Dynamic Modelling of a CPG-Controlled Amphibious Biomimetic Swimming Robot

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Abstract This paper focuses on the modelling and control problems of a self-propelled, multimodal amphibious robot. Inspired by the undulatory body motions of fish and dolphins, the amphibious robot propels itself underwater by oscillations of several modular fish-like propelling units coupled with a pair of pectoral fins capable of non-continuous 360 degree rotation. In order to mimic fish-like undulating propulsion, a control architecture based on Central Pattern Generator (CPG) is applied to the amphibious robot for robust swimming gaits, including forward and backward swimming and turning, etc. With the simplification of the robot as a multi-link serial mechanism, a Lagrangian function is employed to establish the hydrodynamic model for steady swimming. The CPG motion control law is then imported into the Lagrangian-based dynamic model, where an associated system of kinematics and dynamics is formed to solve real-time movements and, further, to guide the exploration of the CPG parameters and steady locomotion gaits. Finally, comparative results between the simulations and experiments are provided to show the effectiveness of the built control models.

Keywords Biomimetics, Dynamic Modeling, Amphibious Swimming Robot

1. Introduction

The research into land-water propulsors (i.e., amphibious robots) tends to attract engineers due to their fascinating locomotion agility and high performance [1–6]. Their potential applications involve near-shore observatories, search-and-rescue missions and amphibious reconnaissance, etc. In view of engineering implementations, amphibious robots typically adopt wheel-driven [7], legged-type [8] or wheel-leg hybrid mechanisms [9] for locomotion on land. Others may mimic snakes to crawl on land [2,10,11]. The legged-type and snake-like mobile mechanisms are complex, with maintenance problems and partially unexplored sophisticated approaches. The wheel-driven mechanism is relatively more mature and simpler to design and control, which is broadly suitable for uncritical environments. For underwater locomotion, amphibious robots generally utilize propeller impetus [4,9], snake-like anguilliform swimming [2,10,11] or fish-like fin actuators [1,5,12]. The former two offer slower locomotion speeds and less efficient propulsion. As a fast, efficient and high-mobility underwater propulsor, fish-like swimming takes advantage of conventional rotary propeller-based propulsion, which achieves impressive propulsion efficiency by coordinated oscillations of the body and/or
fins. As a distinctive feature, a dolphin propels itself by dorsoventral oscillations as distinct from the oscillations of fish tails, which are confined to the horizontal plane. Remarkably, dolphins display superior performance in terms drag reduction, swimming hydrodynamics and manoeuvrability.

![Figure 1. Mechanism design of AmphiRobot-II. (a) Concept design. (b) Fish-like swimming mode. (c) Dolphin-like swimming mode.](image)

In synthesizing the motion characteristics of both ground mobile robots and underwater robots, this paper mainly presents a novel amphibious robot called “AmphiRobot-II” with a pair of bio-inspired wheel-propeller-fins. It is an improved version of the AmphiRobot-I developed in [12]. As shown in Figure 1, mechanically the robot is composed of a head with wheel-propeller-fins, three fish-like propelling units and a flapping caudal fin. When moving on land, the robot implements wheel-based movements, whereby the wheel-propeller-fins function as wheels; when swimming in water, the robot performs fish-like (lateral oscillations) or dolphin-like (dorsoventral oscillations) propulsion, where the wheel-propeller-fins function as pectoral fins. Specifically, the robot takes fish-like swimming as the primary underwater propulsion by utilizing the travelling waves of body undulation transmitted from head to tail. That is, several fish-like oscillating hinge joints (J1–J4) comprise the drive mechanism (yaw control) while coordinated with the pectoral fins (pitch control). A specially-designed swivelling mechanism is further designed to rotate the posterior body by ±90° (roll control) to switch gaits between fish- and dolphin-like modes.

To generate a fish body wave, Lighthill proposed a carangiform propulsive wave equation in the 1960s [13]. In 1996, Barrett et al. developed an experimental model of the RoboTuna’s swimming kinematics in terms of a set of seven key parameters that can determine swimming performance [14]. These sine-based approaches use simple time-indexed sine-based functions for generating travelling waves. Besides the disadvantages of poor anti-interference capability, abrupt modifications of control parameters (in terms of wavelength, oscillating frequency and amplitude, etc.) will lead to a discontinuousness of propulsive waves, which in turn will produce non-smooth trajectories. In recent years, neurobiological studies have indicated that vertebrates have neural circuits - namely, central pattern generators (CPGs) - in their spinal cords that can produce patterns of the oscillations necessary for locomotion. Due to its robustness and the limit cycle property [16], the CPG-based control approach can be employed well to produce rhythmic movements, such as legged walking, flying and swimming. In addition, a robot swimming underwater will interact with the surrounding fluid. The strong coupling and nonlinearity between them mean that highly effective motion control remains unresolved.

