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The Thermopower of a Quantum Dot Coupled to Luttinger Liquid System

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1. Introduction

With the progress in nanofabrication technique and nanometer scale materials, research on electron transport properties of mesoscopic systems has become a very active field in condensed matter physics. Considerable researches have been mainly focused on charge transport in nanostructures and nanodevices. Besides the charge transport, a detailed understanding of heat transport through mesoscopic systems is of equally importance (Afonin, 1995; Small, 2003) because they can provide additional information on the kinetics of carriers not available in the measurement of current voltage characteristics (Heremans, 2004). For instance, thermoelectric properties are very sensitive to dimensionality, the electronic spectrum near the Fermi level, scattering processes (Koch, 2004), electron-phonon coupling strength (Yang1, 2010) and electron-hole symmetry (Small, 2003). There have been several theoretical studies on the thermopower S, which mainly focused on quantum dot (QD) coupled to the normal Fermi liquid (FL) leads (Boese, 2001; Dong, 2002; Kim, 2003; Krawiec, 2007; Yang1, 2010), denoted hereafter as FL-QD-FL. As for systems containing a quantum dot coupled to one-dimensional (1D) interacting electron leads, although their charge transport phenomena have been investigated (Yang2, 2001; Yang3, 2010), yet there have been much less efforts devoted to the thermoelectric properties of them (Kane, 1996; Krive, 2001; Romanovsky, 2002). It is well known that the 1D interacting electron systems can be described by the Luttinger liquied (LL) theory (Luttinger, 1963), which holds some unique features such as spin-charge separation, suppression of the electron tunneling density of states, power-law dependence of the electrical conductance on temperature and bias voltage, etc.. The LL behaviour has been experimentally reported in single- and multi-wall carbon nanotubes (Bockrath, 1999; Kanda, 2004; Yao, 1999) and fractional quantum Hall edge states (Chang, 1996). Recently, the use of carbon nanotubes as a thermoelectric material has gained great interest due to their 1D structure. The thermopower of single-walled carbon nanotubes have been measured in experiments (Bradley, 2000; Choi, 1999; Collins, 2000; Hone, 1998; Kong, 2005; Small, 2003). For example, Kong *et al*. have shown a linear temperature dependence of the thermopower at low temperature (Kong, 2005). Small *et al*. have observed strong modulations of thermopower as the function of gate voltage Vg in individual Carbon nanotubes (Small, 2003). Dresselhaus *et al*. have found that the low-dimensional

thermoelectric materials performed better than bulk ones (Dresselhaus, 1999). Several theoretical works have been developed to predict the enhancement of thermopower in nanoscaled materials by the intralead electron interaction (Kane, 1996; Krive, 2001; Krive2, 2001; Romanovsky, 2002). Krive *et al*. (Kane, 1996; Krive, 2001) used a phenomenological approach to investigate the thermopower of a LL wire containing an impurity. In spite of the above work, an explicit thermopower formula in the LL leads was not given out. In the following, we use the notation S to denote the thermopower S of systems comprosing LL and S_0 to those comprising noninteracting FL. Theoretically, the thermopower S of a LL with an impurity can be represented by the thermopower S_0 multiplied by an interaction-dependent renormalization factor. Alternatively, one may intentionally introduce a QD into the LL, denoted as LL-QD-LL. Thus, he may attach a QD, instead of an impurity atom, to the end of a carbon nanotube. A quantum dot is experimentally more controllable than an impurity. For instance, the energy of a quantum dot can be tuned by the gate voltage V_g.

In this chapter, we will first give the stationary thermopower formula of a QD coupled to LL leads through tunneling junctions, a system of LL-QD-LL (see Fig.1) by applying the nonequilibrium Green function technique (Haug, 1996) instead of phenomenal theories. And then we later turn our attention to the time-dependent phenomena. The generalized thermopower formula is obtained under time-dependent gate voltage. Although there are many studies on time-dependent nonequilibrium transport, the research on the time-dependent thermopower and the formula under the ac field are still lack. Here we will fill the blanks for the low dimension system.

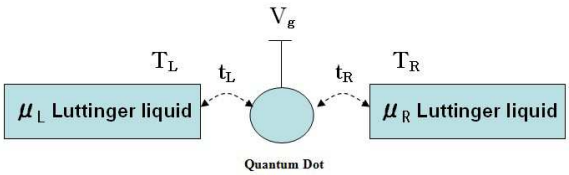

Fig. 1. The two-terminal electron transport through a single-level quantum dot weakly coupled to the Luttinger liquid leads with the chemical potentials μ_L and μ_R. Electrons tunnel from one lead to another by hopping on and off the dot level with the energy ε. The position of the dot levels with respect to the Fermi energy can be uniformly shifted by applying a voltage V_g to the gate electrode.

2. The Model

In the considered LL-QD-LL system, the QD is weakly connected with semi-infinite LL electrodes. The Hamiltonian of this system includes three parts:

$$H = H_{leads} + H_{dot} + H_T, \tag{1}$$

which represents the Hamiltonians of the left and right LL leads ($H_{leads} = H_L + H_R$), the central dot and the tunneling interactions between them H_T, respectively. Firstly, we present a detailed discussion of bosonization for a continuum model of length L with open boundary conditions (Eggert0, 1992; Fabrizio, 1995; Furusaki0, 1994) and electron-electron interaction.

The specify Hamiltonian of the LL leads can be easily written

$$H_{L/R} = H_{0\mathrm{L/R}} + H_{intL/R}, \tag{2}$$

where the first term represents the kinetic energy,

$$H_0 = \sum_{\sigma=\uparrow,\downarrow} \int_0^L dx \psi_s^\dagger(x) \varepsilon(-i\partial_x) \psi_s(x), \tag{3}$$

and the second one describes the electron-electron interaction,

$$H_{int} = \frac{1}{2} \sum_{\sigma\sigma'} \int dx dy \psi_\sigma^\dagger(x) \psi_{\sigma'}^\dagger(y) U_{\sigma\sigma'}(x-y) \psi_{\sigma'}(y) \psi_\sigma(x), \tag{4}$$

ε_k is the dispersion law of the 1D band, and $\psi_\sigma(x)$ is the spin σ electron annihilation operator subject to the open boundary conditions:

$$\psi_\sigma(0) = \psi_\sigma(L) = 0. \tag{5}$$

We apply the boundary conditions Eq.(5) to expand electron annihilation operator ψ which takes the form

$$\psi_\sigma(x) = \sqrt{\frac{2}{L}} \sum_k \sin(kx) c_{\sigma k}, \tag{6}$$

with $k = \pi n/L$, n being a positive integer. Usually a 1D system with periodic boundary conditions has two Fermi points $\pm k_F$. Here we only have single Fermi point given by $k = k_F$. The 1D fermion field ψ_σ can be expanded about the Fermi point k_F in terms of the left moving and right moving fields as

$$\psi_\sigma(x) = e^{ik_F x} \psi_{\sigma R}(x) + e^{-ik_F x} \psi_{\sigma L}(x). \tag{7}$$

