Six-dimensional Hybrid Broadband Vibration Isolation Based on Singular Parallel Mechanisms

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Abstract Six-dimensional broadband vibration isolation is necessary for precision manufacturing, testing and assembly. A six-dimensional nearly zero-stiffness mechanism based on singular parallel mechanisms was proposed, which was taken as the main structure of the vibration isolator to reduce the system's natural frequency. Then control methods of configure maintenance and active vibration isolation were researched using feedback linearization and H-robust control means. Finally six-dimensional active and passive vibration isolation control simulations were carried out respectively. The simulation results show that the vibration isolator has higher six-dimensional broadband vibration isolation capability and feasibility.

Keywords Singular parallel mechanisms, six-dimensional hybrid broadband vibration isolation, H-robust control

1. Introduction

Currently, multi-dimensional vibration isolation is mainly classed as passive, [Ma et al., 2004], semi-active, [Zhu et al., 2008], active, [Chen et al., 2004; Yang et al., 2009; Hoque et al., 2011] and hybrid vibration isolation, [Zhang et al., 2007; Yang et al., 2007; Pu et al., 2010]. Passive vibration isolation decreases the system's natural frequency by use of rubber, air cushion, mechanical spring, etc., but these vibration transmitters have to supply support force of load, and the dilation range of any transmitter is limited, hence the system's natural frequency cannot be very low. Therefore, passive vibration isolation is not effective for low or ultra-low frequency vibration. Semi-active vibration isolation regulates damping force by controlling adjustable damping devices such as a magneto-rheological damper to suppress the harmonic peak, but the system's natural frequency is still not reduced, hence semi-active vibration isolation capability is limited. Active vibration isolation is realized by the use of closed-loop calibration; this method is extremely flexible, but active vibration isolation is effective only for low frequency vibration and its reliability and robustness are also very low.

Over recent years, more and more people have addressed hybrid vibration isolation methods in order to realize broadband vibration isolation. For example, a vibration
isolation system for a micro-manufacturing platform was set up in Zhang et al. [2007], which isolated middle and high frequency vibration by the use of air springs, and isolated low and ultra-low frequency vibration by the use of giant magnetostrictive actuators. A vibration isolation platform with eight pneumatic actuators was built up in Yang et al. [2007], which isolated low frequency vibration by controlling the air springs and isolated high frequency vibration by utilizing the low-pass characteristics of air. A three-dimensional vibration isolation system, actuated by air cylinders and voice coils, was set up in Pu et al. [2010], which realized middle and high frequency vibration isolation by utilizing the low-pass characteristics of air and isolated low frequency vibration by the use of voice coils. But all of these soft transmitters have to support force of load, hence the isolation of the frequency of passive parts of the system cannot be lower and reliability of vibration isolation cannot be higher.

In this paper, a six-dimensional nearly zero-stiffness mechanism is proposed and then a six-dimensional broadband vibration isolator is built up by the use of this mechanism, which can achieve very low natural frequency by holding the mechanism near singular configure, hence it can effectively isolate broadband vibration.

2. Six-dimensional nearly zero-stiffness mechanism

A 3-SS parallel mechanism is shown in Fig.1, the R pair of every branch link is fixed at the base and the S pair is fixed at the support platform. If the support platform plane is coincident with the base plane, which is defined by the three R pairs and parallel with XY-plane, the 3-SS parallel mechanism becomes singular. As shown in Fig.2, the six constraint screws $S_1, S_2, S_3$ are coplanar with each other, so some lines perpendicular to the six constraint screws exist which are parallel with Z-axis. In addition, some lines respectively parallel with X and Y-axis, and coplanar with the six constraint screws also exist. According to screw theory, under this singular configure, rotation motions around Z-axis and translation along X-axis and Y-axis of the support platform lose constraints, hence stiffness of the support platform with respect to the base is transiently zero, and near this singular configure, stiffness of the support platform with respect to the base is close to zero.

Figure 1. 3-RS parallel mechanism

Figure 2. The constraint screws of the support platform

A 6-SS parallel mechanism is shown in Fig.3, the S pair at the end of every branch link is fixed at the base, the S pair at the other end is fixed at the middle suspension fork. The six S pairs on the base must satisfy two conditions: firstly, the two lines connecting any two couples of the S pairs cannot be parallel with each other, secondly, the line connecting any couple of the S pairs cannot be parallel with the middle suspension fork plane. If the six SS branch links are parallel with Z-axis and perpendicular to the middle suspension fork plane, the 6-SS parallel mechanism becomes singular. As shown in Fig.4, the six constraint screws $S_1', S_2', S_3', S_4', S_5', S_6'$ are parallel with each other, hence some lines perpendicular to the six constraint screws and respectively parallel with X-axis and Y-axis exist, and some lines parallel with Z-axis and parallel with the six constraint screws also exist. According to screw theory, under this singular configure, rotation motions around Z-axis and translation along X-axis and Y-axis of the middle suspension fork lose constraints, hence stiffness of the middle suspension fork with respect to the base is transiently zero, and near this singular configure, stiffness of the middle suspension fork with respect to the base is close to zero.

