1. Introduction

Many power electronic system designs focus on energy conversion circuit parameters rather than controller parameters which drive the circuits. Controllers must be designed on the basis of circuit models, which are generally nonlinear systems (Brockett & Wood (1974)), in order to improve performance of controlled systems. The performance of controlled systems depend on nominal models to design the controllers. More broad class of models controllers are designed for, better control performance may be given. Many works (e.g. Kassakian et al. (1991)) design controllers for linearized models because it is not easy to concretely design controllers for the nonlinear models. Controller design in consideration of nonlinear models has been discussed since a work by Banerjee & Verghese (2001) because a research on nonlinear controller design has grown in those times.

This chapter systematically designs a robust controlled power converter system on the basis of its nonlinear model. Concretely, a two-stage power factor correction converter, that is a forward converter (FC) with power factor corrector (PFC), is designed. The systematic controller design clearly analyzes the behavior of nonlinear system to improve the performance.

A work by Orabi & Ninomiya (2003) analyzes a stability of single-stage PFC for variations of controller gain on the basis of its nonlinear model. On the basis of the work by Orabi & Ninomiya (2003) that regards a load of PFC as constant, a work by Dranga et al. (2005) for a two-stage PFC focuses on a point that a load of PFC part is not pure resistive and analyzes a stability of the converter.

A work by Sasaki (2002) discusses an influence between a FC part and a PFC part in a two-stage PFC. A work by Sasaki (2009) clearly shows that a source current reference generator plays an important role in a synthesis for a single-stage PFC. For the works by Sasaki (2002; 2009), this chapter shows how to decide synthesis parameters of robust linear and nonlinear controllers for a two-stage PFC in more detail.

The controller synthesis step, that is this chapter, is organized as follows. First, the converter is divided into two parts which consists of a FC part and a PFC part by considering an equivalent
circuit of transformer. The two parts depend on each other and are nonlinear systems. The FC part has an apparent input voltage which depends on an output voltage in the PFC part. On the other hand, the PFC part has an apparent load resistance which depends on an input current in the FC part. Second, the two parts of converter are treated as two independent converters by analyzing steady state in the converter and deciding a set point. Then, the above input voltage and load resistance are fixed on the set point. Controllers are designed for the two independent converters respectively. For the FC part which is a linear system at the second stage of converter, a robust linear controller is designed against variations of the apparent input voltage and a load resistance. For the PFC part which is a bilinear system, that is a class of nonlinear system, at the first stage, a robust nonlinear controller is designed against variations of the apparent load resistance that mean dynamic variations of the FC part at the second stage. Finally, computer simulations demonstrate efficiencies of the approach. It is also clarified that consideration of nominal load resistance for each part characterizes a performance of the designed robust controlled system.

2. Two-stage power factor correction converter

A controlled system for a two-stage power factor correction converter, that is a forward converter (FC) with power factor corrector (PFC), is systematically constructed as shown in Fig. 1. The systematic controller design clearly analyzes the behavior of controlled converter system. In this section, first, an equivalent circuit of transformer divides the converter into two parts with FC and PFC. Averaged models for the two parts are derived respectively, which depend on each other. Second, steady state in the two parts is analyzed, which is used as a set point. Finally, each of parts is treated as an independent converter respectively.

![Fig. 1. Two-stage converter; forward converter (FC) with power factor corrector (PFC)](image-url)

2.1 Nonlinear averaged models

An equivalent circuit of transformer derives a circuit as shown in Fig. 2 from Fig. 1. A FC part has a variable dc source voltage which depends on a switch $S_2$. A PFC part has a variable load resistance which depends on a switch $S_1$. Fig. 2 gives two nonlinear averaged models ($\Sigma_{S_A}^{fc}$) and ($\Sigma_{S_A}^{pfc}$) for the FC and the PFC parts respectively, which depend on each other. The FC model ($\Sigma_{S_A}^{fc}$) is given as

$$\frac{d}{dt} \begin{bmatrix} \sigma_1 \\ \bar{i}_1 \end{bmatrix} = \begin{bmatrix} -\frac{1}{L_1} & \frac{1}{C_1} \\ -\frac{1}{L_1} & -\frac{1}{L_1} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \bar{i}_1 \end{bmatrix} + \left\{ \bar{a}_2 \begin{bmatrix} 0 \\ N/L_1 \end{bmatrix} \right\} \bar{u}_1$$

