Smooth Jerk-Bounded Optimal Path Planning of Tricycle Wheeled Mobile Manipulators in the Presence of Environmental Obstacles

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Abstract In this work, a computational algorithm is developed for the smooth-jerk optimal path planning of tricycle wheeled mobile manipulators in an obstructed environment. Due to a centred orientable wheel, the tricycle mobile manipulator exhibits more steerability and manoeuvrability over traditional mobile manipulators, especially in the presence of environmental obstacles. This paper presents a general formulation based on the combination of the potential field method and optimal control theory in order to plan the smooth point-to-point path of the tricycle mobile manipulators. The nonholonomic constraints of the tricycle mobile base are taken into account in the dynamic formulation of the system and then the optimality conditions are derived considering jerk restrictions and obstacle avoidance. Furthermore, by means of the potential field method, a new formulation of a repulsive potential function is proposed for collision avoidance between any obstacle and each part of the mobile manipulator. In addition, to ensure the accurate placement of the end effector on the target point an attractive potential function is applied to the optimal control formulation. Next, a mixed analytical-numerical algorithm is proposed to generate the point-to-point optimal path. Finally, the proposed method is verified by a number of simulations on a two-link tricycle manipulator.

Keywords Tricycle mobile manipulator, Potential field method, Jerk-bounded optimal control

1. Introduction

Wheeled mobile manipulators are manipulator systems mounted on mobile platforms. These robots have been applied in a number of applications due to their ability to work in an extended workspace [1-3]. Therefore, there have been numerous techniques to study several aspects
of such systems, especially in the fields of control and path planning [4, 5]. On the other hand, in addition to the classical view of the control of such systems, many heuristic and meta-heuristic algorithms have been developed for mobile robot control, such as fuzzy logic [6, 7], artificial neural networks [8], swarm intelligence [9] and genetic algorithms [10]. Indeed, the path planning of mobile manipulator systems is a complex task, and it becomes more complicated in cases where the point-to-point motion planning of the system in an obstructed environment is the aim. Therefore, the collision-free path planning of such systems is of particular importance, and it has been treated by several researchers over the last three decades. Khatib [11] proposed the potential field method for the obstacle avoidance of mobile robots. This method is based on filling the robot’s workspace with an artificial potential field in which the mobile robot is repulsed away from the obstacles and attracted to its target position. This technique is efficient mathematics and is simple; however, it suffers from some inherent limitations, which were thoroughly studied in [12]. Amato and Wu [13] described a collision avoidance algorithm based on the graphical approach for the path planning of robot manipulators. Here, a graph was constructed via unobstructed straight lines joining through the vertices of obstacles and then a search algorithm was used to find the shortest path among the all the available graphs. Thus, in their method the robot workspace was restricted to a set of paths, but the complicated search and its low efficiency were the main problems of this method. Papadopoulos et al. [14] presented an analytical method for the path planning and obstacle avoidance of nonholonomic mobile manipulators. They used a polynomial as a solution of a path, for which its order was increased so as to avoid the obstacles. However, the need to set a fixed-order polynomial as the solution was the disadvantage of their method. In another research work, Gonzalez et al. [15] studied the dynamic motion analysis of a wheeled mobile robot considering its obstacle avoidance capability. However, in this work only the kinematic model of the system was considered and actuator limitations were not taken into account, which may cause some problems in its application. Selekwa et al. [16] developed a fuzzy logic method for planning the path of a holonomic mobile robot in the presence of a point obstacle. Moreover, a neural network approach was presented in [17] for the obstacle avoidance of mobile robots. The main drawback of these intelligent algorithms is that they are not able to fix the local minima problem; they could not achieve a smooth solution in a complex environment.

