Interception and Rendezvous Between Autonomous Vehicles

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1. Introduction

The problem of pursuit and evasion is a classic problem that has intrigued mathematicians for many generations. Suppose a target or evader is moving along a given curve in the plane. A pursuer or chaser is moving such that its line of sight is always pointing towards the target. The classic problem is to determine the trajectory of the pursuer such that it eventually captures the target. There is extensive literature on this problem, see for example the recent book by Nahin (Nahin, 2007). A special case of the pursuit problem is the case where the curve of the target is a circle. This is a classic problem which was first treated by Hathaway (Hathaway, 1921), see also the book by Davis (Davis, 1962). In the classic problems, the line of sight is always pointing towards the target. This method of pursuit is also known as 'pure pursuit' or 'dog pursuit' or 'courbe de chien' in French. Although there have been some modern works on extensions of the classic problem of pursuit, see for example (Marshall, 2005), here we completely abandon the 'pure pursuit' restriction and we treat the problem where the pursuer is required to capture or intercept the target in a given prescribed time or in minimum time. Also, in the classic problem, the velocities of the target and pursuer are assumed constant and there is no consideration of the physics of the motion, such as thrust forces, hydrodynamic or aerodynamic drag and other forces that might be acting on the target and pursuer.

In an active-passive rendezvous problem between two vehicles, the passive or target vehicle moves passively along its trajectory. The active or chaser vehicle is controlled or guided such as to meet the passive vehicle at a later time, matching both the location and the velocity of the target vehicle. An interception problem is similar to the rendezvous problem, except that there is no need to match the final velocities of the two vehicles. On the other hand, in a cooperative rendezvous problem, the two vehicles are active and maneuver such as to meet at a later time, at the same location with the same velocity. The two vehicles start the motion from different initial locations and might have different initial velocities.

An optimal control problem consists of finding the control histories, i.e., the controls as a function of time, and the state variables of the dynamical system such as to minimize a performance index. The differential equations of motion of the vehicles are treated as dynamical constraints. One possible approach to the solution of the rendezvous problem is to formulate it as an optimal control problem in which one is seeking the controls such as to

minimize the differences between the final locations and final velocities of the vehicles in some mathematical sense, for example in the least squares sense. Here we use a minimax formulation in order to define the objective function that contains the terminal constraints.

The methods of approach for solving optimal control problems include the classical indirect methods and the more recent direct methods. The indirect methods are based on the calculus of variations and its extension to the maximum principle of Pontryagin, which is based on a Hamiltonian formulation. These methods use necessary first order conditions for an optimum, they introduce adjoint variables and require the solution of a two-point boundary value problem (TPBVP) for the state and adjoint variables. Usually, the state variables are subjected to initial conditions and the adjoint variables to terminal or final conditions. **TPBVPs** are much more difficult to solve than initial value problems (IVP). For this reason, direct methods of solution have been developed which avoid completely the Hamiltonian formulation. For example, a possible approach is to reformulate the optimal control problem as a nonlinear programming (NLP) problem by direct transcription of the dynamical equations at prescribed discrete points or collocation points.

Direct Collocation Nonlinear Programming (DCNLP) is a numerical method that has been used to solve optimal control problems. This method uses a transcription of the continuous equations of motion into a finite number of nonlinear equality constraints, which are satisfied at fixed collocation points. This method was originally developed by Dickmanns and Well (Dickmanns, 1975) and used by Hargraves and Paris (Hargraves, 1987) to solve several atmospheric trajectory optimization problems. Another class of direct methods is based on biologically inspired methods of optimization. These include evolutionary methods such as genetic algorithms (Goldberg, 1989), particle swarm optimization methods and ant colony optimization algorithms.

Genetic algorithms (GAs) provide a powerful alternative method for solving optimal control problems. They have been used to solve control problems (Crispin, 2006, 2007), orbital transfer and rendezvous problems (Crispin and Ricour, 2007). GAs use a stochastic search method and are robust when compared to gradient methods. They are based on a directed random search which can explore a large region of the design space without conducting an exhaustive search. This increases the probability of finding a global optimum solution to the problem. They can handle continuous or discontinuous variables since they use binary coding. They require only values of the objective function but no values of the derivatives. However, GAs do not guarantee convergence to the global optimum. If the algorithm converges too fast, the probability of exploring some regions of the design space will decrease. Methods have been developed for preventing the algorithm from converging to a local optimum. These include fitness scaling, increased probability of mutation, redefinition of the fitness function and other methods that can help maintain the diversity of the population during the genetic search.

