Robust Position Estimation of an Autonomous Mobile Robot

Touati Youcef, Amirat Yacine*, Djamaa Zaheer* & Ali-Chérif Arab

University Paris 8 Saint-Denis
France
*University Paris 12 Val de Marne
France

1. Introduction

In the past few years, the topic of localization has received considerable attention in the research community and especially in mobile robotics area (Borenstein, 1996). It consists of estimating the robot’s pose (position, orientation) with respect to its environment from sensor data. Therefore, better sensory data exploitation is required to increase robot’s autonomy. The simplest way to estimate the pose parameters is integration of odometric data which, however, is associated with unbounded errors, resulting from uneven floors, wheel slippage, limited resolution of encoders, etc. However, such a technique is not reliable due to cumulative errors occurring over the long run. Therefore, a mobile robot must also be able to localize or estimate its parameters with respect to the internal world model by using the information obtained with its external sensors. In system localization, the use of sensory data from a range of disparate multiple sensors, is to automatically extract the maximum amount of information possible about the sensed environment under all operating conditions.

Usually, for many problems like obstacle detection, localization or Simultaneous Localization and Map Building (SLAM) (Montemerlo et al., 2002), the perception system of a mobile robot relies on the fusion of several kinds of sensors like video cameras, radars, dead-reckoning sensors, etc. The multi-sensor fusion problem is popularly described by state space equations defining the interesting state, the evolution and observation models. Based on this state space description, the state estimation problem can be formulated as a state tracking problem. To deal with this state observation problem, when uncertainty occurs, the probabilistic Bayesian approaches are the most used in robotics, even if new approaches like the set-membership one (Gning & Bonnifait, 2005) or Belief theory (Ristic and Smets, 2004) have proved themselves in some applications.

SLAM is the technique used by mobile robots to build up a map within an unknown environment while at the same time keeping track of their current position. Several works implementing SLAM algorithms have been studied extensively over the last years in this direction, leading to approaches that can be classified into three well differentiated paradigms depending on the underlying map structure: metric (Sim et al., 2006) (Tardos et
al., 2002), topological (Ranganathan et al., 2006) (Savelli & Kuipers, 2004), or hybrid representations (Estrada et al., 2005) (Kuipers & Byun, 2001) (Dissanayake et al., 2001) (Thrun et al., 2004). These techniques deal mainly with the localization problem using mainly visual features and exteroceptive sensors, such as camera, GPS unit or laser scanner. Localization algorithms have also been developed in sensors networks and applied in a myriad of applications such as intrusion detection, road traffic monitoring, health monitoring, reconnaissance and surveillance. Their main objectives is to estimate the location of sensors with initially unknown location information by using knowledge for absolute positions of a few sensors and their inter-sensor measurements such as distance and bearing measurements (Chong & Kumar, 2003) (Mao et al., 2007).

Ubiquitous computing technology is gradually being used to analyze people’s activities. In this case, several research efforts on localization function have been conducted into recognizing human position and trajectories (Letchner et al., 2005) (Madhavapeddy & Tse, 2005) (Kanda et al., 2007). For example, Liao et al. used locations obtained via GPS with relational Markov model to discriminate location-based activities (Liao et al., 2005). Wen et al. developed an approach for inhabitant location and tracking system in a cluttered home environment via floor load sensors (Liau et al., 2008). In this approach, a probabilistic data association technique is applied to analyze the cluttered pressure readings collected by the load sensors so as to track their movements.

The main idea of data fusion methods is to provide a reliable estimation of robot’s pose, taking into account the advantages of the different sensors (Harris, 1998). The main data fusion applied methods are very often based on probabilistic methods, and indeed probabilistic methods are now considered the standard approach to data fusion in all robotics applications. Probabilistic data fusion methods are generally based on Bayes’ rule for combining prior and observation information. Practically, this may be implemented in a number of ways: through the use of the Kalman and extended Kalman filters, through sequential Monte Carlo methods, or through the use of functional density estimates.

There are a number of alternatives to probabilistic methods. These include the theory of evidence and interval methods. Such alternative techniques are not as widely used as they once were, however they have some special features that can be advantageous in specific problems.

The rest of the presented work is organized as follows. Section 2 discusses the problem statement and related works in the field of multi-sensor data fusion for the localization of a mobile robot. Section 3 describes the global localization system which is considered. We develop the proposed robust pose estimation algorithm in section 4 and its application is demonstrated in section 5. Simulation results and a comparative analysis with standard existing approaches are also presented in this section.