In the case of periodic boundary conditions (Haldane, 1981), these left moving and right moving fields are not independent and satisfy

$$\psi_{\sigma L}(x) = -\psi_{\sigma R}(-x). \tag{8}$$

Then the fermion fields automatically satisfy the boundary conditions

$$\psi_\sigma(0) = 0, \tag{9}$$

whereas the condition

$$\psi_\sigma(L) = 0 \tag{10}$$

implies that the operator $\psi_{\sigma R}(x)$ should obey

$$\psi_{\sigma R}(-L) = \psi_{\sigma R}(L). \tag{11}$$

Therefore, we can actually work with the right moving field only, the left moving one is then defined by the above relation. Thus the field $\psi_{\sigma R}(x)$ can be defined for all x, and obeying the

periodicity condition with the period $2L$:

$$\psi_{\sigma R}(x+2L) = \psi_{\sigma R}(x). \tag{12}$$

In terms of the right moving field, the kinetic energy terms in the Hamiltonian Eq.(1) takes the form

$$H_0 = v_F \sum_{\sigma} \int_{-L}^{L} dx \psi_{\sigma R}^{\dagger}(x)(-i\partial_x)\psi_{\sigma R}(x) \tag{13}$$

where we have linearized the electron spectrum. The single fermion operators for right-moving electrons with spin σ on lead α can be bosonized in the position representation by applying the periodic boundary condition Eq.(12) as

$$\psi_{\sigma R}(x) \approx \frac{\eta_{\sigma}}{\sqrt{2L}} e^{ik_F x} e^{i\phi_{\sigma}(x)}. \tag{14}$$

The operator η_{σ} is real fermion and satisfies the anti-commutation relations

$$\{\eta_{\sigma}, \eta_{\sigma}'\} = \delta_{\sigma\sigma'}, \tag{15}$$

with δ is Delta function. Eq.(15) assure the correct anti-commutation rules for electron operators with different σ. In order to calculate the correlation function, a method of dealing with this was suggested by Luther and Perchel (Luther, 1974). It used a limiting process, where the wave function contained s parameter α', and the limit $\alpha' \to 0$ is taken at the end of the calculation of the correlation function. Using the parameter α', we can represent the electron operator $\psi_{\sigma R}(x)$ as

$$\psi_{\sigma R}(x) \approx \lim_{\alpha' \to 0} \frac{1}{\sqrt{2\pi\alpha'}} e^{ik_F x} e^{i\phi_{\sigma}(x)}. \tag{16}$$

Where α' is a short-distance cutoff of the order of the reciprocal of the Fermi wave number k_F. The phase field $\phi_{\sigma}(x)$ satisfies periodic boundary condition:

$$\phi_{\sigma}(x+2L) = \phi_{\sigma}(x) \tag{17}$$

and can be expressed as follows

$$\phi_{\sigma}(x) = \sum_{q>0} \sqrt{\frac{\pi}{qL}} e^{iqx - \alpha' q/2} a_q + H.c, \tag{18}$$

here, $a_q^{\dagger}$ and a_q are the creation and annihilation operators of bosons. These operators satisfy the canonical bosonic commutation relations $[a_q, a_{q'}^{\dagger}] = \delta(q - q')$. $q = \pi n/L$, n is an integer. The density of right moving electrons is given by

$$\rho_{\sigma R}(x) \approx \frac{\partial_x \phi_{\sigma}(x)}{2\pi} \tag{19}$$

Applying the boundary conditions Eq.(8), we have

$$\rho_{\sigma L}(-x) = \rho_{\sigma R}(x). \tag{20}$$

The bosonized form of the kinetic energy is

$$H_0 = \pi v_F \sum_{\sigma} \int_{-L}^{L} dx : \rho_{\sigma R}(x)\rho_{\sigma R}(x) := v_F \sum_{\sigma q>0} q b^{\dagger}_{\sigma q} b_{\sigma q}, \tag{21}$$

where colon represents the normal order form of the operators.

In order to deal with electron-electron interacting terms in Hamiltonian Eq.(1), we continue to make use of the above bosonization procedure expressing the electron interaction Hamiltonian in terms of the right moving Fermi field $\psi_{\sigma R}$ only.

Before we turn to the interaction effects, we introduce the bosonic variables corresponding to charge and spin excitations:

$$b_{\rho(\sigma)q} = \frac{1}{\sqrt{2}}(b_{\uparrow q} \pm b_{\downarrow q}) \tag{22}$$

and

$$\rho_{\rho(\sigma)} = \frac{1}{\sqrt{2}}(\rho_{\uparrow q} \pm \rho_{\downarrow q}). \tag{23}$$

The interaction part of the Hamiltonian contains several terms classified as: the diagonal terms in the electron densities and the mixing left and right densities term. Consequently, the Hamiltonian becomes

$$H = \sum_{\nu=\rho(\sigma)} \{ \sum_{q>0} v^0_{\nu} q [b^{\dagger}_{\nu q} b_{\nu q} - \frac{g_{2\nu}}{4\pi}(b_{\nu q} b_{\nu q} + b^{\dagger}_{\nu q} b^{\dagger}_{\nu q})] \tag{24}$$

where

$$v_{\nu q} = v_F + \frac{g_{4\nu} + g_{2\nu}}{2\pi}, \tag{25}$$

where v_F is the Fermi velocity, g_4 and g_2 represent forward scatterings; in our work, we will not consider the backscattering interaction. In the absence of backscattering, the Luttinger Hamiltonian, H_{LL}, is exactly soluble using the technique of bosonization. In order to express the Hamiltonian in diagonal form, we introduce the canonically conjugate Boson operators, in a standard way by the Bogolubov rotation,

$$b_{\nu q} = \cosh(\varphi_{\nu})\widetilde{b}_{\nu q} - \sinh(\varphi_{\nu})\widetilde{b}^{\dagger}_{\nu q} \tag{26}$$

where

$$\tanh(2\varphi_{\nu}) = -\frac{\widetilde{g}_{\nu}}{2\pi v^0_{\nu}} \tag{27}$$

The Hamiltonian can be achieved by the canonical transformation in terms of $\widetilde{b}_{\nu q}$ and $\widetilde{b}^{\dagger}_{\nu q}$

$$\widetilde{H} = UHU^{\dagger} = \sum_{\nu q>0} v_{\nu} q b^{\dagger}_{\nu q} b_{\nu q}, \tag{28}$$

where

$$v_{\nu} = \frac{v^0_{\nu}}{\cosh(2\varphi_{\nu})}. \tag{29}$$

The unitary operator U is defined by

$$U = exp\{\frac{1}{2}\sum_{\nu,q>0}\varphi_\nu(b^\dagger_{\nu q}b^\dagger_{\nu q} - b_{\nu q}b_{\nu q})\} \tag{30}$$

In the next step we find how the Fermi operators transform by applying U. Employing the method of Mattis and Lieb (Mattis, 1964), after lengthy but straightforward calculations, we arrive at the expression for the electron annihilation operator in terms of free bosons for the case of the interacting Fermi system with open boundaries:

$$\begin{aligned} U\psi_{\sigma R}(x,t)U^\dagger \approx & \frac{\eta_\sigma}{\sqrt{2\pi\alpha'}}\exp\{i\sum_\nu \varepsilon_{\nu\sigma}[\frac{c_\nu}{\sqrt{2}}\phi_\nu(x-v_\nu t) \\ & - \frac{s_\nu}{\sqrt{2}}\phi_\nu(-x-v_\nu t)]\} \end{aligned} \tag{31}$$

where $\varepsilon_\nu\sigma$ is $+1$ unless $\sigma = \downarrow$ and $\nu = \sigma$ when its value is -1. We have defined

$$c_\nu = \cosh(\varphi_\nu), s_\nu = \sinh(\varphi_\nu). \tag{32}$$

In the continuum limit, the Hamiltonian can be expressed

$$H_{L/R} = \hbar v_c \int_0^\infty k a^\dagger_k a_k \mathrm{d}k. \tag{33}$$

This Hamiltonian describes the propagation of the charge density fluctuations in the leads with renormalized velocity v_c. From Krönig's relation (Krönig, 1935), the kinetic term has been written in a quadratic form of the density operators, because the bosons are defined as excitations above an N particle ground state, Hamiltonian must include terms that include the energy of the different bosonic ground states. These terms are not required for the calculations in this chapter, and are hence omitted.

