A New Fuzzy Inference Technique for Singleton Type-2 Fuzzy Logic Systems

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Abstract A new fuzzy inference technique is presented to replace the conventional fuzzy inference process of type-2 fuzzy logic systems. Because conventional type-2 fuzzy logic systems demand a large amount of memory, they cannot be used by most embedded systems, which do not have enough memory space. To overcome this problem, a new fuzzy inference technique for singleton type-2 fuzzy logic systems is presented in this paper which designs mapping functions from input variables to firing sets and brings out the firing sets directly without using as much memory.

Keywords type-2 fuzzy logic, fuzzy inference, embedded system

1. Introduction

As premiere applications of artificial intelligence, fuzzy logic systems (FLSs) can handle human linguistic variables and reproduce the performance of human experts. Based on the use of linguistic variables and fuzzy reasoning, FLSs can efficiently approximate and describe complex, nonlinear and mathematically intangible systems [1]-[3]. However, type-1 fuzzy logic systems (T1FLSs) cannot handle uncertainties, such as measuring dispersions, linguistic uncertainties and distortions [4], [5]. To overcome the limitation of the T1FLSs, type-2 fuzzy logic systems are proposed, which can handle these uncertainties.

However, type-2 FLSs also suffer certain problems caused by system complexity. The increased dimension of type-2 fuzzy sets increases the complexity and computational cost of the system exponentially. Most researchers of type-2 FLSs have tried to simplify and approximate the original systems using interval sets [6]-[10], improved type-reduction methods [11], [12], and so on [13], [14]. These methods, however, cannot handle the full information of uncertainties and demand a large amount of memory to operate inference processes. If the sensors of the actual control systems come out with 8 bits of data, and the membership grades of fuzzy logic have 8 bit resolutions, then one type-2 fuzzy set demands $2^8 \times 2^8 \times 2^8 = 16\text{ MB}$ of memory for conventional type-2 FLSs. Because of the demand for memory, most embedded systems cannot use conventional type-2 fuzzy sets.

Type-1 fuzzy logic has been applied to many robotic systems because of its advantages in making use of linguistic information in a systematic way [15]-[18].
Moreover, type-2 fuzzy logic, which can describe the uncertainties caused by the non-ideality of sensors, motors and control strategies, can improve the performance of type-1 fuzzy logic systems. However, most robotic systems have a relative lack of memory and computational power, and the implementation of type-2 fuzzy logic in robotic systems is not reliable. In order to apply type-2 fuzzy logic in low-cost systems, this paper introduces a new inference technique that eliminates the need to store all type-2 fuzzy sets. Because the firing sets are obtained directly from the input variables in the fuzzy inference process, the fuzzy logic does not demand as much memory and can be applied to most low-cost systems with only a little memory.

The organization of the research is as follows: Chapter II presents the fuzzy inference engine of singleton type-2 fuzzy logic systems. Chapter III proposes the simple alternative type-2 fuzzy inference method. Chapter IV verifies the performance of the controller through simulation.

2. Fuzzy Inference Engine of Singleton Type-2 Fuzzy Logic Systems

As in the case of type-1 FLSs, type-2 FLSs also handle input signals after mapping to fuzzy sets. The only difference depends on whether or not the fuzzy system describes the uncertainties of the fuzzy sets. The fuzzy set \( A \) of a type-2 FLS is determined at the fuzzifier of the type-2 FLS, and can be denoted as:

\[
A = \frac{\int_{x \in X} \mu_A(x) \, dx}{\int_{x \in X} f_x(u) / u \, dx} \quad J_A \leq 0,1
\]  

(1)

where \( J_A \) is a primary membership of \( X \) and \( f_x(u) \) is a secondary grade. After the fuzzification process, the fuzzy inference engine combines the fuzzy IF-THEN rules to obtain an aggregated fuzzy output. The process of the fuzzy inference can be divided into antecedent, consequent and aggregation operations. This paper introduces a new simple technique to alternate the antecedent operation for singleton type-2 FLSs.

