Attitude and Steering Control of the Long Articulated Body Mobile Robot KORYU

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1. Introduction

Many types of mobile robots have been considered so far in the robotics community, including wheeled, crawler track, and legged robots. Another class of robots composed of many articulations/segments connected in series, such as “Snake-like robots”, “Train-like Robots” and “Multi-trailed vehicles/robots” has also been extensively studied. This configuration introduces advantageous characteristics such as high rough terrain adaptability and load capacity, among others. For instance, small articulated robots can tread through rubbles and be useful for inspection, search-and-rescue tasks, while larger and longer ones can be used for maintenance tasks and transportation of material, where normal vehicles cannot approach. Some ideas and proposal appeared in the literature, to build such big robots; many related studies concerning this configuration have been reported (Waldron, Kumar & Burkat, 1987; Commissariat A l’Energie Atomique, 1987; Burdick, Radford & Chirikjian, 1993; Tilbury, Sordalen & Bushnell, 1995; Shan and Koren, 1993; Nilsson, 1997; Migads and Kyriakopoulos, 1997). However, very few real mechanical implementations have been reported.

An actual mechanical model of an “Articulated Body Mobile Robot” was introduced by Hirose & Morishima in 1988, and two mechanical models of articulated body mobile robot called KORYU (KR for short) have been developed and constructed, so far. KORYU was mainly developed for use in fire-fighting reconnaissance and inspection tasks inside nuclear reactors. However, highly terrain adaptive motions can also be achieved such as: 1) stair climbing, 2) passing over obstacles without touching them, 3) passing through meandering and narrow paths, 4) running over uneven terrain, and 5) using the body’s degrees of freedom not only for “locomotion”, but also for “manipulation”. Hirose and Morishima (1990) performed some basic experimental evaluations using the first model KR-I (a 1/3 scale model compared to the second model KR-II, shown in Fig. 1(a)-(c). Improved control strategies have been continuously studied in order to generate more energy efficient motions.

This chapter addresses two fundamental control strategies that are necessary for long articulated body mobile robots such as KORYU to perform the many inherent motion capabilities cited above. The control issue can be divided in two independent tasks, namely 1) Attitude Control and 2) Steering Control. The underlying concept for the presented
control methods are rather general and could be applied to different mechanical implementations. However, in order to give the reader better understanding of the control issues, the second mechanical model KR-II will be used as example for implementing such controls. For this reason, the mechanical implementation of KR-II will be first introduced, followed by the steering control and attitude control.

The authors believe that not only the financial issue, but the difficulty in the mechanical design and control has limited the progress in this area. However, the realization of this class of robots is still promising, so we expect this work to contribute to the understanding of the many control problems related to it.

(a) Stair climbing  (b) Moving on uneven terrain  (c) Mobile manipulation

Fig. 1. Articulated body mobile robot “KR-II”, (first built in 1990, modified in 1997)

Total weight: 450kg, Height: 1.0m, Width: 0.48m, Wheel Diameter: 0.42m.

2. Mechanical Configuration and Modelling of KR-II

KR-II is a totally self-contained robot, with batteries and controllers installed on-board, and remote controlled through a wireless modem. It is composed of cylindrical bodies numbered 1 to 6, and a manipulator in the foremost segment, numbered no. 0. The cylindrical bodies are in fact modular units we call a “unit segment” that can be easily detached mechanically and electrically from the adjacent segments, so the total system can be disassembled for transportation and easily assembled on-site.
The degrees of freedom (DOF) for these units can be divided in three distinct classes as shown in Fig. 3, say z axis, \( \theta \) axis and s axis.

Fig. 3. KR-II’s “unit segment” and its motion freedoms

### 2.1. Mechanical Details of the z axis

For a conventional translational motion mechanism, the output displacement developed at the front connection plate (Pf) is equal to the input displacement transmitted from the ball screw nut (B\(_N\)). A rack-and-pinion mechanism with two rack gears (G\(_R\)) and one pinion gear (G\(_T\)) as shown in Fig. (b), doubles the translational motion from the ball screw nut.

Fig. 4. KR-II’s z axis mechanical details

### 2.2. Mechanical Details of the \( \theta \) axis

The unit segments have another characteristic concerning the wheel’s heading orientation. For mechanical simplification and weight reduction, a “1/2 angle mechanism” (Fig. 5.), which orients the wheel to half the bending angle of the \( \theta \) axis was introduced. For this reason, wheel sideslips cannot be prevented during general manoeuvrings, but this

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constraint does not degrade the steering motion of KR-II at all. In fact, previous work (Fukushima & Hirose, 1996) it has been shown that using the appropriate steering control method, the energy loss from wheel slippage can be minimized and good trajectory tracking performance achieved.

Fig. 5. KR-II’s $\theta$ axis mechanical details. The “1/2 angle mechanism” orients the wheel to half the bending angle of the $\theta$ axis

![Diagram of KR-II's mechanical details]

Fig. 6. Manipulator mechanism and degrees of freedom

### 2.3. Mechanical Details of the $s$ axis

Each unit segment is equipped with one wheel, such that the odd numbered segments’ wheels are arranged right-sided and the even segments’ left-sided. This arrangement increases the rough terrain adaptability and decreases the total mechanical weight. As
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mentioned in the Introduction, KR-II can pass over obstacles without touching them. In order to ensure static stability for these motions, we need a minimum of 6 basic unit segments connected in series, such that two segments can be lifted up at the same time without losing stability and avoid contact with the obstacles.

2.4. Mechanical Details of the Front Manipulator
An extra connection plate Pf remains in the front part of the segment no. 1. We use it to connect a segment equipped with a manipulator and a wheel, aligned with the segment’s center vertical axis, as shown in Fig. 6. The manipulator part is linked to the wheel so that the heading orientation coincides to it. This manipulator has only one independent degree of freedom in the arm part, say $\phi_1$. However, using the body’s articulations and the motion of the wheels, a higher DOF manipulation tasks can be accomplished. Further details of manipulation are out of the scope of this chapter and will be omitted, but note that considering that the segment front part refers to the manipulator-wheel part, the segments no. 0’s degrees of freedom that are used for steering control are the same as explained above, namely $\theta$ axis and $s$ axis.