The Hamiltonian of the single-level QD takes the form of

$$H_{dot} = \varepsilon d^\dagger d, \tag{34}$$

where ε is the energy of the electron on the dot, and $d^\dagger$ and d are fermionic creation and annihilation operators satisfying canonical commutation relation $\{d, d^\dagger\} = 1$.

The tunneling Hamiltonian is given by the standard expression:

$$H_T = \sum_\alpha (t_\alpha d^\dagger \psi_\alpha + h.c.), \tag{35}$$

where t_α is the electron tunneling constant and $\psi^\dagger_\alpha, \psi_\alpha$ $(\alpha = L/R)$ are the Fermi field operators at the end points of the left/right lead. The operator ψ_α could be written in a "bosonized" form (Furusaki, 1998)

$$\psi_\alpha = \sqrt{\frac{2}{\pi\alpha'}}\exp\left[\int_0^\infty \mathrm{d}q \frac{e^{(-\alpha' q/2)}}{\sqrt{2K_{\rho\alpha}q}}(a_{q\alpha} - a^\dagger_{q\alpha}) + \sigma\int_0^\infty dq \frac{e^{(-\alpha' q/2)}}{\sqrt{2q}}(b_{q\alpha} - b^\dagger_{q\alpha})\right], \tag{36}$$

where and $K_{\rho\alpha} = e^{2\varphi_\nu}$ is the interaction parameter in the "fermionic" form of the LL Hamiltonian (33), which restricts the LL parameter g to vary between 0 and 1. The noninteracting case corresponds to $v_c = v_F$ and $K_{\rho\alpha} = 1$. For repulsive interactions, $K_{\rho\alpha} < 1$. Because of the SU(2) spin symmetry under no magnetic filed, $K_\sigma = 1$. Thus the correlation functions the end point of the left LL lead without the coupling to the quantum dot $\langle \psi_\sigma^\dagger(0,t)\psi_\sigma(0,0)\rangle$ can be obtained after long calculation

$$\langle \psi_\sigma^\dagger(0,t)\psi_\sigma(0,0)\rangle_L = \frac{c_A}{\alpha'}\{\frac{i\Lambda}{\pi T}\sinh[\frac{\pi T(t-i\delta)}{\hbar}]\}^{-1/g_L}. \tag{37}$$

Where c_A is a dimensionless constant of order 1, Λ is a high-energy cutoff or a band width, δ is positive infinitesimal, and $g_L^{-1} = \frac{1}{2}(1/K_{\rho L}+1)$. $\psi_\sigma(0,t) = e^{iH_Lt/\hbar}\psi_\sigma(0,0)e^{-iH_Lt/\hbar}$. Similarly, the correlation function at the end point of the the right lead is obtained as the above method. The electron-electron interaction parameters of the left and right LL leads are assumed equal $g_L = g_R = g$ for convenience.

3. The thermopower formula under no ac field

The charge current J_L flowing from the left lead L into the quantum dot can be evaluated as follows:

$$J_L(t) = -\frac{e}{\hbar}\left\langle \frac{d}{dt}N_L\right\rangle = \frac{ie}{\hbar}\left\langle t_L d^\dagger(t)\psi_L(t) - h.c.\right\rangle. \tag{38}$$

We introduce the time-diagonal parts of the correlation functions: $G_{dL}^<(t,t') = i\langle \psi_L^\dagger(t')d(t)\rangle$ and $G_{Ld}^<(t,t') = i\langle d^\dagger(t')\psi_L(t)\rangle$. With the help of the Langreth analytic continuation rules (Haug, 1996). By means of them, it is easy to express the current as $J_L = 2eRe(t_L^* G_{Ld}^<(t,t'))$. After applying Langreth theorem of analytic continuation, the average current can then be expressed as

$$J_L = \frac{e}{2\pi}|t_L|^2\int d\omega Re[G_d^r(\omega)g_L^<(\omega) + G_d^<(\omega)g_L^a(\omega)]. \tag{39}$$

In terms of a long derivation, we can easily establish an expression for the expectation value of the electric current

$$J_L = \frac{e}{2\pi}|t_L|^2|t_R|^2\int d\omega G_d^r G_d^a\left[g_L^<(\omega)g_R^>(\omega) - g_L^>(\omega)g_R^<(\omega)\right], \tag{40}$$

where $G_d^{r(a)}$ is retarded (advanced) Green function of the quantum dot and $\Gamma_{L/R}$, proportional to $|t_{L/R}|^2$, describes the effective level broadening of the dot. $g_\alpha^{<(>)}(\omega)$ is the Fourier transform of the lesser (greater) Green function at the end point of the left (right) LL lead without the coupling to the QD, which has been obtained by (Furusaki, 1998):

$$g_\alpha^{<,>}(\omega) = \pm i\frac{T_\alpha}{|t_\alpha|^2}\exp[\mp(\omega-\mu_\alpha)/2T_\alpha]\gamma_\alpha(\omega-\mu_\alpha), \tag{41}$$

Now we define the Luttinger liquid distribution functions $F_{L/R}^{<,>}$ as

$$F_\alpha^{<,>}(\omega) = \frac{1}{2\pi}e^{\frac{\mp(\omega-\mu_\alpha)}{2T_\alpha}}\left(\frac{\pi T_\alpha}{\Lambda}\right)^{1/g-1}\frac{|\Gamma[\frac{1}{2g}+i\frac{\omega-\mu_\alpha}{2\pi T_\alpha}]|^2}{\Gamma(1/g)} \quad (\alpha = L/R). \tag{42}$$

The function $F^<(\omega)$ is the electron occupation number for interacting electrons which is analogous to the Fermi distribution function $f(\omega)$ of noninteracting electrons and $F^>(\omega)$ is analogous to $1-f(\omega)$ of FL leads. $T_{L/R}$ is temperature and $\mu_{L/R}$ the chemical potential of the left or right lead where $\mu_L = \mu + \eta V$ and $\mu_R = \mu + (\eta - 1)$. $\Gamma(z)$ is the Gamma function. Following the derivation in Ref. (Yang3, 2010), the current can be obtained as

$$J_L = \frac{e}{2\pi}\int d\omega T(\omega)\left[F_L^<(\omega)F_R^>(\omega) - F_L^>(\omega)F_R^<(\omega)\right], \tag{43}$$

with $T(\omega) = \Gamma_L\Gamma_R G_d^r(\omega)G_d^a(\omega)$ is the transmission probability. If $g = 1$, Eq. (43) will degrade to the usual well-known current expression for a FL-QD-FL system.

