The Identification of Models of External Loads

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1. Introduction

One of the important problems of mathematical modelling of dynamic systems is the coincidence of modelling results with experimental measurements. Such a coincidence is being attained by construction of "correct" mathematical model (MM) of the dynamical system and the choice of a "good" model of external load (MEL). MM of object the motion of which coincides with experimental measurements with acceptable accuracy under action of MEL (or external impact) is understood by us as "correct" model. Thus the degree of "correctness" of MM depends directly on the chosen model of EL and required accuracy of the coincidence with experiment. It is formally possible to write as an inequality for models with the concentrated parameters the following:

\[ F (\mathcal{A}_p z, u_\delta) \leq \varepsilon, \]  

(1)

where \( \mathcal{A}_p \) is an operator of the certain structure which carries out the connection of EL \((z)\) and the response of MM \((u, \mathcal{A}_p z = u)\) and which depends on vectors - parameters \( p \); \( \varepsilon = \text{Const} > 0 \) is the required accuracy of the coincidence of experiment with results of mathematical modelling; \( z \) is the function of model of EL, \( z \in Z \); \( u, u_\delta \) are the vector-functions of the response of researched object on external load, \( u \in U, u_\delta \in U \). One of possible variants of an inequality (1) can be the following inequality

\[ \| \mathcal{A}_p z - u_\delta \|_U \leq \varepsilon, \]  

(2)

where \( \| \cdot \|_U \) there is a norm in functional space \( U \).

Characteristic feature for problems of the considered type is that the operator \( \mathcal{A}_p \) is compact operator (Tikhonov & Arsenin, 1979).

The value \( \varepsilon \) is set a priori and characterizes desirable quality of mathematical modeling. The vector-function \( u_\delta \) is obtained from experiment with the know error \( \delta \):

\[ \| u_T - u_\delta \|_U \leq \delta_0, \]  

(3)

where \( u_T \) is an exact response of object on real EL.

It is obvious that in the case of performance of an inequality (2) operators \( \mathcal{A}_p \) and function \( z \) are connected. It is easy to show that at the fixed operator \( \mathcal{A}_p \) in (2) exists infinite set of various among themselves functions \( z \) which satisfies to an inequality (2) (Tikhonov & Arsenin, 1979). And, on the contrary, at the fixed function \( z \) there are infinite many various
operators for which an inequality (2) is valid. Thus, there are no opportunities of a choice of
good model of system (of process) separately from a choice of correct model of external
load.
As a rule the check of inequality (2) is not executed in the practice of mathematical
modelling, but its performance is meant. The error of the measuring equipment \( \delta_0 \) is
contained in value \( \epsilon \) as obligatory component and therefore the inequality \( \delta_0 \leq \epsilon \) is always
takes place. It occurs for the reason, that the accuracy of experimental measurements is
higher as required accuracy of modelling as a rule. Frequently only qualitative coincidence
of results of mathematical modelling with experiment satisfies.
At research of real dynamic systems the structure of the mathematical description, as a rule,
is fixed. For example, at research of dynamics rolling mills (Menshikov, 1976, 1985), at the
solution of a problem of unbalance diagnostics (Menshikov, 2004) it is possible to use the
models with the concentrated parameters. Proceeding from design features of real systems
or devices, it is possible to determine parameters of the mathematical description
(parameters of operators) precisely enough. However these parameters are believed to be
given approximately. The error of definition of parameters depends on a way of reduction
of dynamic systems to more simple systems, from a various sort of the conditions and
assumptions, from the account of those or other factors (Menshikov, 1994). This error can be
appreciated from above and it does not surpass 10 % as a rule.
Two approaches exist to problem of construction of couple MM and model of EL:
1. MM is given a priori with inexact parameters and then the model of EL is being
determined for which the inequality (2) is valid;
2. Some model of EL is given a priori and then MM is being chosen for which the
inequality (2) is satisfied.
For example for the operators which are given with an error it is necessary to construct
models of external load under use of which the results of mathematical modeling will
coincide with the certain accuracy with results of experiment. Such algorithms of
construction of pair (mathematical description + model of external load) are not unique.