In fact, to increase the productivity and efficiency of the mobile manipulator, it advised that the system transverses on the optimal path. Methods for the obstacle avoidance of mobile manipulators have been researched extensively over the past half century. However, attention to the path planning of such systems in cluttered environments has been increased over the past decade. Wu et al. [18] studied time optimal path planning for a wheeled mobile manipulator, considering its kinematic and dynamic constraints. They first carried out the path planning under kinematic constraints, which resulted in a shortest path composed of circular arcs and straight lines. Then, they generated the time optimal velocity profile under dynamic constraints. The weakness of this method is that the optimal path is limited to a set of geometric profiles and it cannot generate a smooth path near the connection to an adjacent profile. Mohri et al. [19] presented a sub-optimal trajectory planning of a mobile manipulator. Macfarlane and Croft [20] proposed an algorithm for finding the real-time trajectory of a fixed manipulator considering jerk bounds. Constantinescu and Croft [21] presented smooth and time-optimal trajectory planning for industrial manipulators along specified paths. They studied the upper bounds on the rate of torque variation, but in their work the third-order dynamic of the manipulator must be modelled, which may cause more data generation during the optimal process. Korayem and Garibli [22] presented a computational method for the trajectory optimization of flexible mobile manipulators based on the iterative linear programming method. However, the linearizing of the procedure and the converging of the solution presented the significant challenges for their method. Gracia and Tornero [23] presented optimal trajectory planning for wheeled mobile robots based on kinematic singularity. However, they only considered the kinematic model of the system, which may cause problems in practical application because of a lack of inertia and torque constraints. Haddad et al. [24] proposed the optimal control problem for the motion planning of a planar mobile manipulator in a generalized point-to-point task. But since they employed the direct solution of optimal control, they had to parameterize and discretize the optimization problem, which may result in computational expense and numerical explosion. Recently, the trajectory planning of differential-drive mobile manipulators was presented in [25]. In this research work, obstacle avoidance was considered, but since the traditional mobile platform was employed the maneuverability of the system was decreased, especially in cluttered environments. Also, the effect of jerk on the trajectory planning of the system was not considered, which might not be a suitable assumption in real situations. Moreover, an indirect approach to optimal control was successfully employed for the path planning of mobile manipulators [26, 27]. However, in these works the path planning of the end effector were considered and the base trajectory was constrained to movement along the predefined path. Accordingly, the obstacle avoidance of the base was not considered in these works, which again may not be a sound assumption for such systems when working in real environments.
In this paper, a computational algorithm is developed for the smooth path planning of tricycle-type wheeled mobile manipulators in the presence of environmental obstacles. The model has a steerable wheel which makes it possible for the mobile manipulator to have a commendable manoeuvrability in the work space, especially in the presence of obstacles. A kinematic and dynamic model of the tricycle manipulator is presented and the trajectory optimization of system is formulated as an optimal control problem. A new formulation of potential functions is proposed as the cost function, which rectifies the prior limitations of the potential field method. Another contribution of the proposed algorithm is in considering the jerk limitation so as to generate a feasible and smooth path. Next, a solution to the optimal problem is applied based on Pontryagin's minimum principle, which leads to a two-point boundary value problem. The optimality conditions are solved numerically and simulations are performed on a two-link tricycle manipulator.

2. Problem Formulation

In this section, the general formulation of the proposed method for the optimal path planning of the tricycle manipulator in a cluttered environment is presented.

2.1. Formulation of the General Optimal Control Problem

Consider \( X(t) \) and \( U(t) \) as the state vector and control input vector of the system, respectively. The full nonlinear dynamic model of the system in state-space form can be obtained as:

\[
\dot{X}(t) = F(X(t), U(t), t)
\]

The state vector includes the generalized coordinate vector of the system, represented by \( X_1(t) = q \), the generalized velocity vector of the system shown by \( X_2(t) = \dot{q} \), such that the state vector \( X \) can be written as

\[
X = \begin{bmatrix} X_1^T & X_2^T \end{bmatrix}^T.
\]

Let us consider the dynamic equations of the system (Eq. 1) as constraints of the optimal control problem. The problem is to find the optimal value of the state vector \( X^* \) and the optimal value of the input vector \( U^* \), where the given objective function is minimized:

\[
J(X, u) = \int_{t_0}^{t_f} L(X(t), U(t), t) \, dt
\]  

To solve the problem, Pontryagin's minimum principle is employed. This analytical method is based on an indirect solution of the optimal control problem which does not require the discretizing of the problem. Another advantage of the method is that it can use any arbitrary objective function. The solution begins by forming the Hamiltonian function

\[
H(X(t), U(t), t) = L(X(t), U(t), t) + \Omega^T F(X(t), U(t), t),
\]

where \( \Omega \) is the co-state vector. After some analytical effort, the optimal conditions of the system can be stated as [28]:

\[
\dot{X}^*(t) = \frac{\partial H}{\partial \Omega}(X^*, U^*, \Omega^*, t) \quad (3)
\]

\[
\dot{\Omega}^*(t) = - \frac{\partial H}{\partial X}(X^*, U^*, \Omega^*, t) \quad (4)
\]

\[
0 = \frac{\partial H}{\partial U}(X^*, U^*, \Omega^*, t) \quad (5)
\]

where the subscript (*) indicates the optimal solution. Moreover, the application of Pontryagin's minimum principle states that the optimal values must minimize the Hamiltonian function:

\[
H(X^*, U^*, \Omega^*, t) \leq H(X, U, \Omega, t)
\]  

