^{(h)}(0),u^{(h)}(t)))\| \le \rho_C \quad \forall\,(\sigma_j,\sigma_h)\in E_{\mathcal{G}_d}(t))$$

This problem is, in general, very hard to solve analytically. In the next section a simpler discretized model is introduced to evaluate suboptimal solutions.

4. Discretized Model

In order to overcome the difficulty of the problem defined in 3.4, a discretization is performed, both with respect to space, and with respect to time in all the time dependent expressions. The workspace W is then divided into square cells, with resolution (size) l_{res}, obtaining a grid in witch each point c_{rs} is the centre of a cell, and the trajectories are discretized with sample time T_s. This allow to represent the coverage problem as a solvable *Nonlinear Programming Problem* .

4.1 Sensors Discretized Dynamics

The discrete time sensors dynamic is well described by the following equations:

$$\begin{aligned} &q^{(j)}((k+1)T_s) = A_{dj}q^{(j)}(kT_s) + B_{dj}u^{(j)}(kT_s) \\ &x^{(j)}(kT_s) = C_j q^{(j)}(kT_s) \end{aligned} \tag{27}$$

where

$$A_{dj} = e^{A_jT_s} \quad B_{dj} = \int_0^{T_s} e^{A_j\tau}B_j d\tau$$

Representing the $j-th$ sensor input sequence from time $t=0$ to time $t=NT_s$ as:

$$u_N^{(j)} = \begin{pmatrix} u^{(j)}(0) \\ u^{(j)}(T_s) \\ \vdots \\ u^{(j)}((N-1)T_s) \end{pmatrix}$$

and defining the following vectors

$$v_N^{(j)} = \begin{pmatrix} q^{(j)}(0) \\ u_N^{(j)} \end{pmatrix} \qquad H_n^{(j)} = \begin{pmatrix} A_{dj}^n \\ A_{dj}^{n-1} B_{dj} \\ \vdots \\ B_{dj} \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

is possible to write state and output values at time $nT_s \leq NT_s$ as:

$$q^{(j)}(nT_s) = H_n^{(j)^T} v_N^{(j)} = A_{dj}{}^n q^{(j)}(0) + \sum_{k=0}^{n-1} A_{dj}{}^k B_{dj} u^{(j)}((n-1)T_s - kT_s) \tag{28}$$

and

$$x^{(j)}(nT_s) = C_j q^{(j)}(nT_s) = C_j H_n^{(j)^T} v_N^{(j)} \tag{29}$$

State and output sequences, from time $t = 0$ to time $t = NT_s$, can be represented by the following vectors:

$$q_N^{(j)} = \begin{bmatrix} q^{(j)}(0) \\ q^{(j)}(T_s) \\ \vdots \\ q^{(j)}(NT_s) \end{bmatrix} \qquad x_N^{(j)} = \begin{bmatrix} x^{(j)}(0) \\ x^{(j)}(T_s) \\ \vdots \\ x^{(j)}(NT_s) \end{bmatrix}$$

According with 28 and 29 the relations between these sequences and the input one are described by:

$$q_N^{(j)} = H_N^{(j)} v_N^{(j)} = \begin{pmatrix} {H_0^{(j)}}^T \\ {H_1^{(j)}}^T \\ \vdots \\ {H_N^{(j)}}^T \end{pmatrix} v_N^{(i)} \tag{30}$$

and

$$x_N^{(j)} = C_{Nj} H_N^{(j)} v_N^{(j)} = \begin{pmatrix} C_j & 0 & \cdots & 0 \\ 0 & C_j & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & C_j \end{pmatrix} \begin{pmatrix} {H_0^{(j)}}^T \\ {H_1^{(j)}}^T \\ \vdots \\ {H_N^{(j)}}^T \end{pmatrix} v_N^{(j)} \tag{31}$$

As done in 3.1, is possible to define generalized input, state and output sequences of the whole system:

$$v_N = \begin{bmatrix} v_N^{(1)} \\ v_N^{(2)} \\ \vdots \\ v_N^{(m)} \end{bmatrix}$$

$$q_N^{(i)} = \begin{bmatrix} q_N^{(1)} \\ q_N^{(2)} \\ \vdots \\ q_N^{(m)} \end{bmatrix} \qquad x_N^{(i)} = \begin{bmatrix} x_N^{(1)} \\ x_N^{(2)} \\ \vdots \\ x_N^{(m)} \end{bmatrix}$$