2. Interception and rendezvous as optimal control problems

We study interception and rendezvous problems for vehicles moving in an incompressible viscous fluid such as water. The vehicle has a propulsion system that delivers a thrust of constant magnitude T and is controlled by varying the thrust direction, i.e., thrust vectoring. Since the fluid is viscous, a drag force acts on the vehicle, in the opposite direction of the velocity. The motion takes place in a horizontal plane, the (x, z) plane, either at the surface of

the water or at a constant depth. The target vehicle is moving along a circle of radius R. We describe the motion in a cartesian frame of reference (x, z) with its origin at the center of the circle, with x positive to the right and z positive upwards.

Let the angle β denote the orientation of the thrust vector T, which is also the angle of the velocity vector V. The angle $\beta(t)$, which depends on the time t, is measured positive counterclockwise from a reference horizontal line, the positive direction of the x axis. In the examples we are going to present, the motion of the chaser will start from a point located outside the circle and will be moving from right to left, see Figure 2. In this case, it is convenient to use the complementary angle $\gamma(t) = \pi - \beta(t)$ as the control.

The interception problem is an optimal control problem, in which it is required to determine the control function or control history $\gamma(t)$ of the chaser vehicle, such that it will meet the target vehicle at a prescribed location at the terminal time t_f. Since GAs deal with discrete variables, we discretize the values of $\gamma(t)$. The motion of the vehicle is governed by Newton's second law of motion and the kinematic relations between velocity and distance:

$$m dV/dt = T + D \tag{1}$$

$$dx/dt = V \cos\beta(t) \tag{2}$$

$$dz/dt = V \sin\beta(t) \tag{3}$$

where $\beta(t) = \pi - \gamma(t)$ is the angle of velocity vector V, T is the thrust and D is the drag force acting on the body.

$$dV/dt = T/m + D/m \tag{4}$$

Writing this equation for the components of the forces along the tangent to the vehicle's path, we get:

$$dV/dt = T/m - D/m \tag{5}$$

Here V, T and D are the magnitudes of the velocity, thrust and drag vectors, respectively. We introduce the drag coefficient C_D:

$$D = \tfrac{1}{2}\rho V^2 S C_D \tag{6}$$

where ρ is the fluid density, S is a typical cross-section area of the vehicle. The coefficient of drag depends on the Reynolds number $Re = \rho Vd/\mu$ where d is a typical length dimension of the vehicle, e.g., $d = \sqrt{S}$.

Substituting the drag from (6) and writing $T = amg$, where a is the thrust to weight ratio T/mg, equation (5) becomes:

$$dV/dt = ag - \rho V^2 S C_D / 2m \tag{7}$$

Introducing a characteristic length L_c, time t_c and speed v_c as

$$L_c = 2m/\rho S C_D, \quad t_c = \sqrt{L_c/g}, \quad v_c = \sqrt{gL_c} \tag{8}$$

the following non-dimensional variables, denoted by a bar, can be defined:

$$x = L_c\bar{x}, \quad z = L_c\bar{z},$$

$$t = (L_c/g)^{1/2}\bar{t}, \quad V = (gL_c)^{1/2}\bar{V} \tag{9}$$

Substituting in (7), we have:

$$d\bar{V}/d\bar{t} = a - \bar{V}^2 \tag{10}$$

Similarly, the other equations of motion can be written in non-dimensional form as

$$d\bar{x}/d\bar{t} = -\bar{V}\cos\gamma(t) \tag{11}$$

$$d\bar{z}/d\bar{t} = \bar{V}\sin\gamma(t) \tag{12}$$

For each vehicle the initial conditions are:

$$V(0) = V_0, \quad x(0) = x_0, \quad z(0) = z_0 \tag{13}$$

We now define a rendezvous problem between two vehicles. We denote the variables of the first vehicle by a subscript 1 and those of the second vehicle by a subscript 2. We will now drop the bar notation indicating non-dimensional variables. The two vehicles might have different thrust to weight ratios, which we denote by a_1 and a_2, respectively. The equations of motion for the system of two vehicles are:

$$
\begin{aligned}
dV_1/dt &= a_1 - V_1^2 \\
dx_1/dt &= -V_1\cos\gamma_1(t) \\
dz_1/dt &= V_1\sin\gamma_1(t) \\
dV_2/dt &= a_2 - V_2^2 \\
dx_2/dt &= -V_2\cos\gamma_2(t) \\
dz_2/dt &= V_2\sin\gamma_2(t)
\end{aligned}
\tag{14}
$$

