Power Plant Maintenance Scheduling Using Ant Colony Optimization

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1. Introduction

Under the pressure of rapid development around the globe, power demand has drastically increased during the past decade. To meet this demand, the development of power system technology has become increasingly important in order to maintain a reliable and economic electric power supply (Lin et al., 1992). One major concern of such development is the optimization of power plant maintenance scheduling. Maintenance is aimed at extending the lifetime of power generating facilities, or at least extending the mean time to the next failure for which repair costs may be significant. In addition, an effective maintenance policy can reduce the frequency of service interruptions and the consequences of these interruptions (Endrenyi et al., 2001). In other words, having an effective maintenance schedule is very important for a power system to operate economically and with high reliability.

Determination of an optimum maintenance schedule is not an easy process. The difficulty lies in the high degree of interaction between several subsystems, such as commitment of generating units, economical planning and asset management. Often, an iterative negotiation is carried out between asset managers, production managers and schedule planners until a satisfactory maintenance schedule is obtained. In addition, power plant maintenance scheduling is required to be optimized with regard to a number of uncertainties, including power demand, forced outage of generating units, hydrological considerations in the case of hydropower systems and trading value forecasts in a deregulated electricity market. Consequently, the number of potential maintenance schedules is generally extremely large, requiring a systematic approach in order to ensure that optimal or near-optimal maintenance schedules are obtained within an acceptable timeframe.

Over the past two decades, many studies have focused on the development of methods for optimizing maintenance schedules for power plants. Traditionally, mathematical programming approaches have been used, including dynamic programming (Yamayee et al., 1983), integer programming (Dopazo & Merrill, 1975), mixed-integer programming (Ahmad & Kothari, 2000) and the implicit enumeration algorithm (Escudero et al., 1980). More recently, metaheuristics have been favored, including genetic algorithms (GAs) (Aldridge et al., 1999), simulated annealing (SA) (Satoh & Nara, 1991) and tabu search (TS) (El-Amin et al., 2000). These methods have generally been shown to outperform mathematical programming methods and other conventional approaches in terms of the
quality of the solutions found, as well as computational efficiency (Aldridge et al., 1999; Satoh & Nara, 1991).

Ant Colony Optimization is a relatively new metaheuristic for combinatorial optimization problems that is based on the foraging behavior of ant colonies (Dorigo & Stützle, 2004). Compared to other optimization methods, such as GA, ACO has been found to produce better solutions in terms of computational efficiency and quality when applied to a number of combinatorial optimization problems, such as the Traveling Salesman Problem (TSP) (Dorigo & Gambardella, 1997a). Recently, ACO has also been successfully applied to scheduling, including the job-shop, flow-shop and resource-constrained project scheduling problems (Bauer et al., 1999; Colomi et al., 1994; Merkle et al., 2002; Stützle, 1998). Recently, a formulation that enables ACO to be applied to the power plant maintenance scheduling optimization (PPMSO) problem has been introduced by the authors of this chapter (Foong et al., 2005). The formulation was tested on a 21-unit case study and shown to outperform other metaheuristic methods previously applied to the same case study (Foong et al., 2005). In Foong et al. (Accepted for publication), the formulation was further tested on a simplified version of a real hydro PPMSO problem, which was solved again using an improved version of the formulation (Foong et al., 2008).

The overall aim of this chapter is to formalize the ACO-PPMSO formulation presented in Foong et al. (2005) and to extend the testing of the formulation by applying it to three additional case studies. In addition, the utility of a local search strategy and a heuristic formulation when adopting ACO-PPMSO are examined. In section 2, the general formulation of the PPMSO problem is introduced, are the proposed approach for using ACO to solve this problem (ACO-PPMSO) is introduced in section 3. The four problem instances on which the proposed approach has been tested are described in section 4 and the experimental procedures, results and discussion are presented in section 5. In section 6, a summary and conclusions are given.

2. Power Plant Maintenance Scheduling Optimization

PPMSO is generally considered as a minimization problem \((S, f, \Omega)\), where \(S\) is the set of all maintenance schedules, \(f\) is the objective function which assigns an objective function value \(f(s)\) to each trial maintenance schedule \(s \in S\), and \(\Omega\) is a set of constraints. Mathematically, PPMSO can be defined as the determination of a set of globally optimal maintenance schedules \(S^* \subset S\), such that the objective function is minimized \(f(s^* \in S^*) \leq f(s \in S)\) (for a minimization problem) subject to a set of constraints \(\Omega\). Specifically, PPMSO has the following characteristics:

- It consists of a finite set of decision points \(D = \{d_1, d_2, \ldots, d_N\}\) comprised of \(N\) maintenance tasks to be scheduled;
- Each maintenance task \(d_n \in D\) has a normal (default) duration \(\text{NormDur}_n\) and is carried out during a planning horizon \(T_{\text{plan}}\).

Two decision variables need to be defined for each task \(d_n\) including:

1. The start time for the maintenance task, \(\text{start}_n\), with the associated set of options: \(T_{n, \text{start}} = \{t \in T_{\text{plan}}; \text{chdur}_n \in K_n; \text{ear}_n \leq t \leq \text{lat}_n - \text{chdur}_n + 1\}\) where the terms in brackets denote the set of time periods when maintenance of unit \(d_n\) may start; \(\text{ear}_n\) is the earliest time for maintenance task \(d_n\) to begin; \(\text{lat}_n\) is the latest time for maintenance task \(d_n\) to end and \(\text{chdur}_n\) is the chosen maintenance duration for task \(d_n\).
2. The duration of the maintenance task, $chdur_n$, with the associated finite set of decision paths: $K_n = \{0, s_n, 2s_n, ..., NormDur_n - s_n, NormDur_n\}$, where the terms in brackets denote the set of optional maintenance durations for task $d_n$ and $s_n$ is the time step considered for maintenance duration shortening.

A trial maintenance schedule, $s \in S = \{(start_1, chdur_1), (start_2, chdur_2), ..., (start_N, chdur_N)\}$ is comprised of maintenance commencement times, $start_n$, and durations, $chdur_n$, for all $N$ maintenance tasks that are required to be scheduled.

Binary variables, which can take on values 0 or 1, are used to represent the state of a task in a given time period in the mathematical equations of the PPMSO problem formulation. $X_n,t$ is set to 1 to indicate that task $d_n \in D$ is scheduled to be carried out during period $t \in T_{plan}$. Otherwise, $X_n,t$ is set to a value of 0, as given by:

$$X_{n,t} = \begin{cases} 1 & \text{if task } d_n \text{ is being maintained in period } t \\ 0 & \text{otherwise} \end{cases}$$

In addition, the following sets of variables are defined:

- $S_{n,t} = \{k \in T_{n, chdur}, chdur_n \in K_n : t - chdur_n + 1 \leq k \leq t\}$ is the set of start times $k$, such that if maintenance task $d_n$ starts at time $k$ for a duration of $chdur_n$, that task will be in progress during time $t$;

- $D_t = \{d_n : t \in T_n\}$ is the set of maintenance tasks that is considered for period $t$.

### Objectives and Constraints

Traditionally, cost minimization and maximization of reliability have been the two objectives commonly used when optimizing power plant maintenance schedules. Two examples of reliability objectives are evening out the system reserve capacity throughout the planning horizon, and maximizing the total reservoir storage water volumes at the end of the planning horizon, in the case of a hydropower system. An additional objective associated with the more generalized definition of PPMSO is the minimization of the total maintenance duration shortened/deferred (Foong et al., 2008). The rationale behind this objective is that shortening of maintenance duration (i.e. speeding up the completion of maintenance tasks) requires additional personnel and equipment, whereas deferral of maintenance tasks might result in unexpected breakdown of generating units, and in both events, additional costs are incurred by the power utility operator.

Constraints specified in PPMSO problems are also power plant specific. The formulation of some common constraints include the allowable maintenance window, continuity, load, availability of resources, precedence of maintenance tasks, reliability and the minimum maintenance duration required, which are presented in Eqs. 2 to 6.

The timeframes within which individual tasks in the system are required to start and finish maintenance form maintenance window constraints, which can be formulated as:

$$ear_n \leq start_n \leq lat_n - chdur_n + 1$$

where $start_n$ and $chdur_n$ are the start time and maintenance duration, respectively, chosen for task $d_n$.

Load constraints (Eq. 3) are usually rigid/hard constraints in PPMSO problems, which ensure that feasible maintenance schedules that do not cause demand shortfalls throughout the whole planning horizon are obtained:
\[ \sum_{d \in D} P_{n,t} - \sum_{d \in D} X_{n,k} P_{n} \geq L, \text{ for all } t \in T_{\text{plan}}. \] (3)

where \( L_t \) is the anticipated load for period \( t \) and \( P_n \) is the loss of generating capacity associated with maintenance task \( d_n \).

Resource constraints are specified in the case where the availability of certain resources, such as highly skilled technicians, is limited. In general, resources of all types assigned to maintenance tasks should not exceed the associated resource capacity at any time period, as given by:

\[ \sum_{d \in D} X_{n,k} \text{Res}_{n,k} \rho_k \leq \text{ResAvai}^r_t \text{ for all } t \in T_{\text{plan}}, r \in R. \] (4)

where \( \text{Res}_{n,k} \rho_k \) is the amount of resource of type \( r \) available that is required by task \( d_n \) at period \( k \); \( \text{ResAvai}^r_t \) is the associated capacity of resource of type \( r \) available at period \( t \) and \( R \) is the set of all resource types.

Precedence constraints that reflect the relationships between the order of maintenance of generating units in a power system are usually specified in PPMSO problems. An example of such a constraint is a case where task 2 should not commence before task 1 is completed, as given by:

\[ \text{start}_2 > \text{start}_1 + \text{chdur}_1 - 1. \] (5)

where \( \text{start}_n \) is the start time chosen for task \( d_n \).

In the case of maintenance duration shortening, there is usually a practical limit to the extent that the duration can be shortened. Due to the different characteristics of maintenance tasks, minimum maintenance durations may vary with individual tasks:

\[ \text{NormDur}_n \geq \text{chdur}_n \geq \text{MinDur}_n, \text{ for all } d_n \in D. \] (6)

where \( \text{chdur}_n \) is the maintenance duration of task \( d_n \); \( \text{MinDur}_n \) is the minimum shortened outage duration for task \( d_n \); \( \text{NormDur}_n \) is the normal duration of maintenance task \( d_n \).