2. Statement of synthesis of external loads by identification method
Let's consider now opportunities of the first algorithm on an example of dynamic system \( \Sigma \)
with the concentrated parameters, the motion of which is described by ordinary
differential equations of n-order. It is suggested that the records of all external loads
\( f_1(t), f_2(t), \ldots, f_m(t) \) (except only one \( f_1(t) \)) and one state variable, for example \( x_1(t) \), are
obtained in experimental way during motion of system for some interval of time \( t \in [0, T] \).
It is necessary to find the model \( z(t) \) of external load \( f_1(t) \) after the action of which the
mathematical model of system \( \Sigma \) (MM\( \Sigma \)) moves in a such way that the state variable \( x_1(t) \)
coincides with experimental record \( \tilde{x}_1(t) \) of \( x_1(t) \). The rest of external loads coincide with
external loads \( f_2(t), f_3(t), \ldots, f_m(t) \) known from experiment. The problems of such a type
were named the problems of external loads identification (Gelfandbein & Kolosov, 1972),
(Ikeda et al., 1976).
The model \( z(t) \) which was obtained by such a method depends on chosen MM\( \Sigma \) and on
goals of the use at mathematical modeling in future.
If the initial dynamic system does not satisfy the condition as have been specified above then this system can be reduced to system \( \sum \) with the help of additional measurements (Menshikov, 2004).

Let us assume that the MM\( \sum \) is linear and that the connection between unknown function \( z(t) \) and functions \( f_1(t), f_2(t), \ldots, f_m(t), x_i(t) \) has the form:

\[
A_p z = B_p x,
\]

where \( A_p \) is linear integral operator \((A_p : Z \to U)\) which depends continuously on vector parameters \( p \) of mathematical model of system \((MM\sum)\), \( p = (p_1, p_2, \ldots, p_N)^\top, (\cdot)^\top \) is the sign of transposition, \( p \in R^N, R^N \) is the Euclidean vector space with norm \( \| p \|^2 = (p, p) \); \( B_p \) is linear bounded operator \((B_p : X \to U)\) which depends continuously on vector parameters \( p \); \( x = (x_1(t), f_2(t), \ldots, f_m(t))^\top; z \in Z, x \in X; Z, X, U \) are Gilbert spaces. The functions \( x_i(t), f_2(t), \ldots, f_m(t) \) are given with known inaccuracy \( \tilde{x} = (\tilde{x}_1(t), \tilde{f}_2(t), \ldots, \tilde{f}_m(t))^\top \) as these functions had been obtained from experimental measurements:

\[
\left\| x(t) - \tilde{x}(t) \right\|_X \leq \delta,
\]

where \( x(t) \) is the exact vector function of initial data, \( \delta \) – given value.

Besides it is supposed that the vector of parameters \( p \) given inexactly. So vector \( p \) can have values in some closed domain \( D : p \in D \subset R^N \). Two operators \( A_p, B_p \) correspond to each vector from \( D \). The set of possible operators \( A_p \) has been denoted as class of operators \( K_A \), the set of possible operators \( B_p \) has been denoted as class of operators \( K_B \). So we have \( A_p \in K_A, B_p \in K_B \). The maximal deviations of operators \( A_p \) from class \( K_A \) and operators \( B_p \) from class \( K_B \) are equal:

\[
\left\| A_p - A_p \right\|_{Z \to U} \leq h, \quad \left\| B_p - B_p \right\|_{X \to U} \leq d.
\]

Denote by \( Q_{\delta,p} \) the set of the possible solutions of equation (4) with account of experimental measurements inaccuracy only:

\[
Q_{\delta,p} = \{ z : z \in Z, A_p z \in U_{\delta,p}, p \in D \},
\]

where \( U_{\delta,p} = \{ u = B_p x : u \in U, x \in X_{\delta}, p \in D \} \), \( X_{\delta} = \{ x : x \in X, \left\| x - \tilde{x} \right\|_X \leq \delta \} \).