2. Background & related works

The Kalman Filter (KF) is the best known and most widely applied parameter and state estimation algorithm in data fusion methods (Gao, 2002). Such a technique can be implemented from the kinematic model of the robot and the observation (or measurement) model, associated to external sensors (gyroscope, camera, telemeter, etc.). The Kalman filter has a number of features which make it ideally suited to dealing with complex multi-sensor estimation and data fusion problems. In particular, the explicit description of process and
observations allows a wide variety of different sensor models to be incorporated within the basic algorithm. In addition, the consistent use of statistical measures of uncertainty makes it possible to quantitatively evaluate the role each sensor plays in overall system performance. Further, the linear recursive nature of the algorithm ensures that its application is simple and efficient. For these reasons, the Kalman filter has found widespread application in many different data fusion problems (Bar-Shalom, 1990) (Bar-Shalom & Fortmann, 1988) (Maybeck, 1979). In robotics, the KF is most suited to problems in tracking, localisation and navigation; and less so to problems in mapping. This is because the algorithm works best with well defined state descriptions (positions, velocities, for example), and for states where observation and time-propagation models are also well understood.

The Kalman Filtering process can be considered as a prediction-update formulation. The algorithm uses a predefined linear model of the system to predict the state at the next time step. The prediction and updates are combined using the Kalman gain which is computed to minimize the Mean Square Error (MSE) of the state estimate. Figure 1 illustrates the block diagram of KF cycle (Bar-Shalom & Fortmann, 1988), and for further details, refer to (Siciliano & Khatib, 2008).

The Extended Kalman Filter (EKF) is a version of the Kalman filter that can handle non-linear dynamics or non-linear measurement equations. Like the KF, it is assumed that the noises are all Gaussian, temporally uncorrelated and zero-mean with known variance. The EKF aims to minimise mean-squared error and therefore compute an approximation to the conditional mean. It is assumed therefore that an estimate of the state at time $k-1$ is available which is approximately equal to the conditional mean. The main stages in the derivation of
the EKF follow directly from those of the linear Kalman filter with the additional step that the process and observation models are linearised as a Taylor series about the estimate and prediction, respectively. The algorithm iterates in two update stages, measurement and time, see figure 2. Each positioning operation is generated once a new observation is assumed. Localization can be done from odometry or visual input changes. The complete algorithm is implemented for each landmark perception. In this sense, the processing time is saved by reducing covariance matrix function size per landmark. Detailed computations may be found in any number of books on the subject (Samperio & Hu, 2006).

Fig. 2. Flowchart of Extended Kalman filter Algorithm (after Samperio & Hu, 2006)

Various approaches based on EKF have been developed. These approaches work well as long as the used information can be described by simple statistics well enough. The lack of relevant information is compensated by using models of various processes. However, such model-based approaches require assumptions about parameters which might be very difficult to determine (white Gaussian noise and initial uncertainty over Gaussian distribution). Assumptions that guarantee optimum convergence are often violated and, therefore, the process is not optimal or it can even converge. In fact, many approaches are based on fixed values of the measurement and state noise covariance matrices. However, such information is not a priori available, especially if the trajectory of the robot is not elementary and if changes occur in the environment. Moreover, it has been demonstrated in the literature that how poor knowledge of noise statistics (noise covariance on state and measurement vectors) may seriously degrade the Kalman filter performance (Jetto, 1999). In the same manner, the filter initialization, the signal-to-noise ratio, the state and observation processes constitute critical parameters, which may affect the filtering quality. The stochastic Kalman filtering techniques were widely used in localization (Gao, 2002) (Chui, 1987) (Arras, 2001)(Borthwick, 1993) (Jensfelt, 2001) (Neira, 1999) (Perez, 1999) (Borges, 2003). Such approaches rely on approximative filtering, which requires ad hoc tuning of stochastic
modelling parameters, such as covariance matrices, in order to deal with the model approximation errors and bias on the predicted pose. In order to compensate such error sources, local iterations (Kleeman, 1992), adaptive models (Jetto, 1999) and covariance intersection filtering (Julier, 1997) (Xu, 2001) have been proposed. An interesting approach solution was proposed in (Jetto, 1999), where observation of the pose corrections is used for updating of the covariance matrices. However, this approach seems to be vulnerable to significant geometric inconsistencies of the world models, since inconsistent information can influence the estimated covariance matrices.