![Figure 2. Configuration of the CPG model for the AmphiRobot-II.](image)

In this paper, CPG-based locomotion control in conjunction with Lagrangian dynamics is employed to tackle the aquatic propulsion of AmphiRobot-II. In particular, the integration of the Lagrange dynamics and the CPG-based control guides the search for CPG parameters, steady locomotion gaits and preliminary validation, greatly facilitating the creation of versatile gaits. As a result, the amphibious robot equipped with wheel-propeller-fins successfully possesses more novel amphibious gaits, enhancing its motion ability and possible operations.

The rest of the paper is organized as follows. In Section 2, the adopted CPG-based locomotion control is briefly described. The formulated Lagrangian dynamics are detailed in Section 3. The simulated and experimental results are provided in Section 4. Finally, Section 5 concludes the paper with an outline of future work.

2. CPG-based multimodal locomotion control

To estimate the oscillation property of a fish body, Lighthill’s elongated body theory has been widely used to generate a propulsive travelling wave. Many other aspects of fish-swimming behaviour have been studied to shed light on the propulsion mechanism, which demonstrates an implied undulatory wave travelling
from head to tail with increasing amplitude. Meanwhile, neurobiology studies reveal that fundamental rhythmic motor patterns in locomotion are generated by CPGs in various organisms. CPGs are networks of neurons that can produce coordinated oscillatory signals without oscillatory inputs from sensory feedback or higher control centres, which have become widely used in robotics recently [15–18]. The CPG-based controller, in particular, has the quality of distributed control and the ability to deal with redundancies and fast control loops, as well as permitting the modulation of locomotion by simple control signals [15]. As such, it is well-suited for locomotion control in dealing with multi-articulate or multi-DOF-related applications. Since the swimming of fish relies on rhythmic activities through oscillations of a multi-joint body and fins, a CPG is an appropriate tool for modelling the kinematics of the underwater locomotion of the amphibious robot. As the kernel of the CPG model, a phase oscillator with controlled amplitude based upon the Kuramoto model is utilized to achieve the synchronization of multiple oscillators so as to ensure coordinated movements of multiple joints [19,20]:

\[
\begin{align*}
\dot{\phi}_i & = 2\pi f_i + \sum_{j \in T(i)} a_{ij} w_{ij} \sin(\phi_j - \phi_i - \gamma_{ij} + u_i) \\
\dot{a}_i & = -\tau_i \left( A_i - a_i \right) - \dot{\phi}_i \\
\dot{\gamma}_{ij} & = a_{ij} \left[ 1 + \cos(\phi_i - \phi_j) \right],
\end{align*}
\]

(1)

where \(\phi_i\) and \(a_i\) represent the state variables (respectively, the phase and amplitude of oscillator \(i\)), \(f_i\) and \(A_i\) determine its intrinsic frequency and amplitude (see Figure 2b), and \(\tau_i\) is a positive constant determining the speed at which \(a_i\) converges on \(A_i\). Couplings between oscillators are defined by the weights \(w_{ij}\) and phase biases \(\gamma_{ij}\). A positive oscillatory signal \(\dot{x}_i\) for oscillator \(i\) represents the resultant burst serving as the output signal. \(T(i)\) is the discrete set of oscillators inflicting inbound couplings with oscillator \(i\); \(u_i\) is the feedback signal related to oscillator \(i\). Note that the feedback signal is usually an integral component of the differential equations that govern the CPGs. For our amphibious robot, infrared sensors (providing obstacle information), pressure sensors (providing water depth information) and liquid-level sensors (providing presence/absence information in relation to water) can be used to modulate the performance of the CPGs in overcoming the external impact on the dynamic motion. We would like to note that the sensory feedback modulation is not described in this paper due to space limitations.