We consider the case where the second vehicle, which is the target vehicle, is constrained to move along a circular trajectory. In this case, the magnitude of the velocity vector V_2 is

constant. The angular speed ω, the azimuth angle $\theta(t)$ and the required control angle $\gamma_2(t)$ are given by:

$$\omega = V_2/R, \qquad \theta(t) = \omega t, \qquad \gamma_2(t) = \pi/2 - \theta(t) \qquad (15)$$

where R is the radius of the circle. $\theta(t)$ is measured at the center of the circle, positive counter-clockwise, from the positive x direction. The vehicles can start the motion from different locations and at different speeds. The initial conditions are given by:

$$V_1(0) = V_{10}, \quad x_1(0) = x_{10}, \quad z_1(0) = z_{10}$$

$$V_2(0) = V_{20}, \quad x_2(0) = x_{20}, \quad z_2(0) = z_{20} \qquad (16)$$

The rendezvous problem consists of finding the control function $\gamma_1(t)$ such that the two vehicles arrive at a same terminal location on the circle and at the same speed in the given time. The terminal constraints are given by:

$$x_1(t_f) = x_2(t_f), \quad z_1(t_f) = z_2(t_f), \quad V_1(t_f) = V_2(t_f) \qquad (17)$$

In order to fulfill the terminal constraints using a genetic algorithm, we define the following objective function:

$$f[x_1(t_f), x_2(t_f), z_1(t_f), z_2(t_f), V_1(t_f), V_2(t_f)] =$$

$$= \min \max \{(x_1(t_f) - x_2(t_f))^2, (z_1(t_f) - z_2(t_f))^2, (V_{1x}(t_f) - V_{2x}(t_f))^2,$$

$$(V_{1z}(t_f) - V_{2z}(t_f))^2\} \qquad (18)$$

where $V_{1x}(t_f)$, $V_{2x}(t_f)$, $V_{1z}(t_f)$ and $V_{2z}(t_f)$ are the x and z components of the velocity vectors of the two vehicles at the terminal time $t = t_f$. We can also define an interception problem, of the target-chaser type, in which the target vehicle is passive and the chaser vehicle maneuvers such as to match the location of the target vehicle, but not its velocity. Consistent with the above terminal constraints, we define the following objective function for the interception problem:

$$f[x_1(t_f), x_2(t_f), z_1(t_f), z_2(t_f)] = \min \max\{(x_1(t_f) - x_2(t_f))^2, (z_1(t_f) - z_2(t_f))^2\} \qquad (19)$$

We have also tried an objective function defined by the distance between the two terminal points of the two vehicles, but it was found that the mini-max objective function works better in satisfying the terminal constraints. We use standard numerical methods for integrating the differential equations. The time interval is divided into N time steps of duration $\Delta t = t_f/N$. The discrete time is $t_i = i\Delta t$. We used a second-order Runge-Kutta method with fixed time step. We also tried a fourth-order Runge-Kutta method and a variable time step and found that the results were not sensitive to the method of integration.

The control function $\gamma(t)$ is discretized to $\gamma(i) = \gamma(t_i)$ according to the number of time steps N used for the numerical integration. Depending on the accuracy of the desired solution, we can choose the number of bits n^i for encoding the value of the control $\gamma(i)$ at each time step i. The number of bits used for encoding $\gamma(i)$ and the number of time steps N will have an influence on the computational time. Therefore n^i and N must be chosen carefully in order to obtain an accurate enough solution in a reasonable time. We use smoothing of the control function by fitting a third-order polynomial to the discrete values of $\gamma(i)$. The values of the polynomial at the N discrete time points are used as the current values of $\gamma(i)$ and are used in the integration of the differential equations.