(1)
Fig. 2. Two converters; FC and PFC

and the PFC model (\(S_pf_c^{pfc}\)) is

\[
\frac{d}{dt} \begin{bmatrix} \bar{v}_2 \\ \bar{i}_2 \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{L_2} \\ -\frac{1}{L_2} & \frac{1}{L_2} \end{bmatrix} \begin{bmatrix} \bar{v}_2 \\ \bar{i}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L_2} \end{bmatrix} |v_s| \\
+ \begin{bmatrix} \bar{v}_2 \\ \bar{i}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L_2} \end{bmatrix} \bar{I}_2 \bar{p}_1 + \begin{bmatrix} \bar{I}_1 \left( -\frac{N}{C_2} \right) \\ \frac{N}{C_2} \end{bmatrix} \bar{p}_2 \\
\bar{I}_1 \end{bmatrix} \bar{p}_1
\]  

(2)

where \(\bar{v}_1\) and \(\bar{v}_2\) are averaged capacitor voltages, \(\bar{I}_1, \bar{I}_2\) averaged inductor currents, \(C_1, C_2\) capacitances, \(L_1, L_2\) inductances, \(r_1, r_2\) internal resistances, \(R\) a load resistance, \(N\) a turns ratio, \(v_s\) a source voltage, \(\bar{p}_1, \bar{p}_2\) averaged switching functions given by \(0 < \bar{p}_1 < 1\) and \(0 < \bar{p}_2 < 1\).

2.2 DC components of steady state

DC components of steady state in two parts of converter are derived, which are used as a set point for controller design. Given an average full-wave rectified voltage \(V_s := (\omega/2\pi) \int_0^{2\pi} |v_s(t)| dt = 2\sqrt{2}V_c/\pi\) of a source voltage \(v_s = \sqrt{2}V_c \sin \omega t\), and specify averaged switching functions \(\bar{p}_1 = \bar{p}_{1s}, \bar{p}_2 = \bar{p}_{2s}\), which are called a set point, then dc components of voltages and currents of steady state are given by

\[
\bar{v}_{1s} := \bar{p}_{1s} \frac{1}{1 + \frac{R}{\bar{p}_{1s}}} E(\bar{p}_{2s}),
\]

(3)

\[
\bar{I}_{1s} := \frac{1}{R} \bar{v}_{1s} = \bar{p}_{1s} \frac{1}{1 + \frac{R}{\bar{p}_{1s}}} \frac{E(\bar{p}_{2s})}{R},
\]

(4)

\[
\bar{v}_{2s} := \frac{1}{1 - \bar{p}_{2s} + \frac{1}{1 - \bar{p}_{2s}} \frac{r_2}{R_2(\bar{p}_{1s})}} V_s,
\]

(5)

\[
\bar{I}_{2s} := \frac{1}{1 - \bar{p}_{2s}} \bar{p}_{1s} N \bar{I}_{1s} = \frac{1}{1 - \bar{p}_{2s}} \frac{\bar{v}_{2s}}{R_2(\bar{p}_{1s})}
\]

(6)

where

\[
E(\bar{p}_{2s}) := N \bar{v}_{2s}, \quad R_2(\bar{p}_{1s}) := \frac{1 + \frac{R}{\bar{p}_{1s}}}{N^2 R}.
\]

(7)
The above steady states in two parts depend on each other through (7). It is easily clarified by assuming \( r_1 = r_2 = 0 \) that the equations (3), (4) give a steady state in buck converter and the equations (5), (6) give a steady state in boost converter.