Figure 3. 6-SS parallel mechanism

Figure 4. The constraint screws of the middle suspension fork
3. Six-dimensional hybrid vibration isolation scheme

While a definite load is placed on the support platform as shown in Fig.5, the support platform declines and deviates from zero-stiffness configuration. In order to hold the support platform near zero-stiffness configuration, as shown in Fig.6, a P pair is appended onto every branch link of the mechanism, thus the mechanism can be held near zero-stiffness and six-dimensional broadband vibration isolation can be realized by controlling the motion of all P pairs.

4. Dynamics model

Since the load mass is greater than the mass of all of the branch links and actuators, their mass can be neglected, and then only the mass and inertia of the support platform and middle suspension fork, and load are taken into account. Deformation of the support platform and middle suspension fork is also neglected and only deformation of all of branch links is taken into account.

According to Lagrange’s equation, neglecting the 2nd order coupling items of micro-motion and velocity, the dynamics model can be described as follows:

\[ \begin{align*}
M \ddot{x} + G + \Delta = J^T_s u + f_d \\
\dot{c} \ddot{c} + k \dot{c} \epsilon = -u \\
M \ddot{x} + cJ^T_s J_s \dot{x} + kJ^T_s J_s x = -cJ^T_s J_s \dot{x}_0 - kJ^T_s J_s x_0 - G - \Delta + f_d
\end{align*} \]

where \( x = [x, y, z, \alpha, \beta, \gamma, z_1, \alpha_1, \beta_1]^T \) is the coordinate vector of the support platform with respect to the inertial frame, here, \( x, y, z, \alpha, \beta, \gamma \) are the six-dimensional micro-motion coordinates of the support platform with respect to the inertial frame, \( z_1 \) is the micro-translation coordinate of the support platform with respect to the middle suspension fork along Z-axis, \( \alpha_1, \beta_1 \) are respectively the micro-rotation coordinates of the support platform with respect to the middle suspension fork around X-axis and Y-axis, \( u = [u_1, u_2, \ldots, u_9]^T \) is the input vector of the nine actuated joints, \( \epsilon = [\epsilon_1, \epsilon_2, \ldots, \epsilon_9]^T \) is the elastic deformation vector of the nine branch links, \( x_0 = [\delta_1, \delta_2, \delta_3, \vartheta_1, \vartheta_2, \vartheta_3]^T \) is the micro-motion coordinate vector of the base with respect to the inertial frame, \( M \) is the inertia matrix, \( G \) is the gravity vector, \( \Delta \) is the uncertainty of the system, \( f_d = [f_{d_1}, f_{d_2}, f_{d_3}, f_{d_4}, f_{d_5}, f_{d_6}, f_{d_7}, f_{d_8}, f_{d_9}]^T \) is the direct disturbance vector imposed onto the support platform, \( c \) is the damping coefficient of material of the branch links, \( k = E \lambda_0 / L_0 \) is the stiffness of material of the branch links, \( E \) is the elastic modulus of material of the branch links, \( \lambda_0, L_0 \) respectively are the sectional area and original length of the branch links, \( J_s \) is the Jacobian matrix between the driving velocity vector \( \dot{s} = [\dot{s}_1, \dot{s}_2, \ldots, \dot{s}_9]^T \) and \( \dot{x} \), \( J_{s0} \) is Jacobian matrix between the velocity vector \( \dot{x} \) and \( \dot{x}_0 \).
5. H- robust control

While the support platform of the six-dimensional vibration isolator is near the singular configuration, its stiffness is close to zero, and very great control force should be supplied. Its position accuracy is hard to be ensured and the direct disturbance will also make the support platform more unstable. Hence in this paper a H-robust control method is researched in order to achieve above multiple control objectives.

As for Eq. (1), suppose the position $x$ of the support platform is measurable and then define $J^* u$ as follows:

$$ J^* u = Mv + G $$

(4)

where $v$ is the aid control input of the system.