3. Steering Control Problem

3.1 Control variables
The segment’s vertical axes (z-axes) are controlled to be always parallel to the gravitational force field, by an attitude control scheme that will be discussed later. As the attitude control works independently from the steering control, the control variables for the steering control can be modeled on the x-y plane as shown in Fig. 7. Nonetheless, this model is valid not only for locomotion on flat terrain, but also for locomotion on uneven terrain as well. The variables used to model KR-II are as follows:

$\theta_a$  
$\theta$ axis bending angle: the relative bending angle of segment no. $n$’s front part relative to the rear part.

\[
\theta_a = \Theta_a - \Theta_{a+1}, \quad (n = 0 \sim 6)
\]  

$\Theta_a$  
Segment vector orientation: segment no. $n$’s front part orientation in the Global Reference Frame $\Sigma_{GRF}$. This orientation is the same as segment no. $(n-1)$’s rear part orientation. Note that $\Theta_6$ is the orientation of segment no. 6’s rear part.

$\Phi_a$  
Wheel heading orientation vector: segment no. $n$’s wheel heading orientation in the global reference frame. For the foremost segment $\Phi_0 = \Theta_0$.

\[
\Phi_a = \Theta_a - \frac{\Theta_a - \Theta_{a+1}}{2} = \frac{\Theta_a + \Theta_{a+1}}{2}, \quad (n = 1 \sim 6)
\]  

$L$  
Intersegment length: adjacent segments center to center distance. KR-II has constant length $L = 0.48$ m for all segments.
Segment center to wheel distance: distance between the segment center to the wheel-ground contact point in the horizontal plane. For the basic unit segment of KR-II, \( B = 0.23 \text{m} \), and for the segment no. 0, \( B = 0 \).

\[ \phi_n \quad \text{wheel heading orientation in GRF} \]

\[ \theta_n \quad \text{segment direction vector in GRF} \]

\[ \theta_n \quad \text{θ axis bending angle in local reference frame} \]

**Fig. 7. KR’s steering control variables**

### 3.2 Steering Control Objectives

The main issue on KR’s steering control is, given from a remote human operator the velocity and orientation commands for the foremost segment, to automatically generate joint commands for all the following segments, such that they follow the foremost segment’s trajectory.

For KR’s teleoperation scheme as shown in Fig. 2, a remote human operator sends steering control commands for the foremost segment only, namely velocity \( v_0 \) and heading orientation \( \theta_0 \), and the on-board computer calculates the joint commands for the following segments. Note that the bending angle of the manipulator-wheel part (segment no. 0) relative to the front part of the segment no. 1, i.e. \( \theta_0 \), is used as the command for changing the orientation of the foremost segment (i.e. segment no. 0). This is because \( \theta_0 \) coincides with the angular displacement of the wheel displayed on the remote operator’s monitor TV, having a camera set on segment no. 1’s front part.

Concerning the control of KR-II’s wheels (\( s \)-axes) and the bending angles between the segments (\( \theta \)-axes), we have systematically investigated some basic steering control methods in previous work (Fukushima et al., 1995, 1996, 1998). The main criteria for evaluating the performance of each method were: i) trajectory tracking performance, ii) energy consumption performance and iii) amount of wheel sideslip. From these results, we demonstrated that for articulated body mobile robots with short intersegment lengths, e.g., the earlier snake-like robot ACM-III (Umetani & Hirose, 1974), a “Shift Control Method” which simply shifts the bending angle command from the foremost segment to the foremost segment to the following segments, synchronized to the locomotion velocity can be effective.

However, for articulated mobile robots such as KR-II which has large intersegment length, this method introduces a large sideslip in the foremost segment’s wheel, because the motion of the foremost segment is shifted to the next only after moving a certain distance, during which there is a difference between the foremost wheel’s heading orientation and its actual motion direction. To attenuate this problem, two other methods were derived: 1) “Moving Average Shift Control”, which takes the average value of the foremost segment’s control angle \( \theta_0 \) over the time to travel a distance \( L \) as the next segment command \( \theta_1 \), and then shifts \( \theta_1 \) to the following segments according to the moved distance, and 2) “Geometric...
Trajectory Planning Method”, which calculates all the $\theta$ axis bending angle commands from the geometric relationship that results when each segment center is considered to travel over a given desired trajectory. From the evaluation of these methods, the last one presented the best trajectory tracking performance and energy efficiency. This is because exact joint commands were calculated using equations in literal form. However, for a real-time implementation in the on-board computer, the second best method “Moving Average Shift Control” has been chosen. In reality, this method combined with a technique consisting of setting a small position control gain for each $\theta$ axis was considered the best steering control method for KR-II for some time.

For the $s$-axes, the so called “Body Velocity Control by Wheel Torque Compensation”, in which the velocity of the body is controlled by equally distributed torques for all the wheels, presented the best performance in combination with any $\theta$-axes steering method.

Fig. 8. Trajectory representation for the inertial reference frame method

3.3 The Inertial Reference Frame Steering Control Method

The “Inertial Reference Frame Method” introduced in this section can be considered as belonging to the same category as the “Geometric Trajectory Planning Method”, in the way it is based on calculating the joint commands ($\theta$-axes) from a trajectory described in an inertial reference frame. From a biomechanical point of view (Umetani and Hirose, 1974), the methods based on shift control that generate all the steering commands considering joint space variables only, are though to be more suitable for controlling snake-like robots because it is improbable that joint commands generation in real snakes are performed considering inertial reference frame. However, as already demonstrated in previous work, the inertial reference frame based methods offers many advantages such as good trajectory tracking performance, energy efficiency and as will be demonstrated in the following sections, it can be easily extended for use in a “W-Shaped Configuration”, which increases the stability of the robot for motion on uneven terrain. The only drawback of the method based on geometric calculation was the large computation time, which has been overcome by the method here explained. The basic algorithm is constructed by the following steps:

Step 1. Estimate the initial trajectory for the foremost segment (once at power-on)

Step 2. Update the trajectory moved by the foremost segment.

