A New Modeling of the Non-Linear Inductances in MATLAB

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1. Introduction

In this chapter, we present a new approach for modeling the non-linear inductances by an analytic expression under the MATLAB-SIMULINK® code.

The current representation of these nonlinear inductances under MATLAB-SIMULINK® based on the introducing point by point (by a Lookup Table bloc in SIMULINK) [3]-[7] the values of $\Phi (i)$ deduced from the values of the magnetizing curve B-H and the geometric parameters of the corresponding portion of the magnetic circuit.

This new approach is based on a determination of analytic expressions of the non linear inductances deduced by a new analytic fitting of the nonlinear B-H curve. We have validate this new approach using the high voltage circuit of the power supply for magnetrons simulated by MATLAB-SIMULINK® code at the nominal state using the ferromagnetic material SF19. The comparison between experimental and simulated currents and voltages shows that the simulation curves match nearly with the experimental measurements (an error approximately one percent: 1%) [6].

This approach allowed us to give a general equivalent model for an eventual HV power supply (single and three phases) of the microwaves generators with N magnetrons (N = 1, 2,... ….10) for the industrial applications. In other words, we were able to modeling a HV transformer (single and three phases) with magnetic shunts able to supplying N magnetrons on its secondary. Thus, to define a general strategy for multicriteria optimization of this HV power supply with N magnetrons. This approach can also solve many problems of modeling, simulation and optimization of the electrics machines in general [8].

To summarize, this chapter is organized as follows. In the second section, we remind of the actual modeling of the nonlinear inductances under MATLAB-SIMULINK code. Section III presents the new model based on a hybrid method of fitting the magnetizing curve B-H in
order to derive analytic expressions of the nonlinear inductances. In the last one a conclusion.

2. Actual modelling of the nonlinear inductances under MATLAB-SIMULINK code

In the following of this chapter, we take as example the equivalent circuit of the HV power supply of microwaves generators used for the industrial applications. The three nonlinear inductances (primary, secondary and shunt) of the special HV power transformer of this alimentation (model in Fig.1) are function of the reluctances of the magnetic circuit portion which it represents. Each one of them is represented by its characteristic $\Phi(i)$ outcome of the relation, $L(i) = (n_2^2 \Phi(i) / i)$ which can be determined from the magnetization curve $B(H)$.

![Figure 1. Equivalent circuit of HV power supply for microwaves generators](image)

Not having analytical expression representing this $B(H)$ curve for high values of the magnetic field $H$, we have introduced point by point the values of this curve using a linear interpolation between two consecutive points in the iterations of the code under SIMULINK. However, this method of interpolation can lead in case of temporal simulations including the traditional models of transformers to a numerical instability as quoted in [1]-[9]. A specific routine was elaborated in MATLAB to deduce the values couple $(i, \Phi)$ from those $(H, B)$ and geometrical data of three inductances. The implementation of each nonlinear inductance of this circuit under SIMULINK software was realized by using the following blocks (Fig. 2):

![Figure 2. Implementation of each nonlinear inductance under SIMULINK, a: Voltage measurement, b: Integrator, c: Lookup Table, d: current source](image)
- An integrator to derive the flux from the voltage.
- A Lookup Table function, which contains a big number of N points (N=100 points) relating to flux and currents deduced from the code previously quoted.
- A source of current imposed.

To ensure the convergence of nonlinear model simulation under SIMULINK, the simulation steps which is a crucial problem in the numerical simulations has been appropriately chosen $T_e=0.01\text{ ms}$ after many trials.

We superimpose in Fig.3A, Fig.3B the simulation results obtained by SIMULINK code with those simulated by EMTP under the same conditions. These shapes resulting from the two codes under nominal operation ($U_1=220\text{ V et } f=50\text{ Hz}$) are consistent with those obtained in
Figure 3. A. Simulation with EMTP and SIMULINK code: Forms of voltage waves (at the nominal operation); B. Simulation with EMTP and SIMULINK code: Forms of current waves (at the nominal operating)

practice (Fig.7), especially the magnetron current curves which respect the maximum current magnetron constraint (I_{peak}<1.2A) recommended by the manufacturer. Precisely, the current patterns resulting from SIMULINK are closer to practice than EMTP current patterns. Indeed, The current magnetron peak value reaches approximately -0.96 A (Fig.3.B.3) using SIMULINK code which is near -1 A from experimental results (Fig.7) while the peak value obtained from EMTP code equals -0.92 A. In general, between peak to peak values (Fig.3.A, Fig.3.B, Fig.7), the relative variations never exceed 8 % for EMTP code while those resulting from SIMULINK do not exceed 4%. The accuracy of the outcome resulting from SIMULINK can be justified by the large number of points
N=100=2x50 (including the negative values) used to feature B (H) in the table (see lookup table in Fig.2) while in our EMTP code version [18], we were limited to use a restricted number N=17. Taking into account the precision of the various data and acceptable tolerances on operation of the magnetron, modelling was considered to be satisfactory with the two codes.