### 2.3 Two independent models

For two parts of converter with FC and PFC, the steady state analysis derives two independent converters. It means that the FC has an apparent source voltage \( E(\bar{\mu}_2s) \) and the PFC has an apparent load resistance \( R_2(\bar{\mu}_1s) \) which are fixed on a set point respectively.

Then, two independent averaged models of the converters are given as

a linear FC model \((\Sigma_{A0}^{fc})\) of the form

\[
\frac{d}{dt} \begin{bmatrix} \bar{\sigma}_1 \\ \bar{\ell}_1 \end{bmatrix} = \begin{bmatrix} -K_{1s} & 1 \\ -\frac{1}{L_1} & -\frac{r_1}{L_1} \end{bmatrix} \begin{bmatrix} \bar{v}_1 \\ \bar{\ell}_1 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{E(\bar{\mu}_2s)}{L_1} \end{bmatrix} \bar{\mu}_1
\]  \tag{8}

and a nonlinear PFC model \((\Sigma_{A0}^{pfc})\) of the form

\[
\frac{d}{dt} \begin{bmatrix} \bar{\sigma}_2 \\ \bar{\ell}_2 \end{bmatrix} = \begin{bmatrix} -K_{2s}(\bar{\mu}_1s)C_2 & \frac{1}{C_2} \\ -\frac{1}{L_2} & -\frac{r_2}{L_2} \end{bmatrix} \begin{bmatrix} \bar{v}_2 \\ \bar{\ell}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L_2} \end{bmatrix} |v_s| + \begin{bmatrix} \bar{v}_2 \\ \bar{\ell}_2 \left[ \frac{1}{L_2} \right] + \bar{\ell}_2 \left[ \frac{\bar{v}_2}{L_2} \right] \right) \bar{\mu}_2. \]  \tag{9}

The FC is modeled on a linear system and the PFC is modeled on a bilinear system that is a class of nonlinear system (Brockett & Wood (1974); Mohler (1991)).

Here derived is an amplitude of source current of steady state in the PFC, which is used for source current reference generator in Section 3.3. Given a source voltage \( v_s = \sqrt{2}V_e \sin \omega t \) [V], assume an output voltage \( \bar{v}_2 = v_{2r} \) [V], then dc components of steady state in the PFC gives a source current \( \bar{i}_2 = \sqrt{2}I_e \sin \omega t \) [A], which is sinusoidal and in phase with the source voltage, given by

\[
I_e = \frac{V_e}{2r_2} - \sqrt{\frac{V_e^2}{4r_2^2} - \frac{v_{2r}^2}{r_2R_2(\bar{\mu}_1s)}}. \tag{10}
\]

This decision process is similar to that is discussed in works by Escobar et al. (1999); Escobar et al. (2001).

### 3. Robust controller design

Controllers for two parts with FC and PFC are designed on the basis of two independent converter models \((\Sigma_{A0}^{fc})\) and \((\Sigma_{A0}^{pfc})\) respectively. First, the models \((\Sigma_{A0}^{fc})\) and \((\Sigma_{A0}^{pfc})\) are moved to set points given in Section 2.2. Next, controller design models, that are called as generalized plants, are derived which incorporate weighting functions in consideration of controller design specifications. Finally, feedback gains are given to guarantee closed-loop stability and reference tracking performance.
Lyapunov-Based Robust and Nonlinear Control for Two-Stage Power Factor Correction Converter

3.1 Models around set points

Converter models around set points are derived from the averaged models \((\Sigma_{A0}^{fc})\) and \((\Sigma_{A0}^{pfc})\). Moving the state to a specified set point given by \(\bar{\vartheta}_1 := \bar{\vartheta}_1 - \bar{\vartheta}_1s, \bar{r}_1 := \bar{r}_1 - \bar{r}_1s, \bar{\mu}_1 := \bar{\mu}_1 - \bar{\mu}_1s, \bar{\vartheta}_2 := \bar{\vartheta}_2 - \bar{\vartheta}_2s, \bar{r}_2 := \bar{r}_2 - \bar{r}_2s, \bar{\mu}_2 := \bar{\mu}_2 - \bar{\mu}_2s\) derives averaged models around the set point respectively for the two parts, which are given by