Substituting Eq. (4) into Eq. (1) yields

$$ \ddot{x} = v + M^{-1}F_d $$

(5)

where $F_d = f_d - A$

Adopt a coordinate transformation as follows:

$$ \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} x \\ \dot{x} \end{bmatrix} $$

(6)

where $\lambda = \text{diag} \{ \lambda_1, \lambda_2, \cdots, \lambda_6 \}$, $\lambda_1, \lambda_2, \cdots, \lambda_6 > 0$

Substituting Eq. (6) into Eq. (5) yields

$$ \dot{z} = Az + Bv + EF_d $$

(7)

where

$$ z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}, \quad A = \begin{bmatrix} -\lambda & I \\ -\lambda & \lambda \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ I \end{bmatrix}, \quad E = \begin{bmatrix} 0 \\ M^{-1} \end{bmatrix} $$

Define the aid control input $v$ and output $y$ as follows:

$$ v = Kz, \quad y = Cz $$

and then substituting them into Eq.(7) yields

$$ \begin{cases} \dot{z} = (A + BK)z + EF_d \\ y = Cz \end{cases} $$

(8)

**Definition** [9]. As for the following linear system

$$ \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} A & \tilde{B} \\ 0 & C \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} \tilde{C} \end{bmatrix} $$

(9)

where $\ddot{x}$ is the state vector of the system, $w$ is the external disturbance vector, $\dot{y}$ is the output vector. Then, the transfer function of the system can be described as follows:

$$ G_{yr}(s) = \tilde{C}(sI - \tilde{A})^{-1} \tilde{B} $$

Define $\text{H}-\text{norm}$ of $G_{yr}(s)$ as follows:

$$ \| G_{yr}(s) \|_\infty = \sup_{\alpha} \sigma_{\text{max}}(G_{yr}(j\alpha)) $$

it is actually peak of maximal singular value of frequency response of the system.

**Lemma** [10]. As for the linear system (9), the following two conclusions are equivalent:

1) The system is asymptotic stable, and $\| G_{yr}(s) \|_\infty < \gamma$;

2) A symmetric positive definite matrix $P$ exists which satisfies the following inequality:

$$ \tilde{A}^T P + P\tilde{A} + \gamma^{-2}P \tilde{B} \tilde{B}^T P + \tilde{C}^T \tilde{C} < 0 $$

**Theorem 1.** If the closed-loop system (8) is asymptotic stable and $\| G_{yr}(s) \|_\infty < \gamma$, a symmetric positive definite matrix $P$ and an appropriate dimension matrix $Y$ exist which satisfy the following inequality:

$$ \begin{bmatrix} AQ + QA^T + BY + Y^T B^T + \gamma^{-2}EE^T & QC \\
CQ & -I \end{bmatrix} < 0 $$

(10)

Then, the $\text{H}-\text{robust control law}$ of the system is as follows:

$$ v = Kz = YQ^{-1}z $$

(11)

**Proof:** according to the above lemma, if the closed-loop system (8) is asymptotic stable and $\| G_{yr}(s) \|_\infty < \gamma$, a symmetric positive definite matrix $P$ and an appropriate dimension matrix $K$ exist which satisfy the following inequality:

$$ P(A + BK) + (A + BK)^T P + \gamma^{-2}P \tilde{E} \tilde{E}^T P + C^T C < 0 $$

(12)

For the two ends of inequality (12), respectively left-handed and right-handed multiplying by $P^{-1}$ yields

$$ AP^{-1} + P^{-1}A^T + BK P^{-1} + P^{-1}K^T B^T + \gamma^{-2}EE^T + P^{-1}C^T CP^{-1} < 0 $$

(13)
Define \( Q = P^{-1} \), \( Y = KP^{-1} \) and then substituting them into inequality (13) yields

\[
AQ + QA^T + BY + Y^TB^T + \gamma^2 EE^T + QC^T CQ < 0
\]  \hspace{1cm} (14)

According to the Schur additional lemma, inequality (14) can be written as inequality (10). Proof is over.