These sequences are related by:

$$q_N = H_N v_N = \begin{pmatrix} H_N^{(1)} & 0 & \cdots & 0 \\ 0 & H_N^{(2)} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & H_N^{(m)} \end{pmatrix} v_N \tag{32}$$

And

$$x_N = C_N H_N v_N = \begin{pmatrix} C_{N1} H_N^{(1)} & 0 & \cdots & 0 \\ 0 & C_{N1} H_N^{(2)} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & C_{Nm} H_N^{(m)} \end{pmatrix} v_N \tag{33}$$

4.2 Coverage Problem Formulation

Using the coverage model defined in 3.2 and the communication model in 3.3, it is possible to formulate the coverage problem as a nonlinear programming problem.

4.2.1 Objective Function

The objective functional defined in 3.4.1 became, after the discretization, a function of the vector v_N:

$$J_\Xi(v_N) = \sum_\xi \sum_r \sum_s \hat{d}_\xi(C_N H_N v_N, c_{rs}, \rho_\xi) \tag{34}$$

4.2.2 Nonlinear Programming Problem

Defining geometric, dynamic and communication constrains as in 3.4 is possible to write the coverage problem for a dynamic sensor network as a tractable constrained optimization problem:

$$\min_{v_N} J_\Xi(v_N) = \sum_\xi \sum_r \sum_s \hat{d}_\xi(C_N H_N v_N, c_{rs}, \rho_\xi)$$

$$\begin{gathered}
x_{min} \leq C{H_n^{(j)}}^T v_N^{(j)} \leq x_{max} \ \forall j,\ \forall n = 0, 1, \cdots, N \\
\|C{H_n^{(j)}}^T v_N^{(j)} - C{H_n^{(h)}}^T v_N^{(h)}\| \geq \rho_B \ j \neq h,\ \forall n = 0, 1, \cdots, N \\
({H_0^{(j)}}^T V_N^{(j)} = q_{start}^{(j)} \ \forall j) \\
({H_N^{(j)}}^T V_N^{(j)} = q_{end}^{(j)} \ \forall j)
\end{gathered}$$

$$\begin{gathered}
|B^T {H_n^{(j)}}^T v_N^{(j)}| \leq v_{max} \ \forall j,\ \forall n = 0, 1, \cdots, N \\
|u^{(j)}(nT_s)| \leq u_{max} \ \forall j,\ \forall n = 0, 1, \cdots, N-1
\end{gathered}$$

$$\begin{gathered}
\|C_j {H_n^{(j)}}^T v_N^{(j)} - C_h {H_n^{(h)}}^T v_N^{(h)}\| \leq \rho_C \text{ if } \forall\, (j,h) \,\|\, (\sigma_j, \sigma_h) \in E_{\mathcal{T}}(nT_s) \ \forall n \\
\text{or} \\
(\|C_j {H_n^{(j)}}^T v_N^{(j)} - C_h {H_n^{(h)}}^T v_N^{(h)}\| \leq \rho_C \text{ if } \forall\, (j,h) \,\|\, (\sigma_j, \sigma_h) \in E_{\mathcal{G}_d}(nT_s) \ \forall n)
\end{gathered}$$

Suboptimal solutions can be computed using numerical methods. In the simulations performed, the SQP (Sequential Quadratic Programming) method has been applied.

5. Simulation Results

In this section simulation results for different cases are presented to show the effectiveness of the proposed methodology. At first two simulations for the single sensor case are presented to show the *quality* of the computed trajectories that are, anyway, suboptimal. The first case considered is the one of one sensor asked to measure a magnitude ξ, defined a circular area, within a time interval $\Theta = 15\ sec$. Sensor dynamic parameters are:

$$u_{max} = 0.5 \qquad v_{max} = 1.5$$

Sensor starts from position $[0, 0]$ with zero speeds. Sensor radius of measure is $\rho_\xi = 1.5$. Simulation result are showed in figure 4.