The antecedent operation of type-2 FLSs derives the firing set \( \mu_A(x_1, \ldots, x_p) \), which can be denoted as:

\[
\mu_A(x_1, \ldots, x_p) = \prod_{i=1}^{p} \mu_{\tilde{A}_i}(x_i) \prod_{i=1}^{p} \mu_{\tilde{A}_i}(x_i) \prod_{i,j=1}^{p} \mu_{\tilde{A}_{ij}}(x_i) \prod_{i,j,k=1}^{p} \mu_{\tilde{A}_{ijk}}(x_i)
\]  

(2)

when the \( l \)-th type-2 fuzzy rule is:

\[
R_l: \text{IF } x_1 \text{ is } \tilde{A}_{l1} \text{ and } \cdots \text{ and } x_p \text{ is } \tilde{A}_{lp}, \quad \text{THEN } y \text{ is } \tilde{G}_l \quad l = 1, \ldots, M.
\]  

Moreover, the fuzzifier of the singleton type-2 FLS maps crisp inputs \( x = (x_1, \ldots, x_p) \) into a singleton type-2 fuzzy set, such as:

\[
\mu_{\tilde{A}}(x) = \begin{cases} 
1/1 & \text{if } x = x' \\
1/0 & \text{if } x \neq x'.
\end{cases}
\]  

(4)

From (2) and (4), the firing set for the crisp input \( x = (x_1', \ldots, x_p') \) of the singleton type-2 FLS can be rewritten as:

\[
\mu_{\tilde{A}}(x_1', \ldots, x_p') = \left[ \mu_{\tilde{A}_1}(x_1') \prod_{i=2}^{p} \mu_{\tilde{A}_i}(x_i') \right] \left[ \prod_{i=1}^{p} \mu_{\tilde{A}_i}(x_i') \right] = \left[ (1/1) \prod_{i=1}^{p} \mu_{\tilde{A}_i}(x_i') \right] \left[ \prod_{i=1}^{p} \mu_{\tilde{A}_i}(x_i') \right] = \prod_{i=1}^{p} \mu_{\tilde{A}_i}(x_i')
\]  

(5)

which is a vertical slice for \( x = (x_1', \ldots, x_p') \) and a form of a type-1 fuzzy set.

3. Alternative Simple Method of Type-2 Fuzzy Inference

3.1 Uncertainty Representation of Type-2 Fuzzy Sets

The membership function of type-1 FLSs, which does not have any uncertainties, can be determined as shown in Fig. 1(a). However, the membership function of type-2 FLSs should be able to represent uncertainties, as shown in (1). The secondary grade, \( f_x(u) \), presents the uncertainties of mean, variance and scale factor. Assume that the primary membership function is a Gaussian function, as shown in Fig. 1(a), and that the secondary grade, \( f_x(u) \), does not have any uncertainties. Then, the type-2 membership function can be depicted as shown in Fig. 2:

If the secondary grades for these uncertain parameters are determined as shown in Figs. 1(b-d) respectively, the type-2 fuzzy sets that handle these uncertainties can be described as shown in Fig. 3. The images to the left are a 3-dimensional pictorial representation while the images to the right are the footprint of uncertainty (FOU) of their respective type-2 fuzzy sets.
3.2 Alternative Method of Type-2 Fuzzy Inference

As a firing level alters the consequent set for a fired rule in type-1 fuzzy logic, a firing set alters the consequent set for a fired rule in type-2 fuzzy logic. The firing set in type-2 fuzzy logic can be obtained by minimum or product t-norm complementary operation. The type-2 fuzzy logic with the previous inference technique should store all the type-2 fuzzy sets in order to obtain the firing set. The main issue of this paper is to obtain the firing set using less memory. This paper introduces a method to design mapping functions from the inputs X to the output firing sets. Using the mapping function, the type-2 fuzzy logic can obtain the firing set directly from input variances.

From (5), the derived firing sets of singleton type-2 FLSs are forms of type-1 fuzzy sets, and can be regarded as the vertical slice at \( x = (x'_1, \cdots, x'_p) \). When \( x = 7.0 \), the vertical slices of type-2 FLSs, which have the primary membership functions of Fig. 1(a) and the uncertainties of Figs. 1(b-d), can be described as shown in Fig. 4. If the input values of the system are given by \( x = 7.0 \), the mapping function in this paper should be determined as shown in Fig. 4.
The Gaussian primary membership function can be denoted as:

$$\mu_a(x) = \alpha \exp \left[ -\frac{(x-m)^2}{2\sigma^2} \right]$$  \hspace{1cm} (6)

where \(m\) is a certain mean, \(\sigma^2\) is a certain variance and \(\alpha\) is a certain scale factor of amplitude. The primary membership function can have three uncertainties of mean, variance and scale factor. Corresponding to the values of the uncertainties, the secondary grade can be designed as shown in Figs. 1(b-d), and the mapping functions from the input \(X\) to the firing sets can be induced as stated below.