As illustrated in Figure 2, the CPG network for AmphirRobot-II comprises tail CPGs and pectoral CPGs. The pectoral fin on each side corresponds to a nonlinear oscillator, \(O9\) and \(O10\) separately, with the output signal as the control command to drive the pectoral fin. The actuated joint angle signals of the pectoral fins can be expressed as \(\phi_{\text{r}i} = X_{\text{o}i} - a_{\text{r}i}\), \(\phi_{\text{r}i} = X_{\text{o}i} - a_{\text{r}i}\), which are modulated to be positive-negative. The tail CPGs are made up of eight oscillators - i.e., \(O1\)–\(O8\), every two of which constitute a CPG unit for each oscillating joint (1–4). The subtraction of the output of the left and right body oscillators in each oscillating joint is utilized to actuate the homologous joint (i.e., \(\phi_{\text{l}i} = X_{\text{i}i} - X_{\text{i}i}\) \((i=1, \ldots, 4)\). \(d_{\text{l}i}\) and \(d_{\text{r}i}\) represent the input drive received by the left and right body of the robot, which decide the oscillation frequency and amplitude of the oscillators in the left and right parts, as illustrated in Figure 2). Hereafter, \(d_{\text{l}i}\) and \(d_{\text{r}i}\) are abbreviated as \(d_i\) and \(d_i\). Two saturation functions - i.e., \(f = g_i(d)\) and \(A = g_i(d)\) - have been designed to determine the oscillation frequency and amplitude once the input drive \(d\) is given. Again, due to space limitations - for more details about the adopted CPG model - please refer to [21]. It is worthwhile to note that, compared with the CPG model in [19], bio-inspired low-cut-type frequency and amplitude saturation functions are imported. That is, an active oscillatory length-based control strategy loosely inspired from the speed regulation observed in cyprinoids is used to achieve the effect whereby different lengths of the body participate in the oscillations at various speeds. Notice also that the CPG parameter's determination is a somewhat complex and tedious process. With our method, a trial-and-error procedure based on dynamic simulation was first employed to obtain the parameters, offering an opportunity to find a preliminary parameter set. Subsequent physical testing on the robot was conducted in order to refine the preliminary parameter space until a satisfactory performance result reached. Table 1 lists the coupling weights used between the CPG elements. Notice that two types of weights (see the notation 0/30 in Table 1) are preset from the tail CPG onto the pectoral CPG. When the robot swims slowly, the weight will be 0. During fast swimming, the weight will be set as 30.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Symbol</th>
<th>Tail CPG</th>
<th>Pectoral CPG</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coupling in (downwards)</td>
<td>([w_{09}, 10, \pi/4])</td>
<td>([0/30, 3\pi/4])</td>
<td></td>
</tr>
<tr>
<td>tail CPG</td>
<td>([w_{09}, \gamma_{91}, \gamma_{92}])</td>
<td>([0/30, 2\pi/4])</td>
<td></td>
</tr>
<tr>
<td>(contralateral)</td>
<td>([w_{09}, 10, \pi])</td>
<td>([0/30, \pi/4])</td>
<td></td>
</tr>
<tr>
<td>Coupling from the tail CPG to the pectoral CPG</td>
<td>([w_{09}, \gamma_{91}, \gamma_{92}])</td>
<td>([0/30, \pi/4])</td>
<td></td>
</tr>
<tr>
<td>([w_{04}, \gamma_{41}, \gamma_{42}])</td>
<td>([0/30, 0])</td>
<td></td>
<td></td>
</tr>
<tr>
<td>([w_{04}, 10, \pi/4])</td>
<td>([0/30, \pi/4])</td>
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<tr>
<td>([w_{04}, 10, \gamma_{41}, \gamma_{42}])</td>
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<td>([0/30, \pi/4])</td>
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<tr>
<td>Coupling in the pectoral CPG</td>
<td>([0/30, \pi])</td>
<td></td>
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</tr>
</tbody>
</table>

Table 1. Coupling weights between the CPG elements.
3. Hydrodynamic modelling

3.1 Kinematic analysis based on the multi-link serial mechanism

The determination of an optimal locomotion gait for AmphiRobot-II present an acutely challenging problem, involving the robot’s body kinematics and the hydrodynamics of the surrounding water. The intractability of the hydrodynamics of a flexible body involves various uneasy-enveloped factors, such as added mass, trailing vortex effects and leading-edge suction, etc. For the purpose of simulation, a simple physical model of AmphiRobot-II is created to test various locomotion gaits.

Figure 3a shows a simplified five-link mechanism and pectoral fins where the first link corresponds with the head attached by the first modular propelling unit and the last with the caudal fin. The world rectangular coordinate system (WRCS) XOY is established with the origin placed at the starting point of the second link. At the initial moment, the robot body is adjusted to a line while the X-axis directs towards the head with the Y-axis towards the left side of the robot. Specifically, \( \theta_i \) is the angle between the \( i \)-th link and the X-axis \((i = 1, 2, ..., 5)\). \( \phi_i \) is the joint angle between the left pectoral fin and the horizontal plane, while \( \phi_k \) serves for the right pectoral fin. Similarly, \( \phi_i \) denotes the angle between the \((i+1)\)-th link and the extension line of the \( i \)-th link \((i = 1, ..., 4)\) - i.e., the relative joint angle of the \((i+1)\)-th link. Note that the joint angles \( \theta_i \), \( \phi_i \) and \( \phi_k \) serve as the input control signal to the system, and are derived from the above CPG model such that all joint angles are anti-clockwise positive. \((x_{ix}, y_{iy})\) and \((x_{iy}, y_{iy})\) are, individually, the starting point and the end point of the \( i \)-th link. \((x_{il}^i, y_{il}^i)\) and \((x_{il}^i, y_{il}^i)\) are the centroid (centre of figure) and barycentre of the \( i \)-th link separately. \( l_i \) and \( l_i \) are the lengths between \((x_{ix}, y_{iy})\) and \((x_{il}^i, y_{il}^i)\), \((x_{il}^i, y_{il}^i)\), respectively.
the left pectoral fin separately. The symbols for the right pectoral fin are just empathetic, denoted by \((x_{R_i}^l, y_{R_i}^l)\) and \((x_{R_i}^r, y_{R_i}^r)\). Unless otherwise stated, the abbreviations of \(L\) and \(R\) represent the left and right sides of the robot.