Thus, using the bounding values of the actuators, according to Eqs. 5 and 6, the input optimal condition can be rewritten as:

\[
U = \begin{cases} U^- & U^- < U^- \ \text{or} \ U^+ \leq U^- < U^* \ \text{or} \ U^+ < U^* \end{cases}
\]

The optimization problem is completed by defining the boundary conditions which are the initial and final configurations of the mobile manipulator. The aforementioned equations lead to the transformation of the problem of optimal control into a nonlinear two point boundary value problem. Numerical libraries offer numerous powerful and competent commands for solving such nonlinear problems. These commands operate by employing methods such as shooting, collocation and finite difference solving for the problem. In this research work, the bvp4c command in Matlab®, which is based on the collocation method, is employed to solve the problem.

2.2. Defining of the Objective Function

To optimal path planning of the mobile manipulator in the presence of the obstacles, determining the objective function is critical. In the proposed method, the objective function consists of four parts:

- The objective function related to the angular velocities of the joints, \( \| \dot{X}_2 \|_W = \| \dot{X}_2^T W X_2 \| \), where \( W \) is the symmetric, semi-definite weighting matrix.
The objective function related to the input vector $U \in \mathbb{R}^n$, where $R$ is the symmetric, definite weighting matrix.

The objective function defining the attractive potential function which enforces the end effector to a place at the final configuration, $L_{\text{att}}^2_{w_f}$. The objective function includes repulsive potential functions to ensure the avoidance by the mobile manipulator of obstacles, $L_{\text{rep}}^2_{w_{i,j}}$.

According to the above description, the proposed objective function is formed as:

$$J = \frac{1}{2} \int_{t_0}^{t_f} \left( \|X\|^2_{w_f} + \|U\|^2_{w_f} + \|L_{\text{att}}\|^2_{w_f} + \|L_{\text{rep}}\|^2_{w_{i,j}} \right) dt$$

By defining the distance between the end effector $X_E(X_E, Y_E)$ and the target point of the motion $X_{E,f}(x, y, t)$ as $d_f$, the attractive potential function can be formed as:

$$\|L_{\text{att}}\|^2_{w_f} = w_f d_f^2$$

where $w_f$ is the corresponding weighting coefficient and $d_f$ is defined as:

$$d_f = \left( (x_{E,f} - x_E)^2 + (y_{E,f} - y_E)^2 \right)^{1/2}$$

Now, by defining the distance between the $i^{th}$ environmental obstacle and the $j^{th}$ part of the mobile manipulator as $d_{i,j}$, the repulsive potential function is formed as:

$$\|L_{\text{rep}}\|^2_{w_{i,j}} = w_{i,j} d_{i,j}^2$$

where $w_{i,j}$ is the corresponding weighting coefficient. As is obvious from Eq. 11, the presented repulsive potential function is proportional to the inverse of the second order of the relative distance between the mobile robot and the obstacle. Therefore, decreasing the relative distance leads to an increase of the potential function and causes obstacle avoidance in the robot movement.

In this paper, a two-link manipulator mounted on top of the tricycle mobile platform - as depicted in Figure 1 - is studied for simulation. This mechanism consists of three parts: a mobile base and two links. To determine the distance between the $i^{th}$ obstacle and the mobile base, it is assumed that the mobile base is modelled by an artificial circle with the centre coordinate $G(x, y)$ and the radius $r_m$. So, for collision avoidance between the $i^{th}$ obstacle and the mobile platform, the distance $d_{i,1}$ is defined as:

$$d_{i,1} = \left( (x - x_{obi})^2 + (y - y_{obi})^2 \right)^{1/2} - r_m - r_{obi}$$