3. ACO for Power Plant Maintenance Scheduling Optimization (ACO-PPMSO)

Ant Colony Optimization (ACO) is a metaheuristic inspired by the foraging behavior of ant colonies (Dorigo & Stützle, 2004). By marking the paths they have followed with pheromone trails, ants are able to communicate indirectly and find the shortest distance between their nest and a food source when foraging for food. When adapting this search metaphor of ants to solve discrete combinatorial optimization problems, artificial ants are considered to explore the search space of all possible solutions. The ACO search begins with a random solution (possibly biased by heuristic information) within the decision space of the problem. As the search progresses over discrete time intervals, ants deposit pheromone on the components of promising solutions. In this way, the environment of a decision space is iteratively modified and the ACO search is gradually biased towards more desirable regions of the search space, where optimal or near-optimal solutions can be found. Readers are referred to Dorigo & Stützle (2004) for a detailed discussion of ACO metaheuristics and the benchmark combinatorial optimization problems to which ACO has been applied. Due to its robustness in solving these problems, ACO has recently been applied to, and obtained some
encouraging results for real-world engineering problems, such as the design of optimal water distribution systems (Maier et al., 2003) and in the area of power systems (Gomez et al., 2004; Huang, 2001; Kannan et al., 2005; Su et al., 2005). As is the case with other metaheuristics, ACO can be linked with existing simulation models of power systems, regardless of their complexity, when solving a PPMSO problem. In addition, the unique way in which ACO problems are represented by using a graph makes ACO inherently suitable for handling various constraints that are commonly encountered in PPMSO problems. In this section, the novel formulation that enables ACO to be applied to PPMSO problems (herein referred to as ACO-PPMSO) introduced by Foong et al. (2005) is formalized.

3.1 Problem representation

Before the PPMSO problem can be optimized using ACO, it has to be mapped onto a graph shown in Fig. 1, which is expressed in terms of a set of decision points consisting of the N maintenance tasks that need to be scheduled $D = \{d_1, d_2, d_3, \ldots, d_N\}$.

Figure 1. Proposed ACO-PPMSO graph

In accordance with the formulation introduced, there are three variables that need to be defined $V = \{v_1, v_2, v_3\}$ for each maintenance task:

- **Variable 1, $v_1$**: the overall state of the maintenance task under consideration (i.e. if maintenance currently being carried out or not),
- **Variable 2, $v_2$**: the duration of the maintenance task, and
- **Variable 3, $v_3$**: the commencement time for the maintenance task.
For maintenance task $d_n$, a set of decision paths $DP_{c,n}$ is associated with decision variable $v_{c,n}$ (where subscript $c = 1, 2$ or $3$) (shown as dashed lines in Fig. 1). For decision variable $v_{1,n}$, these correspond to the options of carrying out the maintenance tasks $d_n$ at normal duration, shortening the maintenance duration and deferring maintenance tasks. For decision variable $v_{2,n}$, these correspond to the optional shortened durations available for the maintenance tasks. For decision variable $v_{3,n}$, these correspond to the optional start times for maintenance tasks $d_n$. It should be noted that, as the latest finishing time of maintenance tasks is usually fixed, there are different sets of start time decision paths, each corresponding to a maintenance duration decision path (Fig. 1). This graph can then be utilized to construct trial solutions using the ACO-PPMSO algorithm introduced in section 3.2.2.

### 3.2 ACO-PPMSO Algorithm

The new formulation proposed for power plant maintenance scheduling using Ant Colony Optimisation is implemented via an ACO-PPMSO algorithm, represented by the flowchart given in Fig. 2. The mechanisms involved in each procedure of the proposed ACO-PPMSO algorithm are detailed in sections 3.2.1 to 3.2.6.

![Figure 2. ACO-PPMSO algorithm](image)

#### 3.2.1 Initialization

The optimisation process starts by reading details of the power system under consideration (e.g., generating capacity of each unit, daily system demands, time step for duration shortening etc.). In addition, various ACO parameters (e.g., initial pheromone trail concentrations ($\tau_0$), number of ants, pheromone evaporation rate etc.) need to be defined.
3.2.2 Construction of a trial maintenance schedule

A trial maintenance schedule is constructed using the ACO-PPMSO graph shown in Fig. 1. In order to generate one trial maintenance schedule, an ant travels to one of the decision points (maintenance tasks) at a time. At each decision point, a three-stage selection process that corresponds to the three decision variables, \( v_{1,n}, v_{2,n}, \) and \( v_{3,n} \), is performed.

At each stage, the probability that decision path \( opt \) is chosen for maintenance of task \( d_n \) in iteration \( t \) is given by:

\[
p_{n,opt}(t) = \frac{\tau_{n,opt}(t)^{\alpha} \eta_{n,opt}(t)^{\beta}}{\sum_{y \in \text{DP}_c} \tau_{n,y}(t)^{\alpha} \eta_{n,y}(t)^{\beta}}.
\]

Subscripts \( c = 1, 2 \) and 3 refer to the three decision variables, \( v_{1,n}, v_{2,n}, \) and \( v_{3,n} \); \( \tau_{n,opt}(t) \) is the pheromone intensity deposited on the decision path \( opt \) for task \( d_n \) in iteration \( t \); \( \eta_{n,opt} \) is the heuristic value of decision path \( opt \) for task \( d_n \); \( \alpha \) and \( \beta \) are the relative importance of pheromone intensity and the heuristic, respectively.

It should be noted that the term \( opt \) in Eq. 7 represents the decision path under consideration, of all decision paths contained in set \( \text{DP}_c,n \). When used for stages 1, 2 and 3, respectively, the terms \( opt \) and \( \text{DP}_c,n \) are substituted with those associated with the decision variable considered at the corresponding stage (Table 1). The pheromone level associated with a particular decision path (e.g. deferral of a particular maintenance task) is a reflection of the quality of the maintenance schedules that have been generated previously that contain this particular option. The heuristic associated with a particular decision path is related to the likely quality of a solution that contains this option, based on user-defined heuristic information. The following paragraphs detail the three-stage selection process for decision point (maintenance task) \( d_n \), including the adaptations required when using Eq. 7 for each stage.

<table>
<thead>
<tr>
<th>( c )</th>
<th>Stage 1</th>
<th>Stage 2</th>
<th>Stage 3</th>
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<tr>
<td>( opt )</td>
<td>( \text{stat} \in \text{DP}_{1,n} )</td>
<td>( \text{dur} \in \text{DP}_{2,n} )</td>
<td>( \text{day} \in \text{DP}_{3,n,chdur} )</td>
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<td>( \text{DP}_{c,n} )</td>
<td>( \text{DP}_{1,n}=[\text{normal, shorten, defer}] )</td>
<td>( \text{DP}_{2,n} = {0, s_n, 2s_n, \ldots, \text{NormDur}_n} )</td>
<td>( \text{DP}_{3,n,chdur} = {\text{chdur}<em>n \in \text{DP}</em>{2,n}; \text{car}_n, \text{car}_n+1, \ldots, \text{lat}_n - \text{chdur}_n + 1} )</td>
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<tr>
<td>( \tau_{n,opt} )</td>
<td>( \tau_{n,\text{stat}} )</td>
<td>( \tau_{n,\text{dur}} )</td>
<td>( \tau_{n,\text{chdur},\text{day}} )</td>
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<td>( \eta_{n,opt} )</td>
<td>( \eta_{n,\text{defer}} \leq \eta_{n,\text{shorten}} &lt; \eta_{n,\text{normal}} )</td>
<td>( \eta_{n,\text{dur}} = \text{dur} )</td>
<td>( \eta_{n,\text{chdur},\text{day}} = \eta_{n,\text{chdur},\text{day}} \times \eta_{n,\text{chdur},\text{day}} )</td>
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Table 1. Adaptations for Eq. 7 in stages 1, 2 and 3 of the selection process

**Stage 1**: In stage 1, a decision needs to be made whether to perform the maintenance task under consideration at normal or shortened duration, or to defer it (decision variable \( v_{1,n} \) in Fig. 1). In this case, \( c = 1 \) and \( opt = \text{stat} \in \text{DP}_{1,n}=[\text{normal, shorten, defer}] \) is the set of decision paths associated with decision variable \( v_{1,n} \) for task \( d_n \). The probability of each of these
options being chosen is a function of the strength of the pheromone trails and heuristic value associated with the option (Eq. 7). For the PPMSO problem, the heuristic formulation should generally be defined such that normal maintenance durations are preferred over duration shortening, and deferral is the least favored option (Eq. 8). However, real costs associated with duration shortening and deferral options can be used if the extra costs incurred associated with these options are quantifiable and available. The adaptations required for Eq. 7 to be used at the stage 1 selection process are summarized in Table 1. It is suggested that values of the heuristics should be selected such that:

\[ \eta_{n,\text{defer}} < \eta_{n,\text{shorten}} < \eta_{n,\text{normal}} \]  

**Stage 2:** Once a decision has been made at stage 1, the selection process proceeds to stage 2 (decision variable \( v_{2,n} \) in Fig. 1), where the duration of the maintenance task under consideration, \( d_{n} \), is required to be selected from a set of available decision paths \( DP_{2,n} = \{0, s_{n}, 2s_{n}, \ldots, \text{NormDur}_{n}\} \). The symbols \( s_{n} \) and \( \text{NormDur}_{n} \) denote the time step for maintenance duration shortening, and the normal maintenance duration, respectively. For Eq. 7 to be used at stage 2, the terms \( c \) and \( \text{opt} \) in the equation are substituted by the values \( 2 \) and \( \text{dur} \in DP_{2,n} \), respectively. It should be noted that if the ‘normal’ or ‘defer’ options were chosen at stage 1, the normal duration of the maintenance task, or a duration of 0, respectively, are automatically chosen for the task. In the case of duration shortening, a constraint is normally specified where each maintenance task has a minimum duration at which the completion of the task cannot be further accelerated due to limitations, such as the availability of highly specialized technicians. This constraint can be addressed at this stage such that only feasible trial maintenance schedules (with regard to this constraint) are constructed (see section 3.3 for details of such constraint-handling techniques). The pheromone trails and heuristic values associated with optional durations are used to determine the probability that these durations are chosen. In order to favor longer maintenance durations (i.e. the smallest amount of shortening compared with the normal maintenance duration), it is suggested that the heuristic value associated with a decision path should be directly proportional to the maintenance duration (Eq. 9).

\[ \eta_{n,\text{dur}} \propto \text{dur} \]  

The substitutions for the various terms in Eq. 7 when used in stage 2 are summarized in Table 1.