An appropriate range for $\gamma(i)$ is $\gamma \in [-\pi, \pi]$. We choose $N=30$ or 40 as a reasonable number of time steps. We now need to choose the parameters associated with the Genetic Algorithm. First, we select the lengths of the "genes" for encoding the discrete values of $\gamma(i)$. A choice of $n^i = 8$ bits for $\forall i \in [0, N-1]$ was made. A reasonable size for the population of solutions is typically in the range $n_{\text{pop}} \in [50, 200]$. For this problem, there is no need for a particularly large population, so we select $n_{\text{pop}} = 50$. The probability of mutation is set to a value of 5 percent $p_{\text{mut}} = 0.05.$

3. Chaser-target interception

We present an example of a chaser-target interception problem between two vehicles. The motion takes place in the horizontal plane *(x, z)*. The first vehicle is active (vehicle 1) and the second vehicle is the passive or target vehicle (vehicle 2) and moves at a constant speed V_2 along a circular trajectory. The target vehicle starts from a point on the circle and keeps moving along its circular trajectory. The chaser vehicle starts at a point outside the circle and the interception occurs at a point on the circle at the final time. The interception point is not known a priori. The parameters and initial conditions for this example are given below.

$$t_0 = 0, \quad t_f = 35, \quad a_1 = 4a/9, \quad a_2 = a = 0.05, \quad \gamma_1 \in [-\pi, \pi] \tag{20}$$

The initial conditions are:

$$x_1(0) = 4, \quad z_1(0) = 0, \quad x_2(0) = 1, \quad z_2(0) = 0$$

$$V_1(0) = \sqrt{a_1} = 0.2236, \quad V_2(0) = \sqrt{a_2} = 0.1491 \tag{21}$$

Since this is an interception problem, we do not require matching between the final velocities. The objective function is defined by equation (19). The parameters of the genetic algorithm are summarized in Table 1.

N_v	n_{pop}	n^i	N	p_{mut}	N_{gen}		$\gamma_{1\text{min}}$	$\gamma_{1\text{max}}$
2	50	8	40	0.05	100		$-\pi$	π
a	t_0	t_f	(x_{01}, z_{01})	(x_{02}, z_{02})	(V_{01}, V_{02})		x_f	z_f
0.05	0	35	(4,0)	(1,0)	$(\sqrt{4a/9}, \sqrt{a})$		free	free

Table 1. Parameters for the interception problem with $t_f = 35$

The results of this example are displayed in Figures (1-4). Figure 1 displays the control function of the chaser vehicle as a function of time for a terminal time of 35 units.
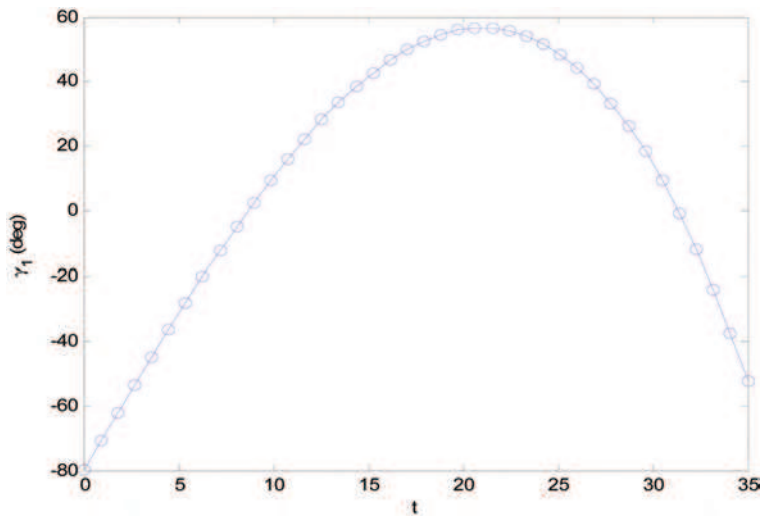