Step 3. Search each segment’s center position over the trajectory.

Step 4. Calculate each segment’s bending angle ($\theta_s$)
Step 5. Repeat steps 2)~4).

This method is characterized by using the moved distance over the trajectory as a parameter for representing the trajectory in an inertial reference frame (as shown in Fig. 8), so that the position of the segments center \( s_{seg} \) can be tracked in a numerical fashion, by iteratively searching for the \( s_{seg} \) that satisfies the geometrical constraint between the segments (i.e., the intersegment length \( L \) is constant). Here are the details of each step.

1) Initial Trajectory Generation At the time the robot is powered-on, the trajectory between the last segment and the foremost segment is unknown. Therefore, the robot cannot initiate the steering motion because segments no. 1 to 6 does not have a trajectory to follow. This trajectory can be estimated by interpolating the segments center position at initialization time. For instance, a cubic spline interpolation function was implemented in the actual KR-II control.

2) Trajectory Update In case a trajectory is specified \textit{a priori} or it is generated by an autonomous path planning algorithm, this step simply updates the position of the foremost segment \( s_0 \) over the known trajectory. However, as we assume that a human operator is manually maneuvering the robot, the trajectory must be calculated online. The natural choice is an odometric approach. This method calculates both the new position at time \( t + \Delta t \) by estimating the moved distance \( \delta \), and the new orientation of the foremost segment from time \( t \) to \( t + \Delta t \). Let \( v_{ina}(t) \) be the actual velocity of the foremost segment measured from the wheel rotation velocity, the distance moved during the interval of one sampling time \( \Delta t \) is estimated by

\[
\delta = v_{ina}(t) \Delta t. \tag{3}
\]

Next, from the measured bending angle of the foremost segment \( \theta_{ina}(t) \), and the segment vector orientation of the segment no. 1 \( \Theta_1(t) \), the foremost wheel heading orientation at time \( (t + \Delta t) \) is estimated as follow.

\[
\Theta_0(t + \Delta t) = \Theta_1(t) + \theta_{ina}(t) \tag{4}
\]

With \( \delta \) and \( \Theta_0(t + \Delta t) \) estimated from real measurements, the position of the foremost segment at time \( t + \Delta t \) can be calculate by

\[
\text{Traj}(s_0(t + \Delta t)) = \text{Traj}(s_0(t)) + E^{n(t+\Delta t)} \begin{bmatrix} \delta \\ 0 \end{bmatrix}, \tag{5}
\]

where

\[
s_0(t + \Delta t) = s_0(t) + \delta, \quad \text{and} \quad E^\alpha = \begin{bmatrix} \cos(\Theta) & -\sin(\Theta) \\ \sin(\Theta) & \cos(\Theta) \end{bmatrix}.
\]
Although the above computations are performed at each sampling time, for practical purposes the array $\text{Traj}(s_n) = (x(s_n), y(s_n))$ holds coordinates of the trajectory separated by a constant interval $\Delta \tau$. For the actual KR-II computation $\Delta \tau$ is normally set to 1mm.

3) Segments Center Position Search By setting a condition that: “the minimum bending radius of the trajectory is greater than the intersegment length $L$ for all intervals”, the position of each segment center can be tracked by searching continuously in the forward direction of the trajectory. This means that for a forward movement, none of the following segments make a backward movement. The distance $L_{\text{calc}}$ between two adjacent segment centers is calculated from equation (6).

$$L_{\text{calc}} = \sqrt{(x(s_n) - x(s_{n-1}))^2 + (y(s_n) - y(s_{n-1}))^2}$$

Thus, considering a margin of error, the position $s_n$ that satisfies equation (7) can be searched for, from segment no. 1 to no.6 ($n = 1 \sim 6$).

$$(0.99L)^2 \leq L_{\text{calc}} \leq (1.01L)^2$$

4) Segments Bending Angle Calculation Knowing the coordinates of all segment center positions, the direction vectors between these segments, $\Theta_\theta$ are calculated by

$$\Theta_\theta = \tan^{-1} \frac{y(s_n) - y(s_{n-1})}{x(s_n) - x(s_{n-1})}.$$  

However, the direction vector of the last segment $\Theta_7$ does not depend on the position of any segment center, so a direction that orients the last segment’s wheel to the tangent of the trajectory is chosen to minimize energy loss due to wheel sideslips. The wheel orientation is calculated by

$$\Phi_\delta = \tan^{-1} \frac{y(s_n) - y(s_{n-1} - \delta)}{x(s_n) - x(s_{n-1} - \delta)},$$

and $\Theta_\theta$ by

$$\Theta_\theta = 2\Phi_\delta - \Theta_\theta.$$  

Finally, the $\theta$ axis bending angles are calculated.

$$\theta_n = \Theta_\theta - \Theta_\theta^{\text{ori}} \quad n = 1 \sim 6.$$  

3.4 Validity of the Trajectory Updating Method

The actual robot KR-II does not have sensors to measure directly its position and orientation with respect to the inertial reference frame. For this reason, an odometric approach has been used to estimate the foremost segment’s position and orientation at each new time step. In
order to verify the estimation accuracy a right angle cornering motion with a bending radius \( R = 500 \text{mm} \) is simulated. First, the \( \theta \) axis commands \( \theta_0 \) to \( \theta_4 \) (shown by the solid lines in Fig. 9) are generated from geometric relationship (i.e., analytically). Using the foremost segment’s command \( \theta_0 \) generated by this method as the input to equation (4), and considering velocity \( v_{in}(t) = 100 \text{ mm/s} \), and sampling time \( \Delta t = 10 \text{ ms} \), results in the dashed lines in the same figure. These results, shows that the presented method generates joint commands extremely close to the analytical method’s exact commands. The actual trajectories for both methods are also coincident.