**Stage 3:** Once a maintenance duration has been selected, the solution construction process enters stage 3 (decision variable \( v_{3,n} \) in Fig. 1), where a start time for the maintenance task is selected from the set of optional start times available \( DP_{3,n,\text{chdur}} = \{\text{chdur}_{n} \in DP_{2,n}: \text{ear}_{n}, \text{ear}_{n} + 1, \ldots, \text{lat}_{n} - \text{chdur}_{n} + 1\} \), given a chosen duration of \( \text{chdur}_{n} \). In order to utilize Eq. 7 at stage 3, adjustments are made such that \( c = 3 \) and \( \text{opt} = \text{day} \in DP_{3,n,\text{chdur}} \). It should be noted that this stage is skipped if the ‘defer’ option is chosen at stage 1. The probability that a particular start day is chosen is a function of the associated pheromone trail and heuristic value. The suggested heuristic formulation for selection of the maintenance start day is given by Eqs. 10 to 15.

\[ \eta_{n,\text{chdur},\text{day}} = (\eta_{n,\text{chdur},\text{day}})^{\text{bias}} \cdot \eta_{n,\text{chdur},\text{day}}^{\text{load}} \]
\[ \eta_{n,chdur,day}^{Res} = \frac{\sum_{k \in J_{n,chdur,day}} Y_{ResV(k)} \cdot R_{n,chdur,day}(k)}{\sum_{k \in J_{n,chdur,day}} Y_{ResV(k)} \cdot R_{n,chdur,day}(k)}. \] (11)

\[ \eta_{n,chdur,day}^{Load} = \frac{\sum_{k \in J_{n,chdur,day}} Y_{LoadV(k)} \cdot C_{n,chdur,day}(k)}{\sum_{k \in J_{n,chdur,day}} Y_{LoadV(k)} \cdot C_{n,chdur,day}(k)}. \] (12)

\[ Y_{ResV(k)} = \begin{cases} 1 & \text{if no violation of resource constraints in time period } k \\ 0 & \text{otherwise} \end{cases} \] (13)

\[ Y_{LoadV(k)} = \begin{cases} 1 & \text{if no violation of load constraints in time period } k \\ 0 & \text{otherwise} \end{cases} \] (14)

\[ w = \begin{cases} 1 & \text{if resource constraints are considered} \\ 0 & \text{otherwise} \end{cases} \] (15)

where \( \eta_{n,chdur,day}(t) \) is the heuristic for start time \( day \in DP_{3,n,chdur} \) for task \( d_n \) given a chosen duration \( chdur_n \); \( R_{n,chdur,day}(k) \) represents the prospective resources available in reserve in time period \( k \) if task \( d_n \) is to commence at start time \( day \) and takes \( chdur_n \) to complete (less than 0 in the case of resource deficits); \( C_{n,chdur,day}(k) \) is the prospective power generation capacity available in reserve in time period \( k \) if task \( d_n \) is to commence at start time \( day \) and takes \( chdur_n \) to complete (less than 0 in the case of power generation reserve deficits); \( J_{n,chdur,day} = \{ day \in DP_{3,n,chdur} : day \leq k \leq day + chdur_n - 1 \} \) is the set of time periods \( k \) such that if task \( d_n \) starts at start time \( day \), that task will be in maintenance during period \( k \).

As mentioned above, the heuristic formulation in Eq. 10 includes a resource-related term, \( \eta_{n,chdur,day}^{Res} \), and a load-related term, \( \eta_{n,chdur,day}^{Load} \). These two terms are expected to evenly distribute maintenance tasks over the entire planning horizon, which potentially maximizes the overall reliability of a power system. For PPMSO problem instances that do not consider resource constraints, the value of \( w \) in Eq. 10 can be set to 0 (Eq. 15). In order to implement the heuristic, each ant is provided with a memory matrix on resource reserves and another matrix on generation capacity reserves prior to construction of a trial solution. This is updated every time a unit maintenance commencement time is added to the partially completed schedule.

The three-stage selection process is then repeated for another maintenance task (decision point). A complete maintenance schedule is obtained once all maintenance tasks have been considered.

### 3.2.3 Evaluation of trial maintenance schedule

Once a complete trial maintenance schedule, \( s \in S \), has been constructed by choosing a maintenance commencement time and duration at each decision point (i.e. for each maintenance task to be scheduled), an ant-cycle has been completed. The trial schedule's
Objective function cost (OFC) can then be determined by an evaluation function, which is a function of the values of objectives and constraint violations:

\[ OFC(s) = f(obj_1(s), obj_2(s), \ldots, obj_{Z_T}(s), vio_1(s), vio_2(s), \ldots, vio_{C_T}(s)) \]  

(16)

where \( OFC(s) \) is the objective function cost associated with a trial maintenance schedule, \( s \); \( obj_1(s) \) is the value of the first objective; \( vio_1(s) \) is the degree of violation of the first constraint; \( Z_T \) is the total number of objectives; \( C_T \) is the total number of constraints that cannot be satisfied during the construction of trial solutions.

It should be noted that not all constraints specified in a problem are accounted for using Eq. 16. Maintenance windows, precedence and minimum duration constraints, just to name a few, can be satisfied during the construction of a trial solution and would not appear in Eq. 16. In other words, a complete trial solution would have satisfied these constraints already before the evaluation process is carried out. On the other hand, load constraints can only be checked upon completion of a complete trial solution and therefore the violations of these constraints, if there are any, can only be reflected through penalty terms in the objective function (Eq. 16). Detailed categorizations of constraints commonly encountered in PPMSO problems, as well as the appropriate methods of handling them, are presented in section 3.3.

In general, the trial schedule has to be run through a simulation model in order to calculate some elements of the objective function and whether certain constraints (those accounted for through penalty terms) have been violated.

After \( m \) ants have performed procedures 3.2.2 and 3.2.3, where \( m \) (the number of ants) is predefined in procedure 3.2.1, an iteration cycle has been completed. At this stage, a total of \( m \) maintenance schedules have been generated for this iteration. It should be noted that all ants in an iteration can generate their trial solutions concurrently, as they are working on the same set of pheromone trail distributions in decision space.

3.2.4 Local search

Recently, local search has been utilized to improve the optimisation ability of ACO. While it has been found to result in significant improvements in some applications (den Besten et al., 2000; Dorigo & Gambardella, 1997b), little success has been obtained in others (Merkle et al., 2002). Local search has also been found useful for some problems (Foong et al., 2008) where the formulation of heuristics is difficult (Dorigo & Stützle, 2004).

In this formulation, local search is coupled with ACO to solve the PPMSO problem. The local search operator proposed in this chapter is called PPMSO-2-opt, which is a modification of the 2-opt strategy used when solving the Travelling Salesman Problem (TSP) (Stützle et al., 1997), where two edges of connected cities are exchanged. In PPMSO-2-opt, ‘neighbor maintenance schedules’ are generated by exchanging the maintenance start times of a pair of randomly selected tasks of the ‘target maintenance schedule’. It should be noted that the maximum number of possible ‘neighbor maintenance schedules’ formed based on a ‘target maintenance schedule’ \( \binom{N}{2} \) \( \frac{N!}{2!(N-2)!} \) can be specified as the termination criterion of the local search. Otherwise, a smaller number of local solutions can be defined as the stopping criterion.
3.2.5 Pheromone updating

Two mechanisms, namely pheromone evaporation and pheromone rewarding, are involved in the pheromone updating process. Pheromone evaporation reduces all pheromone trails by a factor. In this way, exploration of the search space is encouraged by preventing a rapid increase in pheromone on frequently-chosen paths. Pheromone rewarding is performed in a way that reinforces good solutions.

Despite its original inspiration from the foraging behaviour of ant colonies, various ACO algorithms have evolved, such as Elitist-Ant System (EAS) (Dorigo (1992); Dorigo et al. (1996)) and Max-Min Ant System (MMAS) (Stützle & Hoos, 1997; Stützle & Hoos, 2000). These algorithms are distinguished from each other in the way pheromone updating is performed. In the ACO-PPMSO formulation, pheromone updating is performed on the pheromone matrices used for the three-stage selection process. A general pheromone updating formulation (regardless of the ACO algorithm adopted) is introduced for this purpose:

\[\tau_{*,n}(t+1) = \rho \cdot \tau_{*,n}(t) + \Delta \tau_{*,n}(t).\] (17)

\[\Delta \tau_{*,n}(t) = \sum_{* \in \text{Sol}_{\text{update}}(t)} \begin{cases} Q \frac{\text{OFC}(s_{\text{update}})}{\text{OFC}(s_{\text{update}})} & \text{if } * \in s_{\text{update}} \\ \text{otherwise} & \text{otherwise} \end{cases}\] (18)

where \(t\) is the index of iteration; \((1 - \rho)\) is the pheromone evaporation rate; the subscript asterisk * of \(\tau\) denotes the element of the pheromone matrix under consideration (\(\tau_{n,\text{opt}}\), \(\tau_{n,\text{dur}}\) and \(\tau_{n,\text{dur,day}}\) for decision variables \(v_1\), \(v_2\) and \(v_3\), respectively); \(s_{\text{update}}\) is any trial schedule contained in \(\text{Sol}_{\text{update}}(t)\), which is the set of trial schedules chosen to be rewarded in iteration \(t\); \(\Delta \tau_{*,n}(t)\) is the amount of pheromone rewarded to pheromone trail \(\tau_{*,n}\) at the end of iteration \(t\); \(\text{OFC}(s_{\text{update}})\) is the objective function cost associated with the trial schedule \(s_{\text{update}}\) that contains element *; \(Q\) is the reward factor (a user-defined parameter).

As EAS and MMAS are utilized in solving the PPMSO case study systems presented in section 4, the following additional specifications are made according to the general pheromone updating rules:

(A) *Elitist-Ant System (EAS)*

In EAS, only the least-OFC schedule(s) in every iteration is/are rewarded (Eq. 19).

\[\text{Sol}_{\text{update}}(t) = s_{\text{iter-best}}(t).\] (19)

where \(s_{\text{iter-best}}(t)\) is the best maintenance schedule evaluated in iteration \(t\).

(B) *Max-Min Ant System (MMAS)*

Similarly to EAS, MMAS only rewards iteration-best trial solution(s) (Eq. 19). Additionally, upper and lower bounds are imposed on the pheromone trails in order to prevent premature convergence and greater exploration of the solution surface. These bounds are given by:

\[\tau_{\text{max}}(t+1) = \frac{1}{1 - \rho} \cdot \frac{Q}{\text{OFC}_{\text{iter-best}}(t)}.\] (20)
\[ \tau_{c,\text{min}}(t+1) = \frac{\tau_{\text{max}}(t+1)}{(1-p_{\text{best}}^n_c) \left( \frac{\text{avg}_c}{1} \right)} \]

where \( n_c \) is the number of decision points for decision variable \( v_c \); \( \text{avg}_c \) is the average number of decision paths available at each decision point for decision variable \( v_c \); subscript \( c = 1, 2 \) and 3 refers to the three decision variables considered in procedure 3.2.2; \( p_{\text{best}} \) is the probability that the paths of the current iteration-best-solution, \( s_{\text{iter-best}}(t) \), will be selected, given that non-iteration best-options have a pheromone level of \( \tau_{\text{min}}(t) \) and all iteration-best options have a pheromone level of \( \tau_{\text{max}}(t) \).