Any function \( z \) from set \( Q_{\delta,p} \) simulates of the motion of dynamic system \( MM\sum \) with the inaccuracy of experimental measurements only.

The operator \( A_p \) in equation (4) is a completely continuous operator for overwhelming majority of cases and so the set \( Q_{\delta,p} \) is unbounded set in space \( Z \) as a rule (ill-posed problem) (Tikhonov et al., 1990).
The regularization method for equations (4) was used for obtaining of stable solutions of denoted above problems (Tikhonov & Arsenin, 1979).

Let us consider the stabilizing functional $\Omega[z]$ which has been defined on set $Z_1$, where $Z_1$ is everywhere dense in $Z$ (Tikhonov & Arsenin, 1979). Consider now the extreme problem I:

$$\Omega[z_p] = \inf_{z \in \Omega_1 \cap Z_1} \Omega[z], \ p \in D. \quad (6)$$

It was shown that under certain conditions the solution of the extreme problem I exists as unique and stable to small change of initial data $\vec{x}_i(t), \delta$ (Tikhonov & Arsenin, 1979).

The function $z_p$ is named the stable model of EL after taking into account experimental measurements inaccuracy only. Such a model can be used for modeling of initial system motion with operators $A_p, B_p$ only.

3. Synthesis of external loads for class of mathematical descriptions

But according to the first approach it is necessary to take into account the inaccuracy of operators $A_p, B_p$. Let us now consider the problem of EL identification in this case.

The set of possible solutions of equation (4) $Q_{\delta,p}$ has to expand to the set $Q_{\delta,D}$ if we additionally take into account the inaccuracy of operators $A_p, B_p$:

$$Q_{\delta,h,d} = \{ z : z \in Z, A_p \in K_A, B_p \in K_B, \| A_p z - B_p x_0 \| \leq \delta b_0 + d \| x_0 \|_X + h \| z \|_Z \},$$

where $b_0 = \sup_{p \in D} \| B_p \|$.

Any function $z$ from set $Q_{\delta,h,d}$ simulates the motion of initial system with the inaccuracy of experimental measurements and inaccuracy of operators $A_p, B_p$.

The set $Q_{\delta,h,d}$ is unbounded for any $\delta > 0, h > 0, d > 0, p \in D \subset R^N$ (Tikhonov & Arsenin, 1979).

The regularization method for equations with inexact given operators was used for an obtaining stable solutions of denoted above problems (Tikhonov et al., 1990).

Consider now the extreme problem II:

$$\Omega[\vec{z}] = \inf_{z \in Q_{\delta,h,d} \cap Z_1} \Omega[z]. \quad (7)$$

It was shown that under certain conditions the solution of the extreme problem II exists as unique and stable to small change of initial data $\vec{x}_i(t), \delta, d, h, A_p, B_p$ (Tikhonov et al., 1990).

A problem of finding $\vec{z} \in Q_{\delta,h,d}$ was named as problem of synthesis of external load for a class of models (Menshikov, 2002, 2004).

Let's consider the union of sets of the possible solutions $Q_{\delta,p}$ with fixed operators $A_p B_p$: www.intechopen.com
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\[ Q^*_p = \bigcup_{p \in D} Q_{p, \delta} \quad (\cup \text{ is the sign of union}). \]  

(8)

In some cases as the solution \( z_{\min} \) of a problem of synthesis of external load for a class of models we shall accept the stable element of set \( Q^*_p \) instead the set \( Q_{b, h, d} \) (extreme problem III):

\[ \Omega[z_{\min}] = \inf_{z \in Q^*_p \cap Z_l} \Omega[z]. \]  

(9)

This problem can be reduced to more simple extreme problem:

\[ \Omega[z_{\min}] = \inf_{p \in D} \inf_{z \in Q^*_p} \Omega[z]. \]  

(10)

The model of EL \( z_{\min} \) will give the results of mathematical modeling which coincide with given function \( B_p \tilde{x} \) with inaccuracy \( \delta b_0 \).