Fig. 1. Control function for the chaser vehicle in a chaser-target interception on a circle with a terminal time of 35 units.
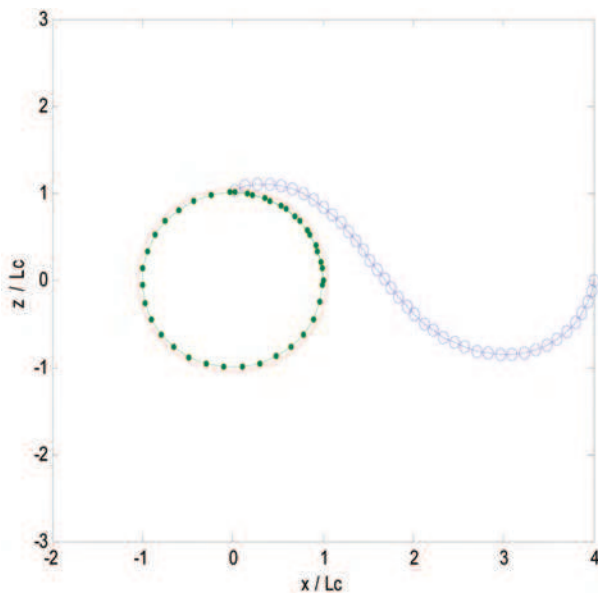

Fig. 2. Trajectories for a chaser-target interception on a circle with a terminal time of 35 units.
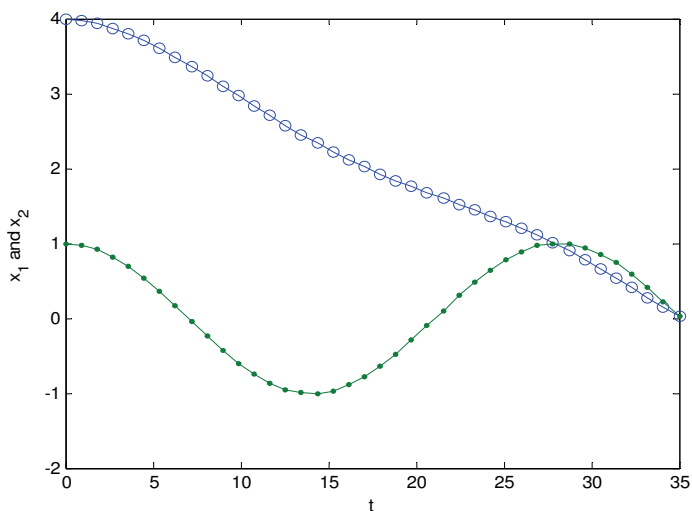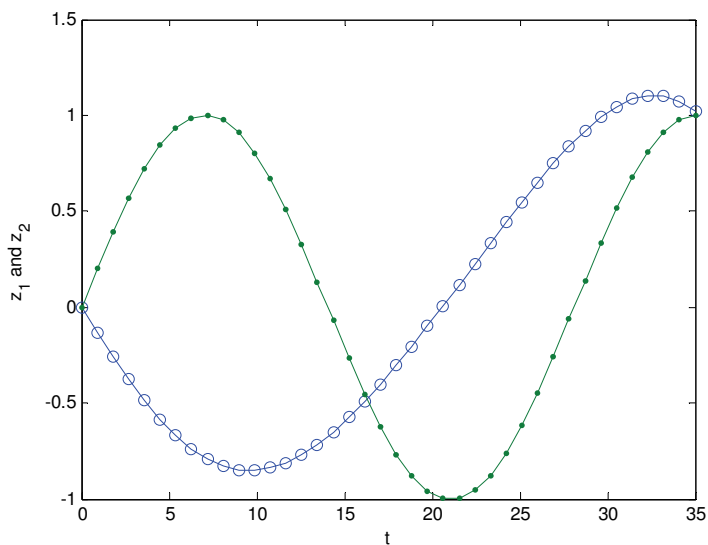
Fig. 3. Trajectories *x(t)* for a chaser-target interception on a circle with a terminal time of 35 units.

Fig. 4. Trajectories *z(t)* for a chaser-target interception on a circle with a terminal time of 35 units.

The trajectories of the target and chaser vehicles are shown in Figure 2. The target starts the motion at the point (1,0) on the circle and the chaser starts at the point (4,0) outside the circle. The line with dots is the trajectory of the target. The line with circles is the trajectory

of the chaser. The continuous curve without dots marks the circle. Figures 3 and 4 show the trajectories of the target and the chaser vehicles in parametric form, with the time as a parameter. Figure 3 displays the $x(t)$ coordinates of the two vehicles as a function of time and Figure 4 displays the $z(t)$ coordinates for the two vehicles. The lines with dots are the target trajectories and the lines with circles are the chaser trajectories. It can be seen that the final boundary conditions are fulfilled, that is, the coordinates of the two vehicles are equal at the final time.