The lower and upper bound of pheromone are applied to all decision paths in the search space:

\[ \tau_{c,\text{min}}(t) \leq \tau_{c,\text{opt}}(t) \leq \tau_{\text{max}}(t) ; \text{opt} \in \text{DP}_{c,n} \quad c = 1, 2, 3 \text{ for all } t, n. \]  \( \text{(22)} \)

3.2.6 Termination of run

Procedures 3.2.2 to 3.2.5 are repeated until the termination criterion of an ACO run is met, e.g. either the maximum number of evaluations allowed has been reached or stagnation of the objective function cost has occurred. A set of maintenance schedules resulting in the minimum OFC is the final outcome of the optimisation run.

3.3 Constraints Handling

ACO is an unconstrained optimisation metaheuristic. As constraints are inevitable in PPMSO problems, there is a need to find ways of incorporating constraints during optimisation. In this research, two different constraint handling techniques are adopted. In order to decide which of the two techniques should be used, constraints encountered in PPMSO problems have been characterized using the following classification scheme:

**Direct vs. indirect constraints**: Constraints can be characterized based on the earliest stage at which they can be addressed during optimisation. The maintenance window (Eq. 2), precedence (Eq. 5) and minimum maintenance duration (Eq. 6) constraints can be addressed when trial solutions are being generated during ant cycles (procedure described in section 3.2.2). On the other hand, the violation of load (Eq. 3) and resource (Eq. 4) constraints often cannot be identified from a partially built trial maintenance schedule. As part of the classification scheme introduced in this paper, the former constraints are referred to as direct constraints and the latter as indirect constraints.

**Rigid vs. soft constraints**: Constraints can also be classified based on their “rigidity”. For rigid constraints, such as maintenance windows, minimum maintenance duration, precedence and load constraints, even the slightest violations are generally intolerable. On the other hand, constraints, such as resource constraints, may be able to be violated to a degree specified by decision makers and are therefore referred to as “soft” constraints.

The two constraint handling techniques used in the ACO-PPMSO formulation and the constraint types they are able to accommodate include:

**Graph-based technique**: This technique utilizes candidate lists during ant cycles when trial solutions are being constructed (Fig. 1). Given a partially built trial schedule, a candidate list consists of the optional start times that are available for a maintenance task, such that the constraints under consideration are not violated. Direct and some rigid constraints, such as the maintenance window, precedence and minimum duration constraints, can be accounted
for using this technique. During the construction of a trial maintenance schedule, an ant incrementally adds start times to a partially built schedule. By dynamically updating the candidate lists of ‘unvisited units’, only start times that would result in solutions that satisfy the maintenance window and precedence constraints are considered.

**Penalty-based technique:** In ACO-PPMSO, penalty functions, which transform a constrained optimisation problem into an unconstrained problem by adding or subtracting a value to/from the objective function cost based on the degree of constraint violation (Coello Coello, 2002), are used to address indirect or potentially soft constraints, such as the availability of personpower to perform the maintenance and load constraints. When dealing with soft constraints, penalty factors may be varied to reflect the amount of constraint violation that may be tolerated. Penalty costs also have to be used to account for indirect constraints, as the degree of constraint violation is not known until a complete trial solution has been constructed, as discussed earlier. In such cases, the degree of violation generally has to be obtained with the aid of a simulation model.

The ability to implement direct and some rigid constraints using the graph-based technique is one of the attractive features of using ACO for PPMSO. Firstly, by preventing the generation of infeasible solutions, the number of simulation model runs required is reduced. This is advantageous for real-world PPMSO problems, as the number of times the simulation model has to be run is a major source of computational overhead. Moreover, there are difficulties associated with the use of penalty-based techniques that remain unresolved at the time of writing, in spite of extensive research into this area (Coello Coello, 2002). For example, hand tuning is required for assigning appropriate penalty factors to each constraint and objective term in the objective function.

### 4. Problem Instances

In order to test the utility of the proposed ACO-PPMSO formulation, it is applied to 4 problem instances, including 21- and 22-unit benchmark case studies from the literature and modified versions of these case studies. The 21- and 22-unit case studies have been chosen as they enable comparisons to be made with results obtained in previous studies. However, as these case studies can be solved without the need for maintenance shortening and deferral, modifications to the case studies are introduced in this chapter to test this feature of the proposed formulation. Details of the four problem instances are given below.

#### 4.1 21-unit system

The first case study considered in this research is the 21-unit power plant maintenance problem investigated by Aldridge *et al.* (1999) and Dahal *et al.* (1999; , 2000) using a number of metaheuristics. This case study is a modified version of the 21-unit problem introduced by Yamaye *et al.* (1983), and consists of 21 generating facilities, of which 20 units are thermal and one is hydropower. Due to space constraints, system details are not presented here but can be found in Aldridge *et al.* (1999). All of the machines are to be scheduled for maintenance either in the first or second half of a year’s planning horizon, which results in a combinatorial optimisation problem with approximately $5.18 \times 10^{28}$ total possible solutions. The objective of the problem is to even out reserve generation capacity over the planning horizon, which can be achieved by minimizing the sum of squares of the reserve (SSR) generation capacity in each week.
Constraints to be satisfied include:
1. Maintenance window constraints: The earliest start time and latest finish time of maintenance tasks for each machine are detailed in Aldridge et al. (1999).
2. Resource constraints: A limit of 20 maintenance personpower is available each week.
3. Demand constraints: A single peak load of 4739 MW has to be met.

Problem formulation
Mathematically, this optimisation problem can be defined as the determination of maintenance schedule(s) such that SSR, which is defined as the sum of square of reserve generation capacity within the planning horizon, is minimized:

$$
\text{Min } SSR = \sum_{i \in T_{\text{plan}}} \left( \sum_{n=1}^{N} P_n - \sum_{d \in D, i \in S_n} X_{n,k} P_n - L_t \right)^2.
$$

(23)

where $$P_n$$ is the generating capacity of unit $$d_n$$; $$L_t$$ is the anticipated load for period $$t$$, subject to the maintenance window, load and personpower constraints, as given by:

$$
ear_n \leq \text{start}_n \leq \text{lat}_n - \text{NormDur}_n + 1 \quad \text{for all } d_n \in D.
$$

(24)

$$
\sum_{d \in D, i \in S_n} X_{n,k} \text{Res}_{n,k} \leq \text{ResAvai}_i \quad \text{for all } t \in T_{\text{plan}}.
$$

(25)

$$
\sum_{n} P_n - \sum_{d \in D, i \in S_n} X_{n,k} P_n \geq L_t \quad \text{for all } t \in T_{\text{plan}}.
$$

(26)

where $$\text{ear}_n$$ is the earliest start time for unit $$d_n$$; $$\text{lat}_n$$ is the latest start time for unit $$d_n$$; NormDur,$$_n$$ is the outage duration (week) for unit $$d_n$$; $$\text{start}_n$$ is the maintenance start time for unit $$d_n$$ and $$\text{ResAvai}_i$$ is the personpower available at period $$t$$.

It should be noted that personpower is considered as a type of resource constraint. The maintenance window constraints are taken into account by the construction graph-based technique (section 3.3), whereas both load and personpower constraints are indirect and are therefore taken into account by using penalty-based techniques (section 3.3).

When applying the ACO-PPPMOSO formulation to this case study, the heuristic developed as part of this research (Eqs. 10 to 15) was used together with pheromone for selection of start times when generating trial maintenance schedules. It should be noted that the value of $$\omega$$ in Eq. 10 was set to 1, as utilization of resource (personpower) constraints is considered in this case. Upon completion of a trial maintenance schedule, a simulation model was used to calculate the SSR value and any violations of personpower or load constraints associated with schedule $$s$$. The quality of individual maintenance schedules in this problem is given by an objective function cost (OFC), which is a function of the value of SSR and the total violation of personpower and load constraints (Eq. 27).

$$
\text{OFC}(s) = \text{SSR}(s) \cdot (\text{ManVio}_{\text{tot}}(s) + 1) \cdot (\text{LoadVio}_{\text{tot}}(s) + 1).
$$

(27)

where OFC($$s$$) is the objective function cost ($) associated with schedule $$s$$; SSR($$s$$) is the sum of squares of reserve generation capacity (MW) associated with schedule $$s$$; ManVio,$$_{\text{tot}}(s)$$ is the total personpower shortfall (person) associated with schedule $$s$$; LoadVio,$$_{\text{tot}}(s)$$ is the total demand shortfall (MW) associated with schedule $$s$$. 
The calculation of constraint violations is given in Eqs. 28 to 31. For a trial maintenance schedule, the total personpower shortfall associated with schedule $s$, $ManVio_{tot}(s)$, is given by summation of the personpower shortage in all periods within the planning horizon:

$$ManVio_{tot}(s) = \sum_{t \in T_{MV}} \left( \sum_{n,d,k} \sum_{k \in S_n} Res_{n,k} - \text{ResAvai}_t \right).$$  \hspace{1cm} (28)

where $T_{MV}$ is the period where personpower constraints are violated, and is given by:

$$T_{MV} = \{ t : \sum_{n,d,k} \sum_{k \in S_n} Res_{n,k} > \text{ResAvai}_t \}.$$  \hspace{1cm} (29)

The total demand shortfall associated with schedule $s$, $LoadVio_{tot}(s)$, is the summation of demand shortfall in all periods within the planning horizon. The calculation of this value may be represented by the following equation.

$$LoadVio_{tot}(s) = \sum_{n \in T_{LV}} \left( \sum_{n,d,k} P_n - \sum_{n,d,k} X_{n,k} P_n \right).$$  \hspace{1cm} (30)

where $T_{LV}$ is the period where load constraints are violated, and is given by:

$$T_{LV} = \{ t : \sum_{n} P_n - \sum_{n,d,k} X_{n,k} P_n < L_t \}.$$  \hspace{1cm} (31)

The OFC can be viewed as the virtual cost associated with a maintenance schedule.

### 4.2 22-unit system

The 22-unit power plant maintenance scheduling optimisation problem was first solved by Escudero et al. (1980) using an implicit enumeration algorithm and later by El-Amin et al. (2000) using tabu search. In this problem, each generating unit is required to be scheduled for maintenance once within a planning horizon of 52 weeks. Details of the system can be found in Escudero et al. (1980). The objective when scheduling for maintenance is to even out reserve generation capacity over the planning horizon subject to the following constraints:

1. The maintenance window constraints specify that all units can be maintained anytime within the planning horizon and have to finish maintenance by week 52, except for unit 10, which can only be taken offline between weeks 6 and 22.
2. Load constraints require peak demands (see Escudero et al., 1980) to be met.
3. The reliability constraint requires a minimum reserve of 20% of the peak demand throughout the planning horizon.
4. The two precedence constraints specify that maintenance of units 2 and 5 has to be carried out before that of units 3 and 6, respectively.
5. Units 15 and 16, as well as units 21 and 22, cannot be maintained simultaneously due to personpower constraints.