### 3. New modelling of the nonlinear inductances

The field problem solution involving ferromagnetic materials is complicated by the nonlinear relationship between B and H. One of the problems encountered is the absence of single mathematical expression, to represent the magnetization curve characteristic over a wide range of magnetic fields, having a smooth variation of the incremental permeability. As well, in our previous works [2]-[3], the B-H curve introduced in the π old model of this special transformer of the HV power supply for magnetrons and based on a set of measurement data, is approximated under software tools (EMTP and SIMULINK) by several straight line segments connecting the points of measurements. However, the B-H curve obtained is not smooth at the joints of the segments and the slopes of the straight lines representing the permeability are discontinuous at these joints [4]-[5]. Hence, the accuracy of permeability computed using such an approximation of the B-H curve is limited by the number of the straight-line segments. In order to improve this π model of this special transformer, we present a more accurate modelling of the nonlinear B-H curve of ferromagnetic material used for fabrication (SF19). In fact, the nonlinear B-H curve is represented by analytic expressions considering two parts of this curve: the first one is the linear region which is fitted by a non integer power series and the second one representing the saturation region is approximated by polynomial representation.

#### 3.1. Fitting H-B curve by a hybrid method

A set of N discrete measurements data In and Φn or Hn-Bn of this special transformer (n=1,2,3,...N=100) is given as depicted in Fig.4. Two parts are considered for this analytic representation:

- The linear region is fitted by a non-integer power series [1]. It is based on selected powers of B (not generally integer) with positive coefficients giving a power series. This linear part of this curve can be expressed by the power curve:

  \[ H = \sum_{i} k_i B^{n_i} \text{ with } k_i>0, n_i>0, \text{ for all } i. \]  

An adequate procedure of determining initial ki and ni values from logarithmic plots is adopted ensuring that these initial estimates of the parameters will be positive. Once an initial estimate of parameters (ki, ni) has been obtained, the parameters are optimized using regression analysis to get the best fit which is defined as corresponding to the minimum sum of squares of absolute errors. The resulting analytic expression of H is given by:
for $H \leq H_{1B}$ \[ H(B) = 220.65B^{0.96} + 19.5B^{11} \] (2)

- The saturation region is fitted by a polynomial representation based on the method of least squares [5]-[1].

for $H > H_{1B}$ \[ H = a_1B + a_3B^3 + a_5B^5 \ldots + a_{2p+1}B^{2p+1} \] (3)

Using the ‘polyfit’ function in MATLAB boxes, we determine all parameters $a_i$. The best fit is obtained by choosing $p=3$. The resulting expression of $H$ is given as follows:

\[ H(B) = 62967B - 59157B^3 + 17475B^5 - 1409B^7 \] (4)

The choice of a hybrid method consisting of two analytic expressions is more adequate than only one analytic expression for fitting the whole magnetizing curve of this material, according to our attempts to estimate these coefficients. It is due to the fact that the fitting is better and uses a small number of terms in analytic expressions to be identified. To illustrate the accuracy of this procedure for modelling nonlinear characteristic of this ferromagnetic material (SF19), we superpose in Fig.4 the simulate curve $B-H$ obtained by the two analytic expressions and data measurements. It shows no discrepancies between the two curves.

![Figure 4. Magnetization curve SF19: ° ° ° data measurements, - - - - curve represented by non integer power series, - - - - curve represented by polynomial function.](image)

### 3.2. Analytic expressions of inductances

Finally, we determine the analytic expressions of inductances from this analytic approach for modelling the nonlinear characteristic $B-H$ and the transformer data. Two analytic expressions deduced for each inductance of this new model are given using the formulae:

For $B \leq 1.7225$ T

\[ L'_p = L_s = \frac{n_2\Phi_p}{i_p} = \frac{n_2^2BS_1}{Hl_p} = \frac{n_2^2S_1}{l_p} \cdot \frac{B}{H(B)} \]

\[ L'_p = L_s = 106338,46 \frac{B}{220.65B^{0.96} + 19.5B^{11}} \] (5)
For $B \geq 1.7225 \, T$

\[
L_p' = \frac{106338.46 - \frac{B}{62967B - 59157B^3 + 17475B^5 - 1409B^7}}{59157B^2 - 1409B^4}
\]

It is the same for the shunt inductance

For $B \leq 1.7225 \, T$

\[
L_{sh}' = \frac{n_2 \Phi_3}{H_{sh}} = \frac{n_2 \cdot 2 \cdot \Phi_{sh}}{H_{sh}} = \frac{2n_2 B S_3}{H_{sh}}
\]

\[
L_{sh}' = 138693 \frac{B}{H(B)}
\]

(6)

For $B \geq 1.7225 \, T$

\[
L_{sh}' = 138693 \frac{B}{62967B - 59157B^3 + 17475B^5 - 1409B^7}
\]

(7)

In the simulation of new model, the three nonlinear inductances (primary, secondary and shunt) are function of induction or flux. The implementation of each nonlinear inductance with its analytic expressions under SIMULINK software was realized by using the blocks shown in Fig.5. This model (or bloc) is valid for any nonlinear inductance. It is validated in [14], and shows a good agreement with the experimental results.