In the literature, the localization problem is often formulated by using a single model, from both state and observation processes point of view. Such an approach, introduces inevitably modelling errors which degrade filtering performances, particularly, when signal-to-noise ratio is low and noise variances have been estimated poorly. Moreover, to optimize the observation process, it is important to characterize each external sensor not only from statistic parameters estimation perspective but also from robustness of observation process perspective. It is then interesting to introduce an adequate model for each observation area in order to reject unreliable readings. In the same manner, a wrong observation leads to a wrong estimation of the state vector and consequently degrades the performance of localization algorithm. Multiple-Model estimation has received a great deal of attention in recent years due to its distinctive power and great recent success in handling problems with both structural and parametric uncertainties and/or changes, and in decomposing a complex problem into simpler sub-problems, ranging from target tracking to process control (Blom, 1988) (Li, 2000) (Li, 1993) (Mazor, 1996).

This paper focuses on robust pose estimation for mobile robot localization. The main idea of the approach proposed here is to consider the localization process as a hybrid process which evolves according to a model among a set of models with jumps between these models according to a Markov chain (Djamaa & Amirat, 1999) (Djamaa, 2001). A close approach for multiple model filtering is proposed in (Oussalah, 2001). In our approach, models refer here to both state and observation processes. The data fusion algorithm which is proposed is inspired by the approach proposed in (Dufour, 1994). We generalized the latter for multi mode processes by introducing multi mode observations. We also introduced iterative and adaptive EKFs for estimating noise statistics. Compared to a single model-based approach, such an approach allows the reduction of modelling errors and variables, an optimal management of sensors and a better control of observations in adequacy with the probabilistic hypotheses associated to these observations. For this purpose and in order to improve the robustness of the localization process, an on line adaptive estimation approach of noise statistics (state and observation) proposed in (Jetto, 1999), is applied for each mode. The data fusion is performed by using Adaptive Linear Kalman Filters for linear processes and Adaptive EKF for nonlinear processes.

3. Localization system description

This paper deals with the problem of multi sensor filtering and data fusion for the robust localization of a mobile robot. In our present study, we consider an autonomous robot equipped with two telemeters placed perpendicularly, for absolute position measurements of the robot with respect to its environment, a gyroscope for measuring robot’s orientation, two drive wheels and two separate encoder wheels attached with optical shaft encoders for
odometry measurements. The environment where the mobile robot moves is a rectangular room without obstacles, see figure 3.

![Mobile robot description and its evolution in the environment](image)

**Fig. 3. Mobile robot description and its evolution in the environment with Nominal trajectory**

The aim is not to develop a new method for environment reconstruction or modelling from data sensors; rather, the goal is to propose a new approach to improve existing data fusion and filtering techniques for robust localization of a mobile robot. For an environment with a more complex shape, the observation model which has to be employed at a given time, will depend on the robot’s situation (robot’s trajectory, robot’s pose with respect to its environment) and on the geometric or symbolic model of environment. Initially, all significant information for localization is contained in a state space vector. The usefulness of an observer in a localization system evokes the modelling of variables that affect the entire behaviour system. The observer design problem relies on the estimation of all possible internal states in a linear system.

### 3.1 Odometric model

Let $X_o(k) = [x(k), y(k), \theta(k)]^T$ be the state vector at time $k$, describing the robot’s pose with respect to the fixed coordinate system.

The kinematic model of the robot is described by the following equations:

$$x_{k+1} = x_k + l_k \cdot \cos(\theta_k + \Delta \theta_k / 2)$$  \hspace{1cm} (1)
\[ y_{k+1} = y_k + l_k \sin(\theta_k + \Delta \theta_k / 2) \] (2)

\[ \theta_{k+1} = \theta_k + \Delta \theta_k \] (3)

with: \( l_k = (l^r_k + l^l_k) / 2 \) and \( \Delta \theta_k = (l^r_k - l^l_k) / d \). \( l^r_k \) and \( l^l_k \) are the elementary displacements of the right and the left wheels; \( d \) the distance between the two encoder wheels.

### 3.2 observation model of telemeters
As the environment is a rectangular room, the telemeters measurements correspond to the distances from the robot location to walls (Fig. 3.).

Then, the observation model of telemeters is described as follows:

for \( 0 \leq \theta(k) < \theta^l \), according to X-axis:

\[ d(k) = (\sqrt{d^x - x(k)})/\cos(\theta(k)) \] (4)

for \( \theta^l \leq \theta(k) \leq \theta^m \), according to Y-axis:

\[ d(k) = (\sqrt{d^y - y(k)})/\sin(\theta(k)) \] (5)

With \( d^x \) and \( d^y \), respectively the length and the width of the experimental site; \( \theta^l \) and \( \theta^m \), respectively the angular bounds of observation domain with respect to X and Y axes; \( d(k) \) is the distance between the robot and the observed wall with respect to X or Y axes at time \( k \).