![Fig. 9. Calculation of \( \theta \) axis steering commands](image)

**3.4.1 Errors in the Odometric Approach**

Note that the odometric approach introduces many errors in the position estimation, due to the terrain irregularities, tire pressure changes, and sensor offsets. These errors are cumulative. For this reason this position estimation is not well suited for autonomous navigation or for autonomous mapping of unknown environments. However, for the steering control purposes, the error introduced in the trajectory estimation affects only the interval from the foremost segment to the last segment (about 3.3 m). Moreover, as we consider teleoperation by a human operator and the motion of the foremost segment is controlled relative to the next segment, the difference between the estimated position and the actual position do not affect the overall steering control performance.

**3.5 W-Shaped Configuration**

The method discussed so far, was intended for the steering control in a “Straight-Line Configuration” (shown in Fig. 11(a)), i.e., when all the segment centers pass through the same trajectory travelled by the foremost segment. This configuration is very important for locomotion in narrow and meandering passes, but as KR-II has the wheels mounted right-sided for odd numbered segments and left-sided for even numbered segments, its stability becomes critical for locomotion over rough terrain. To overcome this problem, a “W-Shaped Configuration” as shown in Fig. 11(b), which enlarges the wheel-to-wheel width by a \( W \)-shape width \( B_w \) is introduced. The “W-Shaped Configuration” is defined as:

“The configuration where the centers of the odd numbered segments follow a trajectory that is
continuous and “parallel” to the trajectory followed by the centers of the no. 0 and even numbered segments.”

Note that the word “parallel” is used with the same meaning as parallel rails of a rail-way where the rails do not cross and maintain a constant distance between them. For the “W-Shaped Configuration”, the distance between the rails (left and right trajectories), denoted as $B_w$, will be the parameter to be controlled, and for the particular case when $B_w = 0$, the two rails coincide and the configuration becomes the same as the “Straight-Line Configuration”.

(Fig. 10. Basic Steering formation for KR-II)

### 3.5.1 Steering Control for the W-Shaped Configuration

The steering control for the “W-Shaped Configuration” is accomplished in a very straightforward way: the trajectory travelled by the foremost segment in the “Straight-Line Configuration” is set as the left trajectory $\text{Traj}_L(s)$ to be followed by the even numbered segments. The odd numbered segments follow the right trajectory $\text{Traj}_R(s)$, which is parallel to $\text{Traj}_L(s)$.

The basic algorithm for the “Straight-Line Configuration” is extended to be used in the W-Shaped Configuration, as follows.

1) **Initial Trajectory Generation** In order to estimate the initial trajectory for a W-Shaped configuration, first the left trajectory $\text{Traj}_L(s)$ is generated from a spline interpolation using the segments no. 0,2,4,6 center positions as control points, and the right trajectory $\text{Traj}_R(s)$ from the segments no. 1,3,5. However, to generate more parallel left and right trajectories, auxiliary control points can also be used. The steps of this algorithm are detailed in Fig. 11. Note that the initial width is calculated by

$$B_{\text{init}} = \frac{1}{5} \sum_{i=1}^{4} L \sin \left( \frac{\theta_i}{2} \right).$$  \hspace{1cm} (12)

2) **Trajectory Update** The left trajectory $\text{Traj}_L(s)$ travelled by the foremost and the even numbered segments can be updated using the same equation (5) used for the line
configuration. And from the definition of W-Shaped configuration, the right trajectory \( \text{Traj}_R(s) \) is generated parallel to \( \text{Traj}_L(s) \).

\[
\text{Traj}_R(s_0(t + \Delta t)) = \text{Traj}_L(s_0(t + \Delta t)) + \mathbf{E}^0\psi_{(s_0+\Delta s)} \begin{bmatrix} 0 \\ -B_w \end{bmatrix}
\]

(13)

3) Segments Center Position Search  
Given the correct left and right trajectories for each segment, equations (6)(7) can be used to calculated each segment center position.

4) Segments Bending Angle Calculation  
In this step, the line configuration’s basic equations are also valid, given the correct trajectory for each segment.

(a) Estimate the W-Shape width \( B_{W_{initial}} \) for the initial configuration from the average of the W-Shape widths of segments no.1 to no.5.

(b) Generate an auxiliary trajectory \( \text{Traj}_{Laux}(s) \) from the segments no. 0,2,4,6, center positions interpolation.

(c) Set auxiliary control points 0',2',4',6', to be located \( -B_{W_{initial}} \) in the normal direction relative to the trajectory \( \text{Traj}_{Laux}(s) \) from segments no.0,2,4,6 center positions.

(d) Generate the right trajectory \( \text{Traj}_R(s) \) from the interpolation of the seven control points set by the segments no. 1,3,5 center positions and the auxiliary points 0',2',4',6'.

(e) Set auxiliary control points 1',3',5', to be located \( +B_{W_{initial}} \) in the normal direction relative to the trajectory \( \text{Traj}_R(s) \) from segments no.1,3,5 center positions.

(f) Generate the left trajectory \( \text{Traj}_L(s) \) from the interpolation of the seven control points set by the segments no. 0,2,4,6 center positions and the auxiliary points 1',3',5'.