a FC model \((\Sigma_A^{fc})\) of the form

\[
\frac{d}{dt} \begin{bmatrix} \bar{\vartheta}_1 \\ \bar{r}_1 \end{bmatrix} = \begin{bmatrix} -\frac{1}{RC_1} & \frac{1}{C_1} \\ -\frac{1}{L_1} & -\frac{R_1}{L_1} \end{bmatrix} \begin{bmatrix} \bar{\vartheta}_1 \\ \bar{r}_1 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{E(\bar{\mu}_1)}{L_1} \end{bmatrix} \bar{\mu}_1
\]

\[
=: A_p^{fc} x_p^{fc} + B_p u_1
\]

and a PFC model \((\Sigma_A^{pfc})\) of the form

\[
\frac{d}{dt} \begin{bmatrix} \bar{\vartheta}_2 \\ \bar{r}_2 \end{bmatrix} = \begin{bmatrix} -\frac{1}{RC_2(\bar{\mu}_1s)} & \frac{1}{C_2} (1 - \bar{\mu}_2s) \\ -\frac{1}{L_2} (1 - \bar{\mu}_2s) & -\frac{R_2}{L_2} \end{bmatrix} \begin{bmatrix} \bar{\vartheta}_2 \\ \bar{r}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L_2} \end{bmatrix} (|\bar{v}_s| - Vs)
\]

\[
+ \begin{bmatrix} -\frac{L_2}{c_2} & 0 \\ \bar{r}_2 & \frac{L_2}{c_2} \end{bmatrix} \bar{\mu}_2
\]

\[
=: A_p^{pfc} x_p^{pfc} + B_p^{pfc} w_p^{pfc} + (B_p^{pfc} + \left(\bar{v}_s^{pfc} N_p^{pfc} \right) u_2
\]

where \(-\bar{\mu}_1s < \bar{\mu}_1 < 1 - \bar{\mu}_1s\) and \(-\bar{\mu}_2s < \bar{\mu}_2 < 1 - \bar{\mu}_2s\). The following discussion constructs controllers on the basis of the averaged models \((\Sigma_A^{fc})\) and \((\Sigma_A^{pfc})\) around the set point.

3.2 Linear controller design for FC

A robust controller for the FC is given by linear \(H^\infty\) control technique because the averaged model \((\Sigma_A^{fc})\) is a linear system. The control technique incorporates weighting functions \(W_{z1}(s) := k_{z1} / (s + \epsilon_1)\) to keep an output voltage constant against variations of load resistance and apparent input voltage as shown in Fig. 3, where \(k_{z1}, \epsilon_1\) are synthesis parameters. Then, the linear \(H^\infty\) control technique gives a linear gain \(K_1\) of the form

\[
u_1 = K_1 x^{fc} := -(D_{12}^{fc} D_{12}^{fc})^{-1} B_2^{fc} Y^{fc} x^{fc}
\]

where \(x^{fc} := \begin{bmatrix} x_p^{fc} \\ x_w^{fc} \end{bmatrix}, B_2^{fc} := \begin{bmatrix} B_p^{fc} \\ 0 \end{bmatrix}, D_{12}^{fc} := \begin{bmatrix} 0^T & W_u 1^T \end{bmatrix}^T\) and \(x^{fc}\) denotes state of weighting function \(W_{z1}(s)\) as shown in Fig.3. In the figure, matrices \(W_{z1}\) and \(W_{u1}\) are weighting coefficients of a performance index in the linear \(H^\infty\) control technique and used only in the controller synthesis.