where $P(x_{obi}, y_{obi})$ and $r_{obi}$ (see Figure 1) are the coordinate and the radius of the $i^{th}$ obstacle in the Cartesian space, respectively. It is obvious that if $d_{i,1} = 0$, the mobile base collides with the obstacle. Let each manipulator link model as a line. Then, the distance between the $i^{th}$ obstacle and the first and the second link are obtained as:

$$d_{i,2} = \left| \begin{array}{c} l_1 \cos(\theta_0 + \theta_1) - x_{obi} \\ l_1 \sin(\theta_0 + \theta_1) - y_{obi} \\ l_2 \end{array} \right|$$

$$d_{i,3} = \left| \begin{array}{c} l_1 \cos(\theta_0 + \theta_1) - x_{obi} \\ l_1 \sin(\theta_0 + \theta_1) - y_{obi} \\ l_2 \end{array} \right|$$

2.3. Smooth-Jerk Optimal Path

In mobile robots, an increasing amount of jerk results in the slippage of the wheels, tracking errors and mechanical shocks to the actuators during point-to-point motion. Thus, introducing jerk constraints and imposing a jerk limit on the planning problem can increase the accuracy and efficiency of the solution. In this paper, in addition to the conditions defined by the cost function in Eq. 8, the optimal path of the mobile manipulator is planned considering the jerk criterion. Accordingly, an iterative algorithm - as shown in Figure 2 - is designed to consider the upper bound limits of jerk in the generation of the optimal path.

Since the jerk is the second derivation of the velocity of the system, it can be formulated as:

$$\Sigma = \dot{\dot{V}}$$
where \( \Sigma \) is the jerk vector and \( \mathbf{v} \) is the velocity vector of the system. In our proposed method, using first the optimal control defined in Eqs. 3-7 and according to the cost function obtained in Eq. 8, an initial optimal path in planned numerically. Then, the iterative algorithm is employed to include the effect of jerk in the planning process. For each iteration of the algorithm, the finite difference procedure is employed to determine the jerk of the system and the bounded formulation is used to compare the obtained value with the maximum allowable value of jerk.

Consider the value of the \( i \)th velocity variable of the system and its corresponding jerk as \( \Sigma_i \) and \( \Sigma_j \), respectively. According to Eq. 15, \( \Sigma_i \) can be obtained as

\[
\Sigma_i = \frac{d^2 v_i}{dt^2}.
\]

Now, we can rewrite this equation in the iterative form as:

\[
\Sigma_i(j) = v_i(j - 2) - 2v_i(j - 1) + v_i(j) \frac{(\Delta t_j)^2}{(\Delta t_j)^2}
\]

(16)

where \( j = 0, 1, \ldots \) is the step number of the iteration and \( \Delta t_j \) is the step size of time. Finally, in order to include the jerk effect in the path planning process, the jerk value obtained in the above backward difference method should bind the allowable maximum values of jerk:

\[
\Sigma_i(j) = \begin{cases}
-\Sigma_{i,\text{max}} & \text{if } \Sigma_i(j) < -\Sigma_{i,\text{max}} \\
\frac{v_i(j - 2) - 2v_i(j - 1) + v_i(j)}{\Delta t_j^2} & \text{if } -\Sigma_{i,\text{max}} \leq \Sigma_i(j) \leq \Sigma_{i,\text{max}} \\
\Sigma_{i,\text{max}} & \text{if } \Sigma_{i,\text{max}} < \Sigma_i(j) 
\end{cases}
\]

(17)

Note that in the above formulation, the maximum bound of the jerk, \( \Sigma_{i,\text{max}} \), is indicated by the designer. Figure 2 illustrates the flowchart of our proposed method.

3. Dynamic Modelling of the Tricycle Mobile Manipulator

In this section, a full dynamic model of a tricycle-type wheeled mobile manipulator consisting of a two-link manipulator mounted on a tricycle platform - as shown in Figure 3 - is presented. The platform has two fixed wheels and a centred orientable wheel. The right and left wheel are fixed to the mobile base but the centre wheel can rotate around a vertical axle passing through the centre of the wheel.
\[ \beta \] is the angle of rotation of the centred orientable wheel about the vertical axis. Also, the angle \( \alpha \) is defined as the angle between the axle \( X_0 \) and a line from the centre of each wheel to the mass centre \( G \). The necessary parameters of the wheels are summarized in Table 1:

<table>
<thead>
<tr>
<th>Wheels of tricycle</th>
<th>( \alpha )</th>
<th>( \beta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Right fixed wheel</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Left fixed wheel</td>
<td>( \pi )</td>
<td>0</td>
</tr>
<tr>
<td>Centred orientable wheel</td>
<td>( \frac{\pi}{2} )</td>
<td>( \beta )</td>
</tr>
</tbody>
</table>

Table 1: Parameters of the tricycle mobile platform

Each wheel of the tricycle platform has the following nonholonomic constraints:
- Each wheel can only move in the direction of its plane.
- Each wheel must exhibit pure rolling without any slippage.