Fig. 11. Initial trajectory generation algorithm for the W-Shaped configuration

3.5.2 Foremost segment’s control angle compensation  
For the basic remote control scheme, the bending angle and velocity command for the
foremost segment is sent from a remote human operator controlling a joystick. For a straight forward motion the operator should hold the joystick $\theta$ axis command $\theta_{\text{joy}}$ at the zero (neutral) position. Furthermore, the operator should be relieved from the task of compensating for the change in cornering radius due to change in the W-Shaped width $w_B$, which changes the apparent intersegment length and results in smaller cornering radius. Taking in account these factors the bending angle $\theta_b$ is compensated as:

$$\theta_b = \theta_{\text{joy}} \cos(\theta_{\text{sv}}) - \theta_{\text{sv}},$$

where $\theta_{\text{sv}}$ is an offset angle due to the W-Shaped configuration.

$$\theta_{\text{sv}} = \frac{\theta_{\text{w}}} {2} = \sin^{-1}\left(\frac{B_s} {L}\right)$$

### 3.5.3 Shift Between Straight-Line and W-Shaped Configurations

The shift between the configurations can be accomplished by smoothly changing the value of $w_B$ according to the moved distance. The configuration shift algorithm must also take in account the trajectories of segments no. 0 and no. 1. There are several ways to perform this transition, and some examples are shown below.

### 3.6 Conclusions about Steering Control

For the real robot KR-II, the introduced method demonstrated good energy efficiency and trajectory tracking performance as well as real-time control feasibility. This method was successfully extended for use in the “W-Shaped Configuration”, and it can be considered the best steering control scheme for articulated body mobile robots with long intersegment lengths. Some experimental results are shown in Fig. 12, 13 and 14.
4. Optimal Attitude Control

Optimal force distribution has been an active field of research for multifingered hand grasping, cooperative manipulators and walking machines. The articulated body mobile robot “KORYU” composed of cylindrical segments linked in series and equipped with many wheels have a different mechanical topology, but it forms many closed kinematic chains through the ground and presents similar characteristics as the above systems. This section introduces an attitude control scheme for the actual mechanical model “Koryu-II (KR-II)”, which consists of optimization of force distribution considering quadratic object functions, combined with attitude control based on computed torque method. The validity of the introduced method is verified by computer simulations and experiments using the actual mechanical model KR-II.

4.1 Attitude control problem description

KR-II is composed of cylindrical units linked in series by prismatic joints which generate vertical motion between adjacent segments. The simplest solution to control the vertical motion would be position control. However, as shown in Fig. 15(a), this method cannot be used for locomotion on uneven terrain. The vertical prismatic joints should be force
controlled so that each segment vertical position automatically adapts to the terrain irregularities, as shown in Fig. 15(b). The simplest implementation of force control is to make these joints free to slide. However, in this case the system acts like a system of wheeled inverted pendulum carts connected in series and is unstable by nature, as shown in Fig. 16(a) (b). Thus, an attitude control scheme to maintain the body in the vertical posture is demanded. This work introduces a new attitude control based in optimal force distribution calculation using quadratic programming for minimization of joint energy consumption.

Fig. 15. Comparison between position and force control in the z axis

Fig. 16. Force control alone cannot maintain the posture of the robot, so an attitude control is needed

The presented method shares similarities with force distribution for multifingered hands, multiple coordinated manipulators and legged walking robots. In this section, the background on optimal force distribution problem is described, the optimal force distribution formulation and shows an efficient algorithm to solve this problem is introduced. Furthermore, the mechanical modelling of KR-II for the attitude control is presented and a feedback control is introduced.

4.2 Background on Optimal Force Distribution Problem

Many types of force distribution problems have been formulated for multifingered hands, multiple coordinated manipulators and legged walking robots. A brief review of the fundamental concepts and similarities with formulation of balance equation and equations of motion of multibody systems are hereafter described.
4.2.1 Balance Equations for Reference Member
Multifingered hands, multiple coordinated manipulators and legged walking robots can be modeled as one reference member with \( k \) external contact points as shown in Fig.17(a). Consider the reference member parameters given by: mass \( m_0 \); linear and angular acceleration at the center of mass \( \alpha_0, \omega_0 \in \mathbb{R}^3 \); inertia tensor at the center of mass coordinate \( H_0 \in \mathbb{R}^{3 \times 3} \); force \( F_i \in \mathbb{R}^3 \) and moment \( M_i \in \mathbb{R}^3 \) acting on the \( i \) th contact point; position of the contact point with respect to the center of mass coordinate \( p_i = [p_{i1}, p_{i2}, p_{i3}]^T \in \mathbb{R}^{3 \times 1} \).

The resulting force and moment at the center of mass is given by \( F_0 = \sum_{i=1}^{k} F_i \in \mathbb{R}^3 \) and \( M_0 = \sum_{i=1}^{k} (M_i + p_i \times F_i) \in \mathbb{R}^3 \). The balance equations is given below, where the gravitational acceleration \( g \) which in principle is an external force, was included in the left term for simplicity of notation.

\[
m_0 \alpha_0 = F_0 + m_0 g
\]

\[
H_0 \omega_0 + \omega_0 \times (H_0 \omega_0) = M_0
\]

The inertial terms can be grouped as \( \mathbf{Q} \in \mathbb{R}^6 \), and the external force terms into the matrix \( \mathbf{P} \) and vector of contact points \( \mathbf{N} \).

\[
\mathbf{P} = \begin{bmatrix}
I_3 & 0 & \ldots & I_3 & 0
\end{bmatrix} \in \mathbb{R}^{6 \times 6}
\]

\[
\mathbf{I}_3 \in \mathbb{R}^{3 \times 3} : \text{identity matrix}
\]

\[
\bar{\mathbf{p}}_i = \begin{bmatrix}
0 & -p_{i3} & p_{i2} \\
p_{i3} & 0 & -p_{i1} \\
-p_{i2} & p_{i1} & 0
\end{bmatrix} \in \mathbb{R}^{3 \times 3}
\]
Thus the balance equations are described by the following linear relation.

\[ Q = P^* N \]  \hspace{1cm} (20)

### 4.2.1 General solution

The general solution of equation (20) with respect to \( N \) is given by

\[ N = P^* Q + (I - P^* P) \hat{\lambda} , \]  \hspace{1cm} (21)

as described in Walker et al., 1988, where \( P^* \) is a generalized pseudo-inverse matrix.