Define

\[
\Gamma = \begin{bmatrix} J^T u - (Mu + G) \\ u \end{bmatrix}
\]

Take the optimum solution \( u^* \) of the following form (15) as the control input of the system (1).

\[
\min_{u} \Gamma^T N \Gamma
\]  \hspace{1cm} (15)

Where \( N \) is the weight matrix,

\[
N = \text{diag}\{N_1, N_2, \cdots, N_{18}\}, \quad N_1, N_2, \cdots, N_{18} > 0
\]

Thus, the optimum solution \( u^* \) must satisfy the following equation:

\[
\frac{\partial (\Gamma^T N \Gamma)}{\partial u} = 0
\]

then

\[
2(J^T u - (Mu + G))^T N_1 J^T u + 2u^T N_2 u = 0
\]  \hspace{1cm} (16)

here

\[
N_1 = \text{diag}\{N_1, N_2, \cdots, N_6\}, \quad N_2 = \text{diag}\{N_{10}, N_{11}, \cdots, N_{18}\}
\]

Handling Eq. (16) yields the optimum control law:

\[
u^* = (J^T N_1 J^T + N_2)^{-1} J^T N_1 (Mu + G)
\]  \hspace{1cm} (17)

6. Active vibration isolation simulation with direct disturbance

The structural and physical parameters of the six-dimensional vibration isolator are as follows:

- the radius of circumscribed circle of the support platform is \( r_s = 300 \text{mm} \), the radius of circumscribed circle of the middle suspension fork is \( R_b = 4000 \text{mm} \), the sectional area of every branch link is \( A_b = 1200 \text{mm}^2 \), the original lengths of the nine branch links are respectively \( L_1 = L_2 = L_3 = 3164.1 \text{mm} \), \( L_4 = 1000 \text{mm} \), \( L_5 = 600 \text{mm} \), \( L_6 = 1100 \text{mm} \), \( L_7 = 1200 \text{mm} \), \( L_8 = 770 \text{mm} \), \( L_9 = 850 \text{mm} \), the mass of the support platform and load is \( m_o = 100 \text{kg} \) and the rotary inertia of the support platform and load is \( J_m = J_o = J_w = 8 \text{ kg m}^2 \). The mass of the middle suspension fork is \( m_b = 30 \text{kg} \), the rotary inertia of the middle suspension fork is \( J_{b_1} = J_{b_2} = 2 \text{ kg m}^2 \), the elastic modulus and damping coefficient of material of every branch link are respectively \( E = 220 \text{GPa} \), \( c = 70 \text{N m}^{-1} \text{s}^{-1} \).

As for inequality (10), set

\[
N_1 = I_o, \quad N_2 = 0.0001I_o, \quad \gamma = 0.8
\]

and using the feasp solver of Matlab/LMI toolbox, solving inequality (10) yields

\[
K = Y Q^{-1} = \begin{bmatrix} 75.36 I_o & -606.31 I_o \end{bmatrix}
\]  \hspace{1cm} (18)

Combining Eq. (18) with Eqs. (11) and (17) can yield the active controller of the six-dimensional vibration isolator.

Impose the following six-dimensional direct disturbance onto the support platform:

\[
\begin{align*}
\text{Along X-axis, } f_{ax} &= 3 \sin 2 \pi t \text{ N} \\
\text{Along Y-axis, } f_{ay} &= 3 \sin 2 \pi t \text{ N} \\
\text{Along Z-axis, } f_{az} &= 3 \sin 2 \pi t \text{ N} \\
\text{Around X-axis, } f_{ax} &= 2 \sin 2 \pi t \text{ N m} \\
\text{Around Y-axis, } f_{ay} &= 2 \sin 2 \pi t \text{ N m} \\
\text{Around Z-axis, } f_{az} &= 2 \sin 2 \pi t \text{ N m}
\end{align*}
\]

Additionally, impose the vibration \( \delta_x, \delta_y, \delta_z, \theta_x, \theta_y, \theta_z \) from the ground onto the base along six-dimensional directions, whose linear amplitude is \( A_x = 5 \text{mm} \), angular amplitude is \( A_y = \frac{\pi}{180} \text{ rad} \), frequency is respectively 0.25Hz, 2Hz, 10Hz, 30Hz. By use of numerical simulation, vibration response of the support platform and actuated force needed are respectively shown in Fig.7 and Fig.8.

From Fig.7, vibration components \( x, y, z, \alpha, \beta, \gamma \) of the support platform with respect to the inertial frame are close to zero, hence it shows that good vibration isolation effects are achieved. Since the support platform and load need a larger supporting force, the coordinate \( z_i \) of the support platform with respect to the middle suspension fork approaches -2mm, but this error can’t rapidly make the stiffness of the system arise.

From Fig.8, actuated forces \( u_x, u_y, u_z \) of the three actuated joints of the 3-RPS mechanism part of the six-dimensional vibration isolator are about 10N, and actuated forces \( u_{4}, u_5, u_6, u_7, u_8, u_9 \) of the six actuated joints of the 6-SPS mechanism part are about 10N, hence the control forces needed for vibration isolation can be accepted.
**Figure 7.** The displacement response of the support platform under active vibration isolation.