### 3.3 observation model of gyroscope
By integrating the rotational velocity, the gyroscope model can be expressed by the following equation:

\[ \theta_f(k) = \theta(k) \] (6)

Each sensor described above is subject to random noise. For instance, the encoders introduce incremental errors (slippage), which particularly affect the estimation of the orientation. For a telemeter, let’s note various sources of errors: geometric shape and surface roughness of the target, beam width. For a gyroscope, the sources of errors are: the bias drift, the nonlinearity in the scale factor and the gyro’s susceptibility to changes in ambient temperature.
So, both odometric and observation models must integrate additional terms representing these noises. Models inaccuracies induce also noises which must be taken into account. It is well known that odometric model is subject to inaccuracies caused by factors such as: measured wheel diameters, unequal wheel-diameters, trajectory approximation of robot between two consecutive samples. These noises are usually assumed to be Zero-mean white Gaussian with known covariance. This hypothesis is discussed and reconsidered in the proposed approach.

Besides, an estimation error of orientation introduces an ambiguity in the telemeters measurements (one telemeter is assumed to measure along X axis while it is measuring along Y axis and vice-versa). This situation is particularly true when the orientation is near angular bounds \( \theta^l \) and \( \theta^m \). This justifies the use of multiple models to reduce measuring errors and efficiently manage robot’s sensors. For this purpose, we have introduced the concept of observation domain (boundary angles) as defined in equations (4) and (5).

### 4. Proposed approach for mobile robot localisation

As mentioned in (Touati et al., 2007), we present our data fusion and filtering approach for the localization of a mobile robot. In order to increase the robustness of the localization and as discussed in section 2, the localization process is decomposed into multiple models. Each model is associated with a mode and an interval of validity corresponding to the observation domain; the aim is to consider only reliable information by filtering erroneous information. The localization is then considered as a hybrid process. A Markov chain is employed for the prediction of each model according to the robot mode. The multiple model approach is best understandable in terms of stochastic hybrid systems. The state of a hybrid system consists of two parts: a continuously varying base-state component and a modal state component, also known as system mode, which may only jump among points, rather than vary continuously, in a (usually discrete) set. The base state components are the usual state variables in a conventional system. The system mode is a mathematical description of a certain behavior pattern or structure of the system. In our study, the mode corresponds to robot’s orientation. In fact, the latter parameter governs the observation model of telemeters along with observation domain. Other parameters, like velocity or acceleration, could also be taken into account for mode’s definition. Updating of mode’s probability is carried out either from a given criterion or from given laws (probability or process). In this study, we assume that each Markovian jump (mode) is observable (Djamaa 2001) (Dufour, 1994). The mode is observable and measurable from the gyroscope.

#### 4.1 Proposed filtering models

Let us consider a stochastic hybrid system. For a linear process, the state and observation processes are given by:

\[
X_e(k/k-1, \alpha_k) = A(\alpha_k) \cdot X_e(k-1/k-1, \alpha_k) + B(k, \alpha_k) \cdot U(k-1, \alpha_k) + W(k, \alpha_k)
\]

\[
Y_e(k, \alpha_k) = C(\alpha_k) \cdot X_e(k/k-1, \alpha_k) + V(k, \alpha_k)
\]

(7)

(8)
For a nonlinear process, the state and observation processes are described by:

\[
X_e(k) = F(X_e(k-1), U(k-1)) + W(k)
\]

\[
Y_e(k) = G_e(X_e(k), U(k)) + V(k)
\]

where \(X_e\), \(Y_e\) and \(U\) are the base state vector, noisy observation vector and input vector; \(\alpha_k\) is the modal state or system mode at time \(k\), which denotes the mode during the \(k^{th}\) sampling period; \(W\) and \(V\) are the mode-dependent state and measurement noise sequences, respectively.

The system mode sequence \(\{\alpha_k\}\) is assumed for simplicity to be a first-order homogeneous Markov chain with the transition probabilities, so for \(\forall \alpha_i, \alpha_j \in S\) :

\[
P(\alpha_{k+1} = \alpha_j | \alpha_k = \alpha_i) = \pi_{ij}
\]

Where \(\alpha_k\) denotes that mode \(\alpha_j\) is in effect at time \(k\) and \(S\) is the set of all possible system modes, called mode space.

The state and measurement noises are of Gaussian white type. In our approach, the state and measurement processes are assumed to be governed by the same Markov chain. However, it’s possible to define differently a Markov chain for each process. The Markov chain transition matrix is stationary and well defined.

### 4.2 Statistics parameters estimation

It is well known that how poor estimates of noise statistics may lead to the divergence of Kalman filter and degrade its performance. To prevent this divergence, we propose an adaptive algorithm for the adjustment of the state and measurement noise covariance matrices.