The matrix \(Y^{fc}\) that constructs the gain \(K_1\) is given as a positive-definite solution satisfying a Lyapunov-based inequality condition.
A robust nonlinear controller design for the PFC, that is a nonlinear system, consists of the following two steps. First, a nonlinear gain is given to guarantee closed loop system stability and reference tracking performance for source current and output voltage against variations of the apparent load resistance. Second, a source current reference generator is derived to adjust an amplitude of current reference to variations of source voltage and load resistance. At the first step, as shown in Fig. 4 incorporated are a weighting function \( W_{i2}(s) = k_{i2}\omega_{i2}^2/(s^2 + 2\zeta\omega_{i2}s + \omega_{i2}^2) \) for a source current to be sinusoidal and be in phase with a source voltage and a function \( W_{v2}(s) = k_{v2}/(s + \epsilon_2) \) for an output voltage to be kept constant against those variations, where \( k_{i2}, \zeta, \omega_{i2}, k_{v2}, \epsilon_2 \) are synthesis parameters. Then, a nonlinear \( H^\infty \) control technique in a work by Sasaki & Uchida (1998) gives a nonlinear gain of the form

\[
K_2(x_p^{pfc}) := -(D_{12}^{pfc} D_{12}^{pfc} )^{-1}B_2^{pfc}(x_p^{pfc}) Y_{pfc}^{-1}
\]

as shown in Fig.4, where \( x_p^{pfc} = \begin{bmatrix} x_p^{pfc} & x_w^{pfc} \end{bmatrix}^T \), \( D_{12}^{pfc} = [0 W_{i2}^T]^T \), \( B_2^{pfc}(x^{pfc}) = \begin{bmatrix} B_{p_2}^{pfc} \\ 0 \end{bmatrix} \) + \( x_1^{pfc} \begin{bmatrix} B_{p_2}^{pfc} \\ 0 \end{bmatrix} + x_2^{pfc} \begin{bmatrix} B_{p_2}^{pfc} \\ 0 \end{bmatrix} \), \( B_{p_21}^{pfc} = [0 \ 0]^T \), \( B_{p_22}^{pfc} = [-\frac{1}{\epsilon_2} \ 0]^T \) and \( x_w^{pfc} \) denotes state of weighting functions. In the figure, matrices \( W_{i2} \) and \( W_{u2} \) are weighting coefficients of a performance index in the nonlinear \( H^\infty \) control technique. A block named as Generator is not used for the synthesis and is discussed at the following second step.
The matrix $Y^{pfc}$ is given as a positive-definite solution satisfying a state-depended Lyapunov-based inequality condition

$$
\begin{bmatrix}
    A^{pfc}Y^{pfc} + Y^{pfc} A^{pfcT} - B_2^{pfc}(x)(D_{12}^{pfc} D_{12}^{pfcT})^{-1} B_2^{pfc}(x)^T Y^{pfc} C_1^{pfcT} \\
    + \gamma^{pfc^{-2}} B_1^{pfc} B_1^{pfcT} C_1^{pfc} Y^{pfc} \\
    -I
\end{bmatrix} < 0 \quad (18)
$$

where $x$ is a state of a generalized plant given by Fig.4, matrices $A^{pfc}, B_1^{pfc}, B_2^{pfc}(x), C_1^{pfc}, D_{12}^{pfc}$ are coefficients of the plant and $\gamma^{pfc}$ is a synthesis parameter in the nonlinear $H_{\infty}$ control technique.

For any state $x$, which is current and voltage in a specified domain, the matrix $Y^{pfc}$ satisfying the inequality (18) is concretely given by solving linear matrix inequalities at vertices of a convex hull enclosing the domain as shown in a work by Sasaki & Uchida (1998).

Next, given is a mechanism to generate a source current reference $i_{2r}$. As shown in Fig.4 the reference generator consists of a feedforward loop given by steady state analysis in Section 2.3 and a feedback loop with a voltage error amplifier. The amplifier $K(s)$ is given by $K(s) = k_p + k_1/s + k_Ds$ where $k_p, k_1$ and $k_D$ are constant parameters decided by system designers as discussed in a work by Sasaki (2009). The feedback loop is the same structure as a conventional loop used in many works (e.g., Redl (1994)). Note that the amplifier $K(s)$ works only for variations from the nominal values in the circuit, because the feedforward loop gives the effective value of the source current as shown in Section 4.4.