#### Problem formulation

In order to even out reserve generation capacity, the formulation used in both Escudero et al. (1980) and El-Amin et al. (2000) for the 22-unit problem was designed to minimize the summed deviation of generation reserve from the average reserve over the entire planning horizon.
horizon, LVL. Mathematically, the optimisation of this case study can be described as the minimization of the sum of the deviation of generation reserve from the average reserve over the planning horizon (Eqs. 32 to 34):

$$\text{Min} \{ LVL = \sum_{t \in T_{plan}} \text{Res}_{avg} - \text{Res}_t \}.$$  (32)

where the generation reserve ($\text{Res}_t$) and average reserve ($\text{Res}_{avg}$) are given by:

$$\text{Res}_t = \sum_{n=1}^{N} P_n - \sum_{d \in D, k \in S_n} X_{n,k} P_n - L_t.$$  (33)

$$\text{Res}_{avg} = \frac{\sum_{t \in T_{plan}} \text{Res}_t}{T_{plan}}.$$  (34)

where $L_t$ is the anticipated load demand for period $t$; $P_n$ is the generating capacity of unit $d_n$; $T$ is the total number of time indices, subject to the following constraints:

$$\text{ear}_d \leq \text{start}_d \leq \text{lat}_d - \text{NormDur}_d + 1 \quad \text{for all } d \in D.$$  (35)

$$\left\{ \sum_{n} P_n - \sum_{d \in D, k \in S_n} X_{n,k} P_n \right\} \geq L_t \quad \text{for all } t \in T_{plan}.$$  (36)

$$\left\{ \sum_{n} P_n - \sum_{d \in D, k \in S_n} X_{n,k} P_n \right\} \geq 1.2L_t \quad \text{for all } t \in T_{plan}.$$  (37)

$$\text{start}_3 > \text{start}_2 + \text{NormDur}_2 - 1$$

$$\text{start}_4 > \text{start}_3 + \text{NormDur}_3 - 1$$

$$X_{i,k} = 0 \text{ for } k = [\text{start}_{i} + \text{NormDur}_{i} - 1]$$

It is interesting to note that, given the same objective, the objective function formulations used by Escudero et al. (1980) and El-Amin et al. (2000) are quite different from that of Aldridge et al. (1999).

As there is no resource utilization throughout the planning horizon, there is no need for the inclusion of the resources term in the heuristic formulation (Eq. 10) for this case study (thus $w$ may be set to 0). The precedence and maintenance window constraints of this system are direct and rigid constraints, which can be incorporated by using the graph-based technique, whereas the load and reliability constraints need to be taken into account using penalty functions. The objective function cost (OFC) used in this case study is a function of the reserve generation capacity LVL value and the total violation of load and reliability constraints (Eq. 40).
\[ OFC(s) = LVL(s) \cdot \left( \text{LoadResViol}(s) + 1 \right). \]  

(40)

where \( OFC(s) \) is the objective function cost ($) associated with schedule \( s \); \( LVL(s) \) is the level of reserve generation capacity (MW) associated with schedule \( s \); \( \text{LoadResViol}(s) \) is the total demand and reserve shortfall (MW) associated with schedule \( s \).

It should be noted that the inclusion of a load constraint violation term in Eq. 40 is not necessary because violation of load constraints would be reflected as violation of reserve constraints. The calculation of constraint violations is given by Eqs. 41 and 42. The total load and reserve shortfall associated with schedule \( s \), \( \text{LoadResViol}(s) \), is the summation of load and reserve shortfall in all periods within the planning horizon:

\[ \text{LoadResViol}(s) = \sum_{n} \left( \sum P_n - \sum_{d \in D} \sum_{k \in S_n} X_{n,k} P_n \right). \]  

(41)

where \( T_{LV} \) is the period where load and reserve constraints are violated, and is given by:

\[ T_{LV} = (t: \sum_{n} P_n - \sum_{d \in D} \sum_{k \in S_n} X_{n,k} P_n < 1.2L_t). \]  

(42)

4.3 Modified 21-unit system

The 21-unit case study system described in section 4.1 was modified in the following ways in order to ensure that maintenance task shortening and/or deferral are required to satisfy load constraints:

1. The original system load (4739MW) is increased by 5% throughout the whole planning horizon, and another 5% increment for weeks 15 to 25.

2. While all maintenance tasks have the option of being deferred, some maintenance tasks can be carried out in durations shorter than the original outage duration (shown in Table 2). The personpower requirements for shortened durations are also detailed in Table 2.

<table>
<thead>
<tr>
<th>Unit No., ( n )</th>
<th>Optional Outage Duration, (weeks)</th>
<th>Personpower required for each week, ( \text{Res}<em>{n,wk}\text{uk}=1,2,...,\text{NormDur}</em>{n} ) (person)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>10, 10, 10, 8, 5</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>15, 14, 14</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>15, 15, 10</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>17, 17, 16</td>
</tr>
<tr>
<td>8</td>
<td>4</td>
<td>13, 13, 13, 6</td>
</tr>
<tr>
<td>9</td>
<td>8</td>
<td>3, 3, 3, 2, 2, 3, 3, 3</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>4, 4, 3, 3, 4, 4</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>6, 5, 5, 6</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>11, 11</td>
</tr>
<tr>
<td>10</td>
<td>2</td>
<td>15, 15</td>
</tr>
<tr>
<td>14</td>
<td>2</td>
<td>20, 20</td>
</tr>
<tr>
<td>20</td>
<td>2</td>
<td>20, 20</td>
</tr>
</tbody>
</table>

Table 2. Personpower utilization for the modified 21-unit case study system
Despite the possibility of shortening and deferral options in this case study, they are unfavorable from both an economic and operations point of view. Therefore, the objective function used for the original version of this case study (Eq. 27) has been modified to:

\[ OFC(s) = SSR(s) \cdot (ManVio_{tot}(s) + 1) \cdot (LoadVio_{tot}(s) + 1) \cdot (DurCut_{tot}(s) + 1). \]  

(43)

where \( OFC(s) \) is the objective function cost ($) associated with schedule \( s \); \( SSR(s) \) is the sum of squares of reserve generation capacity (MW\(^2\)) associated with schedule \( s \); \( ManVio_{tot}(s) \) is the total personpower shortfall (person) associated with schedule \( s \); \( LoadVio_{tot}(s) \) is the total demand shortfall (MW) associated with schedule \( s \); \( DurCut_{tot}(s) \) is the total reduction in maintenance duration (weeks) due to shortening and deferral associated with schedule \( s \).

While the calculation of total demand shortfall associated with schedule \( s \), \( LoadVio_{tot}(s) \), total personpower shortfall associated with schedule \( s \), \( ManVio_{tot}(s) \), and the sum of squares of reserve generation capacity associated with schedule \( s \), \( SSR(s) \), are detailed in section 4.1, the value of \( DurCut_{tot}(s) \) is given by:

\[ DurCut_{tot}(s) = \sum_{n=1}^{21} (NormDur_n - chdur_n(s)). \]  

(44)

where \( NormDur_n \) is the normal duration of maintenance task \( d_n \) and \( chdur_n(s) \) is the maintenance duration (week) of task \( d_n \) associated with schedule \( s \).

It should be noted that by using Eq. 43 to direct the search during an ACO run, a trial maintenance schedule that includes shortened and/or deferred maintenance tasks is being assigned a higher \( OFC \), which represent an unfavorable solution to ACO during pheromone update.

As part of the modified case study, the minimum-duration constraints can be addressed during the stage-2 selection process when a trial solution is being constructed (section 3.2.2) by allowing only optional durations that are greater than the minimum duration for each maintenance task. In this way, trial solutions constructed will not violate the minimum duration constraints. For example, machine unit 1 that normally requires 7 days to be maintained, can be shortened to 5 or 3 days, or be deferred altogether (Table 2).

### 4.4 Modified 22-unit system

The 22-unit case study detailed in section 4.2 was modified as follows in order to ensure that maintenance task shortening and/or deferral are required to satisfy load constraints:

1. The weekly loads for the modified 22-unit case study system are increased by 60%.
2. Maintenance tasks 1 to 13 are allowed to be performed within the first half of the planning horizon, while the remainder of the tasks have to be performed in the second half (except for unit 10 as in original case study).
3. While all maintenance tasks can be deferred, the maintenance tasks listed in Table 3 can be shortened to the optional duration(s) specified.

<table>
<thead>
<tr>
<th>Unit No., ( n )</th>
<th>1</th>
<th>5</th>
<th>6</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>14</th>
<th>15</th>
<th>16</th>
<th>17</th>
<th>18</th>
<th>22</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optional shortened durations (weeks)</td>
<td>4, 2</td>
<td>4, 2</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>10, 8, 6, 4</td>
<td>2</td>
<td>6, 4</td>
<td>4</td>
<td>3</td>
<td>4</td>
<td>3</td>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>

Table 3. Details of the modified 22-unit system
**Problem formulation**

The objective function used for the original 22-unit case study (Eq. 40) has been modified to accommodate the options of shortening and deferral, and is given by:

\[
OFC(s) = LVL(s) \cdot (\text{LoadResVio}_{tot}(s) + 1) \cdot (\text{DurCut}_{tot}(s) + 1).
\]  

(45)

where \(OFC(s)\) is the objective function cost ($) associated with schedule \(s\); \(LVL(s)\) is the level of reserve generation capacity (MW) associated with schedule \(s\); \(\text{LoadResVio}_{tot}(s)\) is the total load constraint violation (MW) associated with schedule \(s\); \(\text{DurCut}_{tot}(s)\) is the total reduction in maintenance duration (weeks) due to shortening and deferral associated with schedule \(s\).

The calculation of the total load constraint violation associated with schedule \(s\), \(\text{LoadResVio}_{tot}(s)\), and the level of reserve generation capacity associated with schedule \(s\), \(LVL(s)\) have been detailed previously in section 4.2, whereas the value of the total duration shortened and deferred associated with schedule \(s\), \(\text{DurCut}_{tot}(s)\), is given by:

\[
\text{DurCut}_{tot}(s) = \sum_{n=1}^{22} (\text{NormDur}_{n} - \text{chdur}_{n}(s)).
\]  

(46)

where \(\text{NormDur}_{n}\) is the normal duration (weeks) of maintenance task \(d_{m}\) and \(\text{chdur}_{n}(s)\) is the maintenance duration (weeks) of task \(d_{n}\) associated with schedule \(s\).