In the above example, a non-dimensional terminal time of 35 units was used. We now look at the problem of finding the minimum time for interception for the same set of parameters and initial conditions as in the previous example. We decreased the final time to 30 units and then to 25 units and were able to obtain a solution, i.e., the terminal constraints are fulfilled for the shorter times. We then tried to decrease the final time to a value of 23 units but we could not obtain a solution that fulfills the terminal constraints. We then tried a slightly higher value of 24 units and were able to obtain a solution. We therefore conclude that the minimum time for interception in this case is very close to 24 units . Since we have shorter times, we decreased the number of discrete time points from $N=40$ to $N=30$. The results for this example are given in Figures (5-8). Figure 5 shows the control function of the chaser vehicle. Comparing with Figure 1, we can see that the control looks much different because of the shorter final time. Figure 6 shows that the interception occurs as a head-on collision, a phenomenon that occurs also in the classic problem of pursuit. From Figure 7, it can be seen that the horizontal distance between the chaser and the interception point is reduced linearly at maximum speed, because of the minimum final time. In this case, it can also be seen from Figures 7 and 8, that the final boundary conditions are fulfilled.
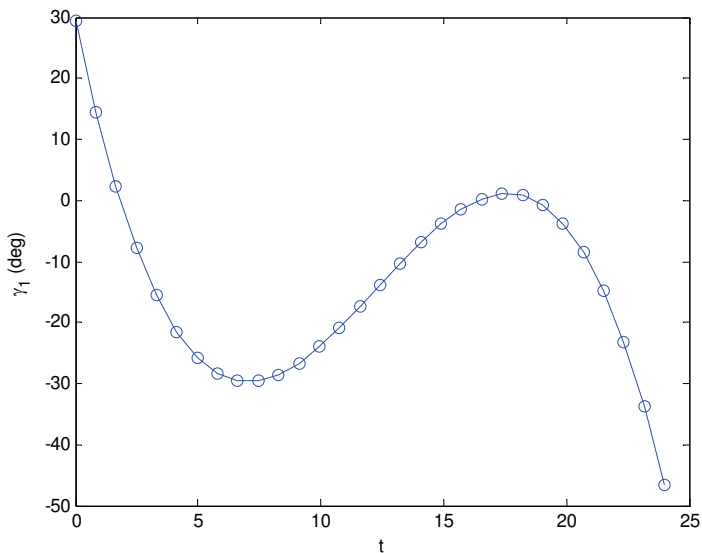

Fig. 5. Control function for the chaser vehicle in a chaser-target interception on a circle with a minimum terminal time of 24 units.
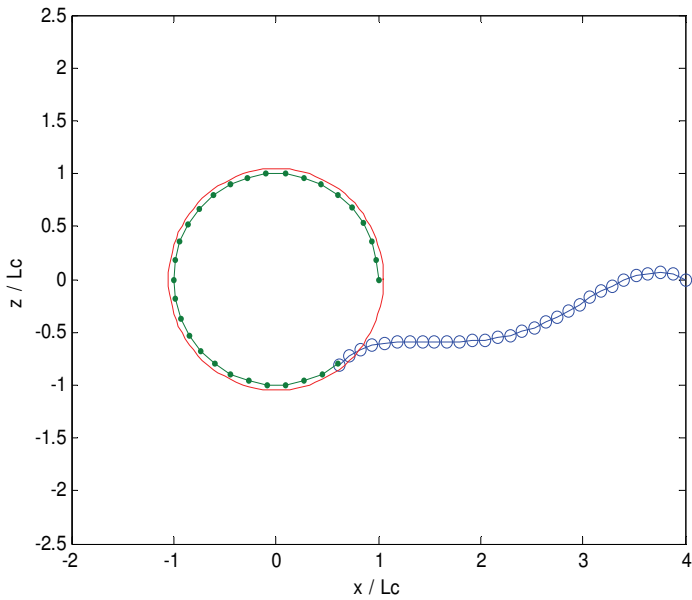
Fig. 6. Trajectories for a chaser-target interception on a circle with a minimum terminal time of 24 units.
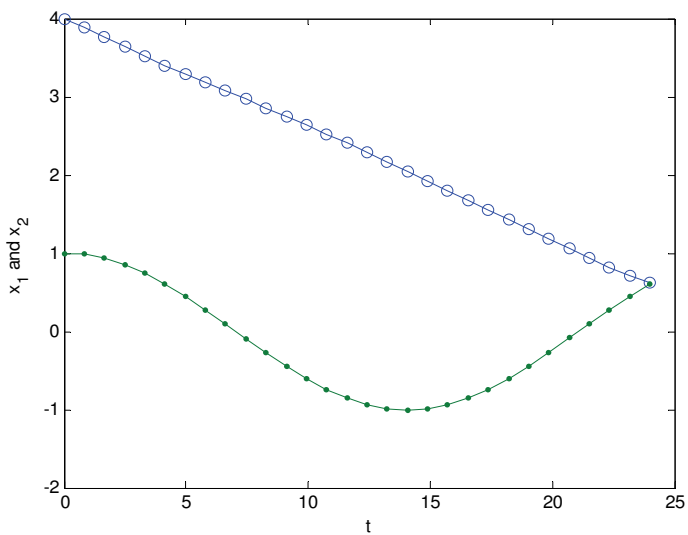