Our goal is to find the general thermopower formula of the model described by the Hamiltonian Eq. (62). The thermopower S is defined in terms of the voltage V generated across the quantum dot when temperature gradient $\Delta T = T_L - T_R$ is much less than T_L and T_R and when current J is zero (Cutler, 1969):

$$S \equiv -\lim_{\Delta T\to 0}\frac{V}{\Delta T}|_{J=0} = -\frac{1}{eT}\frac{L_{12}}{L_{11}}. \tag{44}$$

where L_{12} and L_{11} are linear response coefficients when the current J_L is presented by small bias voltages and small temperature gradients ΔT:

$$J_L = L_{11}\frac{\delta\mu}{T} + L_{12}\frac{\delta T}{T^2} = \frac{e}{2\pi}\int d\omega T(\omega)\left\{\left[\frac{\partial F(\omega)}{\partial\mu}\right]_T\delta V + \left[\frac{\partial F(\omega)}{\partial T}\right]_\mu\delta(\Delta T)\right\}, \tag{45}$$

where $F(\omega) = F_L^<(\omega)F_R^>(\omega) - F_L^>(\omega)F_R^<(\omega)$. Comparing both sides of the Eqs.(78) (let $e = 1, \hbar = 1$), we obtain

$$L_{11} = \frac{T}{2\pi}\int d\omega T(\omega)\left[\frac{\partial F(\omega)}{\partial\mu}\right]_T, \tag{46}$$

$$L_{12} = \frac{T^2}{2\pi}\int d\omega T(\omega)\left[\frac{\partial F(\omega)}{\partial T}\right]_\mu. \tag{47}$$

The formulas Eqs.(46) and Eqs.(47) are independent of the approximation adopted in deriving the retarded (advanced) Green function. However, the partial derivatives $\frac{\partial F}{\partial\mu}$ and $\frac{\partial F}{\partial T}$ are not yet expressed evidently. In the following we will show the explicit expression for L_{11} and L_{12}. The linear expansion of the Luttinger liqiud distribution function becomes

$$F_\alpha(\omega) = F(\omega) + \frac{\partial F_\alpha(\omega)}{\partial\mu_\alpha}|_{\mu_\alpha=\mu,T_\alpha=T}(\mu_\alpha - \mu) + \frac{\partial F_\alpha(\omega)}{\partial T_\alpha}|_{\mu_\alpha=\mu,T_\alpha=T}(T_\alpha - T). \tag{48}$$

In order to achieve a compact expression, we define $F_1 = F_L^<F_R^>$ and $F_2 = F_R^<F_L^>$, and expand them to the first order derivatives:

$$F_1 = F_1(\mu,T) + \frac{\partial F_1}{\partial V}|_T\delta V + \frac{\partial F_1}{\partial(\Delta T)}|_\mu\delta(\Delta T) \tag{49}$$

and

$$F_2 = F_2(\mu,T) + \frac{\partial F_2}{\partial V}|_T\delta V + \frac{\partial F_2}{\partial(\Delta T)}|_\mu\delta(\Delta T), \tag{50}$$

where $F_1(\mu,T)$ and $F_2(\mu,T)$ are the equilibrium LL distribution functions, and $F_1(\mu,T) = F_2(\mu,T)$. Then

$$F_1 - F_2 = \frac{\partial(F_1-F_2)}{\partial V}|_T \delta V + \frac{\partial(F_1-F_2)}{\partial(\Delta T)}|_\mu \delta(\Delta T)$$
$$= \frac{\partial F}{\partial V}|_T \delta V + \frac{\partial F}{\partial(\Delta T)}|_\mu \delta(\Delta T). \quad (51)$$

Substituting of Eq.(51) into Eq.(43)enables one to obtain the expressions of $\frac{\partial F}{\partial V}$ and $\frac{\partial F}{\partial(\Delta T)}$ required in Eqs. (46) and (47). We arrive at that

$$\frac{\partial F_L^{<,>}}{\partial V} = \eta\left\{\pm\frac{1}{2T}F^{<,>} - \frac{i}{2\pi T}\Psi(\frac{1}{2g}+i\frac{\omega-\mu}{2\pi T})F^{<,>} + \frac{i}{2\pi T}\Psi(\frac{1}{2g}-i\frac{\omega-\mu}{2\pi T})F^{<,>}\right\}, \quad (52)$$

and

$$\frac{\partial F_R^{<,>}}{\partial V} = (\eta-1)\left\{\pm\frac{1}{2T}F^{<,>} - \frac{i}{2\pi T}\Psi(\frac{1}{2g}+i\frac{\omega-\mu}{2\pi T})F^{<,>} + \frac{i}{2\pi T}\Psi(\frac{1}{2g}-i\frac{\omega-\mu}{2\pi T})F^{<,>}\right\}, \quad (53)$$

In derivation we have used the relation $|\Gamma(x+iy)|^2 = \Gamma(x+iy)\Gamma(x-iy)$ and $\Gamma'(z) = \psi(z)\Gamma(z)$ with $\psi(z)$ is the Digamma function. Then substituting the Eqs.(52) and Eqs.(53) into the Eq. (51), we obtain

$$\frac{\partial F}{\partial V} = \frac{1}{T}F^>F^<. \quad (54)$$

With the same deriving process, we obtain the partial derivation with respect to temperature as

$$\frac{\partial F}{\partial(\Delta T)} = \frac{\omega-\mu}{T^2}F^>F^<. \quad (55)$$

It follows from Eqs. (54), (55), (46) and (47) that

$$L_{11} = \frac{T}{h}\int d\omega T(\omega)\frac{1}{T}F^>F^< \quad (56)$$

and

$$L_{12} = \frac{T^2}{h}\int d\omega T(\omega)\frac{\omega-\mu}{T^2}F^>F^<, \quad (57)$$

with $T(\omega)|_{\delta V=0,\delta(\Delta T)=0}$. And they become functions related to the QD density of states and LL distribution function. We stress that Eqs. (56) and (57) are the linear response coeffcients in a LL-QD-LL system. These equations will naturally degrade to those of a FL-QD-FL system if $g = 1$. The thermopower can be obtained by the equation Eq. (44) in which the current equals to zero. Substituting Eqs. (56) and (57) into Eq. (44), we have

$$S = \frac{1}{T}\frac{\int d\omega(\omega-\mu)T(\omega)F^>F^<}{\int d\omega T(\omega)F^>F^<}. \quad (58)$$

As shown in Eq. (78), when temperature difference between the leads is zero, conductance is then given by $G = e^2 L_{11}/T$. Comparison of the explicit expressions of the conductance G and thermopower S exhibits that the latter contains information different from the former.

In calculation, the Green functions of the QD are required as shown in Eq. (40). The retarded Green function is defined by $G^r(t) = -i\theta(t)\langle\{d(t), d^\dagger(0)\}\rangle$ and can be derived by means of the equation of motion method. Its analytical expression is

$$G_d^r(\omega) = \frac{1}{\omega - \varepsilon - \Sigma^r(\omega)}, \tag{59}$$

where the retarded self-energy is originated from the tunneling into the leads and is given by:

$$\Sigma^r(\omega) = -\frac{i}{2}\sum_{\alpha=L,R}\Gamma_\alpha[F_\alpha^<(\omega) + F_\alpha^>(\omega)]. \tag{60}$$

In the next section we will give our numerical results and discuss the thermoelectric properties.

4. Numerical results

The expressions (56) and (57)enable us to calculate numerically the conductance and thermopower as functions of the applied voltage and temperature. It is assumed that the system is of structural symmetric: $\Gamma_L = \Gamma_R = \Gamma$. In calculation we take the coupling strength Γ as the energy unit and set the Fermi level of the lead to be zero. Then the energy level ε of the QD represents the gate voltage Vg. No other bias is applied, *i.e.*, we always consider the zero bias case.