4. Computer simulations

This section finally shows efficiencies of the approach through computer simulations. It is also clarified that consideration of nominal load resistance for each part characterizes a performance of the designed controlled system. A software package that consists of MATLAB, Simulink and LMI Control Toolbox is used for the simulations.
Parameters of the circuit shown in Fig. 1 are given by Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_1$</td>
<td>1 [mΩ]</td>
</tr>
<tr>
<td>$r_2$</td>
<td>1 [mΩ]</td>
</tr>
<tr>
<td>$L_1$</td>
<td>10 [mH]</td>
</tr>
<tr>
<td>$L_2$</td>
<td>1 [mH]</td>
</tr>
<tr>
<td>$C_1$</td>
<td>450 [µF]</td>
</tr>
<tr>
<td>$C_2$</td>
<td>1000 [µF]</td>
</tr>
<tr>
<td>$N$</td>
<td>1/36</td>
</tr>
</tbody>
</table>

Table 1. Circuit parameters of two-stage power factor correction converter as shown in Fig. 1

### 4.1 Design specification

Control system design specification for the two-stage power factor correction converter is given by

1. An effective value of source voltage is $82 \sim 255$ volts, and its frequency is $45 \sim 65$ hertz;
2. An output voltage is kept be 5 volts, and its error of steady state is within $\pm 0.5\%$;
3. An output current is $0 \sim 30$ amperes;
4. A source current is approximately sinusoidal and is phase with source voltage.

The above specifications are treated for controller design as the following respective considerations;

1. A nonlinear gain for PFC is designed for a source voltage whose nominal effective value is $V_e = \frac{82 + 255}{2} = 168.5$ volts (then, average full-wave rectified voltage is $V_s = 151.7$ volts). Moreover, the gain is designed to be robust against source voltage variations by treating that as a disturbance $w_{pf}$ as shown in Fig. 4.
2. Incorporated is a weighting function $W_{v1}(s)$ whose magnitude is high at low frequencies.
3. A load resistances $R$ varies between $1/6 \sim \infty$ ohms at an output voltage of 5 volts.
4. Incorporated is a weighting function $W_{i2}(s)$ whose magnitude is high at frequencies in the range of $45 \sim 65$ hertz for the source current to be approximately sinusoidal at the frequencies.

### 4.2 Nominal load resistance

Now, a nominal value of load resistance $R$ need be decided to design controllers. The nominal value influences a performance of closed-loop system controlled by the designed controllers. Root loci of linearized converters for the two parts as shown in Figs. 5 and 6, which are pointwise eigenvalues of the system matrices for variations of load resistance $R$, show that the larger the load resistance $R$ is, the more oscillatory the behavior of FC is and the slower the transient response in PFC is. Controllers, here, are constructed in consideration of undesirable behavior of the converter system. Therefore, a FC controller is designed for a system with oscillatory behavior, that is for a large load resistance, so that an output voltage does not oscillate. A PFC controller is designed for a system with fast response, that is for a small load resistance, so that a source current is not distorted by the controller responding well to source voltage variations.
4.3 Linear controller design parameters for FC

A FC controller is designed for a large nominal load resistance \( R = 100 \) ohms. Consider a set point \( \bar{v}_{1s} = 0.5 \) for a capacitor voltage of \( v_{2s} = 360 \) volts. DC components of steady state are given by \( \bar{v}_{1s} = 4.99 \) volts and \( i_{1s} = 0.0499 \) amperes. For a model around the above set point, incorporated are weighting functions with parameters given by \( K_v = 20, \epsilon_1 = 0.001 \) \( W_v = 5 \), \( W_u = 1.5 \) and \( \gamma^{fc} = 0.95 \) where \( \gamma^{fc} \) denotes control performance level given in the linear \( H^\infty \) control technique. The weighting function \( W_v(s) \) with the above parameter gives bode plots whose characteristics is like an integrator as shown in Fig.7.