3.3 Uncertain Mean

From (6), the certain mean can be rewritten as:

$$m = x \pm \sqrt{-2\sigma^2 \ln \left( \frac{\mu_a}{\alpha} \right)}.$$  \hspace{1cm} (7)

Substituting (7) into the Gaussian function of Fig. 1(b), which means the secondary grade for the uncertainty of the mean variable, the output firing sets for the input variable \(X\) can be rewritten as:

$$\mu_{f_a}(x) = \exp \left[ -\frac{(x \pm \sqrt{-2\sigma^2 \ln \left( \frac{\mu_a}{\alpha} \right)} - m)^2}{2\sigma^2} \right]$$  \hspace{1cm} (8)

where \(m_a\), \(\sigma_a^2\) and \(\alpha_a\) are the mean, variance and scale factor of the secondary grade for the uncertainty of the mean variable, respectively.

3.4 Uncertain Variance

From (6), the certain variance can be rewritten as:

$$\sigma^2 = -\frac{(x-m)^2}{2\ln \left( \frac{\mu_a}{\alpha} \right)}.$$  \hspace{1cm} (9)

Substituting (9) into the Gaussian function of Fig. 1(c), which means the secondary grade for the uncertainty of the variance variable, the output firing sets for the input variable \(X\) can be rewritten as:

$$\mu_{f_\sigma}(x) = \exp \left[ -\frac{(x-m)^2 \ln \left( \frac{\mu_a}{\alpha} \right) - m_a)^2}{2\sigma_a^2} \right]$$  \hspace{1cm} (10)

where \(m_\sigma\), \(\sigma_\sigma^2\) and \(\alpha_\sigma\) are the mean, variance and scale factor of the secondary grade for the uncertainty of the variance variable, respectively.

3.5 Uncertain Scale Factor

From (6), the certain scale factor can be rewritten as:

$$\alpha = \frac{\ln \left( \frac{\mu_a}{\alpha} \right)}{\exp \left[ -\frac{(x-m)^2}{2\sigma^2} \right]}.$$  \hspace{1cm} (11)

Substituting (11) into the Gaussian function of Fig. 1(d), which means the secondary grade for the uncertainty of the scale factor variable, the output firing sets for the input variable \(X\) can be rewritten as:

$$\mu_{f_\alpha}(x) = \exp \left[ -\frac{(x-m)^2 \ln \left( \frac{\mu_a}{\alpha} \right) - m_a)^2}{2\sigma_a^2} \right]$$  \hspace{1cm} (12)

where \(m_\alpha\), \(\sigma_\alpha^2\) and \(\alpha_\alpha\) are the mean, variance and scale factor of the secondary grade for the uncertainty of the scale factor variable, respectively.

4. Simulation

The concept of type-2 fuzzy sets (T2FSs) was first introduced as an extension of the well-known ordinary fuzzy set, i.e., type-1 fuzzy sets (T1FSs). The type-1 fuzzy controllers are unable to directly handle rule uncertainties because they use T1FSs that are certain. On the other hand, T2FSs are very useful in circumstances where it is difficult to determine an exact membership function. However, T2FSs demand significant memory to operate inference processes. The suggested fuzzy inference technique for singleton type-2 fuzzy controllers relieves the demand for memory. This section confirms the good performance of the suggested fuzzy inference technique, as well as the usefulness of the type-2 fuzzy controller.

4.1 Simulation Design

With reference to a commonly used ball-cart control system, the simulations are executed with MATLAB 2009a. Figure 5 shows the graphical description of the ball-cart simulator and control parameters. In the simulations, a ball is placed on a curved surface of a moving cart and the objective of the controller is to move the cart so that the ball is balanced on top of the cart and the cart is simultaneously placed at a predetermined position.

In the simulations of the type-2 fuzzy controller, only the fuzzy sets which describe the ball angle of Fig. 5 are determined by type-2 fuzzy sets, as shown in Fig. 6. Because of the incompleteness of the angle-measuring sensors, the measured ball angle may be subject to certain undesired errors, such as bias error or random noise error. In this case, the measured ball angle can be described by the type-2 fuzzy sets with uncertain mean, as shown in Fig.
6. The primary fuzzy sets of Fig. 6 denote linguistic meanings - such as L: Left, C: Centre, R: Right - and are determined as the Gaussian function whose variance value is 2.0 degrees. Further, the secondary fuzzy sets, which describe the uncertain mean of the measured ball angle, are determined as those Gaussian function whose variance value is 0.4 degrees.

Figure 5. The ball-cart simulator and control parameters.

Figure 6. Representation of the type-2 fuzzy sets which describe the ball angle: (a) 3-dimensional pictorial representation of the type-2 fuzzy sets, (b) footprint of uncertainty (FOU) of the type-2 fuzzy sets.