Figures 1(a) and (b) show the gate voltage dependence of the conductance and thermopower, respectively. The conductance varies smoothly, which is in agreement with the previous scanning gate microscopy experiments (Small2, 2003; Woodside, 2002). The thermopower S varies rapidly with the variation of the gate voltage and can reach a large absolute value at low temperature. Obviously, the gate voltage violates electron-hole asymmetry and its value tunes the thermopower. Experiments did show the features (Small, 2003; Small3, 2004).

From Fig. 1 it is seen that the conductance is an even function of ε , while the thermopower is an odd function: $S(\varepsilon) = -S(-\varepsilon)$, which is coincide to experiments (Staring, 1993). It is easily understood that the Hamiltonian in this paper possesses electron-hole symmetry: as V_g is changed to $-V_g$, the form of the Hamiltonian remains unchanged if the electrons are simultaneously converted to holes. This is the foundation of discussing the symmetry relations for the dependence of the G and S on V_g. When both bias voltage V and the current J_L change their signs, the sign of the conductance $G = dJ_L/dV$ remains unchanged. Subsequently, $G(-\varepsilon) = G(\varepsilon)$. On the other hand, the temperature difference is irrelevant to the change of the current carriers, *i.e.*, the kinetic coefficient $L_{12} = dJ_L/d(\delta T)$, changes sign: $L_{12}(\varepsilon) = -L_{12}(-\varepsilon)$ when V_g changes sign. Thus we conclude that $G(-\varepsilon) = G(\varepsilon)$. Note that G is proportional to L_{11}. Therefore, the thermopower is also an odd function $S = -\frac{1}{e^2T}\frac{L_{12}}{L_{11}}$ of the gate voltage: $S(\varepsilon) = -S(-\varepsilon)$. The result is qualitative agreement with the experiments (Dzurak, 1997; Egger, 1997; Moller, 1998) where there was no evidence of other significant contributions of transport mechanisms, such as phonon drag, to the observed thermopower.

Comparing the curves G in Fig. 1(a) and S in Fig. 1(b), we find that the sign of S coincides with dG/dV_g. The reason of the lowering of the conductance with the energy level increasing of the quantum dot is ascribed to the change of the electron tunneling from the resonant tunneling to sequential behavior.

Now we turn to the effect of temperature on the conductance and thermopower. Figure 2 plots their curves. Figure 2(a) shows that at low temperature the conductance scales as power laws with respect to temperature, $G(T) \propto T^{\alpha}$ where $\alpha = 2/g - 2$ ($g < 1$). This functional form and the power index are in good agreement with experimental results (Bockrath, 1999; Kong, 2005; Yao, 1999). Some theoretical works with respect to impurity-contained systems gained the same conclusion (Dresselhaus, 1999; Krive, 2001; Krive2, 2001; Romanovsky, 2002). The temperature-dependent power-law scalings of conductance is associated with the suppression of tunneling to a LL in which the density of states vanishes as a power law in the energy of the tunneling electron, and the suppression becomes stronger with the decrease of g, which manifests a signature for electron-electron correlations (Harman, 2002; Kane, 1996). With increasing temperature, the mechanism of electron transport gradually turns from a resonant tunneling-like process to a sequential process. At higher temperature, the conductance is inverse to the temperature. This reflects that the effect of the electron-electron interaction on transport mechanism decays. In the temperature range between, there appears a conductance peak.

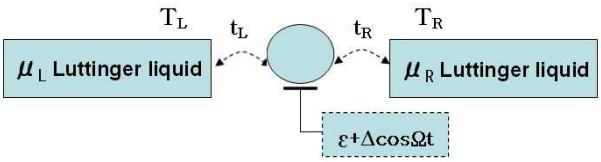

Fig. 2. The schematic picture of the two-terminal electron transport through a single-level quantum dot weakly coupled to the Luttinger liquid leads with the chemical potentials μ_L and μ_R . The position of the dot levels with respect to the Fermi energy can be uniformly shifted by applying a voltage V_g to the gate electrode.

In Fig. 2(b), the thermopower S shows linear behavior as temperature rises from zero. This is because in the low temperature regime electron tunneling transport mechanism is dominant. This behavior is the same as that of a LL containing an impurity (Krive2, 2001) and in agreement with experiments (Hone, 1998; Kong, 2005). With the electron-electron interaction enhancement, the thermopower is also increased, which has been proved in experiment (Lerner, 2008). We fit numerically the thermopower relation between the thermopower S of the LL and S_0 of Fermi liquid at low temperature with $S = (\frac{3}{2g} - \frac{1}{2})S_0(T)$ where $S_0(T) \propto T$. The electron-electron interaction in LL systems enhances and renormalizes the thermopower. In the limit of strong interaction $g << 1$, this thermopower S can be expressed as $S(T) \propto S_0(T)/g$. In this case, the thermopower of the LL is enhanced by a factor of order of magnitude of $3/(2g)$. Figure 2(b) reveals that at a fix temperature, a smaller g results in a lager S value. Hence, a larger slope of the $S - T$ curve at low temperature means a stronger interaction in LL leads. It is worth to note that the thermopower S of LL is much greater than the value S_0 of FL ($g = 1$), which reflects that the intralead electron interactions in the LL enhance the electron-hole asymmetry. With further increasing temperature a peak-like

structure emerges. This is due to the mechanism at low temperature of electron transport switching from a tunnelling process to a diffusive process at high temperature. It is worth mentioning that the result is qualitatively agreement with the works (Romanovsky, 2002) with respect to an impurity in the LL lead connected to noninteracting electrons or a FL. At low temperature, a small potential barrier can strongly influence the transport properties of a LL system, so that the thermopower induced by electron backscattering dominates. This behavior is similar the case of an impurity (Romanovsky, 2002), where the impurity backscattering is considered to be a main origin of the thermopower. The impurity can also be modeled as tunnelling junctions between two decoupled semi-infinite LLs (Collins, 2000), and the tunneling junction between impurity and the LL is described by the tunneling Hamiltonian (Barnabe, 2005; Goldstein, 2010)[45-47].

At high temperature, the thermomotion of electrons become predominant and the interaction between them will be less important. Thus discrepancy between the LL and FL systems will disappear. As a result, in the high temperature limit, S becomes identical to S, as shown in Fig. 2(b). We recall that in a weak interaction system, the thermopower S_M is related to conductance G by Mott's formula (Kane, 1996):

$$S_M = -\frac{\pi^2}{3}\frac{k_B^2 T}{e}\frac{\partial lnG}{\partial \mu}, \tag{61}$$

which was originally derived for bulk systems. Note that this approximation is independent of the specific form of the transmission probability $T(\omega)$. The quantity S_M is different from S_0 of a noninteraction FL. Dependence of the zero bias conductance G on the chemical potential can be in practice measured under the variation of the gate voltage V_g. Since the gate voltage shifts the energy levels of the QD, one may assume that $\frac{\partial lnG}{\partial \mu}$, $\frac{\partial lnG}{\partial V_g}$. Then Eq. (26) becomes $S_M = -\frac{\pi^2}{3}\frac{k_B^2 T}{e}\frac{dlnG}{dV_g}|_{E_F}$. Figure 3 shows the variation of the thermopower S obtained from Eq. (23) and S_M from Mott relation Eq. (26) at $T = 1.0$ for four electron-electron interactions. It is seen that the relation between and G holds qualitatively for weak electron-electron interaction (Appleyard, 2000; Kane, 1996; Krive, 2001). However, even in the noninteraction case $g = 1$, there is some quantitative difference between S and S_M. The difference is obviously enhanced by the strong electron-electron interaction. Experiments (Bockrath2, 2001) evidenced the deviation from the Mott formula Eq. (26). We interpret it as a manifestation of many-body effects in the 1D electron gas. The intralead electron interactions affect the thermopower through the dependence of the transmission probability on electron-electron interaction.