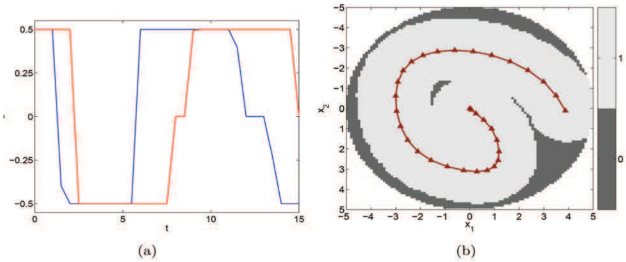


Fig. 4. One sensor covering a circular area. (a) Control components evolution. (b) Speed components evolution. (c) Sensor trajectory and coverage status of the set of interest.

In the second case, showed in figure 5, the constraint of making a cyclic trajectory is added. Cyclic trajectories are very useful for surveillance tasks. Time interval is extended to $\Theta = 25\ sec$.

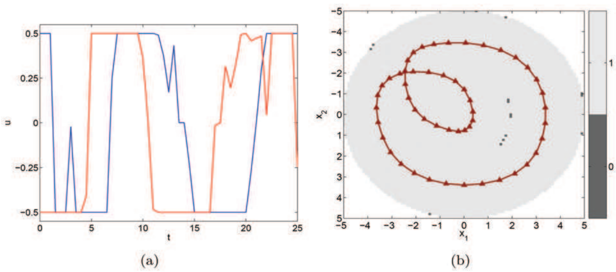


Fig. 5. One sensor covering a circular area making a cyclic trajectory. (a) Control components evolution. (b) Speed components evolution. (c) Sensor trajectory and coverage status of the set of interest

The third case considered (figure 6) is the one of an homogeneous sensor network, with three nodes, covering a box shaped workspace within a time interval $\Theta = 15\ sec.$. Communication between two nodes is assumed to be reliable within a maximum range of

$$\rho_c = 5.5$$

Sensors dynamic parameters are:

$$u_{max} = 1.5 \qquad v_{max} = 1.5$$

Collisions avoidance and connectivity maintenance constraints are considered.

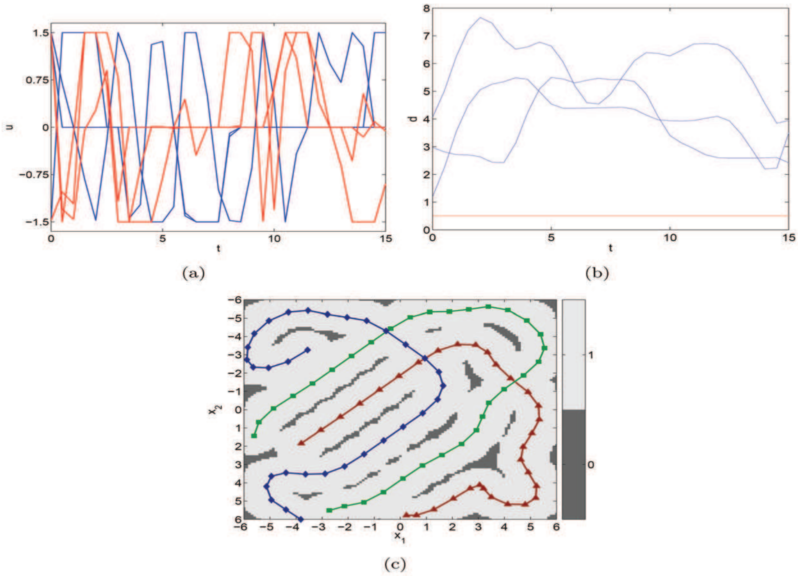


Fig. 6. Coverage of a box shaped workspace with a dynamic sensor network with three homogeneous nodes. (a) Control components evolution. (b) Relative distances between all vehicles, the red line represents minimum distance for collisions avoidance (rho_B). (c) Sensors trajectories and coverage status of the set of interest.