Basic formulation of force distribution and description about internal forces concepts were first addressed by Kerr & Roth, 1986. The fundamental concepts are: i) the general solution can be divided in two orthogonal vectors \( N = N_e + N_i \); ii) the partial solution \( N_e \) can be solved by \( N_e = P^* Q \) using the pseudo-inverse matrix; iii) the partial solution \( N_i \) resides in the null-space of \( P \) and corresponds to the internal forces; iv) \( I - P^* P \) is a matrix formed by orthonormal basis vectors which span the null space of \( P \), and \( \hat{\lambda} \) corresponds to the magnitude of the internal forces. Kerr and Roth used these concepts to formulate a linear programming problem which took in account friction forces at contact points and also joint driving forces.

Force distribution problem for gripper and hands usually results in searching optimal values for \( \hat{\lambda} \). Nakamura et al., 1989, were the first to formulate a nonlinear problem using quadratic cost function \( \| N \| \) to solve the internal forces. Efficient solutions using linear programming were also analyzed by other authors (Cheng & Orin, 1991; Buss et al., 1996). Nahon & Angeles, 1992, showed that minimization of internal forces and joint torques can both be formulated in an efficient quadratic programming method, and Goldfarb and Idnami method (Goldfarb & Idnami, 1983) can be used to solve this problem. Other efficient formulations are also been investigated (Chen et al., 1998; Markefka & Orin, 1998).

### 4.2.2 Multibody systems

Multibody systems differ from multifingered hands as shown in Fig.17(a)(b) not only by the fact that in general they have no common reference member, but also because that forces and moments \( F, M \), acting in the contact points arises from different physical natures. In system (a) the external forces are exerted from the fingers, manipulators or legs. However, system (b) can not have external forces exerted from the ground. Instead, the forces and moments at the contact points are originated from the gravitational acceleration and internal motion of the system itself. However, balance equations and equations of motion for these systems present similar characteristics as described next.

#### 4.2.2.1 Balance equations

Let the variables \( k, F, M, p \) be defined the same in Fig.17(a)(b), the equations (18)(19) are valid for both systems, but equations (16)(17) due to inertial forces are not. However, the
total force acting on the system’s center of mass can be derived as $\dot{Q}(q, \dot{q}, \ddot{q})$, i.e., as function of generalized coordinate $q, \dot{q}, \ddot{q}$. Hence, the balance equations can be described by

$$\dot{Q}(q, \dot{q}, \ddot{q}) = P N.$$  \hspace{1cm} (22)

Equations (22) and (20) are mathematically equivalent, and therefore the fundamental theory discussed for systems having common reference member, can be applied to multibody systems as well.

### 4.2.2.2 Equations of motion

For the system in Fig.17 (a), the problem of finding optimal values for contact forces $N$ and joint forces $\tau$ can be independently formulated. However, the equations of motion can also be grouped as equation (23) for optimization of joint forces of the entire system (Walker et al., 1988).

$$\tau = Hq + C + G_g + J^T N$$  \hspace{1cm} (23)

The above equation has the same structure for robot manipulators and multibody-systems where $H$ is the inertial term, $C$ the coriolis and centrifugal term, $G_g$ the gravitational term, and $J$ is the Jacobian matrix. Therefore, the equations of motion for systems in Fig.17 (a) and (b) are mathematically equivalent.

### 4.3 Efficient Algorithm for Solving Optimal Force Distribution Problem

#### 4.3.1 Cost function

Electrical motor’s energy consumption at low speed but high output torque operation can be estimated by the power loss in the armature resistance. Hence, the sum of squares of joint forces $\tau$ can be used as the cost function to be minimized.

$$S(\tau) = \tau^T W \tau$$  \hspace{1cm} (24)

Note that $W$ is a symmetric positive definite matrix. Now, let, $H_q = Hq + C + G_g$, $G = 2JWJ^T$ and $d = 2JW H_q$ be defined as auxiliary variables. Substituting equation (23) into (24) results in a new cost function depending on the variable $N$.

$$S(\tau) = H_q^TWH_q + d^TN + \frac{1}{2} N^TG N ,$$  \hspace{1cm} (25)
4.3.2 Quadratic problem formulation
The first term in the right side of equation (25) does not depend on $N$, so the new cost function can be described by a new equation (26), and a general quadratic programming problem can now be formulated as equations (26) and (27).

$$\min_N S(N) = d^T N + \frac{1}{2} N^T G N$$

subject to:

$$\begin{cases}
P_i N = Q_i \\
P_i N \geq Q_i
\end{cases}$$

Equations (27) are linear constraint equations, with equality constraints given by equations (20) or (22), and inequality constraints given by the system’s friction, contact and joint force limitations. A positive definite matrix $W$ guarantees this problem to be strictly convex, thus having efficient solution algorithms (Goldfarb & Idnami, 1983).

4.3.3 Solution considering equality constraints
The partial problem when considering only equality constraints can be solved as

$$N_e = P_i^+ Q_i - H_e d,$$

where the generalized pseudo-inverse matrix $P_i^+$ defined as

$$P_i^+ = G^{-1} P_i G (P_i G^{-1} P_i^+)^{-1},$$

and auxiliary matrix $H_e$ defined as

$$H_e = (I - P_i^+ P_i) G^{-1}$$

From the observation that the first term of equation (28) corresponds to the norm of $N$, i.e., the solution which minimizes $N^T G N$, the second term is the partial solution which minimizes the norm of $r$. Note that although it resides in the null-space of $P$, an analytic solution is available.

4.3.4 Solution considering inequality constraints
Problems with inequality constraints usually do not have analytical solutions but use some kind of search algorithms (Nakamura et al., 1989; Nahon & Angeles, 1992; Goldfarb & Idnami, 1983). In order to achieve better real-time performance, only negative contact forces will be considered in this formulation. This is valid for hands, grippers, walking machines and mobile robots in general, that can exert positive forces, i.e., “push”, but can not exert negative forces, i.e., “pull”. The proposed method introduces a new equality constraints term $P_d N = Q_d \in \mathbb{R}^d$ into the balance equation $P N = Q$. 
The basic idea is to transform the problem with inequality constraints into a problem with only equality constraints that can be solved efficiently by equation (28). This is accomplished by the algorithm described below, and illustrated in Fig. 18. Note that the variable \( d \) represents the number of contact points included in the equality constraints.