**a. Measurement noise variance**

Let \(R = \{\sigma^2_{\gamma_i}(k)\} (i = 1:n_0)\), be the measurement noise variance at time \(k\) for each component of the observation vector. Parameter \(n_0\) denotes the number of observers (sensors number).

Let \(\hat{\beta}(k)\) the squared mean error for stable measurement noise variance, and \(\gamma(k)\) the innovation, thus:
\[ \hat{\beta}(k) = \frac{1}{n} \sum_{j=0}^{n} \gamma_j^2(k-1) \]  

(12)

For \( n + 1 \) samples, the variance of \( \hat{\beta}(k) \) can be written as:

\[ E(\hat{\beta}(k)) = \frac{1}{n+1} \sum_{j=0}^{n} \left( C_i(k-j) \cdot P(k-j,k-j-1) \cdot C_i^T(k-j) + \sigma_{i,j}^2 \right) \]  

(13)

Then, we obtain the estimation of the measurement noise variance:

\[ \hat{\sigma}_{i,j}^2 = \max \left\{ \frac{1}{n} \sum_{j=0}^{n} \left( \gamma_i^2(k-j) - \frac{n}{n+1} \cdot C_i(k-j) \cdot \Psi \right) \right\} \]  

(14)

The restriction with respect to zero is related to the notion of variance. A recursive formulation of the previous estimation can be written:

\[ \hat{\sigma}_{i,j}^2(k) = \max \left\{ \hat{\sigma}_{i,j}^2(k-1) + \frac{1}{n} \left( \gamma_i^2(k) - \gamma_i^2(k-(n+1)) - \frac{n}{n+1} \cdot \Psi \right) \right\} \]  

(15)

where:

\[ \Psi = C_i(k) \cdot P(k,k-1) \cdot C_i^T(k-1) - C_i(k-(n+1)) \cdot P(k-(n+1),k-(n+1)-1) \cdot C_i^T(k-(n+1)) \]  

(16)

**b. State noise variance**

To estimate the state noise variance, we employ the same principle as in subsection a. One can write:

\[ \hat{Q}_s(k) = \hat{\sigma}_{i,j}^2(k) \cdot Q_d \]  

(17)

By assuming that noises on the two encoder wheels measurements obey to the same law and have the same variance, the estimation of state noise variance can be written:
\[
\hat{\sigma}_{n,i}^2(k) = \max \left\{ \gamma_i^2(k-1) - C_i(k+1) \cdot P(k+1,k) \cdot \right. \\
\left. \frac{C_i(k+1)^T - \hat{\sigma}_{n,i}^2(k+1)}{C_i(k+1) \cdot Q_i \cdot C_i(k+1)^T}, 0 \right\} 
\]

(18)

with:

\[
\hat{Q}_d(k) = B(k) \cdot B(k)^T 
\]

(19)

By replacing the measurement noise variance by its estimate, we obtain a mean value given by the following equation:

\[
\hat{\sigma}_n^2(k) = \max \left\{ \frac{1}{(m+1) \cdot n_0} \sum_{j=1}^{n_0} \sum_{i=1}^{m} \hat{\sigma}_{n,i}^2(k-j), 0 \right\} 
\]

(20)

Where, the parameter \( m \) represents the sample number.

The algorithm proposed above carries out an on line estimation of state and measurement noise variances. Parameters \( n \) and \( m \) are chosen according to the number of samples used at time \( k \). The noises variances are initialized from an “a priori” information and then updated on line. In our approach, variances are updated according the robot’s mode and the measurement models.

For an efficient estimation of noise variances, an ad hoc technique consisting in a measure selection is employed. This technique consists of filtering unreliable readings by excluding readings with weak probability like the appearance of fast fluctuations. For instance, in the case of Gaussian distribution, we know that about 95% of the data are concentrated in the interval of confidence \([m-2\sigma, m+2\sigma]\) where \( m \) represents the mean value and \( \sigma \) the variance.

The sequence in which the filtering of the state vector components is carried out is important. Once the step of filtering completed, the probabilities of each mode are updated from the observers (sensors). One can note that the proposed approach is close, on one hand, to the Bayesian filter by the extrapolation of the state probabilities, and on the other hand to the filter with specific observation of the mode.

5. Implementation and results

The proposed approach for robust localization was applied for the mobile robot described in section 2. The nominal trajectory of the mobile robot includes three sub trajectories T1, T2 and T3, defining respectively a displacement along X axis, a curve and a displacement along Y axis, see figure 4.
Fig. 4. Mobile robot in moving in the environment with Nominal trajectory T1, T2 and T3.