**Figure 8.** The actuated forces needed under active vibration isolation.
7. Passive vibration isolation simulation

As for Eq. (3), suppose the direct disturbance \( f_j \) and uncertainty \( \Delta A \) are effectively suppressed, and the effects of gravity \( G \) are neglected, then Eq. (3) can be rewritten as follows:

\[
\dot{x} + cJ_{\alpha}^TJ_{\alpha} \dot{x} + kJ_{\alpha}^TJ_{\alpha} x = -cJ_{\alpha}^TJ_{\alpha} \dot{x}_a - kJ_{\alpha}^TJ_{\alpha} x_a
\]

(19)

As for the vibration isolator, \( x = 0, y = 0, z = z_1 = 0, \alpha_i = 0, \beta_i = 0 \) is the transient singular point, under the singular point, stiffness of the system is zero, but its stiffness is increasingly higher in the fields which deviate from the singular point, hence researching the vibration transmission characteristics of the vibration isolator while the system deviates from the singular point is necessary and can provide the theory basis for holding the configuration to reduce the vibration transmission ratio. In this paper four kinds of configurations deviating from the singular point are given. Since the \( z_i \) error caused by the gravity of the load and support platform is the main component of the configure error of the vibration isolator, hence the four kinds of configurations are respectively defined as \( z_i = 5\text{mm}, z_i = 10\text{mm}, z_i = 20\text{mm}, z_i = 50\text{mm} \), the other configuration parameters are respectively defined as \( x_i = 0.5\text{mm}, y_i = 0.5\text{mm}, z_i = 1\text{mm} \), \( \alpha = \beta = \gamma = \alpha_i = \beta_i = \frac{\pi}{180} \text{rad} \) all the time. In addition, then linearizing Eq.(19) yields the vibration transmission functions of the passive part of the vibration isolator under the four configurations as follows:

\[
\frac{x(j\omega)}{x_0(j\omega)} = \frac{J_{\alpha 0}^T J_{\alpha 0} (\omega - cJ_{\alpha 0}^T J_{\alpha 0})}{\sum \frac{A_i A_i^T}{-m_i \omega^4 + c_i \omega + k_i}}
\]

(20)

where \( H_{\alpha d} \) is the vibration transmission function from the \( d \)th component of the six-dimensional vibration \( \delta_x, \delta_y, \delta_z, \theta_x, \theta_y, \theta_z \) of the base to the \( r \)th coordinate of the position \( x, y, z, \alpha, \beta, \gamma \) of the support platform, \( A_i \) is the \( r \)th order fundamental vibration mode of the system, \( m_r \) is the \( r \)th order fundamental mass, \( k_r \) is the \( r \)th order fundamental stiffness, \( c_r \) is the \( r \)th order fundamental damping coefficient, \( J_{\alpha 0} \) is the linearization matrix of \( J_{\alpha} \), \( J_{\alpha 0} \) is the linearization matrix of \( J_{\alpha 0} \), \( \omega \) is frequency of the excitation vibration, \( j = \sqrt{-1} \) is unit of complex number.

![Figure 9. The vibration transmission ratios of the vibration isolator under passive vibration isolation](www.intechopen.com)

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According to the transmission function as expressed in Eq. (20), the vibration transmission ratios under the four kinds of configurations are illustrated in Fig.9 using a numerical simulation. It can be seen that the vibration transmission ratios are increasingly higher with increasing error of the $z$ coordinate, above all, when $z=50\text{mm}$, the increasing vibration transmission ratios are very obvious and within the frequency range beyond 1 Hz, passive vibration isolation effects under the four kinds of configurations are also very obvious. The minimum of attenuation ratios of their vibration responses reaches 10dB, therefore, when the other configuration parameters are invariable, if only ensuring $-30\text{mm} \leq z \leq 0$, broadband vibration isolation efficiency and reliability of the six-dimensional vibration isolator can obviously rise.

8. Conclusions

The $H_\infty$ robust control method proposed in this paper can achieve multiple objectives such as reducing configure control errors, decreasing actuated forces and suppressing direct disturbance, at the same time, it can realize low frequency vibration isolation.

By holding the six-dimensional vibration isolator near the singular configuration, the low frequency vibration isolation capability of the passive part of the system greatly increases, so the efficiency and reliability of the six-dimensional vibration isolator can obviously arise.

9. References