From Figure 3, the position of the centroid in each link can be expressed as:

\[
\begin{align*}
(x_i^l, y_i^l) &= (x_i - (l_i - l_i^l)\cos\theta_i, y_i - (l_i - l_i^l)\sin\theta_i) \\
(x_i^r, y_i^r) &= (x_i + l_i^r\cos\theta_i, y_i + l_i^r\sin\theta_i) \\
(x_i^l, y_i^r) &= (x_i + \sum_{j=1}^{i-1} l_j^l\cos\theta_j + l_i^l\cos\theta_i, y_i + \sum_{j=1}^{i-1} l_j^l\sin\theta_j + l_i^l\sin\theta_i)
\end{align*}
\]

(2)

where \(\theta_i = \theta_2 - \phi_i\), \(\theta_2 = \theta_2 + \sum \phi_i\) and \(i = 3, 4, 5\).

We can get the velocities of the \(i\)th link relative to water - i.e., \(x_i^l\) and \(y_i^l\) within the space-fixed coordinate system. The velocities are further resolved into components parallel and perpendicular to the link, namely:

\[
\begin{align*}
\dot{x}_i^l &= \frac{dx_i^l}{dt} = \frac{dx_i}{dt} \sin\theta_i - \frac{dy_i}{dt} \cos\theta_i \\
\dot{y}_i^l &= \frac{dy_i^l}{dt} = \frac{dx_i}{dt} \cos\theta_i + \frac{dy_i}{dt} \sin\theta_i
\end{align*}
\]

(3)

As distinct from the links oscillating in the horizontal plane, the pectoral fins keep oscillating in the vertical plane. As shown in Figures 3a and 3e, the centroid of the left pectoral fin can be obtained:

\[
\begin{align*}
x_i^l &= x_i - (l_i - l_i^l)\cos\phi_i \cos\theta_i + l_i^l\sin\theta_i \\
y_i^l &= y_i - (l_i - l_i^l)\sin\phi_i \sin\theta_i - l_i^l\cos\theta_i
\end{align*}
\]

(4)

Likewise, the velocity of the pectoral fins can be formulated as below:

\[
v_i^l = \dot{x}_i^l \cos\theta_i \sin\phi_i + \dot{y}_i^l \sin\theta_i \sin\phi_i
\]

(5)

3.2 Lagrangian dynamic modelling

The commonly used approaches to fluid-body interaction have focused on twospects. The former involves a reduced Lagrangian formulation, where a fish is taken to be a rigid body with the action of the tail [22]. The latter is based on a highly simplified quasi-static lift and drag model of forces acting on the fish body and tail [23]. Morgansen et al. have extensively presented a modest model of the latter [24]. The designed modular amphibious robot can be regarded as a multi-link serial mechanism coupled with pectoral fins and represented by thin plates, which can be solved by the Lagrangian function with explicit joint angles as input variables. As a bonus offered by our robot, repeatable experiments can easily be conducted to verify the simulated swimming model in the Mathematica environment, which makes the hydrodynamics computation tractable.

For the convenience of theoretical analysis, some suppositions are made below:

**Supposition 1:** The fluid is stationary and the forces acting on a single link are due only to the motion of the link. Meanwhile, the deformation of the robot body can be ignored, except for the motion of the links and pectoral fins.

**Supposition 2:** The pressure differentials in the directions parallel to the moving body are decoupled from pressure differentials perpendicular to the body.

**Supposition 3:** The reference position for the pectoral fins is defined as the location that the pectoral fins are at when they are in the horizontal plane.

The instantaneous force is out of balance in the vertical plane. Taking into account the intrinsic frequency of the oscillating joint and oscillation symmetry, the vertical movement is temporarily suspended. AmphibRobot-II mainly employs fish-like swimming, where the swivelling body device is casually disabled. To lay out a primary design scheme, the overall buoyancy counterbalances the gravity and only the forces in the horizontal direction are taken into consideration (e.g., thrust and resistance). The 3-D dynamic analysis will not be considered for the moment.