In figure 7 simulations are shown for the case of an heterogeneous sensor network covering a box shaped workspace within a time interval $\Theta = 15\ sec$. Three magnitudes of interest are defined,

$$\Xi = \{\xi_1, \xi_2, \xi_3\}$$

The radii within the three magnitudes can be measured are

$$\rho_{\xi_1} = 2 \qquad \rho_{\xi_2} = 1 \qquad \rho_{\xi_3} = 3$$

Nodes dynamic parameters are:

$$u_{max} = 1.5 \qquad v_{max} = 1.5$$

Communication between two nodes is assumed to be reliable within a maximum range of

$$\rho_c = 5.5$$

The sensor network is composed by 4 nodes, with different sensing capabilities.

$$\begin{array}{ll} \Xi_1 = \{\xi_1, \xi_2\} & \Xi_2 = \{\xi_2, \xi_3\} \\ \Xi_3 = \{\xi_1, \xi_2\} & \Xi_4 = \{\xi_2, \xi_3\} \end{array}$$

Collisions avoidance and connectivity maintenance constraints are considered.

In figure 8 scenario similar to the one considered in the previous case is shown for a generic shaped workspace.

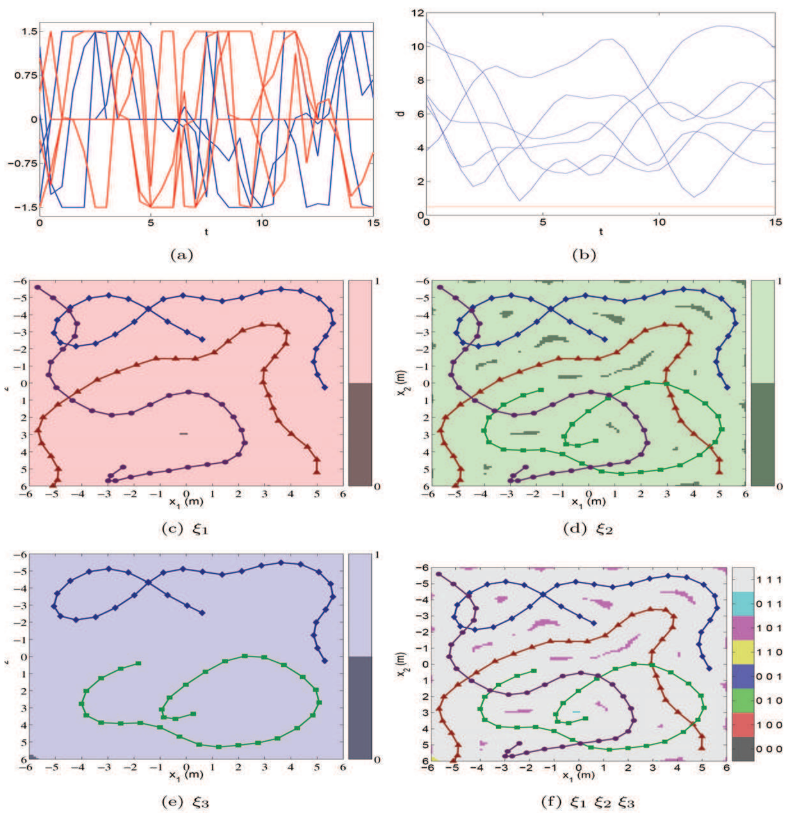
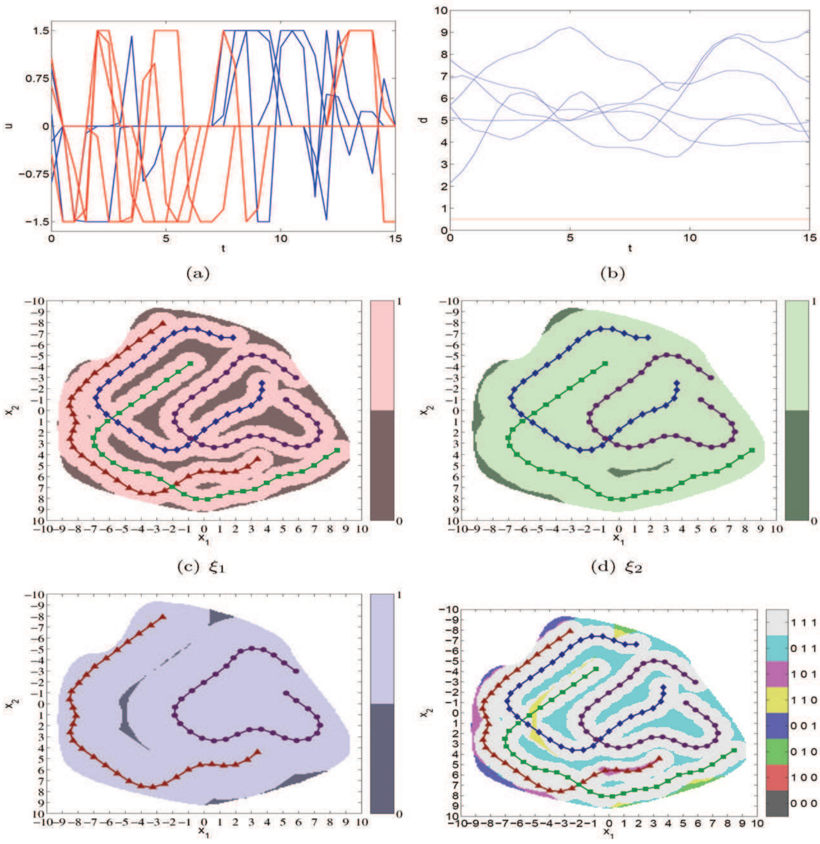