The statement of following problem of model EL construction with the help of identification is possible (extreme problem IV):

\[ \Omega[z_{\max}] = \sup_{p \in D} \inf_{z \in Q^*_p} \Omega[z]. \]  

(11)

The model of EL \( z_{\max} \) will give results of mathematical modeling with inaccuracy \( \delta b_0 \).

The function \( z_{\max} \) gives the evaluation from above of all possible solutions of identification problem for all operators \( A_p, B_p \) from classes \( K_A, K_B \).

Then the stable model \( z_{\text{bel}} \) which gives the evaluation from below of the selected response \( B_p \tilde{x} \) of dynamic system for all possible operators \( A_p, B_p \) can be defined as result of the solution of the following extreme problem V:

\[ \left\| A_{b, a} z_{\text{bel}} \right\|_{U_j}^2 = \inf_{A_{b, a}, b \in K_B, p \in K_A} \inf_{z_p} \left\| A_{b, a} z_p \right\|_{U_j}^2, \quad b, p \in D, \]  

(12)

where \( z_p \) is the solution of extreme problem (6) on set \( Q_{\delta, p} \).

The stable model \( z_{\text{ab}} \) which gives the evaluation from above of the selected response \( B_p \tilde{x} \) of dynamic system for all possible operators \( A_p, B_p \) can be defined as result of the solution of the following extreme problem VI:

\[ \left\| A_{b, a} z_{\text{ab}} \right\|_{U_j}^2 = \sup_{A_{b, a}, b \in K_B, p \in K_A} \sup_{z_p} \left\| A_{b, a} z_p \right\|_{U_j}^2, \quad b, p \in D. \]  

(13)

In some cases it is necessary to synthesize model of external load by a method of identification which gives the best results of mathematical modeling for all possible mathematical descriptions of dynamic system motion. Actually such a problem is the solution of a problem of a choice of the second component (model of external load) for adequate mathematical modeling within of the first approach (Menshikov, 2008). Such kind of identification problems can find applications in different practical areas where the methods of mathematical modeling are used (Menshikov, 2004).
The stable model \( z_{un} \) of external load which gives the best result of motion of dynamic system with guarantee as the solution of the following extreme problem VI is:

\[
\| A_{P_{un}} z_{un} - \tilde{x} \|_U^2 = \inf_{p \in D \cap c_{D}} \sup_{c_{D}} \| A_z p - B c \tilde{x} \|_U^2, \quad p_{un} \in D,
\]

where \( z_p \) is the solution of extreme problem (6) on set \( Q_{\delta, p} \) (Menshikov & Nakonechny, 2005). Function \( z_{un} \in Q^*_{\delta} \) exists and is stable to small change of initial data (function \( \tilde{x} \)), if the functional \( \Omega[z] \) is stabilizing functional and the function \( z_o \) is defined uniquely from (14).

The solution of extreme problem VII was named as unitary mathematical model of external load. If the classes \( K_A, K_B \) consist of a limited number of operators \( K_A = \{ A_1, A_2, \ldots A_N \} = \{ A_i \}, K_B = \{ B_1, B_2, \ldots B_N \} = \{ B_i \}, i = 1, N \), then the algorithm of finding the best unitary model of external load \( z_{un} \) has the form

\[
\inf_{z \in Q_{\delta, p}} \sup_{p \in D} \| A_p z - B_p \tilde{x} \|_U = \| A_{P_{un}} z_{un} - B_{P_{un}} \tilde{x} \|_U = \min_{j} \max_{i} \| A_j z_j - B_j \tilde{x} \|_U,
\]

where

\[
Q_{D,\delta} = \{ z_j : \| A_j z_j - B_j \tilde{x} \|_U = \delta; j, i = 1, 2, \ldots, N \}.
\]

The offered formulations of a problem of identification of external load can not be classified by one name as identification of external load. The additional explanations and additional assumptions are required in the solution of each particular problem. Most likely this set of problems can be united only by principle of synthesis of models - principle of use of experimental data. In other words it is possible to name all these problems as problems in which the principle of identification (comparison of results of calculations with experimental data) is used. The similar situation is present also in a rather conservative area such as identification of parameters (Menshikov, 2006).