### 5. Experimental Procedure, Results and Analysis

#### 5.1 Experimental procedure

Experiments have been conducted on both the original and modified versions of the 21-unit and 22-unit case studies to assess the utility of the proposed ACO-PPMSO formulation. Particular emphasis was given to assessing the usefulness of the heuristics developed, the impact of the local search operator and the overall performance of the proposed ACO-PPMSO formulation.

**A. Usefulness of heuristic formulation**

The effectiveness of the new heuristic formulations for general PPMSO problems (Eqs. 10 to 15) introduced in section 3.2.2 was examined by conducting optimisation runs with and without the heuristics (the latter was achieved by setting the relative weight of the heuristic, \(\beta\), in Eq. 7 to 0). In addition, the sensitivity of optimisation results to increasing values of \(\beta\) was checked. It should be noted that, as a control, the value of \(\alpha\) in Eq. 7 was fixed at 1.

**B. Impact of local search operator**

The impact of local search on the performance of the ACO-PPMSO algorithm was also investigated, both with and without heuristic. The total number of trial solutions evaluated in the ACO runs with local search was identical to those without local search.

**C. Overall performance of ACO-PPMSO**

In order to check the overall utility of the ACO-PPMSO formulation, the results obtained for the two original case studies were compared with those obtained using other optimisation methods in previous studies and the ability to account for maintenance shortening and deferral was assessed on the two modified case studies.

In order to achieve the objectives outlined above, the testing procedure shown in Fig. 3 was implemented separately for each of the four case studies. Items A, B and C mentioned above were investigated at Stages A, B and C in the testing procedure, respectively.
To minimize the impact the ACO algorithms and parameters used have on the evaluation of the effectiveness of the heuristic, local search and overall performance of the ACO-PPMSO algorithm, two ACO algorithms, namely Elitist-Ant System and MMAS, and a range of parameters (shown in the dashed box in Fig. 3) were used to solve the problem instance under consideration. In addition, each run was repeated 50 times with different random number seeds in order to minimize the influence of random starting values in the solution space on the results obtained and to enable Student’s $t$-test to be conducted to determine whether any differences in the results obtained were significant. In total, 3,024 different combinations of parameters, each with 50 different starting random number seeds, were evaluated as part of this study. In order to facilitate fair comparisons, the same number of evaluations per optimisation run were used as in previous studies that investigated the 21-unit case problem (30,000 evaluations). In this research, ‘one ACO run’ is defined as the use of an ACO algorithm with or without using heuristic information, with or without local search and with a defined set of parameters to solve a PPMSO instance. An example of an ACO run is the use of EAS to solve the modified 21-unit case study with heuristic information and local search and a defined parameter set of $m = 200; \rho = 0.9; \tau_0 = 0.1; Q = 500,000; \alpha = 1, \beta = 11$, repeated for 50 random number seeds. The overall performance of a parameter set is then assessed based on the objective function cost (OFC) averaged over the 50 simulations using different random number seeds. An analysis of the results obtained with the testing procedure outlined in Fig. 3 is given in section 5.2.

![Figure 3. Experimental procedure](image)

**5.2 Results and analysis**
The experimental results obtained for the original 21- and 22-unit case studies are summarized in Tables 4 to 7, while those for the modified case studies are presented in Tables 8 to 11.
| Heuristic | Local search | Best OFC (SM) | Average OFC (SM) | Worst OFC (SM) | Std dev. (SM) | Average evaluations | Best parameter settings $[\mu, \rho, \gamma, \beta]$
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>14.84 [8.64%]</td>
<td>140.49 [928.48%]</td>
<td>365.13 [2572.99%]</td>
<td>86.00</td>
<td>28,841</td>
<td>[300; 0.9; 0.01; 0]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>13.68 [0.15%]</td>
<td>13.71 [0.37%]</td>
<td>13.83 [1.39%]</td>
<td>0.03</td>
<td>20,692</td>
<td>[200; 0.9; 0.01; 9]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>13.74 [0.59%]</td>
<td>51.62 [227.89%]</td>
<td>138.80 [916.11%]</td>
<td>33.72</td>
<td>25,494</td>
<td>[300; 0.8; 0.1; 0]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>13.66 [0%]</td>
<td>13.70 [0.29%]</td>
<td>13.82 [1.17%]</td>
<td>0.03</td>
<td>22,434</td>
<td>[200; 0.9; 0.01; 9]</td>
</tr>
</tbody>
</table>

*a* Number of evaluations to reach the best solution in one run averaged over 50 runs with different random starting positions.

*b* Number of ants; $\rho$: pheromone evaporation rate; $\gamma$: initial pheromone trail; $\beta$: relative weight of heuristic in Eq. 7

### Table 4. Results for the 21-unit problem instance given by Elistit-Ant System (EAS)

| Heuristic | Local search | Best OFC (SM) | Average OFC (SM) | Worst OFC (SM) | Std dev. (SM) | Average evaluations | Best parameter settings $[\mu; \rho; \gamma; \beta]$
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>13.86 [1.46%]</td>
<td>16.11 [17.94%]</td>
<td>43.35 [217.35%]</td>
<td>5.95</td>
<td>16,480</td>
<td>[10; 0.3; 0.2; 0]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>13.66 [0%]</td>
<td>13.68 [0.15%]</td>
<td>13.72 [0.44%]</td>
<td>0.01</td>
<td>13,593</td>
<td>[20; 0.4; 0.35; 5]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>13.80 [1.02%]</td>
<td>17.90 [31.04%]</td>
<td>69.04 [405.42%]</td>
<td>10.51</td>
<td>18,089</td>
<td>[50; 0.2; 0.05; 0]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>13.66 [0%]</td>
<td>13.69 [0.22%]</td>
<td>13.78 [0.88%]</td>
<td>0.02</td>
<td>15,867</td>
<td>[50; 0.5; 0.5; 11]</td>
</tr>
</tbody>
</table>

* Number of evaluations to reach the best solution in one run averaged over 50 runs with different random starting positions.

### Table 5. Results for the 21-unit problem instance given by Max-Min Ant System (MMAS) (deviation from best-known OFC of $13.66M$)

| Heuristic | Local search | Best OFC (SM) | Average OFC (SM) | Worst OFC (SM) | Std dev. (SM) | Average evaluations | Best parameter settings $[\mu; \rho; \gamma; \beta]$
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>63.41 [21.80%]</td>
<td>72.27 [38.82%]</td>
<td>81.15 [55.88%]</td>
<td>4.17</td>
<td>29,294</td>
<td>[200; 0.9; 100; 0]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>58.41 [12.20%]</td>
<td>64.31 [23.53%]</td>
<td>73.25 [40.70%]</td>
<td>3.21</td>
<td>28,384</td>
<td>[300; 0.9; 1; 11]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>58.91 [13.16%]</td>
<td>67.03 [28.76%]</td>
<td>79.99 [53.65%]</td>
<td>4.70</td>
<td>25,858</td>
<td>[300; 0.8; 1; 0]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>55.67 [6.93%]</td>
<td>60.35 [16.31%]</td>
<td>67.97 [30.56%]</td>
<td>2.90</td>
<td>26,931</td>
<td>[300; 0.8; 10; 11]</td>
</tr>
</tbody>
</table>

* Number of evaluations to reach the best solution in one run averaged over 50 runs with different random starting positions.

### Table 6. Results for the 22-unit problem instance given by Elistit-Ant System (EAS) (deviation from best-known OFC of $52.06M$)

| Heuristic | Local search | Best OFC (SM) | Average OFC (SM) | Worst OFC (SM) | Std dev. (SM) | Average evaluations | Best parameter settings $[\mu; \rho; \gamma; \beta]$
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>59.91 [15.08%]</td>
<td>66.30 [28.51%]</td>
<td>76.17 [46.31%]</td>
<td>3.67</td>
<td>24,597</td>
<td>[100; 0.9; 0.5; 0]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>55.72 [7.03%]</td>
<td>62.22 [19.52%]</td>
<td>68.65 [31.87%]</td>
<td>2.97</td>
<td>28,433</td>
<td>[200; 0.9; 0.2; 11]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>57.64 [10.72%]</td>
<td>64.81 [24.49%]</td>
<td>76.65 [47.23%]</td>
<td>4.27</td>
<td>27,455</td>
<td>[200; 0.8; 0.5; 0]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>54.56 [4.80%]</td>
<td>59.42 [14.14%]</td>
<td>66.56 [27.85%]</td>
<td>2.87</td>
<td>24,537</td>
<td>[200; 0.8; 0.35; 11]</td>
</tr>
</tbody>
</table>

* Number of evaluations to reach the best solution in one run averaged over 50 runs with different random starting positions.

### Table 7. Results for the 22-unit problem instance given by Max-Min Ant System (MMAS) (deviation from best-known OFC of $52.06M$)
<table>
<thead>
<tr>
<th>Heuristic</th>
<th>Local search</th>
<th>Best OFC ($M$)</th>
<th>Average OFC ($M$)</th>
<th>Worst OFC ($M$)</th>
<th>Std dev. ($M$)</th>
<th>Average $\text{DurCut}_\text{sat}$ (wks)</th>
<th>Average evaluations(a)</th>
<th>Best parameter settings ${\alpha, \beta, \gamma}$(b)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>65.61 [317.63%]</td>
<td>120.39 [666.33%]</td>
<td>209.05 [1230.68%]</td>
<td>39.16</td>
<td>17.6</td>
<td>27,538</td>
<td>(300; 0.9; 0.01; 0)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>16.15 [2.80%]</td>
<td>24.42 [55.44%]</td>
<td>51.06 [97.71%]</td>
<td>5.16</td>
<td>6.4</td>
<td>29,029</td>
<td>(500; 0.9; 0.01; 1)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>68.42 [339.52%]</td>
<td>135.13 [760.15%]</td>
<td>219.07 [1294.46%]</td>
<td>36.67</td>
<td>19.3</td>
<td>28,784</td>
<td>(300; 0.9; 0.01; 0)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>16.12 [2.61%]</td>
<td>26.87 [71.04%]</td>
<td>41.24 [162.51%]</td>
<td>5.17</td>
<td>6.9</td>
<td>28,213</td>
<td>(500; 0.9; 0.01; 1)</td>
</tr>
</tbody>
</table>

*Number of evaluations to reach the best solution in one run averaged over 30 runs with different random starting positions.

$\alpha$: number of ants; $\gamma$: pheromone evaporation rate; $\beta$: initial pheromone trail; $\beta$: relative weight of heuristic in Eq. 7.