Fig. 7. Coverage of a box shaped workspace with an heterogeneous dynamic sensor network. (a) Control components evolutions. (b) Relative distances between all vehicles, the red line represents minimum distance for collisions avoidance (rho_B). (c) $\xi_1 - sensors$ trajectories and area $\xi_1 - covered$. (d) $\xi_2 - sensors$ trajectories and area $\xi_2 - covered$. (e) $\xi_3 - sensors$ trajectories and area $\xi_3 - covered$ status. (f) All nodes trajectories and coverage status of the workspace with respect to the whole magnitudes set Ξ



(a) (b) (c) ξ_1 (d) ξ_2 (e) ξ_3 (f) ξ_1 ξ_2 ξ_3

Fig. 8. Coverage of a generic shaped workspace with an heterogeneous dynamic sensor network. (a) Control components evolutions. (b) Relative distances between all vehicles, the red line represents minimum distance for collisions avoidance (rho_B). (c) $\xi_1 - sensors$ trajectories and area $\xi_1 - covered$. (d) $\xi_2 - sensors$ trajectories and area $\xi_2 - covered$. (e) $\xi_3 - sensors$ trajectories and area $\xi_3 - covered$ status. (f) All nodes trajectories and coverage status of the workspace with respect to the whole magnitudes set Ξ

6. Conclusions

In this chapter the case of heterogeneous mobile sensor networks has been considered. The mobility of the sensors is introduced in order to allow a reduced number of sensors to measure the same field, under the assumption that the temporal resolution of the measures, i.e. the maximum time between two consecutive measures at the same coordinates, is not too small. In addition, each mobile platform representing the nodes of the net has been considered equipped with different sets of sensors, so introducing a non homogeneity in the sensor network. A general formulation of the field coverage problem as been introduced in terms of optimal control techniques. All the constraints introduced by kinematics and dynamic limits on mobility of the moving elements as well as by communications limits (network connectivity) have been considered. A global approach has been followed making use of time and space discretization, so getting a suboptimal solution. Some simulation results show the behaviour and the effectiveness of the proposed solution.

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InTech Europe
University Campus STeP Ri
Slavka Krautzeka 83/A
51000 Rijeka, Croatia
Phone: +385 (51) 770 447
Fax: +385 (51) 686 166
www.intechopen.com

InTech China
Unit 405, Office Block, Hotel Equatorial Shanghai
No.65, Yan An Road (West), Shanghai, 200040, China
中国上海市延安西路65号上海国际贵都大饭店办公楼405单元
Phone: +86-21-62489820
Fax: +86-21-62489821