From the above numerical results, we can observe that both the thermopower and conductance manifest linear temperature-dependent power-law scaling, a behavior the same as that of an impurity-contained LL system (Kane, 1996) at low temperature. The electron-electron interaction in the leads brings a significant improvement of the thermopower, a conclusion similar to that of a LL with an impurity (Krive, 2001; Krive2, 2001). As is well known, in a perfectly electron-hole symmetric system, the thermopower $S = 0$. The strong suppression of thermopower arises from the exact counteraction of the currents of electrons and holes induced by temperature gradient, which results in a zero net electric current. Only when the electron-hole symmetry is broken, the nonzero thermopower emerges. Our numerical fittings show that at low temperature the thermopower S can be expressed by the thermopower S_0 of noninteracting electrons multiplied by an

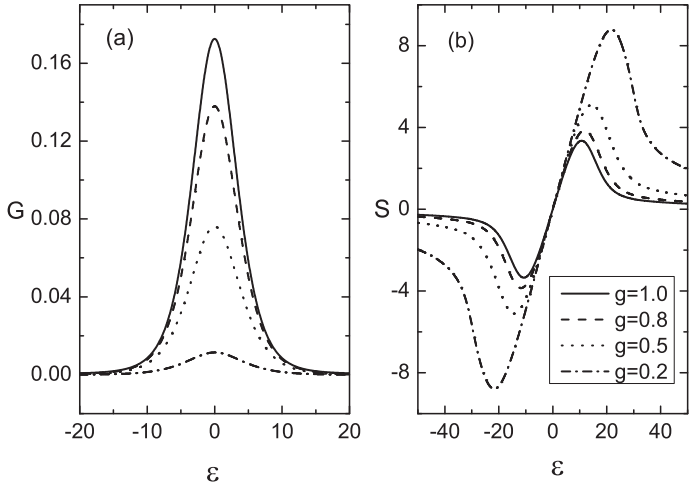

Fig. 3. The dependence of the conductance G and thermopower S on the gate voltage with $T = 1.0$ for g=0.2, 0.5, 0.8 and 1.0. The thermopower is strongly modulated by the gate voltage.

interaction factor as: $S = (3/2g - 1/2)S_0(T)$. In the limit of the strong intralead electron interaction, we have $S(T) \approx 3S_0(T)/2g$ which has an additional 3/2 factor compared to $S(T) \approx S_0(T)/g$ of the impurity-induced thermopower in 1D systems (Romanovsky, 2002). A slight deviation from the electron-hole symmetry will cause a considerable thermopower. In low-dimensional materials, the electron-hole asymmetry is usually strong and can be modulated experimentally by tuning external parameters, such as gate voltage and magnetic field. Our results reveal that how the thermopower of a LL system containing a QD is modulated by tuning the gate voltage.

5. The thermopower formula with a time-dependent gate voltage

The thermopower formula has been derived at the stationary system. Below we will derive an expression for the time-dependent thermopower for the Luttinger liquid leads connected to the central region. It is well known that more rich physics could be exploited if the QD device is subject to a microwave irradiation field. The perturbations of ac fields can give rise to some very interesting phenomena, such as photon-electron pumping effect, the turnstile effect, the sideband effect, and photon-assisted tunneling (Blick, 1995; Kouwenhoven et al., 1997; Tien, 1963). It has been reported that the microwave spectroscopy is a possible tool to probe the energy spectrum of small quantum systems (Wiel, 2002). So the photon-assisted tunneling could provide a new way of understanding the electron-electron influence on the transport properties of the dot. Indeed, the influence of the ac field on the current-voltage characteristics in the strongly correlated interaction model was discussed by some authors. The essential effect of photon-assisted tunneling on transport properties is that the electrons tunneling through the system can exchange energy with microwave fields, opening new inelastic tunneling channels and introducing many effects. The measurement of the thermopower at ac field frequency in the order of GHz regime may offer more value information on the electron interaction. However, the explicit thermopower formula under the time-dependent gate has

been still lacking. Here we will fill the blanks. We start out by introducing a model for a QD coupled to the LL leads under a time-dependent gate voltage. The Hamiltonian of the system (see Fig.2) can be described as follows

$$H = H_{leads} + H_{\rm D} + H_{\rm T}. \tag{62}$$

where $H_{leads} = H_L + H_R$ represents the Hamiltonians of the left and right LL leads and its standard form is given as above, $H_{\rm D} = \varepsilon(t)d^\dagger d$ is the Hamiltonian of the QD, with $\{d^\dagger, d\}$ the creation/annihilation operators of the electron in the QD, $\varepsilon(t) = \varepsilon + \Delta cos\Omega t$, ε is the time-independent single electron energy of the QD without microwave fields, Δ and Ω are the amplitude and frequency of the ac gate voltage, respectively. It causes an alternating current through the dot. H_T is the tunneling Hamiltonian and can be written as

$$H_{\rm T} = \sum_\alpha (t_\alpha d^\dagger \psi_\alpha + h.c.)). \tag{63}$$

by applying a time-dependent canonical transformation (Bruder, 1994)(hereafter $\hbar = 1$) to Hamiltonian H_D

$$U_1(t) = \exp\left[i\int_{-\infty}^{t} dt' \Delta\cos(\Omega t') d^\dagger d\right]. \tag{64}$$

Under this transformation, we obtain $H'_D(t) = U_1(t)H(t)U_1^{-1}(t) - iU_1(t)\partial_t U_1^{-1}(t) = \varepsilon d^\dagger d$, the time-dependence of the gate voltage ε is removed. Instead, the electron tunnel coupling

$$t_\alpha(t) = t_\alpha \exp\left[-i\int_{-\infty}^{t} dt' \Delta\cos(\Omega t')\right]. \tag{65}$$

is now time-dependent. The current operator which describes tunneling from the L lead into the QD at time t is found to be: (in units of $\hbar = 1$))

$$J_L(t) = ie\left[t_L(t)\psi_L^\dagger d - t_L^*(t) d^\dagger \psi_L\right]. \tag{66}$$

Using nonequilibrium-Green-function technique and Langreth theorem of analytic continuation, the current can then be expressed as:

$$J_L(t) = -2eRe\int dt_1 t_L(t)[G^r(t,t_1)g_L^<(t_1,t) + G^<(t,t_1)g_L^a(t_1,t)]t_L^*(t_1). \tag{67}$$

where $G^r(t,t_1)$ and $G^<(t,t_1)$ are the Green's function of the QD. The retarded Green function $G^r(t,t_1)$ and lesser Green function $G^<(t,t_1)$ can be calculated from the following Dyson equation:

$$G^r(t,t_1) = g^r(t,t_1) + \int\int d\tau d\tau' g^r(t,\tau)\Sigma^r(\tau,\tau')G^r(\tau',t_1). \tag{68}$$

and the Keldysh equation

$$G^<(t,t_1) = \int\int d\tau d\tau' G^r(t,\tau)\Sigma^<(\tau,\tau')G^a(\tau',t_1). \tag{69}$$

where $g^r(t,t_1)$ is the free retarded Green function of isolated dot which depends only on the time difference $t - t_1$. $\Sigma^{r/a,<}(\tau,\tau') = \sum_{\alpha=L,R} t_\alpha^*(\tau) g_\alpha^{r/a,<}(\tau,\tau')t_\alpha(\tau')$ is the self-energy.