**Step 0.** Initialization: case \( d_0 > 0 \) make \( d = d_0 \) and include the contact forces \( P \in \mathbb{R}^{d_0} \) into equations (31)(32). Case \( d_0 = 0 \), initialize \( P = \mathbf{P} \) and \( Q = \mathbf{Q} \).

**Step 1.** Calculate the partial solution \( N \) considering only equality constraints from equation (28).

**Step 2.** Let the number of negative contact forces in the solution \( N \) be \( n_d \). Case \( n_d > 0 \) go to Step 3. Otherwise, this is the optimal solution. Calculate joint forces by equation (33). Finish.

\[
\tau_r = H \mathbf{e} + J^T N
\]  

(33)

**Step 3.** Update \( d = d + n_d \). Case \( d \leq (\text{free variables} - \text{balance equations}) \), go to Step 4. Otherwise, the problem cannot be solved. Finish.

**Step 4.** Set the desired contact force at the contact points where resulted in negative forces to zero and include in the equality constraint \( P \in \mathbb{R}^{d_0} \). Return to Step 1.

Although this algorithm is suited for real-time applications, it does not search for all the combination of possible solutions. For this reason, it might finish in Step 3, even a possible solution exists. However, for normal steering control, passing-over pipes and ditches, and...
attitude control of KR-II, a possible solution was always found after a limited number of iterations.

### 4.4 Formulation of KR-II’s Attitude Control

#### 4.4.1 KR-II’s variables

KR-II’s motion freedoms can be grouped as:

- z-axes linear displacements $z = [z_1, z_2, \ldots, z_n]^T \in \mathbb{R}^n$;
- \(\theta\)-axes angular displacements $\theta = [\theta_1, \theta_2, \ldots, \theta_n]^T \in \mathbb{R}^n$;
- wheel’s (s-axes) angular displacements $s = [s_1, s_2, \ldots, s_n]^T \in \mathbb{R}^n$;
- body’s coordinate position $c = [x_0, y_0, z_0]^T$ and attitude $\phi = [\phi_x, \phi_y, \phi_z]^T$ relative to the inertial coordinate. This accounts for 26 degrees of freedom.

In this work, balance and motion equations given by Equations (22)(23) were derived by Newton-Euler method, but other efficient virtual power methods (Thanjavur & Rajagopalan, 1997) could also be used. The inertial parameters for KR-II can be find in Fukushima & Hirose, 2000.

#### 4.4.2 Simplifications

##### 4.4.2.1 Wheel modeling

The wheel will be simplified to a simple model:

1. It is a thin circular plate with constant radius.
2. It contacts a horizontal plane even when moving on slopes.

However, the horizontal plane is set independently for each wheel so that this simplification is effective for motion over uneven terrains. Nonetheless, for stair climbing and step overcoming motions, a better contact point estimation algorithm should be used.

##### 4.4.2.2 Other simplifications

1) **External contact forces**: the wheel’s lateral and longitudinal forces are small because optimal trajectory is planned by the steering control, and also abrupt acceleration and deceleration are avoided. Moreover, moments between the tire and the ground are also negligible. For these reasons, the tire normal force $F_z$ can be considered the only external force acting on the system. Hence the external contact force vector is given by

$$ N = [F_{z_1}, F_{z_2}, \ldots, F_{z_n}]^T \in \mathbb{R}^n $$

(34)

2) **Joint forces**: z-axes and \(\theta\)-axes motions can be independently planned because KR’s z-axes orientations are controlled to be always vertical. In fact, \(\theta\)-axes is position controlled and their desired angular displacements are planned by the steering control. On the other hand, although s-axes motion can be used for the attitude control, it would involve undesirable acceleration and deceleration in the system. For these reasons, z-axes forces will be set as the variables to be optimized.

$$ \tau = [f_{z_1}, f_{z_2}, \ldots, f_{z_n}]^T \in \mathbb{R}^n $$

(35)
Note that $f_{s_0}$ was included just for avoiding singularity in the calculation, but always result in $f_{s_0} = 0$.

3) **Balance equations**: from the above considerations, force balance in the $z$ direction and moment balance around $x$ and $y$ directions are enough to model our system. Hence, the dimension of the balance equation $PN = Q$ becomes $Q \in R^3$ and $P \in R^{3\times7}$.

4) **Generalized accelerations**: only a part of the 26 degrees of freedom of KR-II, $q_t = [z^T \quad \theta^T \quad \dot{\phi}_x \quad \dot{\phi}_y]^T$ and its time derivative $\dot{q}_t = [\ddot{z} \quad \ddot{\theta} \quad \ddot{\phi}_x \quad \ddot{\phi}_y]^T$ is used in the calculation. The acceleration variables are further simplified to

$$\ddot{q}_a = [\ddot{x}_a \quad \ddot{y}_a \quad \ddot{z}_a \quad \ddot{\phi}_x \quad \ddot{\phi}_y]^T \in R^8,$$

and used as $\ddot{q} \equiv \ddot{q}_a$.

5) **Other parameters**: other dimensions are as follows: $H \in R^{2\times8}$; $C \in R^{7\times1}$; $G_j \in R^{7\times4}$; $J \in R^{3\times7}$; $P_d \in R^{5\times7}$; $Q_d \in R^{4\times4}$.