Thus, the tricycle mobile base has six nonholonomic constraints regarding its wheels, which are expressed as [29]:

\[
J_1 R_0 \dot{\vec{\varphi}} + J_2 \dot{\phi} = 0 \\
C_1 R_0 \ddot{\varphi} = 0
\]  

(18)

where \( \vec{\varphi} = [x \ y \ \theta_0]^T \) is the mobile base posture vector, \( \dot{\phi} = [\dot{\phi}_1 \ \dot{\phi}_2 \ \dot{\phi}_3]^T \) is the rotation vector of the wheels around its axle, \( R_0 \) is the rotation matrix, \( J_2 \) is a diagonal matrix of the radii of the wheels, and each row of matrices \( J_1 \) and \( C_1 \) represent the pure rolling and nonlateral slippage, respectively. For each wheel, the \( i \)th rows of these matrices are:

\[
J(i) = \begin{bmatrix}
-\sin(\alpha + \beta) & \cos(\alpha + \beta) & l_0 \cos \beta
\end{bmatrix}
\]

\[
C(i) = \begin{bmatrix}
\cos(\alpha + \beta) & \sin(\alpha + \beta) & l_0 \sin \beta
\end{bmatrix}
\]  

(19)

Figure 2. Algorithm of the proposed method
Thus, using the parameters of Table 1, the matrices introduced in Eq. 18 can be written as:

\[
J_1 = \begin{bmatrix}
0 & 1 & l_0 \\
0 & -1 & l_0 \\
\cos \beta & \sin \beta & l_0 \cos \beta
\end{bmatrix}
\]

\[
J_2 = \begin{bmatrix}
r & 0 & 0 \\
0 & r & 0 \\
0 & 0 & r
\end{bmatrix}
\]

\[
C_1 = \begin{bmatrix}
1 & 0 & 0 \\
-1 & 0 & 0 \\
\sin \beta & -\cos \beta & l_0 \sin \beta
\end{bmatrix}
\]

\[
R_0 = \begin{bmatrix}
\cos \theta_0 & \sin \theta_0 & 0 \\
-\sin \theta_0 & \cos \theta_0 & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

(20)

Considering the kinematic model of the system, the generalized coordinates of the tricycle manipulator are defined as \( q = [x \ y \ \theta_0 \ \phi_1 \ \phi_2 \ \phi_3 \ \theta_1 \ \theta_2]^T \) and the nonholonomic constraints of the system can be rewritten as:

\[
A(q) \ \dot{q} = 0
\]

(21)

where the Jacobian matrix \( A \) represents the nonholonomic constraints of the system and it is equal to:

\[
A = \begin{bmatrix}
J_1 R_0 & 0 & J_2 & 0 & 0 & 0
\end{bmatrix} \begin{bmatrix}
C_1 R_0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}_{6 \times 9}
\]

(22)

For deriving the dynamic equation of motion, the total kinetic energy (\( T \)) and the potential energy (\( U \)) of the system must be computed. Then, by constructing the Lagrangian function (\( L = T - U \)) and following the Lagrangian approach, nonlinear equations of motion can be obtained. The Lagrangian equation can be formed as:

\[
\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} = Q_i - \sum_{j=1}^{6} \lambda_j a_{ji}
\]

(23)

where \( Q_i \) is the generalized force related to the generalized coordinate \( q_i \) and \( \lambda_j \) is the unknown force related to each nonholonomic constraint of the mobile base. Now, using the Lagrangian equation (Eq. 23), the dynamic equations of the system can be obtained in their compact form as:

\[
M \ddot{q} + V(q, \dot{q}) = BU - A^T \lambda
\]