Fig. 7. Trajectories $x(t)$ for a chaser-target interception on a circle with a minimum terminal time of 24 units.
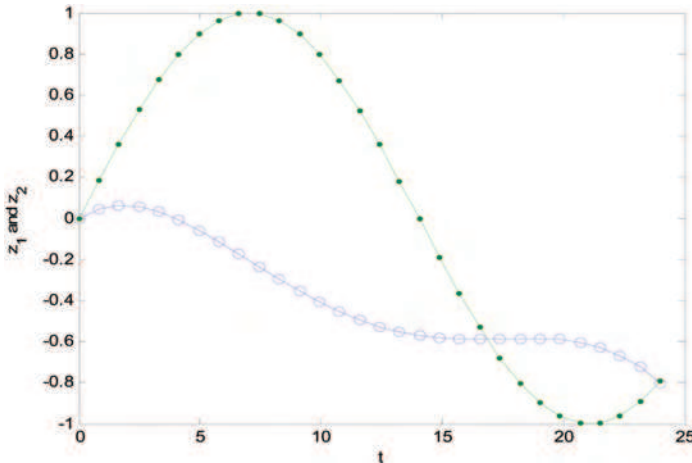
Fig. 8. Trajectories $z(t)$ for a chaser-target interception on a circle with a minimum terminal time of 24 units.

4. Rendezvous between two vehicles

We treat a rendezvous problem between two vehicles where one vehicle is moving along a circular trajectory. The first vehicle starts from a point outside the circle. The second vehicle starts at a point on the circle. The final time is given and the rendezvous point can occur at any point on the circle. The vehicles have the same thrust to weight ratio a. We present results with a final time of 25 units, which is close to the minimum time.

$$t_0 = 0, \ t_f = 25, \ a_1 = a = 0.05, \ a_2 = a = 0.05, \ \gamma_1 \in [-\pi, \pi] \tag{22}$$

The initial conditions are:

$$x_1(0) = 4, \ z_1(0) = 0, \ x_2(0) = 1, \ z_2(0) = 0, \ V_1(0) = \sqrt{a_1}, \ V_2(0) = \sqrt{a_2} \tag{23}$$

The parameters for this test case are summarized in Table 2. The results are given in Figure 9. The curves with dots show the trajectories of the target and the curves with circles show the trajectories of the chaser vehicle. The upper left part of the figure shows the trajectories in the plane (x, z). The chaser vehicle is maneuvering such as to approach the rendezvous point from behind, in order to match the velocity vector of the target vehicle. A similar effect can be seen in Figure 10, which displays results similar to Figure 9, but with the chaser vehicle starting from rest. It is interesting to note that in this case, the z coordinate of the chaser, follows the z coordinate of the target, as can be seen in the bottom right of Figure 10.