The potential energy is constant (\(E\)) according to Supposition 3. The kinetic energy of each link comprises translational kinetic energy within WRCS and the rotational kinetic energy within the centre-of-mass system. Thus, the Lagrangian function will be expressed as follows:

\[
L = \sum_{i=1}^{3} \frac{1}{2} m_i v_i^2 + \sum_{i=1}^{3} \frac{1}{2} I_i \dot{\theta}_i^2 + \frac{1}{2} m_i l_i^2 \dot{\theta}_i^4 + \frac{1}{2} I_k \dot{\theta}_k^2 - E
\]

(6)

where \(m_i\) is the mass of the \(i\)th link, \(v_i\) is the velocity of the \(i\)th link, \(I_i\) is the estimated moment of inertia of the \(i\)th link under the centre-of-mass system, and \(w_i\) is the angular velocity of the \(i\)th link derived by the differential of the angle \(\theta_i\). Note that when - in order to compute \(L - x_i, y_i\) and \(\dot{\theta}_i\) are chosen as the generalized coordinates. Let \(x_i = x_i, y_i = y_i\) and \(\dot{\theta}_k\), we can then rewrite the Lagrangian \(L\) as \(L(X, Y, \Theta)\). Thus, Lagrange's equations of the second kind can be given by:

\[
\text{Rui Ding, Junzhi Yu, Qinghai Yang and Min Tan: Dynamic Modelling of a CPG-Controlled Amphibious Biomimetic Swimming Robot}
\]
\[
\begin{align*}
F_x &= \frac{d}{dt} \frac{\partial L}{\partial \dot{X}} - \frac{\partial L}{\partial X} \\
F_y &= \frac{d}{dt} \frac{\partial L}{\partial \dot{Y}} - \frac{\partial L}{\partial Y} \\
M_{\Theta} &= \frac{d}{dt} \frac{\partial L}{\partial \dot{\Theta}} - \frac{\partial L}{\partial \Theta}
\end{align*}
\]  

where \( F_x \) and \( F_y \) are the compositions of the hydrodynamic forces on the X axis and Y axis, respectively. \( M_{\Theta} \) is the moment about the point \((X, Y)\). Notice that \( X, Y \) and \( M_{\Theta} \) are the functions of time evolution - i.e., \( X(t), Y(t) \) and \( M_{\Theta}(t) \) - which are fundamentally decided by the joint actuation laws.

### 3.3 Hydrodynamic force approximation

The interaction of a robot swimming underwater with the fluid flow is a crucial issue that remains unresolved. The hydrodynamic aspects of swimming have been described in various models [25,26]. In many cases, some simplifications are necessary. To calculate the resultant forces, a large Reynolds number is applied and all of the forces acting on any propulsive element are due to the motion of that element in the fluid. The movement of any object through a stationary fluid causes an increase in pressure in front of the object and a decrease behind it. The different pressures on the two sides also give a net drag force to the object, counteracting the movement. Forces acting on the body from the surrounding water depend upon the speed of the body relative to the water. The movement of robot’s propulsive elements through the fluid causes pressure differentials to develop on either side of the elements, resulting in drag forces in opposition to the motion. Next, each link is acted upon by three forces: pressure on the links, the approach stream pressure and friction drag.

#### 3.3.1 Pressure on the links

While oscillating, the hydrodynamic force acting perpendicular to the surface of the \( i \)th link is the thrust for advancement (see Figure 3b), which is given by:

\[ F_v^i = -\mu v^i \cdot \nu^i_v \]

where \( \nu^i_v \) is the component of the velocity perpendicular to the surface of \( i \)th link relative to the fluid, which can be obtained from (3). Likewise, \( \mu_v = \rho S_i C_i / 2 \) is the drag coefficient with \( C_i \) of a flat plate-type (i.e., \( C_i = 1.28 \) [27]). \( S_i \) is the effective area of the \( i \)th link).

Many fish balance vertical forces by continually swimming, with their pectoral fins flapping and while maintaining a steady swimming speed. Thanks to the pectoral fins mounted on both sides of the head, the robot is more agile for turning or maintaining balance in the water. The propulsive forces generated by the pectoral fins in conjunction with the rear body promote the robot in moving forwards. While oscillating, the pectoral fin produces the perpendicular pressure and parallel friction drag in relation to the surface. The thickness of the pectoral fin is small and the friction drag can be ignored simply by considering the perpendicular component that contributes to propulsion (see Figure 3e). As with the moving links, the pressure on the pectoral fin is given by:

\[ F_v^i = -\mu_\perp v^i \cdot \nu^i \]

where \( \mu_\perp \) is the drag coefficient adopting the flat plate-type, \( S_i \) is the effective area of the left pectoral fin and \( v^i \perp \) is the normal velocity perpendicular to the surface of the left pectoral fin which can be calculated from (5).

Similarly, \( F_L^i \) is resolved into components along the direction of the head and perpendicular to the surface of the pectoral fin. The perpendicular component mainly contributes to an up-and-down locomotion, which will not be considered here. The force along the head can be expressed as below:

\[ F_L^i \sin \phi_v = \left( F_L^i \sin \phi_v \cos \theta_v \right) \hat{i} + \left( F_L^i \sin \phi_v \sin \theta_v \right) \hat{j} \]

where \( \hat{i} \) and \( \hat{j} \) are the unit vectors along the X axis and Y axis, respectively. Corresponding results for the right pectoral fin can be obtained by analogy, as denoted by \( F_R^i \).

#### 3.3.2 Approach stream pressure

During swimming, the robot suffers from a resulting force which acts upon the head in opposition to the motion - i.e., the approach stream pressure (see Figure 3c). Owing to a larger cross-section of the head with small oscillation amplitudes of the rear body, the drag forces acting on the other links can be ignored, except for the head. The drag force on the head is given by:

\[ F_v^i = -\mu_v \cdot \nu^i_v \]

where \( \nu^i_v \) is the component of the velocity parallel to the surface of the head relative to the fluid, which is determined by (3). \( \mu_v = \rho C L S_L / 2 \) is the drag coefficient with \( C_L \) of the bullet-type [25] - i.e., \( C_L = 0.295 \). \( S_L \) is the cross-section area of the head.

#### 3.3.3 Friction drag

Friction drag arises as a result of the viscosity of the fluid flow between the fish body and the boundary layer of the water (see Figure 3d). The movement of the robot’s propelling units through the fluid causes friction drag parallel to the propelling units, resulting in drag forces adverse to the motion. It is empirically evaluated as 20–50% of the approach stream pressure (i.e., \( F_v = (0.2–0.5)F_v^i \)). In view of the matte body of AmphirRobot-II, a factor of 50% is adopted here.

#### 3.3.4 Composition of the hydrodynamic forces

By resolving all of the above forces in the WRCS, the components of the forces in the direction of the X-axis and the Y-axis can be rewritten as:
\[
\begin{align*}
F_x &= \sum_{i=1}^{5} F_{x(i)} + F_{x3} + F_{xL} + F_{xR} + F_{xS} \\
F_y &= \sum_{i=1}^{5} F_{y(i)} + F_{y3} + F_{yL} + F_{yR} + F_{yS} \\
F_{\phi} &= \sum_{i=1}^{5} F_{\phi(i)} + F_{\phi3} + F_{\phiL} + F_{\phiR} + F_{\phiS}
\end{align*}
\]

(8)

where \( F_{x(i)} \) is the net force in the x-direction, \( F_{y(i)} \) is the net force in the y-direction, \( F_{\phi(i)} \) is the net moment about the z-axis, \( \phi_i \) is the position of joint \( i \), and \( i = 1, \ldots, 5 \).

The composition of moment acting on the hinged joint point \( (X, Y) \) is given by:

\[
M_{\Theta} = \sum_{i=1}^{5} [-F_{x(i)}(y_{i} - Y) + F_{y(i)}(x_{i} - X) - F_{x3}(y_{3} - Y) - F_{xL}(y_{L} - Y) - F_{xS}(y_{S} - Y) + F_{x3}(y_{3} - Y) + F_{xL}(y_{L} - Y) + F_{xS}(y_{S} - Y)]
\]

(9)

Based on (7)–(9), a system of partial differential equations of \( X, Y, \Theta, \) and \( t \) can be obtained:

\[
\begin{align*}
\dot{X} &= \sum_{i=1}^{5} F_{x(i)}/m_{i} + F_{x3}/m_{3} + F_{xL}/m_{L} + F_{xR}/m_{R} + F_{xS}/m_{S} \\
\dot{Y} &= \sum_{i=1}^{5} F_{y(i)}/m_{i} + F_{y3}/m_{3} + F_{yL}/m_{L} + F_{yR}/m_{R} + F_{yS}/m_{S} \\
\dot{\Theta} &= \sum_{i=1}^{5} [-F_{x(i)}(y_{i} - Y) + F_{y(i)}(x_{i} - X) - F_{x3}(y_{3} - Y) - F_{xL}(y_{L} - Y) - F_{xS}(y_{S} - Y) + F_{x3}(y_{3} - Y) + F_{xL}(y_{L} - Y) + F_{xS}(y_{S} - Y)]
\end{align*}
\]

(10)

To further solve the above equations, some initial values should be given. Suppose that the robot is at the origin at the initial moment and that the robot body maintains itself motionless in a line - i.e., \( X(0)=0, Y(0)=0, \Theta(0)=0, \dot{X}(0)=0, \dot{Y}(0)=0, \dot{\Theta}(0)=0 \) - and that the hydrodynamic model is then decided by (10). Furthermore, the body parameters such as masses, lengths and lateral areas for all the links and the pectoral fins, are needed. The parameters measured and estimated from the physical robot are listed in Table 2. In particular, the joint angles \( \phi_i \), \( \phi_3 \), and \( \phi_L \) - i.e., oscillation motion laws - are directly obtained from the CPG model. With such parameters, the real-time swimming performance can be numerically estimated interactively. In turn, suitable CPG control parameters and conceived gaits can be searched and eventually better locomotion performance will be acquired.

Notice that, in a similar fashion to the hydrodynamic modelling of fish-like lateral motions, dolphin-like dorsoventral motions can be dynamically simulated with the same kinematic analysis and hydrodynamic force approximation [28]. Hence, a comparison between fish-like and dolphin-like swimming in the same robotic platform becomes available.

<table>
<thead>
<tr>
<th>Terms</th>
<th>( m_i ) (kg)</th>
<th>( l_i ) (mm)</th>
<th>( I_{x1}^i ) (mm²)</th>
<th>( I_{y1}^i ) (mm²)</th>
<th>( L_i ) (10⁻⁴kg⋅m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Head</td>
<td>3.075</td>
<td>324.0</td>
<td>146.0</td>
<td>162.0</td>
<td>31253 (S1)</td>
</tr>
<tr>
<td>L1</td>
<td>0.605</td>
<td>103.5</td>
<td>72.5</td>
<td>58.0</td>
<td>8797.5 5.4</td>
</tr>
<tr>
<td>L2</td>
<td>0.875</td>
<td>123.5</td>
<td>87.5</td>
<td>68.0</td>
<td>10497.5 11</td>
</tr>
<tr>
<td>Peduncle</td>
<td>0.170</td>
<td>56.0</td>
<td>25.0</td>
<td>28.0</td>
<td>1710 0.44</td>
</tr>
<tr>
<td>Caudal fin</td>
<td>0.040</td>
<td>73.5</td>
<td>45.0</td>
<td>37.0</td>
<td>5800 0.18</td>
</tr>
<tr>
<td>Left Pec.</td>
<td>0.095</td>
<td>242</td>
<td>83 (l)</td>
<td>35.0</td>
<td>7956 0.82</td>
</tr>
<tr>
<td>Right Pec.</td>
<td>0.095</td>
<td>242</td>
<td>83 (l)</td>
<td>35.0</td>
<td>7956 0.82</td>
</tr>
</tbody>
</table>

Table 2. Feature parameters of AmphiliRobot-II for the dynamic simulation.

4. Simulations and experiments

4.1 Simulations

By assigning the same drive between the left and right sides of the model - e.g., \( d_1 = d_2 = 3 \) - the corresponding oscillatory frequencies and amplitudes for moving joints can be obtained from the CPG model, which can then be imported into the Lagrangian model. Figure 4a shows the displacement of the point \((X, Y)\), where the robot moved in the negative-direction of the X-axis. The orientation is not strictly in the negative direction, for the reason that the forces acting upon the robot are not absolutely symmetrical at the start time, as illustrated in Figures 4c and 4d. The oscillation curve of \( \Theta \) in Figure 4b partly demonstrates the relative accuracy of the hydrodynamic model. The moment acting on the point \((X, Y)\) is plotted in Figure 4e. It is a vital parameter for the servomotor selection and the maximum moment of the servomotor should satisfy the simulation result. With the simulated results, the average velocity can be estimated by:
every the trajectory. \[ F_x (N) \]

\[ F_y (N) \]

\[ M_0 (N \cdot m) \]

\[ \tau (m/s) \]

\( \frac{d\phi}{dt} \)

\[ \frac{d\psi}{dt} \]

\[ \frac{d\theta}{dt} \]

\[ \frac{d\phi}{dt} \]

\[ \frac{d\psi}{dt} \]

\[ \frac{d\theta}{dt} \]

\[ \tau = \sqrt{(X(t + T) - X(t))^2 + (Y(t + T) - Y(t))^2} / T \]  

Figure 4. Simulated forward gait. (a) Forward swimming trajectory. (b) Oscillatory angle signal of \( \Theta \). (c) Hydrodynamic forces acting on the X-axis. (d) Hydrodynamic forces acting on the Y-axis. (e) Moment acting on \( (X, Y) \). (f) Average velocity at every period.

Figure 5. Simulated turning gait. (a) Actuated joint angle signals from the CPG model. (b) Comparison of turning trajectories.

\[ \tau = \sqrt{(X(t + T) - X(t))^2 + (Y(t + T) - Y(t))^2} / T \]  

where \( t_s \) is an arbitrary time after the simulation is steady and \( T \) is the oscillating period indirectly obtained from the oscillating frequency. The velocity curve obtained in Figure 4f indicates the motion stability after several periods, and the steady average velocity is 0.335 m/s.