(24)
where M is the inertia matrix, V is the vector of Coriolis and centrifugal forces in addition to the gravity effects vector, B is the input transformation matrix and $\lambda$ is the unknown force vector regarding the nonholonomic constraints. It should be noted that the mobile base has two actuators: the first actuator is employed to steer the centre orientable wheel and the second actuator is used for deriving one individual fixed wheel. In addition to the base actuators, each joint of the manipulator has a separate actuator to move in the work space. Accordingly, the matrix B is obtained as:

$$B = \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}_{9 \times 4}$$

(25)

Now, to eliminate the unknown forces from the dynamic equation of the system, the matrix S is defined in such a form that the following equation can be satisfied:

$$\dot{q} = S \dot{v}$$

(26)

where $\dot{v} = [\dot{\eta}_1 \ \dot{\eta}_2 \ \dot{\theta}_1 \ \dot{\theta}_2]^T$ is the velocity vector and the matrix S is represented as:

$$S = \begin{bmatrix}
R^T \Gamma & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
-J_2^{-1}J_1 \Gamma & 0 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}_{9 \times 4}$$

(27)

In the above equation, the vector $\Gamma$ is defined as $\Gamma = \begin{bmatrix} 0 & l_0 \sin \beta & \cos \beta \end{bmatrix}^T$. In addition, from Eq. (21) and (26) we can conclude that

$$AS = 0$$

(28)

Now, by differentiating Eq. 26 and substituting the results in Eq. 30, one can eliminate the vector $\lambda$ from the nonlinear dynamic equation of the system, as shown in Eq. 29:

$$\begin{bmatrix}
S^T & S^T V
\end{bmatrix} + S^T V = S^T B U$$

(29)

Finally, considering the state vector as $X = [q^T \ \dot{v}^T]_{3 \times 1}$, the nonlinear dynamic equations of the system in state-space form are obtained as:

$$\dot{X} = \begin{bmatrix}
S \dot{v}

\end{bmatrix} + \begin{bmatrix}
0

\end{bmatrix} U$$

(30)

4. Numerical Simulation and Discussion of the Results

In this section, numerical analysis is performed for a two-link tricycle mobile manipulator. The resulting two point boundary value problem is coded using the Matlab® library bvp4c [30]. The necessary parameter values of the system are summarized in Table 2:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass of the base</td>
<td>$m_b=6.0$</td>
<td>kg</td>
</tr>
<tr>
<td>Mass of the wheels</td>
<td>$m_w=5$</td>
<td>kg</td>
</tr>
<tr>
<td>Moment of inertia of the base about the Z axis</td>
<td>$I_b=6.61$</td>
<td>kg.m²</td>
</tr>
<tr>
<td>Moment of inertia of the wheels about the rotation axis</td>
<td>$I_w=0.131$</td>
<td>kg.m²</td>
</tr>
<tr>
<td>Length of the links</td>
<td>$l=0.5$</td>
<td>m</td>
</tr>
<tr>
<td>Distance from the wheels to the mass centre G</td>
<td>$l=0.145$</td>
<td>m</td>
</tr>
<tr>
<td>Radius of the wheels</td>
<td>$r=0.075$</td>
<td>m</td>
</tr>
<tr>
<td>Mass of the links</td>
<td>$m_l=5$, $m_m=3$</td>
<td>kg</td>
</tr>
<tr>
<td>Moment of inertia of the links</td>
<td>$I_1=0.416$, $I_1=0.121$</td>
<td>kg.m²</td>
</tr>
</tbody>
</table>

Table 2. Parameters of the tricycle manipulator

The actuator which is most commonly used for medium and small size manipulators is the permanent magnet DC motor. Using [31], the limits of control for such motors can be obtained as:

$$U^+ = k_1 - k_2 \dot{v}$$
$$U^- = -k_1 - k_2 \dot{v}$$

(31)

where the upper limits of the torque-speed are defined as $U^+ = [u_{r,\max} \ u_{c,\max} \ u_{l,\max} \ u_{2,\max}]$ and the lower limits are represented as $U^- = [u_{r,\min} \ u_{c,\min} \ u_{l,\min} \ u_{2,\min}]$. In the simulations carried out in the presented study, the actuator constants are chosen as:
\( K_1 = \begin{bmatrix} 20 & 20 & 34.67 & 12.21 \end{bmatrix} Nm \) 
\( K_2 = \begin{bmatrix} 1.12 & 1.12 & 6.45 & 2.4 \end{bmatrix} Nms/rad \) (32)