Table 8. Results for the Modified 21-unit unit problem instance given by Elitist-Ant System (EAS) [deviation from best-known OFC of $15.71M$]

<table>
<thead>
<tr>
<th>Heuristic</th>
<th>Local search</th>
<th>Best OFC ($M$)</th>
<th>Average OFC ($M$)</th>
<th>Worst OFC ($M$)</th>
<th>Std dev. ($M$)</th>
<th>Average $\text{DurCut}_\text{sat}$ (wks)</th>
<th>Average evaluations(a)</th>
<th>Best parameter settings ${\alpha, \beta, p_{\text{explo}}}$(b)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>28.69 [82.62%]</td>
<td>61.32 [290.32%]</td>
<td>119.15 [658.43%]</td>
<td>19.54</td>
<td>11.8</td>
<td>16,934</td>
<td>(20; 0.2; 0.2; 0)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>15.97 [1.65%]</td>
<td>19.69 [25.33%]</td>
<td>29.03 [64.79%]</td>
<td>4.02</td>
<td>5.6</td>
<td>18,551</td>
<td>(50; 0.2; 0.05; 1)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>33.64 [114.13%]</td>
<td>71.67 [356.21%]</td>
<td>132.10 [740.87%]</td>
<td>24.64</td>
<td>12.6</td>
<td>24,898</td>
<td>(500; 0.1; 0.05; 0)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>15.71 [0%]</td>
<td>22.04 [40.29%]</td>
<td>29.66 [88.80%]</td>
<td>4.86</td>
<td>6.1</td>
<td>23,713</td>
<td>(500; 0.7; 0.05; 1)</td>
</tr>
</tbody>
</table>

$\alpha$: number of evaluations to reach the best solution in one run averaged over 30 runs with different random starting positions.

$\alpha$: number of ants; $\beta$: pheromone evaporation rate; $p_{\text{explo}}$: refer to Eq. 21; $\beta$: relative weight of heuristic in Eq. 7.

Table 9. Results for the Modified 21-unit problem instance given by Max-Min Ant System (MMAS) [deviation from best-known OFC of $15.71M$]

<table>
<thead>
<tr>
<th>Heuristic</th>
<th>Local search</th>
<th>Best OFC ($)</th>
<th>Average OFC ($)</th>
<th>Worst OFC ($)</th>
<th>Std dev. ($)</th>
<th>Average $\text{DurCut}_\text{sat}$ (wks)</th>
<th>Average evaluations(a)</th>
<th>Best parameter settings ${\alpha, \beta, \gamma}$(b)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>2186.22 [138.64%]</td>
<td>2797.85 [205.40%]</td>
<td>4267.31 [365.80%]</td>
<td>410.33</td>
<td>21.9</td>
<td>27,896</td>
<td>(300; 0.9; 0.01; 0)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1365.60 [49.06%]</td>
<td>1756.34 [91.72%]</td>
<td>2153.97 [133.12%]</td>
<td>175.55</td>
<td>13.8</td>
<td>28,648</td>
<td>(500; 0.9; 0.01; 11)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2331.92 [154.54%]</td>
<td>2876.16 [213.95%]</td>
<td>4357.14 [375.61%]</td>
<td>501.14</td>
<td>23.2</td>
<td>26,187</td>
<td>(300; 0.9; 0.01; 0)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1174.10 [28.16%]</td>
<td>1724.37 [88.23%]</td>
<td>2238.34 [144.33%]</td>
<td>172.63</td>
<td>13.7</td>
<td>21,718</td>
<td>(300; 0.9; 0.01; 11)</td>
</tr>
</tbody>
</table>

$\alpha$: number of evaluations to reach the best solution in one run averaged over 30 runs with different random starting positions.

$\alpha$: number of ants; $\gamma$: pheromone evaporation rate; $\beta$: initial pheromone trail; $\beta$: relative weight of heuristic in Eq. 7.

Table 10. Results for the Modified 22-unit unit problem instance given by Elitist-Ant System (EAS) [deviation from best-known OFC of $916.12$]


<table>
<thead>
<tr>
<th>Heuristic</th>
<th>Local search</th>
<th>Best OFC ($)</th>
<th>Average OFC ($)</th>
<th>Worst OFC ($)</th>
<th>Std dev. ($)</th>
<th>Average DurCut$_{tot}$ (wks)</th>
<th>Average evaluations</th>
<th>Best parameter settings</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1489.33</td>
<td>2076.43</td>
<td>3996.67</td>
<td>440.16</td>
<td>26,219</td>
<td>(300; 0.6; 0.2; 0)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1008.13</td>
<td>1489.54</td>
<td>2017.44</td>
<td>280.45</td>
<td>23,329</td>
<td>(20; 0.3; 0.35; 11)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1614.39</td>
<td>2088.8</td>
<td>3596.71</td>
<td>425.87</td>
<td>20,767</td>
<td>(20; 0.3; 0.2; 0)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1301.12</td>
<td>1513.86</td>
<td>2084.59</td>
<td>306.26</td>
<td>21,347</td>
<td>(50; 0.1; 0.35; 11)</td>
<td></td>
</tr>
</tbody>
</table>

*Number of evaluations to reach the best solution in one run averaged over 50 runs with different random starting positions.

*Number of ants, [1-β]: pheromone-evaporation rate, $\mu$: refer to Eq. 21, β: relative weight of heuristic in Eq. 7.

Table 11. Results for the Modified 22-unit unit problem instance given by Max-Min Ant System (MMAS) [deviation from best-known OFC of $916.12$]

Stage A: Impact of heuristic

Overall, the new heuristic formulation for applying ACO to PPMSO problems significantly improved the results obtained for all four case studies, with and without the use of a local search operator and for both ACO algorithms (using a Student’s t-test at a 95% significance level). It can be seen that when the heuristic was used, not only were the average OFCs improved, but the standard deviations of the OFCs were also significantly smaller for all case studies (Tables 4 to 11), indicating that use of the new heuristic formulation enables good solutions to be found consistently.

In order to gain a better understanding of the searching behavior of the ACO algorithms in solving each of the four case studies with and without heuristic, the optimisation process of the ACO runs was examined. The investigation is facilitated by utilizing the following terms to describe a given ACO-PPMSO run:

- Objective function values (SSR, LVL and DurCut$_{tot}$) associated with iteration-best schedules (referred to as IB-SSR, IB-LVL and IB-DurCut$_{tot}$ hereafter)
- Violation of various constraints (demand and power shortfall) associated with iteration-best schedules (referred to as IB-LoadVio$_{tot}$, IB-ManVio$_{tot}$ and IB-LoadResVio$_{tot}$ hereafter)

The optimization process of only one ACO-PPMSO run for the modified 21-unit case system is used for discussion purposes (Fig. 4). Figs. 4a and 4b compare the behaviour of the ACO-PPMSO in solving the case system with and without heuristic. Overall, the ACO-PPMSO algorithm is found to explore the problem search space effectively by minimizing the objective function values (SSR, LVL and DurCut$_{tot}$) for the four case studies investigated. This is illustrated by the decreasing trends of the IB-SSR and IB-DurCut$_{tot}$ curves in Figs. 4a and 4b.

For all case studies, it is found that when the heuristic is used, the IB-SSR and IB-LVL obtained during the early stages of the optimisation runs were substantially lower (compare IB-SSR curves in Figs. 4a and 4b). In addition, it is observed that during the early stages of the ACO runs, fewer trial solutions that violated constraints were constructed when the heuristic was utilized (lower IB-LoadVio$_{tot}$, IB-ManVio$_{tot}$ and IB-LoadResVio$_{tot}$). It is also found that the improvement in OFCs obtained when the heuristic is used for the modified 21- and 22-unit case studies is partly attributed to a significant reduction in duration shortened. This is clearly shown in the comparison between Figs. 4a and 4b by the fact that the IB-DurCut$_{tot}$ curve is consistently lower throughout an ACO run when the heuristic formulation is used.
Figure 4. Modified 21-unit case system - Comparison of the SSR- and total duration shortened values associated with iteration-best schedules during optimisation run (Best-known SSR = $2.62 \times 10^6$ MW$^2$ with 5-week deferral)

‡ Parameter settings used shown in the first row, last column of Table 9 (random number seed = 655)
‡‡ Parameter settings used shown in the second row, last column of Table 9 (random number seed = 655)
IB-SSR: Sum of squares of reserve associated with iteration-best schedules;
IB-DurCut$_{tot}$: Total reduction in outage duration due to shortening and deferral associated with iteration-best schedules
In view of the experimental results, the heuristic formulation is useful for ACO-PPMSO in three ways. Firstly, as the distribution of pheromone intensity within the search space of a problem is uniform at the beginning of an ACO run (assuming a single initial pheromone value is used), the optimisation process initially resembles a random search. During this period, the heuristic formulation can guide the algorithm to search in regions where feasible solutions are located with a higher probability. In this way, the number of infeasible solutions being constructed and rewarded with pheromone can be reduced. Secondly, even if a heuristic is not essential for constructing feasible/near feasible trial solutions (as is the case when the PPMSO problem is not highly constrained), the heuristic can assist with constructing trial solutions that consist of fewer overlapping tasks. In this way, the generation capacities throughout the planning horizon associated with trial maintenance schedules being constructed are more evenly distributed, which is one of the common objectives of PPMSO problems. Thirdly, when shortening and deferral options are allowed, use of the heuristic increases the probabilities that longer outage durations are chosen throughout an entire ACO run. This is particularly useful when shortening and deferral options are frequently chosen at random during the early stage of an ACO run.

In relation to the two ACO algorithms investigated (EAS and MMAS), the results obtained indicate that the heuristic has a significant positive impact on both EAS and MMAS. This is probably due to the ability of heuristic information to identify regions of the search space where high-quality initial solutions lie, reducing the number of low-quality trial solutions being reinforced at the beginning of an optimisation run. In addition, the results indicate that the ACO-PPMSO heuristic has a bigger positive impact on EAS compared to MMAS. EAS tends to stagnate after a number of iterations, which increases the impact of the quality of the initial solutions. The importance of the regions where the ants initially search using EAS is also highlighted by the relatively larger number of ants found for the best parameter settings than those for MMAS (Tables 4, 6, 8 and 10), implying that a search with more ants in each iteration (resulting in a smaller number of iterations during an optimisation run, as the total number of function evaluations is fixed) works better than one with fewer ants (resulting in a larger number of iterations during an optimisation run, as the total number of function evaluations is fixed). On the other hand, relatively smaller ant populations are found to perform best for MMAS (Tables 5, 7, 9 and 11), which might be attributed to the continuous exploration during an MMAS run (Fig. 4b) as a result of the lower and upper bound for pheromone values. It is interesting to observe that despite the expected overall downward trends throughout an optimisation run, the IB-SSR and IB-LVL curves spike occasionally throughout a run when a small population of ants is used (Fig. 4b). This phenomenon is found to be caused by the choice of non-best solutions after a short convergence (stagnation in OFC), which altered the distribution of pheromone over the problem search space. It should be noted that the possibility of having an iteration-best solution that is not the best-so-far solution is higher when a smaller population of ants is used.