4. Practical problems of identification of external load models

4.1 Synthesis of moment of technological resistance on rolling mill

One of the important characteristics of rolling process is the moment of technological resistance (MTR) arising at the result of plastic deformation of metal in the center of deformation. Size and character of change of this moment define loadings on the main mechanical line of the rolling mill. However the complexity of processes in the center of deformation does not allow construct the authentic mathematical model of MTR by usual methods. In most cases at research of dynamics of the main mechanical lines of rolling mills MTR is being created on basis of hypothesis and it is imitated as piecewise smooth linear function of time or corner of turn of the working barrels (Menshikov, 1985, 1994). The results of mathematical modeling of dynamics of the main mechanical lines of rolling mills with such model MTR are different among themselves (Menshikov, 1976).

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In work the problem of construction of models of technological resistance on the rolling mill is considered on the basis of experimental measurements of the responses of the main mechanical system of the rolling mill under real E1 (Menshikov, 1985, 1994). Such an approach allows to carry out in a future mathematical modeling of dynamics of the main mechanical lines of rolling mills with a high degree of reliability and on this basis to develop optimum technological modes.

The kinematics scheme of the main mechanical line of the rolling mill was presented on Fig.1. (a).

\[ \ddot{M}_{12} + \omega_{12}^2 M_{12} - \frac{c_{12}}{\varpi_2} M_{23} - \frac{c_{12}}{\varpi_2} M_{24} = \frac{c_{12}}{\varpi_1} M_{\text{eng}}(t); \]

\[ \ddot{M}_{23} + \omega_{23}^2 M_{23} - \frac{c_{23}}{\varpi_2} M_{12} + \frac{c_{23}}{\varpi_2} M_{24} = \frac{c_{23}}{\varpi_3} M_{\text{Urol}}(t); \]  \hspace{1cm} (16)

\[ \ddot{M}_{24} + \omega_{24}^2 M_{24} - \frac{c_{24}}{\varpi_2} M_{12} + \frac{c_{24}}{\varpi_4} M_{23} = \frac{c_{24}}{\varpi_4} M_{\text{Lrol}}(t); \]

where \( \omega_{ik}^2 = \frac{c_{ik}}{\varpi_i \varpi_k} \), \( \varpi_k \) are the moments of inertia of the concentrated weights, \( c_{ik} \) are the rigidity of the appropriate elastic connection, \( M_{\text{Urol}} \), \( M_{\text{Lrol}} \) are the moments of
technological resistance applied to the upper and lower worker barrel, respectively, $M_{\text{eng}}(t)$ is the moment of the engine.

The problem of synthesis of model of EL can be formulated so: it is necessary to define such models of technological resistance on the part of metal which would cause in elastic connections of model fluctuations identical experimental (in points of measurements) taking into account of an error of measurements for chosen MM of the main mechanical line of rolling mill. The information on the real motion of the main mechanical line of rolling mill is received by an experimental way (Menshikov, 1976, 1976a). Such information is understood as presence of functions $M_{12}(t), M_{23}(t), M_{24}(t)$.

The most typical case of rolling on a smooth working barrels was chosen for processing when the frustration of fluctuations are not observed and when a skid is absent (Menshikov, 1976, 1976a). The records of functions $M_{12}(t), M_{23}(t), M_{24}(t)$ by rolling process are shown on Fig.2.

Let’s consider a problem of construction of models of EL to the upper working barrel. On the lower working barrel all calculations will be carried out similarly. From system (16) the equation concerning required model $M_{\text{rol}}^{U}$ can be received

$$
\int_{0}^{t} \sin \omega_{23}(t-\tau) M_{\text{rol}}^{U}(\tau) d\tau = u_\delta(t) \text{ or } A_p z = u_\delta ,
$$

where $z = M_{\text{rol}}^{U}(\tau)$, $A_p$ is a linear integral operator.