We now make Fourier transformation over the two times t and t' which switches from the time-domain into energy representation through a double-time Fourier-transform defined as (Wang, 1999; Xing, 2004)

$$F(\omega,\omega_1)=\int dtdt_1F(t,t_1)e^{i\omega t}e^{-i\omega_1 t_1} \tag{70}$$

and

$$F(t,t_1)=\int\frac{d\omega}{2\pi}\frac{d\omega_1}{2\pi}F(\omega,\omega_1)e^{-i\omega t}e^{i\omega_1 t_1}. \tag{71}$$

And with the help of the above equation with respect to $\tau=(t+t_1)/2$ and let $t'=t-t_1$.

$$\langle F(t,t_1)\rangle=\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}F(\tau+t'/2,\tau-t'/2)d\tau, \tag{72}$$

we finally obtain following expression for Dyson equation

$$G^r(\omega)=g^r(\omega)++g^r(\omega)\Sigma^r(\omega,\omega_1)G^r(\omega_1,\omega). \tag{73}$$

and the following expression for the lesser Green function from Eq. (69)

$$G^<(\omega)=G^r(\omega)\Sigma^<(\omega)G^a(\omega). \tag{74}$$

Where the time-average greater (lesser)self-energy $\Sigma^>(\omega)$ $(\Sigma^<(\omega))$ is which can be obtained by the time-average double-time self energy

$$\Sigma^{>,<}(\omega)=\sum_{\alpha=L/R,n}J_n^2(\frac{\Delta}{\Omega})|t_\alpha|^2g_\alpha^{>,<}(\omega+n\Omega). \tag{75}$$

After using Langreth theorem of analytic continuation, and taking Fourier transformation over the current equation Eq. (67) and the time-averaged tunneling current can be expressed as,

$$I_L=\frac{e}{2\pi}\sum_{m,n}J_m^2J_n^2|t_L|^2|t_R|^2\int d\omega G_d^rG_d^a\left[g_{Ln}^<(\omega)g_{Rm}^>(\omega)-g_{Ln}^>(\omega)g_{Rm}^<(\omega)\right]. \tag{76}$$

with $g_{Lm/Rm}(\omega)=g_{L/R}(\omega+m\Omega)$.

As the above step, we also introduce the electron occupation number for interacting electrons $F(\omega)$, then we finally obtains the photon-assisted tunneling current

$$J=e\sum_{m,n}J_m^2J_n^2\int\frac{d\omega}{2\pi}\Gamma_L\Gamma_RG^rG^a(F_{Rm}^>(\omega)F_{Ln}^<(\omega)-F_{Rm}^<(\omega)F_{Ln}^>(\omega)), \tag{77}$$

where $J_m(z)$ is the mth-order Bessel function and $F_{Lm/Rm}(\omega)=F_{L/R}(\omega+m\Omega)$. The more detail derivation process of the time-dependent current formula Eq. (77) can be found in the work (Yang3, 2010).

In the next we only give the time-dependent thermopower formula using the above procedure. Under the small bias voltages and small temperature gradients and with the help

of the linear expansion, we have

$$J_L = L_{11}\frac{\delta\mu}{T} + L_{12}\frac{\delta T}{T^2} = \frac{e}{2\pi}\sum_{m,n} J_m^2 J_n^2 \int d\omega T(\omega)\left\{\left[\frac{\partial F(\omega)}{\partial\mu}\right]_T \delta V + \left[\frac{\partial F(\omega)}{\partial T}\right]_\mu \delta(\Delta T)\right\}, \quad (78)$$

where $F_{mn}(\omega) = F^<_{Ln}(\omega)F^>_{Rm}(\omega) - F^>_{Ln}(\omega)F^<_{Rm}(\omega)$. omparing both sides of the Eqs.(78) (let $e = 1, \hbar = 1$), we obtain

$$L_{11} = \frac{T}{2\pi}\sum_{m,n} J_m^2 J_n^2 \int d\omega T(\omega)\left[\frac{\partial F_{mn}(\omega)}{\partial\mu}\right]_T, \quad (79)$$

$$L_{12} = \frac{T^2}{2\pi}\sum_{m,n} J_m^2 J_n^2 \int d\omega T(\omega)\left[\frac{\partial F_{mn}(\omega)}{\partial T}\right]_\mu, \quad (80)$$

where the partial derivatives $\frac{\partial F_{mn}}{\partial\mu}$ and $\frac{\partial F_{mn}}{\partial T}$ are not yet expressed evidently. In the following we will show the explicit expression for L_{11} and L_{12}. In order to obtain L_{11} and L_{12}, we must arrive at $\frac{\partial F^{<,>}_{Lm}}{\partial V}$ and $\frac{\partial F^{<,>}_{Lm}}{\partial T}$. With the help of the linear expansion of the Luttinger liqiud distribution function and expand them to the first order derivatives, the Luttinger liqiud distribution function becomes

$$\frac{\partial F^{<,>}_{Lm}}{\partial V} = \eta\left\{\pm\frac{1}{2T} - \frac{i}{2\pi T}\Psi_{+m} + \Psi_{-m}\right\}F^{<,>}_{Lm}, \quad (81)$$

and

$$\frac{\partial F^{<,>}_{Rm}}{\partial V} = (\eta - 1)\left\{\pm\frac{1}{2T} - \frac{i}{2\pi T}\Psi_{+m} + \Psi_{-m}\right\}F^{<,>}_{Rm}, \quad (82)$$

where $\Psi_{\pm m} \equiv \Psi(\frac{1}{2g} \pm i\frac{\omega + m\Omega - \mu}{2\pi T})$. Substituting of Eq.(81) and Eq.(82) into Eq.(77) enables one to obtain the expressions of $\frac{\partial F_{mn}}{\partial V}$ and $\frac{\partial F_{mn}}{\partial(\Delta T)}$ required in Eqs. (79) and (80). After a long calculation using the same steps above we finally obtain

$$\begin{aligned}\frac{\partial F_{mn}}{\partial V} &= \frac{\partial[F^<_{Lm}F^>_{Rn} - F^>_{Lm}F^<_{Rn}]}{\partial V} \\ &= F^<_m F^>_n\left(\frac{1}{T} + \frac{i}{2\pi T}(\Psi_{-m} - \Psi_{+m} - \Psi_{-n} + \Psi_{+n})\right),\end{aligned} \quad (83)$$

With the same deriving process, we obtain the partial derivation with respect to temperature as

$$\begin{aligned}\frac{\partial F_{mn}}{\partial V} &= \frac{\partial[F^<_{Lm}F^>_{Rn} - F^>_{Lm}F^<_{Rn}]}{\partial T} \\ &= F^<_m F^>_n\left[\frac{(\omega_m + \omega_n)}{2T^2} - \frac{i\omega_m}{2\pi T^2}(\Psi_{+m} - \Psi_{-m}) + \frac{i\omega_n}{2\pi T^2}(\Psi_{+n} - \Psi_{-n})\right],\end{aligned} \quad (84)$$

with $\omega_m = \omega + m\Omega$ and $F_m = F(\omega + m\Omega)$. In terms of the linear expansion of the time-dependent current formula, The coefficients $L_{11/12}$ of the linear response theory can be determined from the corresponding correlation functions.