**B. Impact of local search**

The optimisation results obtained by coupling the PPMSO-2-opt local search operator with the ACO algorithms investigated (Stage B of the testing procedure in Fig. 3) are tabulated in Tables 4 to 11. The unpaired Student's t-test was used to check the significance of the impact of the local search operator in solving the four case studies with and without heuristic (Tables 12 and 13). Overall, the impact of the PPMSO-2-opt local search operator ranges from being insignificant, to significantly improving or degrading the performance of the ACO
algorithm investigated. While having a positive impact on solving the original 22-unit case study, regardless of which of the two ACO algorithms was used, the PPMSO-2-opt local search operator was found to improve only the performance of EAS when the heuristic was not used for solving the original 21-unit case study. As for the modified case studies, the performance of ACO in solving the modified 21-unit case study was reduced significantly when the PPMSO-2-opt local search operator was adopted, while the impact of the local search was not significant when applied to the modified 22-unit case study.

<table>
<thead>
<tr>
<th>Heuristic</th>
<th>21-unit system</th>
<th>22-unit system</th>
<th>Modified 21-unit system</th>
<th>Modified 22-unit system</th>
</tr>
</thead>
<tbody>
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Table 12. Impact of PPMSO-2-opt local search operator with and without heuristic

From the results of the Stage B testing, it is interesting to observe that despite the similarity in the number of generating units for the 21- and 22-unit case study systems, the impact of the PPMSO-2-opt local search algorithm on the optimisation results of these case studies was quite different, which is likely to be caused by the difference in the problem characteristics of the two systems. In order to better understand the results obtained, a series of tests were conducted to investigate the mechanism of PPMSO-2-opt in detail. The satisfaction of constraints associated with iteration-best solutions (target solutions) used for the local search operation and the % of infeasible local solutions generated when using MMAS were examined. It should be noted that the results were obtained using the proposed heuristic formulation.

Figure 5. Infeasible local solutions obtained using PPMSO-2-opt (original 21-unit case study using MMAS)
It can be seen that for the original 21-unit case study (Fig. 5), a large number of infeasible local solutions were generated by PPMSO-2-opt in every iteration, even with feasible iteration-best solutions (target maintenance schedules). A local solution generated by simply exchanging the maintenance start time of two randomly chosen generating units without any guidelines is likely to result in infeasible solutions in such a highly constrained search space. As a result, PPMSO-2-opt seems to have an insignificant or even detrimental impact when coupled with ACO for solving the aforementioned case studies. This is particularly evident for the modified 21-unit case study, where as many as 50% to 80% of the local solutions generated by PPMSO-2-opt in every iteration are infeasible with regard to both load and personpower constraints, which is responsible for the significant decrease in ACO performance. These results suggest that the PPMSO-2-opt local search operator is not well suited to problems with highly constrained search spaces.

On the other hand, the local solutions generated by PPMSO-2-opt in solving the original 22-unit case study are all feasible, as the iteration-best solutions are also feasible. In fact, this is the only case study for which PPMSO-2-opt is found to be effective in improving the optimisation ability of ACO. Compared to the other three case systems, the original 22-unit case system is less constrained. Therefore, the results obtained indicate that PPMSO-2-opt can be useful for solving problems that are not highly constrained.

C. Overall performance of ACO-PPMSO

Original 21-unit and 22-unit case studies

By using the ACO-PPMSO algorithm, a new best-known objective value has been found for both the original 21-unit case study ($SSR = 13.66 \times 10^6 \text{ MW}^2$) and the original 22-unit case study ($LVL = 52.06 \text{ MW}$).

A comparison of the results obtained by ACO-PPMSO with those obtained by various metaheuristics in other studies for the 21-unit case study, including those by Aldridge et al. (1999), who used a simple genetic algorithm (GA), a generational GA (GNGA) and a steady state GA (SSGA), and Dahal et al. (2000), who applied Simulated Annealing (SA) and an Inoculated GA to this problem, is shown in Fig. 6. As mentioned previously, the number of evaluations (trial solutions) allowed in the ACO runs and those of the other metaheuristics was identical. In particular, the best and average results of the metaheuristics were compared. While the best and average results given by the simple GA, SSGA, GNGA, inoculated GA and SA were obtained by 10 runs with different starting positions (Aldridge et al., 1999; Dahal et al., 1999; Dahal et al., 2000), those of EAS and MMAS were obtained using 50 runs.

It can be seen that the EAS and MMAS algorithms have outperformed the algorithms that have been applied to this case study previously. It should be noted that a new best-found solution ($SSR = 13.66 \times 10^6 \text{ MW}^2$) for the 21-unit case study has been found by EAS and MMAS using the new ACO-PPMSO formulation. In addition, it can be seen that the differences between the average and best results of the ACO algorithms are much smaller than those for other metaheuristics (Fig. 6), which indicates a consistent performance of the ACO-PPMSO formulation.

Among the metaheuristics previously used for solving the 21-unit case study, the inoculated GA, where the initial population is generated using a heuristic that ranks the generating units in order of decreasing capacity, was found to perform best in terms of the average results obtained. This highlights the potential benefit of a heuristic in solving PPMSO problems.
As mentioned previously, a new best-found solution \(SSR = 13.66 \times 10^6 \text{ MW}^2\) has been found by the ACO-PPMSO formulation proposed in this chapter. An examination of the solutions obtained for the 21-unit case study found that different maintenance schedules are associated with the new best-found SSR solution. In other words, there is more than one optimal solution in the problem search space.

In Fig. 7, the reserve level across the planning horizon associated with the best-known schedule found by ACO-PPMSO for the original 22-unit case study is compared with those obtained by implicit enumeration (Escudero et al., 1980) and tabu search (El-Amin et al., 2000). It can be seen that the reserve level given by the ACO schedule is more evenly spread out (summed deviation of generation reserve from the average reserve, \(LVL = 52.06 \text{ MW}\)) than those obtained with implicit enumeration (\(LVL = 118.81 \text{ MW}\)) and tabu search (\(LVL = 256.93 \text{ MW}\)). It should be noted that due to insufficient information about the optimum solution in El-Amin et al. (2000), the LVL value of tabu search shown in Fig. 7 was calculated using the best available published information.

**Modified 21-unit and 22-unit case studies**

As the modified versions of the 21- and 22-unit case studies have been introduced in this chapter to test the developed ACO-PPMSO formulation, there are no previous results available for comparison purposes. As can be seen in Tables 7 to 10, the optimized maintenance schedules of both the modified 21- and 22-unit case studies include the shortening and/or deferral of maintenance tasks (average duration shortened/deferred > 0). The best-found objective function costs (OFCs) found for the modified 21-unit case study is $15.71M and $916.12 for the modified 22-unit case study. In the maintenance schedules associated with the best-found OFC for the modified 21-unit case study, the maintenance tasks for generating units 11 and 21 are deferred, while all other tasks are carried out as normal. For the modified 22-unit case study, maintenance tasks for generating units 10, 16
and 17 are shortened by 2, 4 and 2 weeks, respectively. It should be noted that all constraints are satisfied by the best-found schedules.

The results for the modified versions of the 21-unit and 22-unit case studies indicate that the ACO-PPMSO formulation introduced in this chapter is able to identify maintenance schedules that satisfy hard system constraints (e.g., system demands) by shortening and deferring maintenance tasks. More importantly, the shortening and deferral options were only used if necessary, as only a few, but not all, maintenance tasks were shortened/deferred.

![Graph comparing reserve levels obtained using ACO, implicit enumeration, and tabu search](image_url)

**Figure 7.** Comparison of reserve levels obtained using ACO, implicit enumeration (Escudero et al., 1980) and tabu search (El-Amin et al., 2000)

### 5. Summary and Conclusions

In this chapter, a formulation for applying Ant Colony Optimization (ACO) to power plant maintenance scheduling optimization (PPMSO) has been developed and successfully tested using four case studies (original and modified versions of two benchmark case studies from the literature). In particular, the performance of the heuristic formulation developed, the two local search algorithms introduced and the overall utility of the ACO-PPMSO formulation were investigated. The results obtained have shown that the heuristic formulation improves the performance of the ACO-PPMSO algorithm significantly when applied to the four case studies investigated. It was found that while the PPMSO-2-opt local search operator seems to work well for unconstrained problems, it is not suitable for highly-constrained PPMSO problems. Lastly, the results obtained by ACO-PPMSO for the two original case studies were better than those obtained by other optimisation methods, such as various genetic algorithm (GAs) formulations and simulated annealing (SA). For the 21-unit and 22-unit case studies, a new optimal solution has been found by the ACO-PPMSO
formulation. In addition, the results given by ACO-PPMSO were more consistent compared with those obtained using other metaheuristics previously applied to the two benchmark case studies. The maintenance schedules found for the modified case studies have also been examined and it was found that the ACO-PPMSO formulation is able to meet hard system constraints by shortening and deferring maintenance. The results of experiments carried out using the original and modified versions of the 21-unit and 22-unit case studies indicate that the ACO-PPMSO formulation presented in this chapter has potential for solving real-world PPMSO problems.

6. References


In the era globalisation the emerging technologies are governing engineering industries to a multifaceted state. The escalating complexity has demanded researchers to find the possible ways of easing the solution of the problems. This has motivated the researchers to grasp ideas from the nature and implant it in the engineering sciences. This way of thinking led to emergence of many biologically inspired algorithms that have proven to be efficient in handling the computationally complex problems with competence such as Genetic Algorithm (GA), Ant Colony Optimization (ACO), Particle Swarm Optimization (PSO), etc. Motivated by the capability of the biologically inspired algorithms the present book on "Swarm Intelligence: Focus on Ant and Particle Swarm Optimization" aims to present recent developments and applications concerning optimization with swarm intelligence techniques. The papers selected for this book comprise a cross-section of topics that reflect a variety of perspectives and disciplinary backgrounds. In addition to the introduction of new concepts of swarm intelligence, this book also presented some selected representative case studies covering power plant maintenance scheduling; geotechnical engineering; design and machining tolerances; layout problems; manufacturing process plan; job-shop scheduling; structural design; environmental dispatching problems; wireless communication; water distribution systems; multi-plant supply chain; fault diagnosis of airplane engines; and process scheduling. I believe these 27 chapters presented in this book adequately reflect these topics.

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