The maximal deviation of the operators $A_p \in K_A$ from one another is defined by an error of parameters of mathematical model of the rolling mill. The error of definition of values of discrete weights is accepted as 8 %, the error of stiffness values - 5 %, error of values of damping factors - 30 %. The size of the maximal deviation of the operators $A_p \in K_A$ was defined by numerical methods and it is $h = 0.121$, the size of the maximal deviation of the operators $B_p \in K_B$ was defined by numerical methods and it is $d = 0.11$. An error initial data for a case $Z = U = C[0,T]$ is $\delta = 0.0665$ MhM.

Fig. 2. The records of functions $M_{12}(t), M_{23}(t), M_{24}(t)$.

Fig. 3. The diagrams of extreme problem I solutions $M_{\text{rol}}^{U}(\tau), M_{\text{rol}}^{L}(\tau)$.
We shall choose functional
\[ \Omega[z] = \int_0^T (z^2 + z'^2) dt , \]
(18)
as the stabilizing functional (Tikhonov & Arsenin, 1979).
At first the problem of identification of the stable models of external load to working barrels was calculated with account of inaccuracy of experimental data only. The results of calculation are presented on Fig.3.

4.2 Unitary model of external load for rolling mill
It is evident that the results of identification will change under other parameters of mathematical descriptions. So the problem of external load identification was solved as extreme problem VI (unitary model of external load).
In Fig.4 the diagram of function \( z_{un} \) for a typical case of rolling on an upper worker barrel is submitted.

![Diagram of change of models of the moment of technological resistance on the upper worker barrel of rolling mill.](image)

Fig. 4. The diagrams of change of models of the moment of technological resistance on the upper worker barrel of rolling mill.

The solution of extreme problem III is presented on Fig. 4, with the same conditions of initial data and inaccuracy of operators \( A_p, B_p \).
The results of calculations show that the evaluation from above of accuracy of mathematical modeling with model \( z_{un} \) for all \( A_p \in K_A \) and for all \( B_p \in K_B \) does not exceed 11% in the uniform metrics with error of MM parameters of the main mechanical line of rolling mill in average 10 % and errors of experimental measurements 7 % in the uniform metrics.
The calculations of model of EI \( z \) for classes \( K_A, K_B \) on set of the possible solutions \( Q_{ah,d} \) was executed for comparison. The function which is the solution of a problem of synthesis in this case has the maximal deviation from zero equal 0.01 МММ. Such a model does not present interest for the purposes of mathematical modeling as it practically coincides with trivial a model.
In work (Menshikov, 1994), the comparative analysis of mathematical modeling with various known models of external load was executed. The model of external load \( z_{un} \) corresponds to experimental observations in the greater degree.

5. Conclusion
In paper some problems of construction of external load models for dynamic systems with this case has the maximal deviation from zero equal 0.01 МММ. Such a model does not present interest for the purposes of mathematical modeling as it practically coincides with trivial a model.
In work (Menshikov, 1994), the comparative analysis of mathematical modeling with various formulations of such a problem is offered: the stable model for obtaining the best results of of mathematical modeling with guarantee, the stable model for obtaining evaluation of response from above, stable model for obtaining evaluation of response from below, stable model for mathematical modeling of the selected motion with the fixed model of dynamic system, the stable model for mathematical modeling of the selected motion of system for whole class of mathematical descriptions of system.

The offered approach to synthesis of models of external loads on dynamical system can find application in cases when the information about external impacts is absent or scarce and also for check of hypotheses on the basis of which were constructed the known models of external loads.

6. References


This book was conceived as a gathering place of new ideas from academia, industry, research and practice in the fields of robotics, automation and control. The aim of the book was to point out interactions among various fields of interests in spite of diversity and narrow specializations which prevail in the current research. The common denominator of all included chapters appears to be a synergy of various specializations. This synergy yields deeper understanding of the treated problems. Each new approach applied to a particular problem can enrich and inspire improvements of already established approaches to the problem.

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