Note that the proposed approach remains valid for any type of trajectory (any trajectory can be approximated by a set of linear and circular sub trajectories). For our study, we have considered three models. This number can be modified according to the environment’s structure, the type of trajectory (robot rotating around itself, forward or backward displacement, etc.) and to the number of observers (sensors). Notice that the number of models (observation and state) has no impact on the validity of the proposed approach.

To demonstrate the validity of our proposed Adaptive Multiple-Model approach and to show its effectiveness, we’ve compared it to the following standard approaches: Single-Model based EKF without estimation variance, single-model based IEKF. For sub trajectories T1 and T3, filtering and data fusion are carried out by iterative linear Kalman filters due to linearity of the models, and for sub trajectory T2, by iterative and extended Kalman filters.

The observation selection technique is applied for each observer before the filtering step in order to control, on one side, the estimation errors of variances, and on the other side, after each iteration, to update the state noise variance. If an unreliable reading is rejected at a given filtering iteration, this has for origin either a bad estimation of the next component of the state vector and of the prediction of the corresponding observation, or a bad updating of the corresponding state noise variance. The iterative filtering is optimal when it is carried out for each observer and no reading is rejected. In the implementation of the proposed approach, the state noise variance is updated, for a given mode $i$, is carried out according to the following filtering sequence: $x, y$ and then $\theta$. 

www.intechopen.com
Firstly, let’s consider the set of the following notations, table 1:

| $(\epsilon x, \epsilon y, \epsilon \theta)$ | Estimation errors corresponding to $(x, y, \theta)$ |
| $(Ndx, Ndy, Nd\theta)$ | Percentage of selected data for filtering corresponding to $(x, y, \theta)$ |
| $(Ndxe, Ndye, Nd\theta e)$ | Percentage of selected data for estimation of the variances of state and measurement noises, corresponding to $(x, y, \theta)$ |
| SM | $\rightarrow$ | Single-Model based EKF |
| SMI | $^\circ$ | single-model based IEKF |
| AMM | $- -$ | Adaptive Multiple-Model |

Table 1. Set of notations

Several scenarios have been studied according to the variation of statistics parameters, i.e., sensors signal-to-noise ratio, initial state variance, noise statistics (measurement and state variances). Simulations were carried out to analyze the performances of each approach in various scenarios. Thus, in scenarios 1 and 2, we show the influence of measurement and state noises variances estimation on the quality of localization. In scenario 3, it will concern the sensors signal-to-noise ratio.

**Scenario 1:**
- Noise-to-signal Ratio of odometric sensors: right encoder: 4%, left encoder: 4%
- Noise-to-signal Ratio of Gyroscope: 1%
- Noise-to-signal Ratio of telemeters: 2% of the odometric elementary step
- “A priori” knowledge on the variance in initial state: Good
- “A priori” knowledge on measurement noise variances: Good
- State and measurement noise variances estimation: 10 times real average variances of encoders

This scenario is characterized by weak state and measurement noises and by high initial value of state noise variance. One can note that although a bad initialization (10 times the average variance), the AMM approach presents better performances for estimation of the 3 components of state vector (Tables 2-4, figures 5-11). On section $T_1$, (figure 12), the estimated variance remains constant compared to the a priori average variance (10 times the average variance) corresponding to the initial state. Indeed, the algorithm of estimation of variances does not show any evolution because of the high value of variance in the initial state. However, for section $T_2$ and $T_3$, the variance decreases by half compared to the initial variance, and approaches the actual average variance.
Table 2. Average estimation errors

<table>
<thead>
<tr>
<th></th>
<th>SM</th>
<th>SMI</th>
<th>AMM</th>
<th>SM</th>
<th>SMI</th>
<th>AMM</th>
<th>SM</th>
<th>SMI</th>
<th>AMM</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon_x$ (cm)</td>
<td>3.46</td>
<td>6.12</td>
<td>0.64</td>
<td>8.3</td>
<td>6.15</td>
<td>9.6</td>
<td>4.76</td>
<td>3.38</td>
<td>0.72</td>
</tr>
<tr>
<td>$\varepsilon_y$ (cm)</td>
<td>4.58</td>
<td>3.69</td>
<td>0.5</td>
<td>12.3</td>
<td>7.3</td>
<td>9.7</td>
<td>4.64</td>
<td>3.58</td>
<td>1.82</td>
</tr>
<tr>
<td>$\varepsilon^\theta$ (10^{-3} rad)</td>
<td>22.7</td>
<td>30.5</td>
<td>2.7</td>
<td>8.2</td>
<td>11.9</td>
<td>9.7</td>
<td>21.6</td>
<td>29.3</td>
<td>7.9</td>
</tr>
</tbody>
</table>