For the simulations, it is assumed that the tricycle mobile manipulator moves from the initial configuration 
\( X_i(\begin{array}{c} x = 0.5m, \ y = 0.5m, \ \theta_0 = 0 \text{rad}, \ \theta_1 = 0 \text{rad}, \ \theta_2 = \frac{\pi}{2} \text{rad} \end{array}) \) 
to the final configuration 
\( X_f(\begin{array}{c} x = 2m, \ y = 0.4m, \ \theta_0 = \frac{\pi}{8} \text{rad}, \ \theta_1 = \frac{\pi}{4} \text{rad}, \ \theta_2 = \frac{\pi}{2} \text{rad} \end{array}) \) 
at time \( t_f = 6s \). An obstacle is considered in the robot’s workspace with the centre at \( P_{ob} = (0.8, 0.2) \text{ m} \) and a radius of \( r_{ob_i} = 0.05 \text{ m} \). The weighting matrices are assumed as 
\( W = \text{diag}(0 \ ... \ 0 \ 1 \ 1 \ 1)_{13 \times 13} \) and
\( R = \text{diag}(1 \ 1 \ 1 \ 1) \). Also, the weighting coefficient of the attractive potential function is assumed as \( w_F = 1 \).

It is clearly observed in Figure 4 that if the repulsive potential function term assumes zero, the mobile manipulator does not sense the obstacle and, therefore, collide with it. However, when the repulsive potential function considers the nonzero value, the mobile robot can manoeuvre in the presence of the obstacle and does not collide with it.

Figure 5 shows the angular orientation \( \dot{\beta} \) (a) and the rotational velocity \( \dot{\phi}_3 \) (b) of the centred orientable wheel. As shown in this figure, the values of the angular orientation of the wheel in the presence of the obstacle are increased. This results from the fact that the mobile manipulator must manoeuvre more in the presence of the obstacle and that the tricycle mobile base enables the system to do this by changing the angular orientation (the prominence of the tricycle over the traditional mobile base).

The angular velocities of the fixed wheels and the joints of the mobile manipulator are shown in Figure 6. It is observed in the figure that the maximum velocities of the system in the case with a nonzero repulsive potential function are greater. This result is verified by the fact that in this case the mobile manipulator must pass a longer path in a given time in order to reach the final point in the presence of the obstacles.

Figure 4. Optimal path of the tricycle mobile manipulator
Figure 5. Angular orientation and rotational velocity of centred wheel

Figure 6. Optimal velocities of the tricycle mobile manipulator
The desired maximum jerk values of wheels and joints are considered as $\Sigma_{\text{max}} = 40 \text{rad/s}^3$ and $\Sigma_{\text{max}} = 20 \text{rad/s}^3$, respectively. The following figures show the jerk values of the systems:

![Figures showing the jerk values of the systems](image)

Figure 7. Jerk of the wheels and joints of the tricycle mobile manipulator

It is observed in Figure 7 that the maximum jerks of wheels and joints are as appear in the initial and final times of their motions. Also, this figure illustrates that the wheels are exposed to bigger jerks. Finally, the figure shows that the maximum jerks of the actuators are increased in the presence of the obstacle; however, these maximums are less than the desired value.

5. Conclusion

A hybrid approach has been proposed for the smooth optimal path planning of tricycle mobile manipulators in the presence of obstacles, which combines the optimal control theory and the potential field method. The nonlinear dynamic model of the system has been derived with respect to the nonholonomic constraints of the tricycle mobile platform. In the proposed method, the environmental obstacles have been avoided via a repulsive potential function. Moreover, an attractive potential function has been applied to the cost function, so as to ensure that the end effector moves into the desired target position. Also, the effect of the jerk limitation has been considered via a backward difference algorithm in order to generate a smooth path. The effect of the repulsive potential function term on the optimal path planning and obstacle avoidance has been simulated, the results of which have shown that increasing the corresponding parameters causes obstacle avoidance, while the maximum values of the velocities and the jerks of the wheels and joints have increased.
Furthermore, the results show that in the presence of obstacles, the orientable wheel of the tricycle exhibits more manoeuvrability, which demonstrates the applicability of this type of mobile base, especially in cluttered environments.