### 4.4.3 Attitude feedback law

The formulation described so far, solves for joint forces which balance the system in a given desired posture. This is fundamentally an inverse dynamics problem. A feedback control law shown below is added into equation (36).

$$\ddot{\phi}_x = \ddot{\phi}_x - K_{px} \dot{\phi}_x + K_{px} (\dot{\phi}_x - \dot{\phi}_x)$$

$$\ddot{\phi}_y = \ddot{\phi}_y - K_{py} \dot{\phi}_y + K_{py} (\dot{\phi}_y - \dot{\phi}_y)$$

$K_p, K_d$ are proportional and derivative gain and the indexes $d, m$ stands for desired and measured values. This control law is equivalent to the Computed Torque Method (Markiewics, 1973) so that the closed-loop system stability can be analyzed in the same way.

### 4.5 Computer Simulation and Experimental Results

Computer simulation and experiment using the real robot KR-II, is here evaluated for an "obstacle passing over (without touching them)" where a box shape obstacle with width 300mm and height 150mm is considered. This motion was calculated considering a constant forward velocity of 100mm/s. The vertical motion of each segment is shown in Fig.19(a). Note that the calculated displacement includes a 10mm displacement for safety, resulting in a 160mm total vertical displacement.
4.5.1 Continuity of calculated forces
Discontinuities in the calculated forces occur when topological changes are caused by lifting-up or touching-down of the wheels. In this work these discontinuities are avoided by introducing desired contact forces using the equality constraints $P_d N = Q_d$ in Step 0. The desired forces when lifting-up is given as

$$F_{dU} = F_{U,n} \frac{(50 - Z_n)}{50},$$

and when touching-down the ground is given as

$$F_{dD} = F_{D,ONS} \frac{(50 - Z_n)}{50}.$$  \hspace{1cm} (40)

$F_{U,n}$ is the optimal force calculated just before the wheel lift-up, $F_{D,ONS}$ is the optimal force calculated considering that the wheel has completely touch-down the ground, $Z_n$ is the segment vertical displacement as shown in Fig.19(a). These equations are applied only in the interval $Z_n = 0 \sim 50$ mm. The constant 50 was derived considering KR-II’s spring suspension stroke. For vertical displacements above 50, the desired contact force is set to zero. The simulation results shown in Fig.19(b)-(c) demonstrate the validity of the proposed method.

Fig. 19. Obstacle passing over simulation.  \hspace{1cm} Fig. 20. Experiment overview
4.5.2 Experimental results

The experiment was held applying the feedforward command shown in Fig.19(c) and feedback command given by equations (37) and (38). The feedback gains were $K_{sx} = 65$, $K_{sy} = 18$, $K_{sy} = 95$, $K_{sy} = 16$, and all the computation was performed in real-time with a sampling-time of 20ms. The experiment was performed without forward motion, but actually lifting-up the segments to the specified height as shown in Fig.20. Fig.21(a) shows the performance of attitude control. Large attitude changes occur at times when more than one segment is lifted-up at the same time. Fig.21(b) is the plot of the sum of squares of z-axes joint forces measured during the experiment. It has a much higher magnitude than the simulation, but shows the same tendency compared to the plot in Fig.19(d). This experiment is an extreme case for the attitude control, when there are many segments being lifted-up from the ground at the same time. For normal manoeuvring on flat terrain or even on uneven terrain, most all wheels are in contact with the ground, so that the overall stability is much higher.

![Attitude displacements](image)

(a) Attitude displacements

![Sum of squares of z-axes joint forces](image)

(b) Sum of squares of z-axes joint forces

Fig. 21. Obstacle passing over experiment results

![KR-II moving around a shopping center street](image)

Fig. 22. KR-II moving around a shopping center street [1993]
5. Conclusion

The authors have designed and actually built a mechanical prototype of a class of articulated body mobile robots, 3.3m long, and mass of more than 400kg, called KR-II. Attitude control and steering control has been successfully implemented, and the robot can move stably in the outdoors, even on uneven terrain.

The presented steering control method is based on parameter representation for describing trajectories in an inertial reference frame, with travelled distance as a parameter. In doing so, the position of each element (in the case of the articulated body mobile robot KR-II, segment center positions) on the trajectory can be tracked by simple and effective numerical searching algorithms. For the real robot KR-II, the introduced method demonstrated good energy efficiency and trajectory tracking performance as well as real-time control feasibility. This method was successfully extended for use in the “W-Shaped Configuration”, and it can be considered the best steering control scheme for articulated body mobile robots with long intersegment lengths, such as the KR-II.

The presented attitude control scheme is based in optimal force distribution using quadratic programming, which minimizes joint energy consumption. Similarities with force distribution for multifingered hands, multiple coordinated manipulators and legged walking robots were demonstrated. The attitude control scheme to maintain the vertical posture of the robot was introduced inside this force distribution problem.

The validity and effectiveness of proposed methods were verified by computer simulations and also experimentally using the actual mechanical model. Moreover, the introduced attitude control and steering control can be used not only for control of big “snake-like” robots, but even for walking machines at some extent.

Some negative points for developing such big robots, is the complexity in the mechanical design and also the high cost to build it, which makes the advance in this area of research a little bit slow, compared to other mobile robot systems. Nonetheless, sometime passed since the last prototype has been built (1990-2000), and we experienced many advances in actuators, computers, materials technology, so a much better design should be possible now.

The authors hope the mechanical concept and practical results presented in this Chapter, could give enough insights for new developments in the area.

6. References


With the advancement of technology, new exciting approaches enable us to render mobile robotic systems more versatile, robust and cost-efficient. Some researchers combine climbing and walking techniques with a modular approach, a reconfigurable approach, or a swarm approach to realize novel prototypes as flexible mobile robotic platforms featuring all necessary locomotion capabilities. The purpose of this book is to provide an overview of the latest wide-range achievements in climbing and walking robotic technology to researchers, scientists, and engineers throughout the world. Different aspects including control simulation, locomotion realization, methodology, and system integration are presented from the scientific and from the technical point of view. This book consists of two main parts, one dealing with walking robots, the second with climbing robots. The content is also grouped by theoretical research and applicative realization. Every chapter offers a considerable amount of interesting and useful information.

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