4.2 Simulation Result

The simulations are executed with three kinds of fuzzy controllers: type-1, conventional type-2 and suggested type-2 fuzzy controllers. However, only the simulation results with the type-1 fuzzy controller (T1FC) and type-2 fuzzy controller (T2FC) are shown in Figs. 7-9, because the performance of the type-2 fuzzy controller with the suggested fuzzy inference technique is exactly identical to that of the conventional T2FC. The only difference between the conventional T2FC and the suggested T2FC is the size of the demand on memory to store fuzzy sets. The measuring resolution of the ball angle sensor is 136 and the membership grades of fuzzy logic have 8 bit resolutions. The memory demands of the three fuzzy controllers are shown in Table 1. Although the performance of the suggested T2FC is identical to that of the conventional T2FC, the memory demand of the suggested T2FC is much smaller than that of the others because the fuzzy inference technique in this paper does not need to store any antecedent fuzzy sets.

<table>
<thead>
<tr>
<th>Controller</th>
<th>Memory Size (MB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type-1 Fuzzy Controller</td>
<td>0.166</td>
</tr>
<tr>
<td>Conventional Type-2 Fuzzy Controller</td>
<td>8.633</td>
</tr>
<tr>
<td>Suggested Type-2 Fuzzy Controller</td>
<td>0.133</td>
</tr>
</tbody>
</table>

Table 1. Memory demand to store fuzzy sets

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cart Force</td>
<td>-25 kgf.m/s ~ 25 kgf.m/s</td>
</tr>
<tr>
<td>Cart Position</td>
<td>-2.5m ~ 2.5m</td>
</tr>
<tr>
<td>Ball Angle</td>
<td>-50 ° ~ 50 °</td>
</tr>
<tr>
<td>Ball Position</td>
<td>-2.5m ~ 2.5m</td>
</tr>
<tr>
<td>Curvature of Cart</td>
<td>0.5m</td>
</tr>
<tr>
<td>Radius of Ball</td>
<td>0.055m</td>
</tr>
<tr>
<td>Mass of Cart</td>
<td>1kg</td>
</tr>
<tr>
<td>Mass of Ball</td>
<td>1kg</td>
</tr>
</tbody>
</table>

Table 2. Physical data for simulation

Figure 7. The simulation results when the ball angle is measured exactly.

Figure 8. The simulation results when the measured ball angle has a bias error of 1.0 degrees.
Initially, the ball on the cart is placed with a ball angle of 5.0 degrees and the physical data for the simulation is shown on Table 2. Next, the fuzzy controller moves the cart so that the ball is balanced on top of the cart and the cart is simultaneously placed at the zero position. The simulations are executed in three cases: 1) no measuring error, 2) bias error, and 3) random noise error. Figures 7-9 show the simulation results for T1FC and T2FC, and the simulation results are described by the ball position of the ball-cart control system. Because the T2FC can handle the uncertainty in the controller design, the simulation result of the T2FC shows much better control performance than that of the T1FC, as shown in Fig. 7. The settling time of the T2FC is much shorter than that of the T1FC. Moreover, while the ball angle measuring sensor has a bias error of 1.0 degrees, the cart using the T2FC can be placed near the zero position, in contrast to the cart using the T1FC, as shown in Fig. 8. Because the T2FC can handle the uncertainty caused by the bias error of the ball angle measuring sensor, a control system with some bias error can be successfully controlled by the T2FC and the simulation result of the T2FC has a less steady state error. Finally, Fig. 9 shows the simulation results when the measured ball angle has the random noises of Fig. 9(a). In order to check the performances of the type-1 and type-2 controllers clearly, the root mean squares of the simulation results are measured in all simulations, as shown in Table 3. In all simulations, the RMS errors of the type-2 fuzzy controller are smaller than those of the type-1 fuzzy controller.

### Table 3. The Root Mean Square Error for Fuzzy Controllers

<table>
<thead>
<tr>
<th>Simulation</th>
<th>RMS Error for Type-1 Fuzzy Controller (m)</th>
<th>RMS Error for Type-2 Fuzzy Controller (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Measuring Error</td>
<td>0.0213</td>
<td>0.0211</td>
</tr>
<tr>
<td>Bias Error</td>
<td>0.1211</td>
<td>0.0635</td>
</tr>
<tr>
<td>Random Noise Error</td>
<td>0.0216</td>
<td>0.0214</td>
</tr>
</tbody>
</table>

5. Acknowledgments

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6. References