$$L_{11} = \frac{T}{h}\int d\omega T(\omega) \sum_{m,n=-\infty}^{\infty} J_m^2(\frac{\Delta}{\Omega}) J_n^2(\frac{\Delta}{\Omega}) F_m^< F_n^> \Big[\frac{1}{T} + \frac{i}{2\pi T}(\Psi_{-m} - \Psi_{+m} - \Psi_{-n} + \Psi_{+n})\Big] \tag{85}$$

and

$$L_{12} = \frac{T^2}{h}\int d\omega T(\omega) \sum_{m,n=-\infty}^{\infty} J_m^2(\frac{\Delta}{\Omega}) J_n^2(\frac{\Delta}{\Omega}) F_m^< F_n^> \Big[\frac{(\omega_m + \omega_n)}{2T^2} - \frac{i\omega_m}{2\pi T^2}(\Psi_{+m} - \Psi_{-m}) + \frac{i\omega_n}{2\pi T^2}(\Psi_{+n} - \Psi_{-n})\Big]. \tag{86}$$

From the expression of the coefficients $L_{11/12}$, we can see the coefficients $L_{11/12}$ containing a additional term caused by the time-dependent gate voltage.

The time-dependent zero bias conductance is then given by $G(0) = \frac{e^2}{T} L_{11}$, and the time-dependent thermopower can be obtained from the ratio between voltage gradient ΔV and and temperature gradient ΔT between the two reservoirs, when both left and right time-dependent electric currents cancel

$$S = -\frac{\Delta V}{\Delta T}|_{<I(t)=0>}. \tag{87}$$

Thus the conductance and thermopower take the form

$$G = \frac{1}{h}\int d\omega T(\omega) \sum_{m,n=-\infty}^{\infty} J_m^2(\frac{\Delta}{\Omega}) J_n^2(\frac{\Delta}{\Omega}) F_m^< F_n^> \Big[\frac{1}{T} + \frac{i}{2\pi T}(\Psi_{-m} - \Psi_{+m} - \Psi_{-n} + \Psi_{+n})\Big], \tag{88}$$

and

$$S = \frac{\int d\omega T(\omega) \sum_{m,n=-\infty}^{\infty} J_m^2(\frac{\Delta}{\Omega}) J_n^2(\frac{\Delta}{\Omega}) F_m^< F_n^> (\frac{1}{T} + \frac{i}{2\pi T}\psi_{mn})}{T^2 \int d\omega T(\omega) \sum_{m,n=-\infty}^{\infty} J_m^2(\frac{\Delta}{\Omega}) J_n^2(\frac{\Delta}{\Omega}) F_m^< F_n^> [\frac{(\omega_m+\omega_n)}{2T^2} - \frac{i\omega_m}{2\pi T^2}\psi_{\pm m} + \frac{i\omega_n}{2\pi T^2}\psi_{\pm n}]}, \tag{89}$$

where $\psi_{mn} = \Psi_{-m} - \Psi_{+m} - \Psi_{-n} + \Psi_{+n}$, $\psi_{\pm m} = \Psi_{+m} - \Psi_{-m}$ and $\psi_{\pm n} = \Psi_{+n} - \Psi_{-n}$. When no ac filed, the above formula return to the equation (58). This formula describes the time-averaged thermopower through the LL-QD-LL system in the presence of ac fields which contains more information than the equation (58). The numerical results of the time-dependent thermopower will be published in the future.

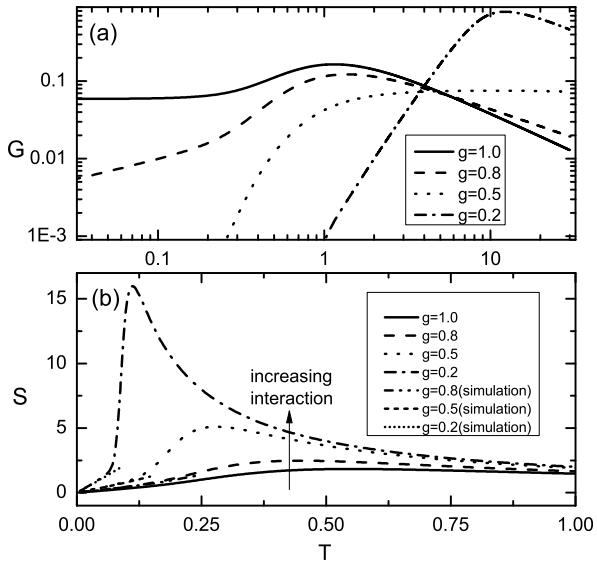

Fig. 4. The conductance (a) and thermopower S (b) as a function of temperature with $\varepsilon = 2.0$ for g=0.2, 0.5, 0.8 and 1.0. At low temperature, the conductance exhibits a power-law dependence of the temperature and the thermopower manifests the linear and positive temperature dependence, respectively. The interaction factor g can be inferred from the slopes of thermopower. With the enhancement of the electron-electron, the thermopower is increased.

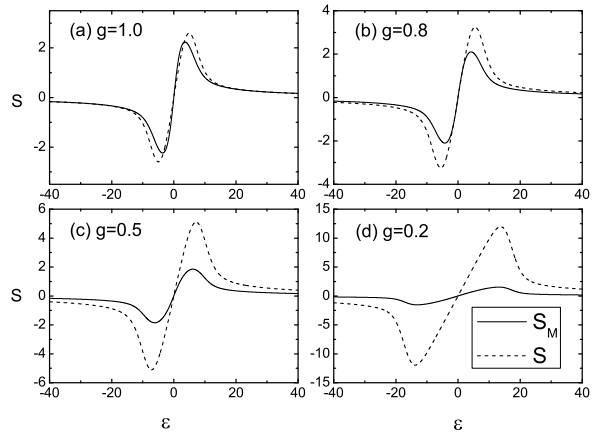

Fig. 5. Thermopowers S calculated by Eq. (23) (dash lines) and SM calculated by Mott relation Eq. (26) (solid lines) at T=1.0 for (a) g=1.0, (b) g=0.8, (c) g=0.5 and (d) g=0.2.

6. Acknowledgment

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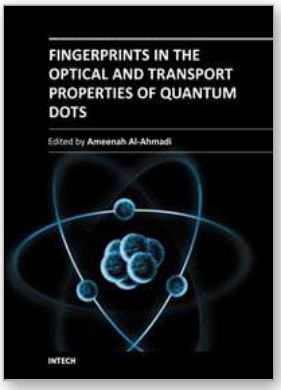

Fingerprints in the Optical and Transport Properties of Quantum Dots

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The book "Fingerprints in the optical and transport properties of quantum dots" provides novel and efficient methods for the calculation and investigating of the optical and transport properties of quantum dot systems. This book is divided into two sections. In section 1 includes ten chapters where novel optical properties are discussed. In section 2 involve eight chapters that investigate and model the most important effects of transport and electronics properties of quantum dot systems This is a collaborative book sharing and providing fundamental research such as the one conducted in Physics, Chemistry, Material Science, with a base text that could serve as a reference in research by presenting up-to-date research work on the field of quantum dot systems.

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