Then, the matrix \( Y^{fc} \) that gives a linear gain (15) is obtained as

\[
Y^{fc} = \begin{bmatrix}
2.00 \times 10^4 & -1.55 \times 10^3 & 5.05 \times 10^2 \\
-1.55 \times 10^3 & 7.68 \times 10^2 & 2.59 \times 10^1 \\
5.05 \times 10^2 & 2.59 \times 10^1 & 2.51 \times 10^1
\end{bmatrix}.
\]
4.4 Nonlinear controller design parameters for PFC

A PFC controller is designed for a small nominal load resistance $R = 10$ ohms. Consider a set point $\bar{\mu}_2 = (v_{2r} - V_s)/v_{2r} = 0.579$ to make a rectified voltage $v_{2r} = 360$ volts. Then, an apparent load resistance in the PFC is given by $R_2(\bar{\mu}_1) = 51845.2$ ohms for a set point $\bar{\mu}_1 = 0.5$ in the FC. Therefore, dc components of steady state are given by $\bar{v}_2 = 359.9$ volts and $\bar{i}_2 = 100$.

Fig. 7. Bode plots of weighting function $W_{v1}(s)$ for output voltage tracking in FC controller design.

Fig. 8. Bode plots of weighting function $W_{i2}(s)$ for achieving unity power factor in PFC controller design.
0.0165 amperes. For a model around the above set point, incorporated are weighting functions with parameters given by $\zeta = 0.001$, $\omega_i = 130\pi$, $K_{i2} = 300/\omega_i$, $\gamma_p = 0.002$, $\varepsilon = 0.001$, $W_{i2}(s)$ with the above parameter gives bode plots that is weighted at frequencies given by the design specification (4). The function $W_{i2}(s)$ has low gains at all frequencies because a voltage tracking fed-back by a controller is not important for the PFC design. It also leads that the $W_{i2}$ is set such that the tracking performance for source current is more weighted than that for output voltage.

Then the matrix $Y_{p}^{pf}$ that gives a nonlinear gain (17) is obtained as

$$Y_{p}^{pf} = \begin{bmatrix}
2.77 \times 10^4 & -5.56 \times 10^4 & 2.93 \times 10^0 & 4.70 \times 10^3 & -1.72 \times 10^4 \\
-5.56 \times 10^4 & 1.89 \times 10^5 & 8.86 \times 10^{-1} & -5.57 \times 10^3 & 4.03 \times 10^4 \\
2.93 \times 10^0 & 8.86 \times 10^{-1} & 9.71 \times 10^8 & -2.60 \times 10^{-2} & 1.89 \times 10^{-1} \\
4.70 \times 10^3 & -5.57 \times 10^3 & -2.60 \times 10^{-2} & 1.10 \times 10^3 & -2.98 \times 10^3 \\
-1.72 \times 10^4 & 4.03 \times 10^4 & 1.89 \times 10^{-1} & -2.98 \times 10^3 & 1.24 \times 10^4 
\end{bmatrix}$$

(20)

by considering a voltage variation of $\pm 10$ volts and a current variation of $\pm 3$ amperes around the set point (i.e., by considering $349.9 \leq v_2 \leq 369.9$ and $-2.9835 \leq i_2 \leq 3.0165$) and solving a convex programming problem as shown in the work by Sasaki & Uchida (1998).

Next, a source current reference generator is constructed with a voltage error amplifier given by $k_P = 8 \times 10^{-2}$, $k_i = 4$, $k_D = 1 \times 10^{-5}$ as shown in Fig.10. The gain is chosen in order to be low around twice the input line frequency.
4.5 Simulation results

Figs. 11–13 show behaviors of the nonlinear averaged models of FC ($\Sigma_{SA}^{fc}$) and PFC ($\Sigma_{SA}^{pfc}$) in the following cases:

(C1) A load resistance $R$ changes from 1000 to 0.25 ohms in steady state for a source voltage $v_s = \sqrt{2} \times 100 \sin 100\pi t$ volts;

(C2) An efficient value $V_e$ of source voltage with 50 hertz changes from 100 to 85 volts for 2 seconds in steady state with a load resistance $R = 0.25 \text{ ohms.}$

Figs. 11–13 show that the controllers works very well.