Table 3. Selected data percentage

<table>
<thead>
<tr>
<th>$Ndx$</th>
<th>$Ndy$</th>
<th>$Nd\theta$</th>
<th>$Ndxe$</th>
<th>$Ndye$</th>
<th>$Nd\theta e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>98.75%</td>
<td>90%</td>
<td>97.5%</td>
<td>98.75%</td>
<td>98.75%</td>
<td>97.5%</td>
</tr>
</tbody>
</table>

Fig. 5. Estimated trajectories by Encoders and, SM, SMI and AMM Filters

Fig. 6. Estimated trajectories (sub trajectory T1)
Fig. 7. Estimated trajectories (sub trajectory T2)

Fig. 8. Estimated trajectories (sub trajectory T3)

Fig. 9. Position error with respect to X axis
Fig. 10. Position error with respect to Y axis

Fig. 11. Absolute error on orientation angle

Fig. 12. Ratio between the estimate of state noise variance and the average variance
Scenario 2:
- Noise-to-signal Ratio of odometric sensors: right encoder: 10%, left encoder: 10%
- Noise-to-signal Ratio of Gyroscope: 3%
- Noise-to-signal Ratio of telemeters: 4% of the odometric elementary step (40% of the state noise)
- “A priori” knowledge on the variance in initial state: Good
- “A priori” knowledge on noise variances (i) telemeters and state: Good; (ii) gyroscope: Bad

The results presented here (Tables 4-5 and figures 13-20) show the influence of signal-to-noise ratio and estimation of noise variances on performances of SM and SMI filters. In this scenario, the initial variance of measurement noise of the gyroscope is incorrectly estimated. Unlike AMM approach, filters SM and SMI do not carry out any adaptation of this variance, leading to unsatisfactory performance.

<table>
<thead>
<tr>
<th></th>
<th>T1</th>
<th>T2</th>
<th>T3</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1</td>
<td>SM</td>
<td>SMI AMM</td>
<td>SM</td>
</tr>
<tr>
<td>$\varepsilon_x$ (cm)</td>
<td>11.7</td>
<td>11</td>
<td>1.8</td>
</tr>
<tr>
<td>$\varepsilon_y$ (cm)</td>
<td>16.7</td>
<td>21</td>
<td>1</td>
</tr>
<tr>
<td>$\varepsilon_\theta$ (10^{-3} rad)</td>
<td>99.3</td>
<td>129</td>
<td>1.5</td>
</tr>
</tbody>
</table>

Table 4. Average estimation errors

<table>
<thead>
<tr>
<th>Ndx</th>
<th>Ndy</th>
<th>Nd$\theta$</th>
<th>Ndx</th>
<th>Ndy</th>
<th>Nd$\theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>87.5%</td>
<td>66%</td>
<td>99.37%</td>
<td>87.5%</td>
<td>82.5%</td>
<td>99.37%</td>
</tr>
</tbody>
</table>

Table 5. Selected data percentage

Figure 20 illustrates the evolution of state noise variance estimate compared to the average variance. Note that the ratio between variances reaches 1.7 on sub trajectory T1, 3.0 on sub trajectory T2, and 3.3 on sub trajectory T3. It is important to mention that the algorithm proposed for estimation of variances estimates the actual value of state noise variance and not its average value. These results are related to the fact that the signal-to-noise ratio is weak both for the odometer and the telemeters.

![Fig. 13. Estimated trajectories by Encoders and, SM, SMI and AMM Filters](www.intechopen.com)
Fig. 14. Estimated trajectories (sub trajectory T1)

Fig. 15. Estimated trajectories (sub trajectory T2)

Fig. 16. Estimated trajectories (sub trajectory T3)
Fig. 17. Position error with respect to X axis

Fig. 18. Position error with respect to Y axis

Fig. 19. Absolute error on orientation angle
Fig. 20. Ratio between the estimate of state noise variance and the average variance

Scenario 3:
- Noise-to-signal Ratio of odometric sensors: right encoder: 8%, left encoder: 8%
- Noise-to-signal Ratio of Gyroscope: 3%
- Noise-to-signal Ratio of telemeter 1: 10% of the odometric elementary step
- Noise-to-signal Ratio of telemeter 2: 10% the odometric elementary step
- “A priori” knowledge on the variance in initial state: Good
- “A priori” knowledge on noise statistics (measurement and state variances): Good