6. References

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>State vector</td>
</tr>
<tr>
<td>U</td>
<td>Input control vector</td>
</tr>
<tr>
<td>F(X(t), U(t), t)</td>
<td>Nonlinear dynamic equations of the system</td>
</tr>
<tr>
<td>q</td>
<td>Generalized coordinate vector of the system</td>
</tr>
<tr>
<td>v</td>
<td>Generalized velocity vector of the system</td>
</tr>
<tr>
<td>J(X, U)</td>
<td>Objective function of optimal control</td>
</tr>
<tr>
<td>( \Omega )</td>
<td>Co-state vector</td>
</tr>
<tr>
<td>U^+</td>
<td>Upper bound of the input vector</td>
</tr>
<tr>
<td>U^-</td>
<td>Lower bound of the input vector</td>
</tr>
<tr>
<td>W</td>
<td>Weighting matrix of the velocity vector</td>
</tr>
<tr>
<td>R</td>
<td>Weighting matrix of the control vector</td>
</tr>
<tr>
<td>L_att</td>
<td>Attractive potential function</td>
</tr>
<tr>
<td>w_f</td>
<td>Weighting coefficient related to the attractive potential function</td>
</tr>
<tr>
<td>L_rep</td>
<td>Repulsive potential function</td>
</tr>
<tr>
<td>w_{i,j}</td>
<td>Weighting coefficient corresponding to the repulsive potential function</td>
</tr>
<tr>
<td>t_0</td>
<td>Initial time of motion</td>
</tr>
<tr>
<td>t_f</td>
<td>Final time of motion</td>
</tr>
<tr>
<td>X_e</td>
<td>End effector coordinate</td>
</tr>
<tr>
<td>X_e,f</td>
<td>Target point of the end effector</td>
</tr>
<tr>
<td>d_f</td>
<td>Distance between the end effector and the target point</td>
</tr>
<tr>
<td>d_{i,j}</td>
<td>Distance between the i^{th} obstacle and the j^{th} part of the mobile manipulator</td>
</tr>
<tr>
<td>G(x, y)</td>
<td>Mass centre of the mobile platform</td>
</tr>
<tr>
<td>P_i(x_{ob_i}, y_{ob_i})</td>
<td>Centre coordinate of the i^{th} obstacle</td>
</tr>
<tr>
<td>r_{ob_i}</td>
<td>Radius of the i^{th} obstacle</td>
</tr>
<tr>
<td>( \theta_0 )</td>
<td>Heading angle of the mobile platform</td>
</tr>
<tr>
<td>( \theta_i )</td>
<td>Angular displacement of the i^{th} joint of the mobile manipulator</td>
</tr>
<tr>
<td>l_i</td>
<td>Length of the i^{th} link of the mobile manipulator</td>
</tr>
<tr>
<td>( \beta )</td>
<td>Rotation of the centred orientable wheel of the tricycle base about the vertical axis</td>
</tr>
<tr>
<td>( \phi_i )</td>
<td>Rotation of the i^{th} wheel around its axle</td>
</tr>
<tr>
<td>R_0</td>
<td>Rotation matrix</td>
</tr>
<tr>
<td>A(q)</td>
<td>Jacobian matrix of the nonholonomic constraints</td>
</tr>
<tr>
<td>T_i</td>
<td>Torque exerted to the i^{th} joint actuator</td>
</tr>
<tr>
<td>S(q, u)</td>
<td>Matrix in null space of A</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>( r )</td>
<td>Radius of each wheel of the mobile base</td>
</tr>
<tr>
<td>( m_w )</td>
<td>Mass of each wheel</td>
</tr>
<tr>
<td>( I_w )</td>
<td>Moment of inertia of each wheel about its axis</td>
</tr>
<tr>
<td>( m_c )</td>
<td>Mass of the tricycle mobile base</td>
</tr>
<tr>
<td>( I_c )</td>
<td>Moment of inertia of the mobile platform</td>
</tr>
<tr>
<td>( l_0 )</td>
<td>Distance from ( G ) to the centre point of the wheel</td>
</tr>
<tr>
<td>( m_i )</td>
<td>Mass of the ( i^{th} ) link</td>
</tr>
<tr>
<td>( I_i )</td>
<td>Moment of inertia of the ( i^{th} ) link about a vertical axis</td>
</tr>
</tbody>
</table>