**Figure 5.** Implementation of the nonlinear inductance under MATLAB-SIMULINK

We superpose in Fig.6A and Fig.6B the simulation results of this new modelling, obtained by SIMULINK code with those obtained from old one by EMTP under the same conditions. These shapes resulting from the two codes under nominal operation ($U_1=220 \, V, \, f=50 \, Hz$) are consistent with those obtained in practice (Fig.7), especially the magnetron current curves which respect the maximum current magnetron constraint ($I_{peak} < 1.2A$) recommended by the manufacturer. Precisely, the current patterns resulting from the new modelling under SIMULINK are closer to practice than EMTP current patterns issues from old modelling (Fig.2). Indeed, The current magnetron peak value reaches approximately -1 A (Fig.6.B.3) using SIMULINK code which is identical to experimental results (Fig.7) while the peak value obtained from EMTP code equals -0.92 A. In general, between peak to peak values (Fig.6.A, Fig.6.B, Fig.7), the relative variations never exceed 8 % for EMTP code (with old model) while those resulting from SIMULINK with the new model do not exceed 1%. The accuracy of the outcome resulting from SIMULINK can be justified by determining analytic inductances of
this model. It leads to continuous slopes representing permeability while with the old model under EMTP introducing the B-H curve point by point and approximating it by several straight line segments, the slopes representing the permeability are discontinuous at the joints. Taking into account the precision of the various data (software) and acceptable tolerances in the operation of the magnetron, the validation of this improved modelling based on analytic inductances is considered to be more satisfactory than the old one.
4. Experimental results

We sought to integrate the transformer new model (with analytic expressions of inductances) in the circuit of the HV supply from the source to the magnetron (Fig. 1),
where we represented the tube microwave by the equivalent diagram deduced from its electric characteristic [2]-[3]-[6] which is formally similar to that of a diode of dynamic resistance $R = \frac{dU}{dI}$ neighbor of 350 Ohms and threshold voltage $E$ of about 3800 Volts. We validate this new model by carrying out tests that have been set up previously [2] on generator microwaves composed of the following elements (Fig.1):

- A high voltage transformer with magnetic shunt ratings: $f=50$ Hz, $S=1650$ VA, $U_1=220$ V, and, no-load $U_2=2330$ V $(r'_1=100 \ \Omega$ and $r_2=65 \ \Omega$, for $n_1=224$ and $n_2=2400$ turns).
- A cell composed of a condenser, its capacity $C=0,9 \ \mu F$ and a diode high voltage DHV.
- A magnetron designed to operate under an approximate voltage of 4000 V. To obtain its nominal power, it needs an average current $I_{moy}=300$ mA, but without exceeding the peak current which might destroy it ($I_{max}<1,2$ A). The experimental curves are presented in Fig.6. The first three curves represent the shapes of voltage quantities while the last ones represent current patterns.

Figure 7. Experimental forms curves of the voltages and currents waves (nominal operating)
5. Conclusion

From the Simulink block modelling a non-linear inductance by introducing point by point its values, we performed a new model in two steps:

**Step 1.** A program M-file was developed and gives us the final results of the B-H curve fitting.

**Step 2.** By performing a function 'MATLAB Function' that replace us the nonlinear inductance of the current passing through this inductance as shown in Fig.4.

This approach can solve many problems of modeling, simulation and optimization of the electrical networks and electric machines also [7].

Thus, an improved procedure for modeling the nonlinear characteristic B-H of the ferromagnetic materiel (SF10) of this special transformer supplying one magnetron has been presented. Starting with a discrete set of measurement data of B and H, we have obtained two analytic expressions of B-H curve in terms of non-integer power series for the linear part of this curve and polynomial representation for the saturation region. Then, these expressions have been used for improving the old modelling of nonlinear inductances into the new one based on the determination of analytic expressions of each inductance. We validate the new π model (with three nonlinear inductances) of the transformer with magnetic shunts, used in HV supply for magnetron by MATLAB-SIMULINK code in nominal mode. A comparison between the numerical results and experimental ones has shown a significant agreement.

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**References**