In this scenario, the telemeters measurement noise is higher than state noise. We remark that performances of AMM filter are better that those of SM and SMI filters concerning x and y components (tables 6-7; figures 21-28). In sub trajectory T3, the orientation’s estimation error relating to AMM filter (Table 6) has no influence on filtering quality of the remaining components of state vector. Besides, one can note that this error decreases in this sub trajectory, see figure 27. In this case, only one gyroscope is used for the prediction and updating the Markov chain probabilities. In sub trajectory T2, we remark that the estimation error along x-Axis for AMM filter is lightly higher than those relating to other filters. This error is concentrated on first half of T2 sub trajectory (figure 25) and decreases then on second half of the trajectory. This can be explained by the fact that on one hand, the estimation variances algorithm rejected 0.7% of data, and on the other hand, the filtering step has rejected the same percentage of data. This justifies that neither the variances updating, nor the x-coordinate correction, were carried out (figure 28). Note that unlike filters SM and SMI, filter AMM has a robust behavior concerning pose estimation even when the signal-to-noise ratio is weak. By introducing the concept of observation domain for observation models, we obtain a better modeling of observation and a better management of robot’s sensors. The last remark is related to the bad performances of filters SM and SMI when the signal-to-noise ratio is weak. This ratio degrades the estimation of the orientation angle, observation matrices, Kalman filter gain along with the prediction of the observations.
Fig. 21. Estimated trajectories by Encoders and, SM, SMI and AMM Filters

Fig. 22. Estimated trajectories (sub trajectory T1)

Fig. 23. Estimated trajectories (sub trajectory T2)
Fig. 24. Estimated trajectories (sub trajectory T3)

<table>
<thead>
<tr>
<th></th>
<th>T1</th>
<th>T2</th>
<th>T3</th>
</tr>
</thead>
<tbody>
<tr>
<td>SM</td>
<td>6.25</td>
<td>13.2</td>
<td>31.9</td>
</tr>
<tr>
<td>SMI</td>
<td>3.23</td>
<td>10.8</td>
<td>31.2</td>
</tr>
<tr>
<td>AMM</td>
<td>2.5</td>
<td>15.3</td>
<td>1.2</td>
</tr>
</tbody>
</table>

Table 6. Average estimation errors (Scenario 1)

<table>
<thead>
<tr>
<th></th>
<th>T1</th>
<th>T2</th>
<th>T3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e_x$ (cm)</td>
<td>13.6</td>
<td>23.9</td>
<td>19.2</td>
</tr>
<tr>
<td>$e_y$ (cm)</td>
<td>16.7</td>
<td>11.9</td>
<td>5.75</td>
</tr>
<tr>
<td>$e_\theta$ (10^{-3} rad)</td>
<td>3.8</td>
<td>39.9</td>
<td>3.23</td>
</tr>
</tbody>
</table>

Table 7. Selected data percentage

<table>
<thead>
<tr>
<th></th>
<th>$Ndx$</th>
<th>$Ndy$</th>
<th>$Nd\theta$</th>
<th>$Ndxe$</th>
<th>$Ndye$</th>
<th>$Nd\theta e$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>99.37%</td>
<td>84.37%</td>
<td>99.37%</td>
<td>97.5%</td>
<td>99.37%</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 25. Position error with respect to X axis
Fig. 26. Position error with respect to Y axis

Fig. 27. Absolute error on orientation angle

Fig. 28. Ratio between the estimate of state noise variance and the average variance

6. Conclusion

This research work introduces a multiple model approach for the robust localization of a mobile robot. The localization method is considered as a hybrid process, which is
decomposed into multiple models. Each model is associated with a mode and an interval of validity corresponding to the observation domain. A Markov chain is employed for the prediction of each model according to the robot mode. To prevent divergence of standard Kalman Filtering and to increase its robustness, we proposed an adaptive algorithm for the adjustment of the state and measurement noise covariance matrices. For an efficient estimation of noise variances, we introduced an ad hoc technique consisting in a measure selection for filtering unreliable readings. The simulation results we obtain in different scenarios show better performances of the proposed approach compared to standard existing filters. Some future research need to be conducted to complete the proposed approach and particularly in probabilistic data fusion through sequential Monte Carlo methods, or through the use of functional density estimates. These investigations into utilizing multiple model technique for robust localization show promise and demand continuing research. Fuzzy logic theory can also be considered to increase robustness of the proposed localization algorithm.

7. References


Letchner, J.; Fox, D. & LaMarce, A. (2005). Large-scale localization from wireless signal strength, Proceedings of the National Conference on Artificial Intelligence (AAAI-05)


www.intechopen.com


This book includes 23 chapters introducing basic research, advanced developments and applications. The book covers topics such as modeling and practical realization of robotic control for different applications, researching of the problems of stability and robustness, automation in algorithm and program developments with application in speech signal processing and linguistic research, system's applied control, computations, and control theory application in mechanics and electronics.

How to reference
In order to correctly reference this scholarly work, feel free to copy and paste the following: