Abstract Ant Colony Optimization (ACO) is a population-based constructive meta-heuristic that exploits a form of past performance memory inspired by the foraging behaviour of real ants. The behaviour of the ACO algorithm is highly dependent on the values defined for its parameters.

Adaptation and parameter control are recurring themes in the field of bio-inspired algorithms. The present paper explores a new approach to diversity control in ACO. The central idea is to avoid or slow down full convergence through the dynamic variation of certain parameters.

The performance of different variants of the ACO algorithm was observed to choose one as the basis for the proposed approach.

A convergence fuzzy logic controller with the objective of maintaining diversity at some level to avoid premature convergence was created. Encouraging results have been obtained on its application to the design of fuzzy controllers. In particular, the optimization of membership functions for a unicycle mobile robot trajectory control is presented with the proposed method.

Keywords Fuzzy Control, Mobile Robot, Ant Colony Optimization

1. Introduction

ACO is inspired by the foraging behaviour of ant colonies and targets discrete optimization problems [8].

The behaviour of the ACO algorithm is highly dependent on the values defined for its parameters and has an effect on its convergence [6] [11]. Often, these are kept static during the execution of the algorithm. Changing the parameters at runtime, at a given time or depending on the search progress may improve the performance of the algorithm [25], [26], [27].
Controlling the dynamics of convergence to maintain a balance between exploration and exploitation is critical for good performance [13].

Early convergence leaves large sections of the search space unexplored. Slow convergence does not concentrate its attention on areas where good solutions are found [28], [31].

Fuzzy control has emerged as one of the most active and fruitful areas of research in the application of fuzzy set theory [1], [17], [24], [35].

The methodology of the fuzzy logic controller is useful when processes are too complex for analysis by conventional quantitative techniques or when the available sources of information are interpreted in a qualitatively inaccurate or uncertain way [40].

Finding the correct parameters for the fuzzy logic controller is a complex problem; it is also a task that consumes considerable time [2], [9], [12], [22]. Because of their ability to solve complex NP problems, ACO can be used for the selection of the optimal fuzzy controller parameters [6].

There is also some interest in using ACO algorithms in mobile robotics [5], [28]. Nowadays, robotic automation is an essential part in the manufacturing process [3], [4], [7]. The autonomous navigation of mobile robots is a challenge [10], [14], [15]. A mobile robot can be useful in unattainable goal situations due to geological conditions or where the human being is endangered. Accordingly, mobile robotics is an interesting subject for science and engineering [16], [18], [19], [36], [37], [39].

This paper explores a new method of diversity control in ACO. The central idea is to prevent or stop total convergence through the dynamic adjustment of certain parameters of the algorithm applied to the design of fuzzy controllers - specifically, to the optimization of membership functions of a trajectory controller for a unicycle mobile robot [21], [29], [30], [32].

The rest of the paper is organized as follows. Section 2 presents an overview of ACO. Section 3 presents a new method of parameter tuning through fuzzy logic. Section 4 describes the optimized fuzzy controller. Section 5 presents the considerations taken in order to implement the ACO algorithm in the optimization of the membership functions. Section 6 describes how the proposal was applied. Sections 7 and 8 present simulation results in relation to the membership functions optimization problem. Finally, section 9 presents some conclusions.

2. Ant Colony Optimization (ACO)

The first ACO algorithm was called Ant System (AS) and its objective was to solve the travelling salesman problem (TSP), the goal of which is to find the shortest route to link a number of cities [20]. In each of the iterations, each ant keeps adding components to build a complete solution; the next component to be added is chosen with respect to the probability that depends on two factors [33]. The pheromone factor reflects the past experience of the colony while the heuristic factor evaluates the interest of selecting a component with respect to an objective function. Both factors are weighted by the parameters $\alpha$ and $\beta$ respectively (1).

$$p_{ij}^k = \frac{[\tau_{ij}]^\alpha [\eta_{ij}]^\beta}{\sum_{i \in N^k} [\tau_{ij}]^\alpha [\eta_{ij}]^\beta} \quad \text{if } j \in N^k$$  \hspace{1cm} (1)

After all of the ants have built their tours, pheromone trails are updated. This is done by lowering the pheromone value on all arcs by a constant factor (2), which prevents the unlimited accumulation of pheromone trails and allows the algorithm to forget bad decisions previously taken.

$$\tau_{ij} \leftarrow (1 - \rho) \tau_{ij}, \quad \forall (i,j) \in L$$ \hspace{1cm} (2)

Moreover, by depositing pheromone on the arcs that ants have crossed in their path (3), the better the tour, the greater the amount of pheromone that the arcs will receive.

$$\tau_{ij} \leftarrow \tau_{ij} + \Delta \tau_{ij} \quad \forall (i,j) \in L$$ \hspace{1cm} (3)

A first improvement on the initial AS was called the elitist strategy for Ant System (EAS). The idea is to provide strong additional reinforcement to the arcs belonging to the best tour found since the start of the algorithm (4) [8].

$$\tau_{ij} \leftarrow \tau_{ij} + \Delta \tau_{ij} + \Delta \tau_{ij}^b, \quad \forall (i,j) \in L$$ \hspace{1cm} (4)

Another improvement over AS is the rank-based version of it (ASRank). In ASRank, each ant deposits an amount of pheromone that decreases with its rank. Additionally, as in EAS, the best-ant-so-far always deposits the largest amount of pheromone during each iteration [8].
Based on previous results, we decided to use ASRank as the basis for our proposed ACO variant. The central idea is to prevent or stop total convergence through the dynamic variation of the alpha parameter.

Alpha has a big effect on diversity. Is recommended to keep $\alpha$ in the range of $0 < \alpha < 1$ \[8\].

A value closer to 1 will emphasize better paths but reduces diversity, while lower $\alpha$ will maintain more diversity but reduces selective pressure \[26\].

However, it appears impossible to fix a universally best $\alpha$. In most approaches it is taken to be 1, so that the selection probability is linear in the pheromone level.

An adaptive parameter control strategy was used in \[34\]; this takes place when there is some form of feedback from the search that is used to determine the direction and/or magnitude of the change to the strategy parameter \[9\]. In our case, this is the average lambda branching factor - this factor measures the distribution of the values of the pheromone trails and provides an indication of the size of the search space effectively explored \[8\].

A convergence fuzzy controller \[38\] to prevent or delay the full convergence of the algorithm was created (Figure 1). Fuzzy control can be seen as the translation of external performance specifications and observations of plant behaviour into a rule-based linguistic control strategy \[40\].

The objective of the controller is the maintenance of the average lambda branching factor at a certain level to avoid premature convergence; as such, its rules were made to fulfil this purpose (Figure 2).

If (error is P) and (error_change is N) then (alpha increment is Z)
If (error is N) and (error_change is P) then (alpha increment is Z)
If (error is Z) and (error_change is Z) then (alpha increment is Z)
If (error is Z) and (error_change is N) then (alpha increment is P)
If (error is Z) and (error_change is P) then (alpha increment is N)

The controller uses as inputs the error and change of error (Figure 3) with respect to an average lambda branching factor reference level and provides as an output an increase in the value of the parameter alpha (Figure 4).

The control goal for the mobile robot is: Given a path $q_d(t)$ and a desired orientation, a fuzzy logic controller must be designed to apply an adequate torque $\tau$, such that measured positions $q(t)$ reach the reference trajectory $q_d(t)$. That is:

$$\lim_{t \to \infty} ||q_d(t) - q(t)|| = 0$$ \hspace{1cm} (6)

We decided to optimize a fuzzy trajectory controller for a unicycle mobile robot to test the developed method in a complex problem. The control goal for the mobile robot is: Given a path $q_d(t)$ and a desired orientation, a fuzzy logic controller must be designed to apply an adequate torque $\tau$, such that measured positions $q(t)$ reach the reference trajectory $q_d(t)$. That is:

If (error is P) and (error_change is P) then (alpha increment is N)
If (error is N) and (error_change is N) then (alpha increment is P)
If (error is P) and (error_change is Z) then (alpha increment is P)
If (error is N) and (error_change is Z) then (alpha increment is N)
If (error is P) and (error_change is P) then (alpha increment is N)
If (error is N) and (error_change is P) then (alpha increment is Z)
The fuzzy system to optimize [23] in this case is of Takagi-Sugeno type. For simplicity, we decided to modify it and convert it into a Mamdani type controller so that the input and output parameters are represented by linguistic variables.

The controller receives as input variables the error in the linear \((e_{v})\) and angular \((e_{\omega})\) speed (Figure 5) - that is, the difference between the predefined desired speed and the actual speed of the plant - and as output variables the right \((\tau_{1})\) and left \((\tau_{2})\) torques of the robot (Figure 6).

The membership functions of the input variables are trapezoidal for the negative (N) and positive (P) linguistic terms and triangular for the zero (Z) linguistic term. The output variables have three membership functions - negative (N), zero (Z), positive (P) - of triangular shape and uses nine fuzzy rules, which are shown below in Figure 7:

1. If \((e_{v} \text{ is } N)\) and \((e_{\omega} \text{ is } N)\) then \((\tau_{1} \text{ is } N)(\tau_{2} \text{ is } N)\)
2. If \((e_{v} \text{ is } N)\) and \((e_{\omega} \text{ is } Z)\) then \((\tau_{1} \text{ is } N)(\tau_{2} \text{ is } Z)\)
3. If \((e_{v} \text{ is } N)\) and \((e_{\omega} \text{ is } P)\) then \((\tau_{1} \text{ is } N)(\tau_{2} \text{ is } P)\)
4. If \((e_{v} \text{ is } Z)\) and \((e_{\omega} \text{ is } N)\) then \((\tau_{1} \text{ is } Z)(\tau_{2} \text{ is } N)\)
5. If \((e_{v} \text{ is } Z)\) and \((e_{\omega} \text{ is } Z)\) then \((\tau_{1} \text{ is } Z)(\tau_{2} \text{ is } Z)\)
6. If \((e_{v} \text{ is } Z)\) and \((e_{\omega} \text{ is } P)\) then \((\tau_{1} \text{ is } Z)(\tau_{2} \text{ is } P)\)
7. If \((e_{v} \text{ is } P)\) and \((e_{\omega} \text{ is } N)\) then \((\tau_{1} \text{ is } P)(\tau_{2} \text{ is } N)\)
8. If \((e_{v} \text{ is } P)\) and \((e_{\omega} \text{ is } Z)\) then \((\tau_{1} \text{ is } P)(\tau_{2} \text{ is } Z)\)
9. If \((e_{v} \text{ is } P)\) and \((e_{\omega} \text{ is } P)\) then \((\tau_{1} \text{ is } P)(\tau_{2} \text{ is } P)\)

5. ACO for membership function optimization

ACO was used to find the membership functions with optimal parameters through their adjustment and by the subsequent evaluation of the system.

The parameters \(a, b, f, j\) and \(k\) corresponding to the membership functions of the input variables remain fixed to simplify the problem. The algorithm will find the optimal values of the parameters \(c\) and \(i\) in a straightforward manner and through the optimum position of the intersection points \((X_1, Y_1), (X_2, Y_2)\), which are the values of the parameters \(d, e, g\) and \(h\) (Figure 8).

Regarding the membership functions of the output variables, the algorithm will search for the optimum centre \((b, h\), except \(e\) that remains fixed for simplicity) and span of each one \((a, c, d, f, g, i)\) (Figure 9).

The application of ACO to optimize membership functions involves certain considerations. First, encode all
of the parameters in a weighted graph. For this purpose, we chose a complete graph of 43 nodes to maintain the similarity of the problem with a classical TSP where a minimum Hamiltonian circuit is searched. The range of each variable was discretized in 22 normalized values in the range [-1 1] and a symmetric data matrix of 43x43 with the distance between nodes was created. The parameters of the membership functions of the fuzzy system are obtained through the distance between two nodes using the relations of Tables 1, 2, 3 and 4.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Relation</th>
</tr>
</thead>
<tbody>
<tr>
<td>c</td>
<td>( c = -1 + 0.475 \left( \frac{d_{13,1} + d_{13,2}}{2} \right) + 0.475 )</td>
</tr>
<tr>
<td>X1</td>
<td>( X_1 = c + \left( \frac{d_{13,2} + d_{13,3}}{2} \right) + C_1 )</td>
</tr>
<tr>
<td>Y1</td>
<td>( Y_1 = 0.5 \left( \frac{d_{13,2} + d_{13,3}}{2} \right) + 0.5 )</td>
</tr>
<tr>
<td>i</td>
<td>( i = 1 - 0.475 \left( \frac{d_{23,1} + d_{23,2}}{2} \right) + 0.475 )</td>
</tr>
<tr>
<td>X2</td>
<td>( X_2 = i + \left( \frac{d_{23,2} + d_{23,3}}{2} \right) + C_2 )</td>
</tr>
<tr>
<td>Y2</td>
<td>( Y_2 = 0.5 \left( \frac{d_{23,2} + d_{23,3}}{2} \right) + 0.5 )</td>
</tr>
</tbody>
</table>

Table 1. Relation variable weight for the linear speed error input of the fuzzy system to be optimized

<table>
<thead>
<tr>
<th>Variable</th>
<th>Relation</th>
</tr>
</thead>
<tbody>
<tr>
<td>b</td>
<td>( b = 0.5 \left( \frac{d_{23,1} + d_{23,2}}{2} \right) - 0.5 )</td>
</tr>
<tr>
<td>span1</td>
<td>( \text{span}<em>1 = 0.475 \left( \frac{d</em>{23,1} + d_{23,2}}{2} \right) + 0.525 )</td>
</tr>
<tr>
<td>span2</td>
<td>( \text{span}<em>2 = 0.475 \left( \frac{d</em>{23,2} + d_{23,3}}{2} \right) + 0.525 )</td>
</tr>
<tr>
<td>h</td>
<td>( h = 0.5 \left( \frac{d_{23,2} + d_{23,3}}{2} \right) + 0.5 )</td>
</tr>
<tr>
<td>span3</td>
<td>( \text{span}<em>3 = 0.475 \left( \frac{d</em>{23,2} + d_{23,3}}{2} \right) + 0.525 )</td>
</tr>
</tbody>
</table>

Table 3. Relation variable weight for the right torque output of the fuzzy system to be optimized

<table>
<thead>
<tr>
<th>Variable</th>
<th>Relation</th>
</tr>
</thead>
<tbody>
<tr>
<td>b</td>
<td>( b = 0.5 \left( \frac{d_{23,1} + d_{23,2}}{2} \right) - 0.5 )</td>
</tr>
<tr>
<td>span1</td>
<td>( \text{span}<em>1 = 0.475 \left( \frac{d</em>{23,1} + d_{23,2}}{2} \right) + 0.525 )</td>
</tr>
<tr>
<td>span2</td>
<td>( \text{span}<em>2 = 0.475 \left( \frac{d</em>{23,2} + d_{23,3}}{2} \right) + 0.525 )</td>
</tr>
<tr>
<td>H</td>
<td>( H = 0.5 \left( \frac{d_{23,2} + d_{23,3}}{2} \right) + 0.5 )</td>
</tr>
<tr>
<td>span3</td>
<td>( \text{span}<em>3 = 0.475 \left( \frac{d</em>{23,2} + d_{23,3}}{2} \right) + 0.525 )</td>
</tr>
</tbody>
</table>

Table 4. Relation variable weight for the left torque output of the fuzzy system to be optimized

\[
MSE = \frac{1}{N} \sum_{k=1}^{N} (y(k) - \hat{y}(k))^2
\]  
(7)

Where:

- \( y(k) \) = Reference value at instant \( k \)
- \( \hat{y}(k) \) = Computed output of the system at instant \( k \)
- \( N \) = Number of samples considered

Since the system is responsible for controlling the linear \( (v) \) and angular \( (w) \) velocities of the plant, the overall error is given by:

\[
MSE_v = \frac{1}{N} \sum_{k=1}^{N} [v(k) - \hat{v}(k)]^2
\]

\[
MSE_w = \frac{1}{N} \sum_{k=1}^{N} [w(k) - \hat{w}(k)]^2
\]

\[
Error_{global} = MSE_v + MSE_w
\]

This was used to represent the entire length of each ant-generated graph.

6. ASRank+ConvCont for the membership functions’ optimization

Due to the nature of the problem, we do not have any features’ heuristic information to balance between the influence of the a priori knowledge that we have of the problem and the pheromone trails that ants have
generated; thus, the dynamic variation of the parameter alpha had a null effect on the convergence of the algorithm when applied to the optimization of membership functions (Figure 10).

It was decided to continue with the same strategy of convergence control, but this time by varying the evaporation rate ($\rho$) and the weight to be given to the amount of pheromone that each ant leaves on its trail ($w$) to control diversity. As such, a fuzzy system was implemented for this task.

The controller uses as inputs the error ($e$) and change of error ($ce$) with respect to an average lambda branching factor reference level (Figure 10) and provides as output the evaporation rate corresponding to arcs which belong ($\rho_{bs}$) and do not belong ($\rho$) to the best-so-far tour, in addition to an increase in the weight that is given to the pheromone increment of the arcs that form part of the best-so-far tour ($\rho_{bs}$) and the remaining arcs ($u$) in ASRank (Figure 12).

![Figure 10. Membership functions of the input variables of the fuzzy system proposed to control the convergence of the ACO algorithm without heuristic information.](image)

Again the rules were created with the intention to keep the average lambda branching factor at some level so as to slow the convergence process, as shown below in Figure 11:

- If (error is P) and (error_change is P) then $(\Delta_{bs} N) (\Delta_u P)$ $(\rho_{bs} P) (\rho N)$
- If (error is N) and (error_change is N) then $(\Delta_{bs} N) (\Delta_u P)$ $(\rho_{bs} P) (\rho N)$
- If (error is P) and (error_change is Z) then $(\Delta_{bs} N) (\Delta_u P)$ $(\rho_{bs} P) (\rho N)$
- If (error is N) and (error_change is Z) then $(\Delta_{bs} N) (\Delta_u P)$ $(\rho_{bs} P) (\rho N)$
- If (error is P) and (error_change is N) then $(\Delta_{bs} N) (\Delta_u P)$ $(\rho_{bs} P) (\rho N)$
- If (error is N) and (error_change is P) then $(\Delta_{bs} N) (\Delta_u P)$ $(\rho_{bs} P) (\rho N)$

![Figure 11. Rules of the proposed fuzzy system to control the convergence of the ACO algorithm without heuristic information.](image)

- If (error is Z) and (error_change is Z) then $(\Delta_{bs} N) (\Delta_u P)$ $(\rho_{bs} P) (\rho N)$
- If (error is Z) and (error_change is P) then $(\Delta_{bs} N) (\Delta_u P)$ $(\rho_{bs} P) (\rho N)$
- If (error is Z) and (error_change is N) then $(\Delta_{bs} N) (\Delta_u P)$ $(\rho_{bs} P) (\rho N)$
- If (error is Z) and (error_change is Z) then $(\Delta_{bs} N) (\Delta_u P)$ $(\rho_{bs} P) (\rho N)$

![Figure 12. Membership functions of the output variables of the fuzzy system proposed to control the convergence of the ACO algorithm without heuristic information.](image)

Thus, equations 2 and 4 corresponding to the evaporation and pheromone deposit process in ASRank become:

$$t_{ij}^{bs} \leftarrow (1 - \rho^{bs}) t_{ij}^{bs} \quad \forall (i,j) \in T^{bs}$$

$$t_{ij} \leftarrow (1 - \rho) t_{ij} \quad \forall (i,j) \in T$$

$$t_{ij} \leftarrow t_{ij}^{bs} + \sum_{r=1}^{w-1} \frac{w - r}{(w - 1)} \Delta t_{ij}^{bs} + u^{bs} b_{ij}^{bs}$$

$$\Delta t_{ij}^{bs} = \frac{1}{c} \text{ and } \Delta t_{ij}^{bs} = \frac{1}{c}$$

7. Simulation in the membership functions’ optimization problem

The model of the mobile robot and the path used in the simulations performed by the ACO algorithm are defined in [23].
The approach described in the previous section was able to maintain diversity in the required level (Figure 13), unlike the convergence controller that was tested in section 5.

Figure 13. Behaviour of the average lambda branching factor during the execution of the developed approach to control the convergence of the ACO algorithm without heuristic information.

30 experiments were performed with the proposed approach (Table 5) to compare the performance of classical approaches with the proposal developed. The following parameters were used:

<table>
<thead>
<tr>
<th>ACO</th>
<th>α</th>
<th>β</th>
<th>γ</th>
<th>m</th>
<th>τ₀</th>
</tr>
</thead>
<tbody>
<tr>
<td>AS</td>
<td>1</td>
<td>0</td>
<td>0.5</td>
<td>n</td>
<td>m/Cₙₙ</td>
</tr>
<tr>
<td>ASRank</td>
<td>1</td>
<td>0</td>
<td>0.1</td>
<td>n</td>
<td>0.5(r-1)/rCₙₙ</td>
</tr>
<tr>
<td>EAS</td>
<td>1</td>
<td>0</td>
<td>0.5</td>
<td>n</td>
<td>(e+m)/ρCₙₙ</td>
</tr>
<tr>
<td>ASRank+CONVCONT</td>
<td>1</td>
<td>0</td>
<td>dynamic</td>
<td>n</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Table 5. Parameters used for each ACO algorithm in the membership function optimization problem.

With the exception of ASRank, the average simulation results obtained were very similar. The proposal obtained the lowest average, but despite this it was EAS which generated the lowest MSE controller (Figure 15) and, therefore, a more accurate trajectory (Figure 14). Table 6 shows the best and average values achieved with the different variants of ACO.

<table>
<thead>
<tr>
<th>ACO</th>
<th>Best</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>AS</td>
<td>0.0015</td>
<td>0.0172</td>
</tr>
<tr>
<td>EAS</td>
<td>0.00013</td>
<td>0.0161</td>
</tr>
<tr>
<td>ASRank</td>
<td>0.00015</td>
<td>0.0572</td>
</tr>
<tr>
<td>ASRank+CONVCONT</td>
<td>0.00029</td>
<td>0.0131</td>
</tr>
</tbody>
</table>

Table 6. Results obtained by the proposal and each approach under review in the membership function optimization problem.

It is difficult to determine whether the proposal exceeded the classical approaches with the above analysis. Thus, a Z test for two samples’ means was performed so as to come to a conclusion (Table 7).

<table>
<thead>
<tr>
<th>Case</th>
<th>Null hypothesis (H₀)</th>
<th>Alternative hypothesis (Hₐ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>μAS ≤ μASRank+CONVCONT</td>
<td>μAS &gt; μASRank+CONVCONT</td>
</tr>
<tr>
<td>2</td>
<td>μEAS ≤ μASRank+CONVCONT</td>
<td>μEAS &gt; μASRank+CONVCONT</td>
</tr>
<tr>
<td>3</td>
<td>μASRank ≤ μASRank+CONVCONT</td>
<td>μASRank &gt; μASRank+CONVCONT</td>
</tr>
</tbody>
</table>

Table 7. Null and alternative hypothesis for the statistical hypothesis testing performed for the membership function optimization problem.

No statistical evidence was found with a significance level of 5% such that the average of AS or EAS is greater than the average of ASRank+CONVCONT (Figures 16.a, 16.b).
Figure 16. Results of the statistical hypothesis testing performed for a) AS vs. ASRank+ConvCont, b) EAS vs. ASRank+ConvCont, c) ASRank vs. ASRank+ConvCont for the membership functions optimization problem.

With a significance level of 5%, only statistical evidence that the average of the results of simulations of AS Rank is greater than AS Rank+CONVCONT was found (Figure 16.c); that is, the proposal was only able to outperform the ASRank variant.

8. ASRank+ConvCont vs. S-ACO

The results obtained with the proposed method were compared with those obtained in [5] which considered the same membership function optimization problem for the same fuzzy trajectory controller and unicycle mobile robot model. The difference lies in S-ACO as strategy used to solve the problem and the directed graph of 12 nodes chosen to represent it.

<table>
<thead>
<tr>
<th>Method</th>
<th>Best</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>ASRank+CONVCONT</td>
<td>0.00029</td>
<td>0.0131</td>
</tr>
<tr>
<td>S-ACO</td>
<td>0.0982</td>
<td>0.1199</td>
</tr>
</tbody>
</table>

Table 8. Performance obtained by ASRank+CONVCONT and S-ACO in the membership function optimization problem.

At first glance, it can be observed that the best result of ASRank+CONVCONT was significantly lower than S-ACO as well as the average of the results obtained in the experiments (Table 8). This is reflected in the path generated by each controller (Figure 17). Therefore, we conclude that its performance is higher.

To support the assertions made above, a t-test for means of two samples was performed, for which it took a random sample of 10 experiments per technique to compare their performance.

The null hypothesis claims that the average of S-ACO is less than or equal to ASRank+CONVCONT.

\[ H_0: \mu_{S-ACO} \leq \mu_{ASRank+ConvCont} \]

\[ H_1: \mu_{S-ACO} > \mu_{ASRank+ConvCont} \]

\[ \alpha = 0.05 \]

Figure 18. Results of the statistical hypothesis testing performed for a) S-ACO vs. ASRank+ConvCont.

Since t is located at the rejection zone with a significance level of 5% and 9 degrees of freedom, there is sufficient statistical evidence to prove that the average of S-ACO is greater than ASRank+CONVCONT (Figure 18); that is, the developed approach outperformed the method used by [5] and, therefore, likewise with AS and EAS.

9. Conclusions

Maintaining diversity is important for good performance in the ACO algorithm. An adaptive control strategy of the parameter alpha for this purpose was used, which was embodied in a diversity fuzzy controller which allows the
avoidance of or else delays total convergence and, thereby, controls the exploration and exploitation capabilities of the algorithm.

An improvement was observed by dynamically changing the parameter alpha value, as was seen in the statistical analysis performed where our approach outperformed the classical strategies.

It was found that the parameter alpha is not appropriate when there is no heuristic information to guide the search, as was the case with the optimization of the membership functions. In this case it is not possible to balance between the previous knowledge of the problem and that generated by the algorithm itself during its execution. Thus controlling the convergence of the algorithm is difficult. As such, it was decided to continue with the same strategy for this kind of problem while varying the evaporation rate and the weight that is given to the amount of pheromone that each ant deposited, allowing the control of the convergence of the algorithm without heuristic information. This modification improved the performance of AS\textsubscript{Rank}. However, since this variant scored the lowest performance, it is probably not appropriate in these cases.

The formulated strategy was outperformed by AS and EAS in the membership functions’ optimization problem but managed to outperform the method developed in [5]. As such, it was concluded that the improvement could not come from the convergence control made and it is attributed to the way in which the problem was encoded.

As future work, we intend to apply convergence control to other variants of the ACO algorithm and modify the reference - and thus diversity - in an intelligent way, depending upon the search progress or some other performance measure. We will look for heuristic information relevant to the membership functions’ optimization problem that drives the search process in early iterations of the algorithm, making it possible to use the strategy of the dynamic variation of the parameter alpha and an analysis in the presence of any noise of the controller generated by the ACO algorithm.

10. References