N_v	n_{pop}	n^i	N	p_{mut}	N_{gen}	γ_{min}	γ_{max}
2	50	8	30	0.05	100	$-\pi$	π
a	t_0	t_f	(x_{01}, z_{01})	(x_{02}, z_{02})	(V_{01}, V_{02})	x_f	z_f
0.05	0	25	$(4, 0)$	$(1, 0)$	$(\sqrt{a_1}, \sqrt{a_2})$	free	free

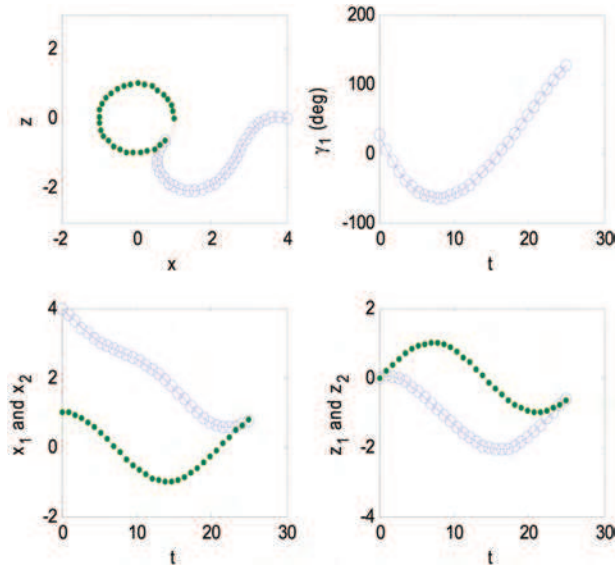Table 2. Parameters for the rendezvous problem

Fig. 9. Rendezvous between two vehicles on the circle when the outer vehicle starts at an initial speed. Top left: Trajectories of the two vehicles. Top right: Control thrust angle of the vehicle starting outside the circle. Bottom left: Horizontal coordinates as a function of time. Bottom right: vertical coordinates as a function of time.
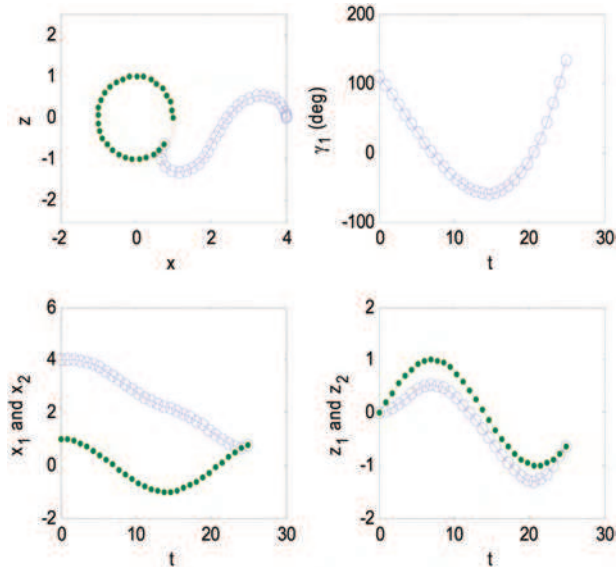

Fig. 10. Rendezvous between two vehicles on the circle when the outer vehicle starts from rest. Top left: Trajectories of the two vehicles. Top right: Control thrust angle of the vehicle starting outside the circle. Bottom left: Horizontal coordinates as a function of time. Bottom right: vertical coordinates as a function of time.

5. Conclusion

The interception and rendezvous problems between two autonomous vehicles moving in an underwater environment has been treated using an optimal control formulation with terminal constraints. The vehicles have a constant thrust propulsion system and use the direction of the thrust vector for steering and control. We use a genetic algorithm to determine directly the control history of the vehicle by evolving populations of possible solutions of initial value problems. In order to fulfill the final boundary conditions as terminal constraints, a mini-max objective function has been defined. An interception problem, where one vehicle moves along a circular trajectory at constant speed and the second vehicle acts as a chaser, maneuvering such as to capture the target in a prescribed time has been solved. The problem of minimum time to interception has also been treated. The rendezvous problem where the target vehicle moves along a circle and the chaser vehicle starts from a point outside the circle either from rest or with an initial velocity has also been solved for a terminal time close to the minimum time for rendezvous. This method can be extended to include multiple autonomous vehicles. Another direction for future research is to include additional realistic effects besides thrust and drag. For example, additional effects can be included to better describe the dynamics of the vehicles, such as finite size, rigid body dynamics, inertia and added mass effects.

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Advances in Robotics, Automation and Control

Edited by Jesus Aramburo and Antonio Ramirez Trevino

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