Meanwhile, turning can be induced when asymmetrical drives are applied between the left and right sides of the CPG model. The robot will turn towards the side receiving the higher drive. The posterior body keeps oscillating at a common frequency in terms of synchronization, which is in vain for modulating the direction. The turning gait will mostly be credited to asymmetrical oscillation amplitudes of all the joints in the posterior body, with the side receiving higher input drive oscillating at higher amplitudes, as shown in Figure 5a. The turning motion simulation is displayed in Figure 5b at \( d_l=1.5 \) and \( d_s=3.5 \), and the average turning radius is around 1.15 m. Notice that the pectoral fins will oscillate in their own intrinsic frequencies due to unattained critical input drive during swimming. The differences of the oscillatory frequencies and amplitudes of the pectoral fins on both sides will also help turning and assist with propulsion. The successful simulations of forward gaits and turning gaits show the effectiveness of hydrodynamic modelling in describing yaw motions.

In addition, by modulating the attack angle of the pectoral fins, the robot is able to achieve an up-and-down motion, termed ‘pitching gait’. Figure 6 shows the comparative results between simulations and experiments on the pitching gait, where the attack angle increases from 0 to 60º, with a step of 10º. The simulated
data largely conforms to the experimental data over the whole range of attack angles, demonstrating the validity of hydrodynamic force analysis in relation to the pitching motions. More interestingly, Figure 7 depicts the comparison results of fish- and dolphin-like modes. The simulation demonstrates that the dolphin-like swimming precedes the fish-like mode, with a higher speed comprising a 6% increase at a maximum input drive $d=d_{d}=5$, though it is less remarkable. In particular, aquatic experiments verify this trend. The established dynamic model that is capable of fish- and dolphin-like swimming simulation further verifies the effectiveness of the whole hydrodynamic analysis when primarily involving three types of forces.

![Figure 6. Comparison of simulated and actual pitching gait via modulating attack angle of the pectoral fins.](image)

**Figure 7.** Comparison of simulated and actual propulsive speeds between fish-like and dolphin-like modes.

### 4.2 Experiments

The experiments were performed in a lake and the drive signals were generated through wireless communication by a human operator. By applying the same drive between the left and right sides of the CPG model, the AmphibRobot could realize forward swimming. As the same drive is increasing and is applied to both sides, the swimming distances and corresponding consuming time are measured at drives ranging from 1.0 to 4.0, with a step of 0.5. As the equal input drives cannot serve as a guarantee of actual locomotion in a straight line - the measurement has been run three times for a specific drive. The average velocity can thus be calculated.

At present, the robot is able to achieve different forward gaits in BCF mode (by oscillating only the body and/or caudal fin) or in PF mode (by oscillating only the pectoral fins), or their combination PF+BCF. Figure 8 demonstrates how the input drive affects the propulsion velocity in different forward gaits. The velocity curves partly confirm the biology-inspired concept that fish typically rely on the BCF mode for propulsion while employing the PF mode for manoeuvring and stabilization. The velocity increases with the drive and attains the maximum velocity when the input reaches the maximum acceptable drive. The experimental results generally agree with the simulated velocity curve despite the existence of small errors. This could be due to an inaccurate model, a simplified mechanical analysis and initial random parameters.

![Figure 8. Propulsion comparison of different forward gaits.](image)

Furthermore, a series of terrestrial and aquatic locomotion gaits were successfully implemented (see Figure 9), which can significantly expand the operational field and capability of the amphibious robot. Currently, the gait transition is primarily induced by modulating the activity of the CPG, the attack angle of the mechanical pectoral fins or else the swivelling mechanism. As a quantitative case, the average turning radius shown in Figure 9d is approximately 0.95 m, which is minor in contrast with Figure 5b. This mainly owes to the larger bending of the rear body and the approach stream pressures produced by the rear body, which are ignored in the dynamic model. In particular, the experimental trajectory of the turning robot exhibits an irregular circular path. This phenomenon might be caused by the drive difference that undermines the balance of the robot body tilting to the side of the small drive.
5. Conclusions

This paper presents the modelling and control issues of an amphibious swimming robot capable of multimodal motions. To generate robust gait control for steady swimming, a CPG-based control law was designed to produce the joint angle signals of the propelling units. A hydrodynamic model based on a Lagrangian formulation has been proposed for underwater locomotion. By introducing the CPG-based control into the hydrodynamic solution, multiple gaits could be numerically estimated. Comparative results have been given to show the effectiveness of the formed dynamic model combined with the CPG-based control model.

Further research will focus on mechanical and gait optimization for improved amphibious performance. Furthermore, sensory feedback information will be integrated into the CPGs to improve amphibious locomotor performance.

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7. References