Fig. 12 shows behaviors by a different controller from that in Figs. 11 and 13, which is designed for desirable load resistances with a small nominal resistance $R = 10 \text{ ohms}$ for the FC and a large $R = 100 \text{ ohms}$ for the PFC.

Therefore, in the FC the output voltage $v_1$ in Fig. 12 oscillates more than in Fig. 11 at the large resistance $R = 1000.$ Also, in the PFC the capacitor voltage $v_2$ in Fig. 12 drops larger than in Fig. 11. Then the source current $i_2$ in Fig. 12 raises faster than in Fig. 11 and then is more distorted.

In the case as shown in Fig. 12, the above control techniques give the following matrices as

$$Y^{fc} = \begin{bmatrix} 2.92 \times 10^4 & -1.28 \times 10^3 & 6.26 \times 10^2 \\ -1.28 \times 10^3 & 8.07 \times 10^2 & 3.57 \times 10^1 \\ 6.26 \times 10^2 & 3.57 \times 10^1 & 2.27 \times 10^1 \end{bmatrix} \quad (21)$$
Fig. 11. (C1) Behaviors when a load resistance $R$ changes from 1000 to 0.25 ohms in steady state and

$$
\gamma_{pfc} = \begin{bmatrix}
2.76 \times 10^4 & -5.56 \times 10^4 & 2.93 \times 10^0 & 4.70 \times 10^3 & -1.72 \times 10^4 \\
-5.56 \times 10^4 & 1.89 \times 10^5 & 8.85 \times 10^{-1} & -5.56 \times 10^3 & 4.03 \times 10^4 \\
2.93 \times 10^0 & 8.85 \times 10^{-1} & 9.71 \times 10^8 & -2.60 \times 10^{-2} & 1.89 \times 10^{-1} \\
4.70 \times 10^3 & -5.56 \times 10^3 & -2.60 \times 10^{-2} & 1.10 \times 10^3 & -2.98 \times 10^3 \\
-1.72 \times 10^4 & 4.03 \times 10^4 & 1.89 \times 10^{-1} & -2.98 \times 10^3 & 1.24 \times 10^4
\end{bmatrix}.
$$
Fig. 12. (C1) Behaviors when a load resistance $R$ changes from 1000 to 0.25 ohms in steady state where a controller is designed for a different nominal load resistance from that in Fig.11.

Entries of the matrix $Y_{fc}$ in (21) are clearly different from those in (19). On the other hand, difference of the $Y_{pfc}$ between the matrix (22) and (20) is much small. Those differences give the different behaviors as mentioned above.

Fig. 12 demonstrates the discussion in Section 4.2 that controllers needs be designed in consideration of undesirable behavior of system. For a large load resistance the output voltage of the FC oscillates and for small load resistance the source current and the capacitor voltage of the PFC respond to variations larger than those in Fig.11.
5. Conclusion

For a two-stage power factor correction converter, a nonlinear controlled converter system was systematically designed on the basis of its nonlinear model. The systematic controller design clearly analyzed the behavior of nonlinear system to improve the performance. The synthesis step were clearly shown. It was clarified that a nominal load resistance characterizes the controller performance. Finally, computer simulations demonstrated efficiencies of the approach. The nonlinear controller synthesis was shown as a natural extension of a well-known Lyapunov-based linear controller synthesis.
6. Acknowledgment

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7. References


A trend of investigation of Nonlinear Control Systems has been present over the last few decades. As a result the methods for its analysis and design have improved rapidly. This book includes nonlinear design topics such as Feedback Linearization, Lyapunov Based Control, Adaptive Control, Optimal Control and Robust Control. All chapters discuss different applications that are basically independent of each other. The book will provide the reader with information on modern control techniques and results which cover a very wide application area. Each chapter attempts to demonstrate how one would apply these techniques to real-world systems through both simulations and experimental settings.

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